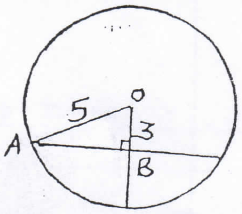
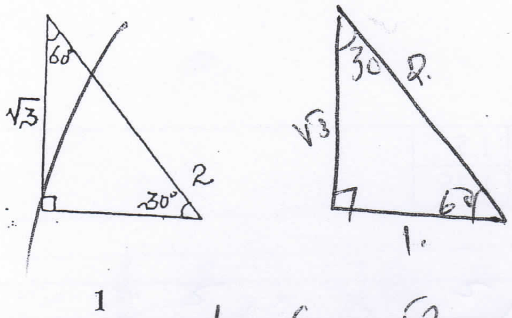
MOMALICHE 4 CYCLE 8

Kenya Certificate of Secondary Education (K.C.S.E.)

121/2
MATHEMATICS
PAPER 2
~~APRIL 9TH~~

1	No	log		
	0.5249^2	$\bar{1}.7200$ X2 $\bar{1}.4400$	M_1	All logs✓
	83.58	$1.9221 +$ 1.3621	M_1	✓addition and subtraction
	$3\sqrt{0.3563}$	$\dfrac{\bar{1}.5518}{3} = \dfrac{\bar{3}}{3} + \dfrac{2.5518}{3}$	M_1	✓division by 3
	$10^1 \times 3.247$ ←	$\bar{1}.8505$ 1.5115	A_1	
	$= 32.5$			CAO
			04	
2	$n^2 = \dfrac{4x^2}{g^2} \times \dfrac{H-K}{3y}$ $n^2 = \dfrac{4x^2H - 4kx^2}{3yg^2}$		M_1	simplification
	$n^2 3yg^2 = 4x^2H - Kx^2$ $n^2 3yg^2 + 4kx^2 = 4x^2H$ $\dfrac{4kx^2}{4x^2} = \dfrac{4x^2H - 3n^2 3yg^2}{4x^2}$		M_1 A_1	✓ grouping
	$K = \dfrac{4x^2H - 3n^2 3yg^2}{4x^2}$			✓ Answer
			03	
3	$2\cos 2\theta + 1 = 0$ $\cos 2\theta = -0.5$ $\Rightarrow 2\theta = 120^0, 240^0, 480^0, 600^0$ $\theta = 60^0, 120^0, 240^0$		B_1 B_1	For all For all

	$\frac{\pi^c}{3}, \frac{2}{3}\pi^c, \frac{4}{3}\pi^c$	B_1	
		03	
4	$4\frac{7}{15} = 4.66667$ Actual $= 4.667$ Truncation $= ~~4.666~~$ 4.466 Error $= ~~4.667 - 4.666~~$ 4·467 − 4·466 = 0·001 $= 0.001$ % Error $= \frac{0.001}{4.667} \times 100$ $\frac{0\cdot001}{4\cdot467} \times 100 = 0\cdot02238638\%$ $= 0.02143\%$	M_1 M_1 A_1	✓ Expression for error C.A.O
		03	
5	$\frac{40X3 + 60X1}{4} = \frac{120+60}{4} = 45l = perkg$ 45 ⟶ 47 ⟵ 50 3 ⟵ 47 ⟶ 2 $= 3 : 2$	M_1 M_1 A_1	**ALT₁** Let the ratio be 1: n $\frac{45x1 + 50xn}{1+n} = 47$ $50n - 47n = 47 - 45$ $3n = 2$ $n = 2/3$ $1 : 2/3$ $\Rightarrow 3:2$
		03	
6	 AB $= \sqrt{5^2 - 3^2}$ $= 4$	B_1	

	Length = 4x2 = 8cm	B_1	
		02	
7	AB = AO + OB $= (2i - j + 8k) + (-3i + 2j - 2k)$ = -i + j + 6k $(AB) = \sqrt{(-1)^2 + (1)^2 + (6)^2}$ $= \sqrt{38}$ = 6.164	B_1 M_1 A_1	✓ Attempt to find magnitude.
		03	
8	 tan 60 = √3 Cos 240 = $-\frac{1}{2}$ $\dfrac{\frac{1}{2}-\frac{1}{2}=\frac{-1}{2\sqrt{3}-1}}{\sqrt{3-\frac{1}{2}}}$ $= \frac{-1}{(2\sqrt{3}-1)} \times \frac{(2\sqrt{3}+1)}{(2\sqrt{3}+1)}$ $= \frac{-2\sqrt{3}-1}{12-1}$ $= \frac{-2\sqrt{3}-1}{11}$	B_1 M_1 M_1 A_1	✓substitution of trig ratio ✓ Attempt to rationalize denomination
		03	
	Total monthly installment = 3200x12 = 38400 = Principal = 30000 – 10000 = 20000 $38400 = 20000\ (1+\frac{r}{100})^{12}$ $\text{Log } 1.92 = 12 \log (1+\frac{r}{100})$ $1.059 = 1+\frac{r}{100}$ r = 5.9% $2\frac{3}{4} = \frac{11}{4}$ $\frac{4}{11}$	B_1 M_1 M_1 A_1	For both ✓ Expression ✓ simplification C.A.O
		04	
10 a	$(\cancel{1-3x})^4 = 1 - 12x + 54x^2 - 108x^3$	M_1	✓ Expression

