8.2 Mathematics Alt. A Paper 2 (121/2)

1.

No.	Log
83.46	1.9215
	+
0.0054	$\bar{3}.7324$
	$\bar{1}.6539$
	-
1.56^2	$\underline{0.3862}$
	$\bar{1}.2677 \div 3$
0.5700	**$\bar{1}.7559$**

(4marks)

2. (a) cost of 1kg of mixture

$$= \frac{120 \times 3 + 90 \times 4 + 6 \times 5}{12}$$

$$= 85$$

b) Cost of 5kg of mixture

$$= \frac{108}{100} \times 85 \times 5$$

$$= 459$$

(4marks)

3.

$$w^3 = \frac{s+t}{s}$$

$$w^3 s = s + t$$

$$w^3 s - s = t$$

$$s = \frac{t}{w^3 - 1}$$

(3marks)

4. (a)

$2x - 5 > -11 \Rightarrow 2x > -6 \Rightarrow x > -3$

$3 + 2x \leq 13 \Rightarrow 2x \leq 10 \Rightarrow x \leq 5$

$\therefore -3 < x \leq 5$

(b) Integral values: -2, -1, 0, 1, 2,3,4,5.

(4marks)

5. $\angle BAD = 30^0 + 40^0 = 70^0$
$\angle BCD = 180^0 - 70^0 = 110^0$

(2marks)

6. (a)

+	7	8	9	10	11
4	11	12	13	14	15
5	12	13	14	15	16
6	13	14	15	16	17
7	14	15	16	17	18
8	15	16	17	18	19

(b) P (sum of at least 17) $= \frac{6}{25}$

(2marks)

7. (a) $T = \begin{pmatrix} 6 \\ -2 \end{pmatrix} - \begin{pmatrix} 4 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ -3 \end{pmatrix}$

(b)

$$OA' = \begin{pmatrix} 1 \\ 2 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$$

$$A' = (3,-1)$$

$$OB' = \begin{pmatrix} 3 \\ 5 \end{pmatrix} + \begin{pmatrix} 2 \\ -3 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$$

$$B'(5,2)$$

(3marks)

8.

$$\sin 45^0 = \frac{1}{\sqrt{2}}$$

$$\frac{\sqrt{8}}{1+\sin 45} = \frac{2\sqrt{2}\left(1-\frac{1}{\sqrt{2}}\right)}{\left(1+\frac{1}{\sqrt{2}}\right)\left(1-\frac{1}{\sqrt{2}}\right)}$$

$$= \frac{2\sqrt{2}-2}{\frac{1}{2}}$$

$$= 4\left(\sqrt{2}-1\right)$$

(3marks)

9. Maximum area $= 4\pi \times 7.5^2$
Minimum area $= 4\pi \times 6.5^2$

$$\text{Absolute error} = \frac{4\pi\left(7.5^2 - 6.5^2\right)}{2}$$

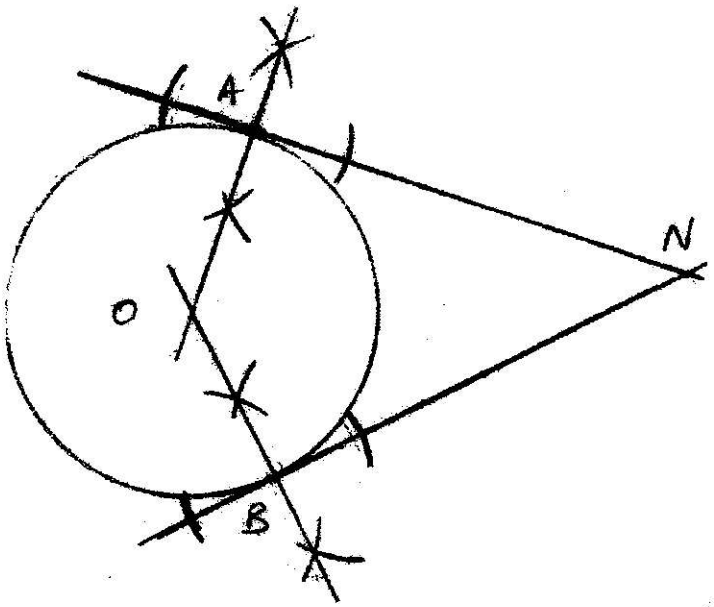
$$= 28\pi$$

$$\% \text{ error} = \frac{28\pi}{4\pi \times 7 \times 7} \times 100\%$$

$$= 14.29\%$$

(4marks)

10.



