KABARAK UNIVERSITY

UNIVERSITY EXAMINATIONS

2009/20010 ACADEMIC YEAR

FOR THE DEGREE OF BACHELOR OF COMPUTER SCIENCE

COURSE CODE: COMP 122

COURSE TITLE: DISCRETE STRUCTURE

STREAM: Y1S2

DAY: FRIDAY

TIME: 9.00 – 11.00 A.M.

DATE: 06/08/2010

INSTRUCTIONS:

Note: - **Part-A** is compulsory, have **30 marks** and from **Part-B**, You can attempt any **two** questions. Each question has **20 marks.**

PLEASE TURNOVER

Part-A

Question One (Marks 30) Compulsory

a) Rewrite the following statements using set notation:
- (i) the element 1 is not a member of A
- (ii) A is a subset of B — Marks 2

b) State the following:
- (i) The principle of extension and
- (ii) The principle of abstraction. — Marks 2

c) List the elements of the following sets; here N= {1, 2, 3……}.
- (i) $A = \{ x: x \in N, 6<x<10\}$
- (ii) $B = \{ x: x \in N, x \text{ is even}, x<11\}$
- (iii) $C = \{ x: x \in N, 4+x = 3\}$
- (iv) $A = \{ x: x \in N, x^2 + 2 = 11\}$ — Marks 4

d) Let $X = \{x: 3x = 6\}$.Explain whether X = 2? — Marks 2

e) Which of these sets are equal: {r,s,t}, {t,s,r}, {s,r,t}, {t,r,s}? — Marks 2

f) Consider the sets:
$\{4, 2\}$, $\{x: x^2 -6x + 8=0\}$, $\{x: x\in N, x \text{ is even}, 1<x<5\}$.
Which of them are equal to B = {2, 4}? — Marks 2

g) Draw the K-Map of the following expression. $Z = f(A,B,C) = \overline{A}\,\overline{B}\,\overline{C} + \overline{A}B + AB\overline{C} + AC$
Marks 2

h) Explain the difference between $A \subseteq B$ and $A \subset B$. — Marks 2

i) Show that A = {2, 3, 4, 5} is not a subset of $B= \{x: x\in N, x \text{ is even}\}$. — Marks 2

j) Suppose A = {1, 2}. Find
- (i) A^2
- (ii) A^4 — Marks 2

k) A class consists of seven men and five women. Find the number m of committees of five that can be selected from the class. — Marks 2

l) Determine the power set P(A) of A={1,2,3,4} — Marks 2

m) Find the truth table of $\sim p \wedge q$ — Marks 2

n) Draw the complete bipartite graphs $K_{2,3}$ — Marks 2

Part-B

Question Two (Marks 20)

a) Consider the following sets:

(I) X= {x: x is an integer, x>1}

(II) Y= {y: y is an positive integer, divisible by 2}

Which of them are subset of w = {2, 4, 6…….}? Marks 2

b) Suppose that A= {1,2,3,4}, B= {2,3,4,5,6,7}, C= {3,4}, D= {4,5,6}and E= {3}

(i) Which of the five sets can equal X if $X \subseteq A$ and $X \subseteq B$?

(ii) Which of the five sets can equal to X if $X \not\subset D$ and $X \subseteq C$?

(iii) Find the smallest set M which contains all five sets.

(iv) Find the largest set N which is a subset of all the five set.

Marks 4

c) Draw a venn diagram of sets A, B, C where A and B have elements in common, B and C have elements in common, but A and C are disjoint. Marks 2

d) Suppose U={1,2,3,……..8,9},A= {1,2,3,4,5}, B={2,4,6,7,8}, and C={3,4,5,6,9}. Find

(i) $(A \cup B) \cup C$ and (ii) $A \cup (B \cup C)$ Marks 4

e) Determine which of the following sets are finite.

(i) B= {state in the union}

(ii) C={+ve integers less than 1} Marks 2

f) A student is to answer seven out of ten questions on an exam. Find the number m of ways that the student can choose the eight questions. Marks 2

g) Verify that the proposition $p \vee \neg (p \wedge q)$ is a tautology. Marks 2

h) Prove the associative law: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ Marks 2

Question Three (Marks 20)

a) Suppose U={1,2,3,……..8,9},A= {1,2,3,4,5}, B={4,6,8}, and C={3,4,5,6}.

Find (i) A^c

(ii) A\B

(iii) B\B Marks 4

b) Suppose U={1,2,3,……..8,9},A= {1,2,3,4}, B={2,4,6,8}, and C={3,4,5,6}.