(3marks)

11.

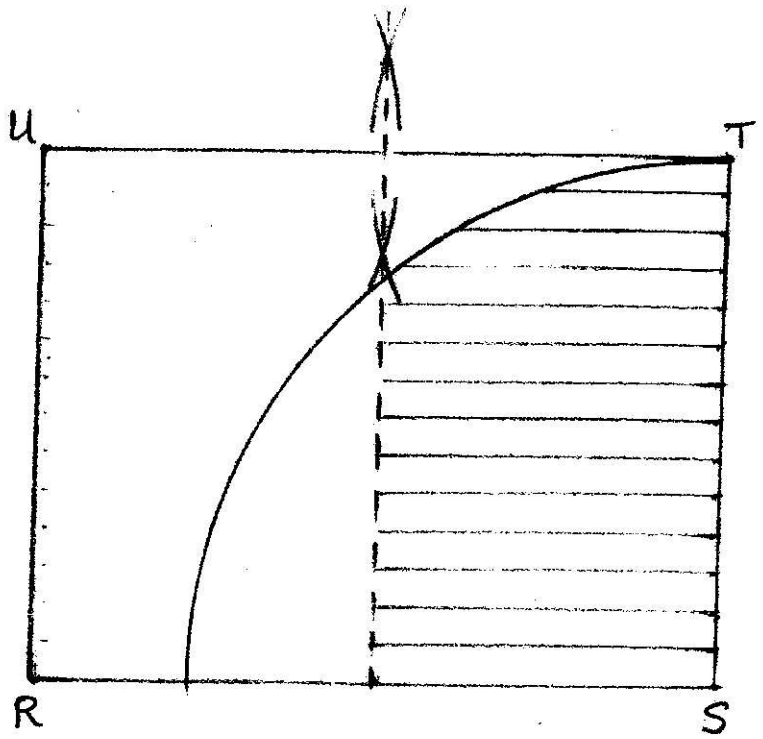
$$\left(a+\frac{1}{2}\right)^4+\left(a-\frac{1}{2}\right)^4=\left[a^4+4a^4\left(\frac{1}{2}\right)+6a^2\left(\frac{1}{2}\right)^2+4a\left(\frac{1}{2}\right)^3+\left(\frac{1}{2}\right)^4\right]-$$

$$\left[a^4+4a^3\left(-\frac{1}{2}\right)+6a^2\left(-\frac{1}{2}\right)^2+4a\left(-\frac{1}{2}\right)^3+\left(-\frac{1}{2}\right)^4\right]$$

$$=2a^4+3a^2+\frac{1}{8}$$

(3marks)

12.



(3marks)

13. **PQ** = -6**i** -**j**- 2**i**+5**j** = -8**i**+4**j**

PN = $\frac{3}{4}(-8i+4j)$ = - 6**i** + 3**j**

(3marks)

14. (a) let longitude difference be θ^0

$\theta \times 60 \cos 60^0 = 630$

$$\theta = \frac{630}{60\cos 60^0}$$

$$= 21^0$$

(b) 21^0 due east of 18^0E is the longitude $(18^0 + 21^0)$ E

Location of N is $(60^0\text{N}, 39^0\text{E})$

(3marks)

15.

$x^2 + y^2 - 6x - 10y + 30 = 0$

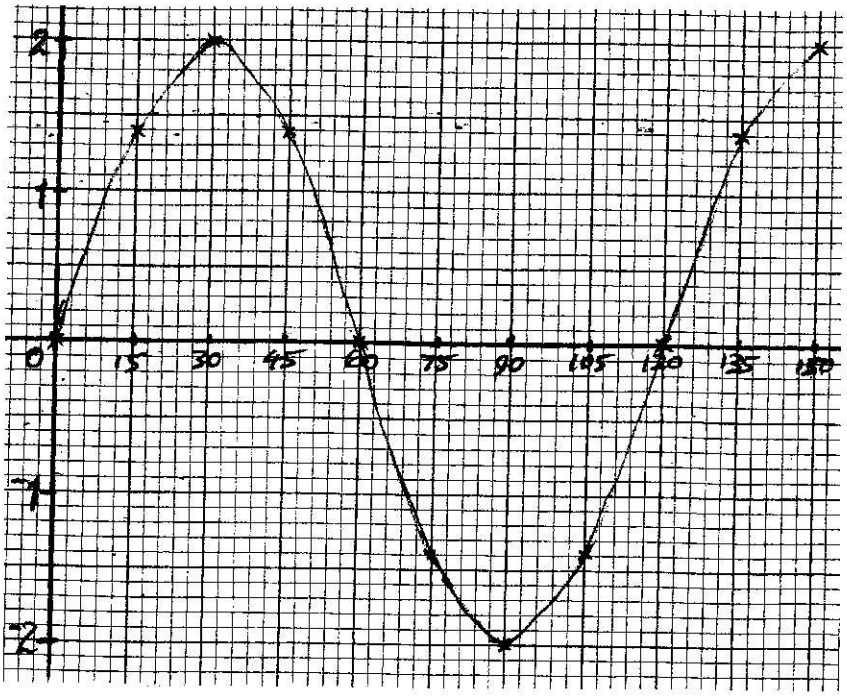
$x^2 - 6x + 9 + y^2 - 10y + 25 = 4$

$(x-3)^2 + (y-5)^2 = 2^2$

$a = 3, b = 5$

(3marks)

16.



Period $= 120^0$

(3marks)

17. (a) (i) $7500 + 11 \times 6000 = 73500$

(ii) $\frac{73500 - 60000}{60000} \times 100 = 22.5\%$

(b) $60000 \times 25 \times 0.95 = 1425000$

(c) Institution X

$73500 \times 25 = 1837500$

Institution Y

$60000 \times 25 \times (1.12)^2 = 1881600$

Difference $= 1881600 - 1837500 = 44100$

(10marks)

18. (a) (i) $64 + 4d = 64r$
$64 + 6d = 64r^2$

(ii) From (i)
$d = 16r - 16$
$64r^2 = 64 + 6(16r - 16)$
$64r^2 = 64 + 96r - 96$
$2r^2 - 3r + 1 = 0$
$(2r - 1)(r - 1) = 0$
$r = ½$ or $r = 1$
For decreasing GP, $r = ½$