Find (i) $(A \cap B) \setminus C$

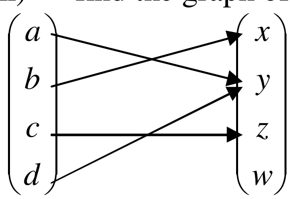
(ii) $(A \setminus B)^c$ Marks 2

c) Prove the commutative laws: (i) $A \cup B = B \cup A$ and (ii) $A \cap B = B \cap A$

Marks 2

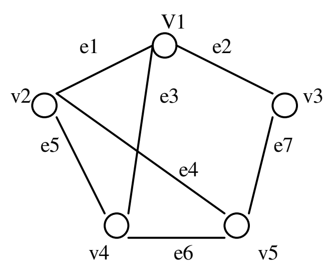
d) Consider the function f from A = { a,b,c,d) into B ={x,y,z,w} defined by figure.

(i) find the image of each element of A;
(ii) find the image of f; and
(iii) find the graph of f Marks 3



e) Draw all trees with five vertices Marks 5

f) Find the adjacency matrix A of the graph G in figure.



Marks 2

g) Translate each of the following statements in to a venn diagram.

a. all students are lazy
b. some students are lazy. Marks 2

Question Four (Marks 20)

a) Find the number of elements in the finite set:

(i) A={2,4,6,8,10}
(ii) B={x: $x^2 = 4$} Mark 1

b) One hundred students were asked whether they had taken courses in any of the three areas, sociology, anthropology, and history. The result were:

43 had taken sociology
36 had taken anthropology
16 had taken history
18 had taken sociology and anthropology
9 had taken sociology and history
5 had taken history and anthropology and
4 had taken all the three subjects.

(i) Draw a venn diagram that will show the results of the survey. Marks 4

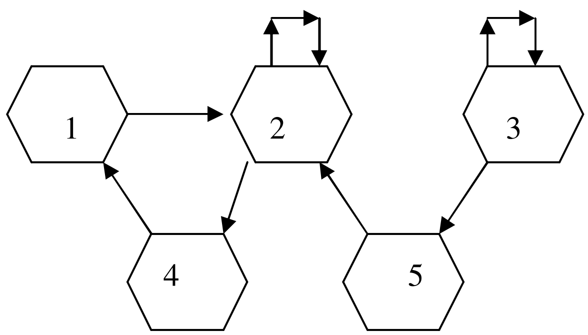
(ii) Determine the number k of students who had taken classes in exactly (1) one of the areas, and (2) two of the areas. Marks 2

c) Let A = {1,2,3} and B = {a,b}. find $A \times B$ Marks 2

d) Given A = {1,2}, B ={x,y,z}, and C = {a,b}
Find $A \times B \times C$ and $n(A \times B \times C)$ by the help of tree diagram. Marks 3

e) Let R be the relation from A = {1,2,3,4}to B = {x,y,z}defined by
R = {(1,y),(1,z),(3,y),(4,x),(4,z)}
(i) determine the domain and range of R
(ii) find the inverse relation R^{-1} of R. Marks 2

f) Let R be the relation on A = {1,2,3,4,5} described by the directed graph in the fig. write R as a set of ordered pairs. Marks 2



g) Draw the graph G whose adjacency matrix A is

$$\begin{pmatrix} 0 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 0 \end{pmatrix}$$

Marks 2

h) Compute $\binom{8}{5}$ Marks 2

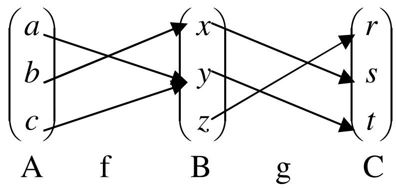
Question Five (Marks 20)

a) Describe the " arrow diagram" of a relation R from a finite set A to a finite set B. Illustrate using the relation R from set A ={1,2,3,4} to set B = {x,y,z}defined by
R = {(1,y),(1,z),(3,y),(4,x),(4,z)} Marks 2

b) Consider the following three relations on the set A= {1,2,3}:
R= {(1,1),(1,2),(1,3),(3,3)}
S = {(1,1),(1,2),(2,1),(2,2),(3,3)}
T = AXA
(i) Determine which of the relations are reflective.

(ii) Determine which of the relations are symmetric.
(iii) Determine which of the relations are transitive. Marks 3

c) Functions f: A $\rightarrow$ B, g: B $\rightarrow$ C
Find the composition function f o g Marks 3



d) Prove the associative law: $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$ Marks 2

e) Use a K-map to find the prime implicants and minimal form for each of the following complete sum-of-products Boolean expressions.
E_1=xyz + xyz' + xy'z + x'yz + x'y'z Marks 3

f) Design a three-input minimal AND-OR circuit L that will have the following truth table:
T= [A=00001111, B= 00110011, C= 01010101, L= 11001101
Marks 3

g)Simplify $\frac{(n+1)!}{(n-1)!}$ Marks 4