Substituting $r = ½$ in (i)
$64 \times ½ = 64 + 4d$
$d = -8$

(b) (i) A.P

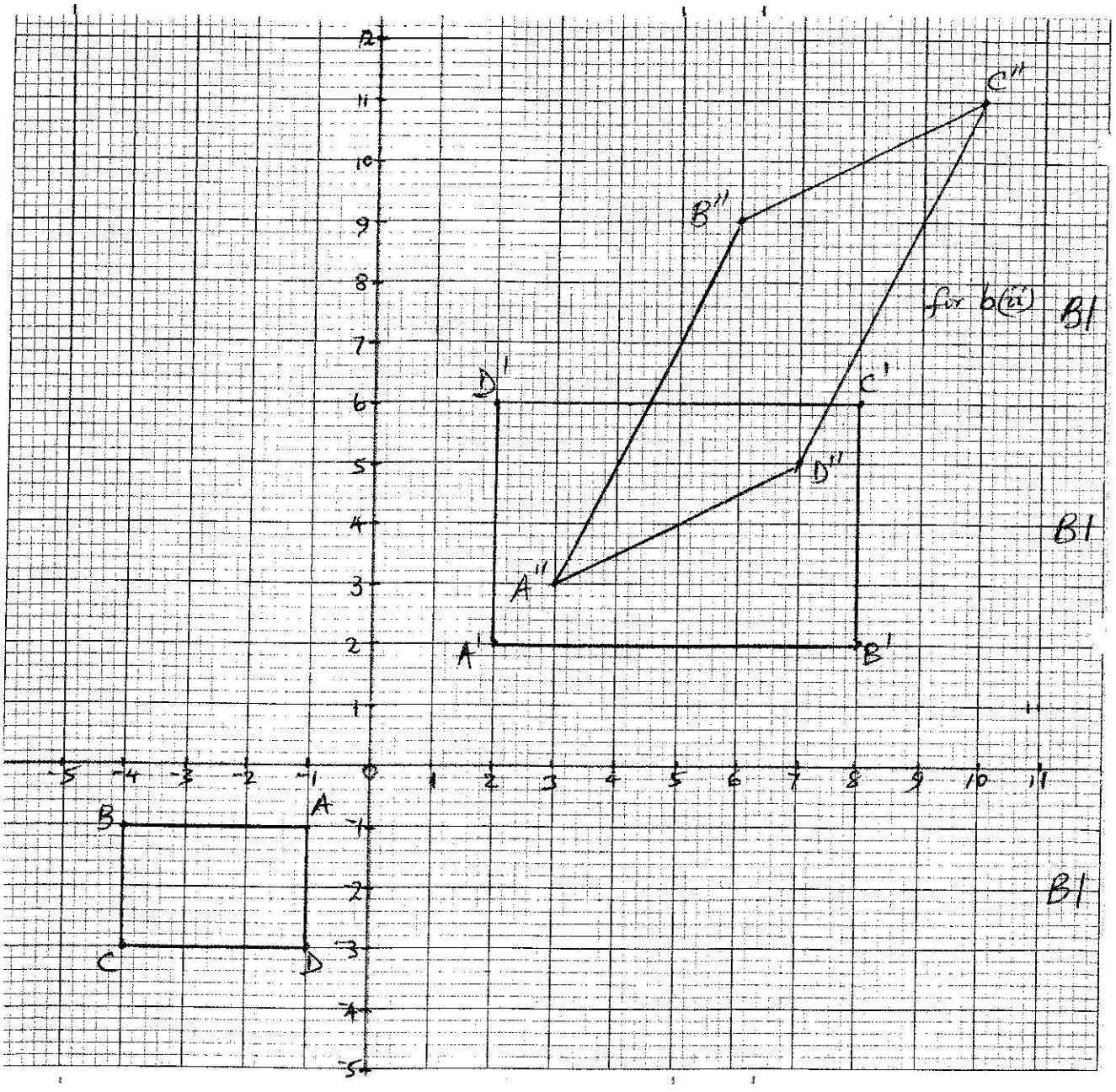
$$S_{10} = \frac{10}{2}\{2 \times 64 + 9 \times -8\} = 280$$

(ii) G.P

$$S_{10} = \frac{64\left(1 - \frac{1}{2^{10}}\right)}{1 - \frac{1}{2}} = \frac{64\left(\frac{1023}{1024}\right)}{\frac{1}{2}} = 127.875$$

(10marks)

19. (a) $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}\begin{pmatrix} -1 & -4 & -4 & -1 \\ -4 & -1 & -3 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 8 & 8 & 2 \\ 2 & 2 & 6 & 6 \end{pmatrix}$



(b)

Coordinates; $A''(3,3), B''(6,9), C''(10,11), D''(7,5)$

(c) Determinant of matrix P $= \frac{1}{2} \times \frac{1}{2} - 1 \times 1$

$= -\frac{3}{4}$

Area of $A''B''C''D'' = \frac{3}{4} \times 6 \times 4 = 18$ sq units

(10 marks)

20. (a) (i) $x-5$

(ii) $[x+(x-5)]\times 2 = 4x-10$

(b) (i)

$(x+20)(x+15) = 15(4x+10)$

$x^2 - 25x + 150 = 0$

$(x-10)(x-15) = 0$

$x = 10$ *or* $x = 15$

(ii) Parents' possible ages

$4\times 10 - 10 = 30$

or $4\times 15 - 10 = 50$

(iii) Possible ages of younger child in 20 years time

$(10-5)+20 = 25$

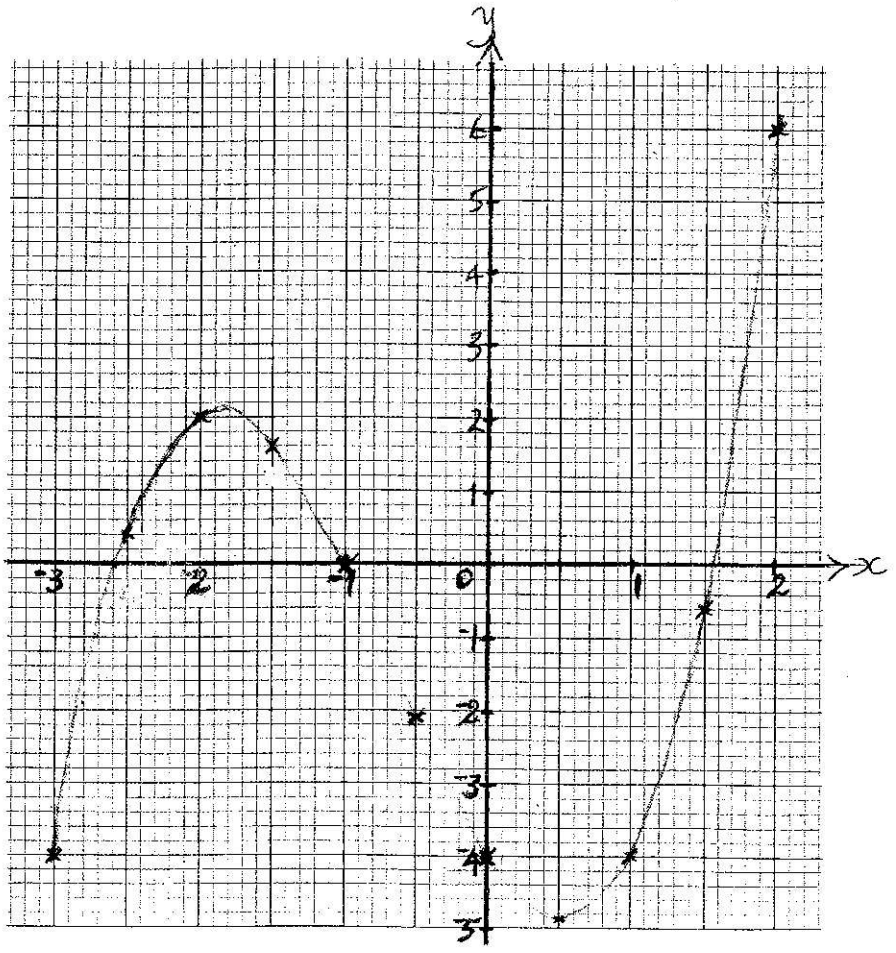
or $(15-5)+20 = 30$

(10marks)

21. (a) (i)

x	-3	-2.5	-2	-1.5	-1	-0.5	0	0.5	1	1.5	2
$y = x^3 + 2x^2 - 3x - 4$			2			-2.1			-4	-0.6	

(ii)

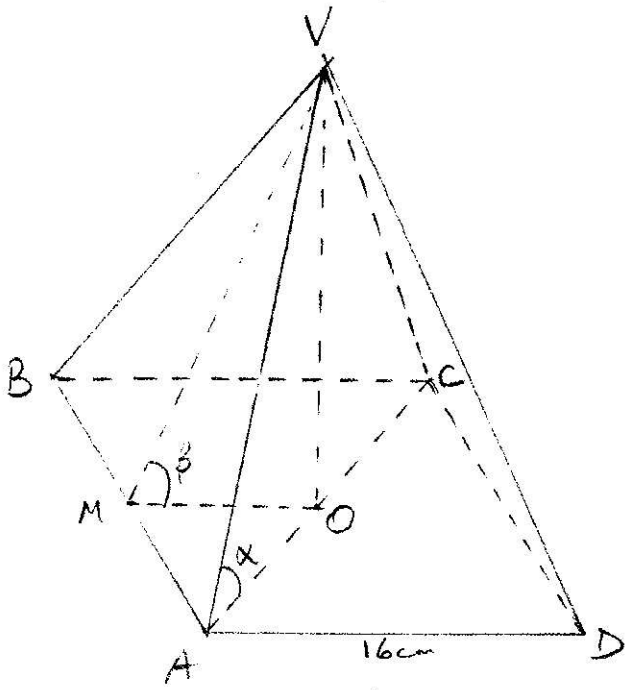


(b) (i) x = - 2.6 ,- 1 , 1.55

(ii) Coordinates of turning points
(-1.85, 2.1) and (0.5, 4.9)

(10marks)

22.



(a) Height VO

$AC^2 = 16^2 + 12^2 = 400$

$AC = 20$

$AO = 20 \div 2 = 10$

$VO = \sqrt{26^2 - 10^2} = 24$

(b) Angle between edge VA and Plane ABCD = angle VAO = α

$\frac{h}{10} = \tan\alpha$

$\alpha = \tan^{-1} 2.4$

$= 67.38^0$

(c) Angle between planes VAB and ABCD = Angle VMO = β

$\frac{h}{8} = \tan\beta$

$\beta = \tan^{-1} 3$

$= 71.57^0$

(10marks)

23. (a) $c = an + \frac{b}{n}$

$135 = 2a + \frac{b}{2}$

$140 = 2a + \frac{b}{3}$

$270 = 4a + b$

$420 = 9a + b$

$150 = 5a \Rightarrow a = 30$

$270 = 120 + b \Rightarrow b = 150$

$c = 30n + \frac{150}{n}$

(b)

$c = 30 \times 10 + \frac{150}{10}$

$= 315$

(c) $756 = 30n + \frac{150}{n}$

$756n = 30n^2 + 150$

$5n^2 - 126n + 25 = 0$

$(5n - 1)(n - 25) = 0$

$n = \frac{1}{5}$ *or* $n = 25$

number of items $= 25$

(10marks)

24. (a)

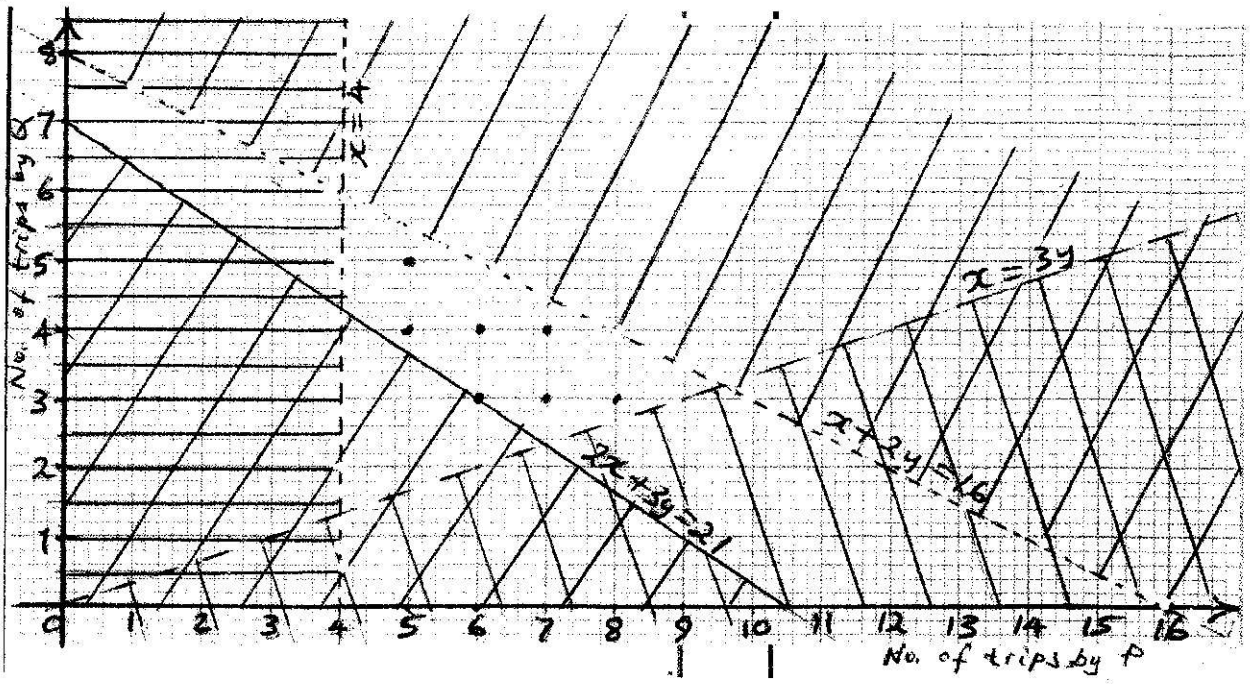
$4x + 6y \geq 42 \Leftrightarrow 2x + 3y \geq 21$

$2x + 4y < 32 \Leftrightarrow x + y < 16$

$x < 3y$

$x > 4$

(b)



(c) $x = 5, y = 5 \Rightarrow 5\times 4 + 5\times 6 = 50$ tons

$x = 6, y = 4 \Rightarrow 6 \times 4 + 4\times 6 = 48$ tons

$x = 7, y = 4 \Rightarrow 7\times 4 + 4\times 6 = 52$ tons

7 trips by P and 4 trips by Q

(10marks)