$(a+\frac{x}{2})^4 = 9^4 + 2a^3x + \cancel{2ya^2x^2 + \frac{1}{2}ax^3}$ $a^4+2a^3x+\frac{3}{2}a^2x^2+\frac{ax^3}{2}+\frac{x^4}{16}$	M_1	✓ identification of co-efficient of x^3 and equation
$\Rightarrow -108x^3 + \frac{1}{2}ax^3 = -107x^3$		
$-108 + \frac{1}{2}a = -107$ $\frac{a}{2} = 1$	A_1	
$a = 2$ $a = 2$		
b) $2^4 + 2(2^3)x + 24(2^2)x^2 + \frac{1}{2}(2)x^3$ $2^4 + 2^4x + \frac{3}{2}\times 4x^2 + x^3$	B_1	
$= 16 + 16x + 96x^2 + x^3$ $16+16x+{}^{2}6x^2+x^3$		
	04	

11

Det $= 0$		
$x(x+1) - 2 = 0$	M_1	
$x^2 + x - 2 = 0$		
$(x+1)(x-2) = 0$		
$x = -1$ or 2	A_1	Both values ✓

12

X	-1.5	-0.5	0.5	1.5
Y	1.75	3.75	3.75	1.75

B_1 — for all values

Area $= 1(1.75 + 3.75 + 3.75 + 1.75)$ — M_1

$= 11$ sq.units — A_1

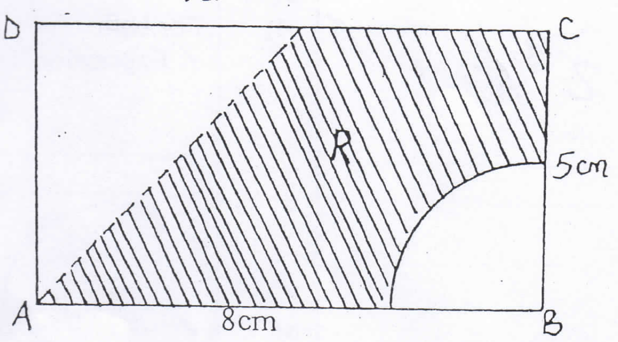
03

13

Log a = -0.35 a = 0.45 $(x-4)^2+(y+2)^2-4=0$	B_1	
$\log b = \frac{0.35}{2}$ Centre $(4,-2)$	M_1	Expression of grad
$\log b = 0.175$ b = 1.496 (Accept 1.5) radius $=\sqrt{4^2+(-2)^2+4}$	A_1	For b
$M = ab^T$ When T=4 $M = 0.45 \times 1.496^4$ $\sqrt{24}$ units		
= 2.254 (Accept 2.278)	B_1	For m
	04	

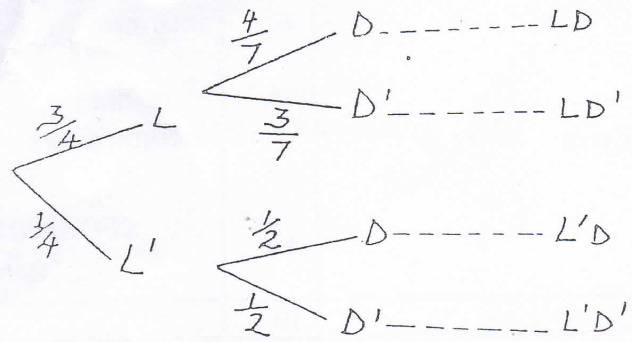
14

1 cm rep 1m



B_1	✓ arc constructed
B_1	Bisecting<BAD
B_1	Shading the region
04	

15



M1

A1

b) $P(LD^1 \text{ or } L^1D^1)$

$= P(LD^1) + P(L^1D^1)$

$= \left(\frac{3}{4} \times \frac{3}{7}\right) + \left(\frac{1}{4} \times \frac{1}{2}\right)$

$= \frac{25}{56}$

03

16

a, ar, ar^2

$a, a + 3d, a + 9d$

$\Rightarrow ar = a + 3d$

$6r = 6 + 3d$

$r = 1 + \frac{1}{2}d$

$a(1 + d + \frac{1}{4}d^2) = a + 9d.$

$\frac{1}{4}d^2 - \frac{1}{2}d = 0$

$d = 2$

M1 — ✓ expressions of G.P and A.P

M1 — For equation

A1

$r = \cancel{\frac{4}{8}}\ \frac{8}{4} = 2$ B1

$S_n = \frac{a(r^n - 1)}{r - 1} = \frac{4(2^8 - 1)}{1}$ M1

4×255

1020 A1

03

SECTION II

a) Gross tax = Net tax + Deductions (reliefs)

= 2336 + 1056

= Shs. 3392

M1
A1

b) Gross tax p.a = 339 x 12

= 40,704/=

Income p.a	Tax rate	Tax
121,968	10%	12196.80
114,912	15%	17236.80
56,352	20%	11270.40
293,232		40704.00

M1
M1
M1

$\frac{20}{100} X = 112\,70.40$

$X = 11270.40 \ \frac{10}{2}$

$= 56,352/=$

M1 — ✓ Attempt to find x

~~24~~ 17

WORKING

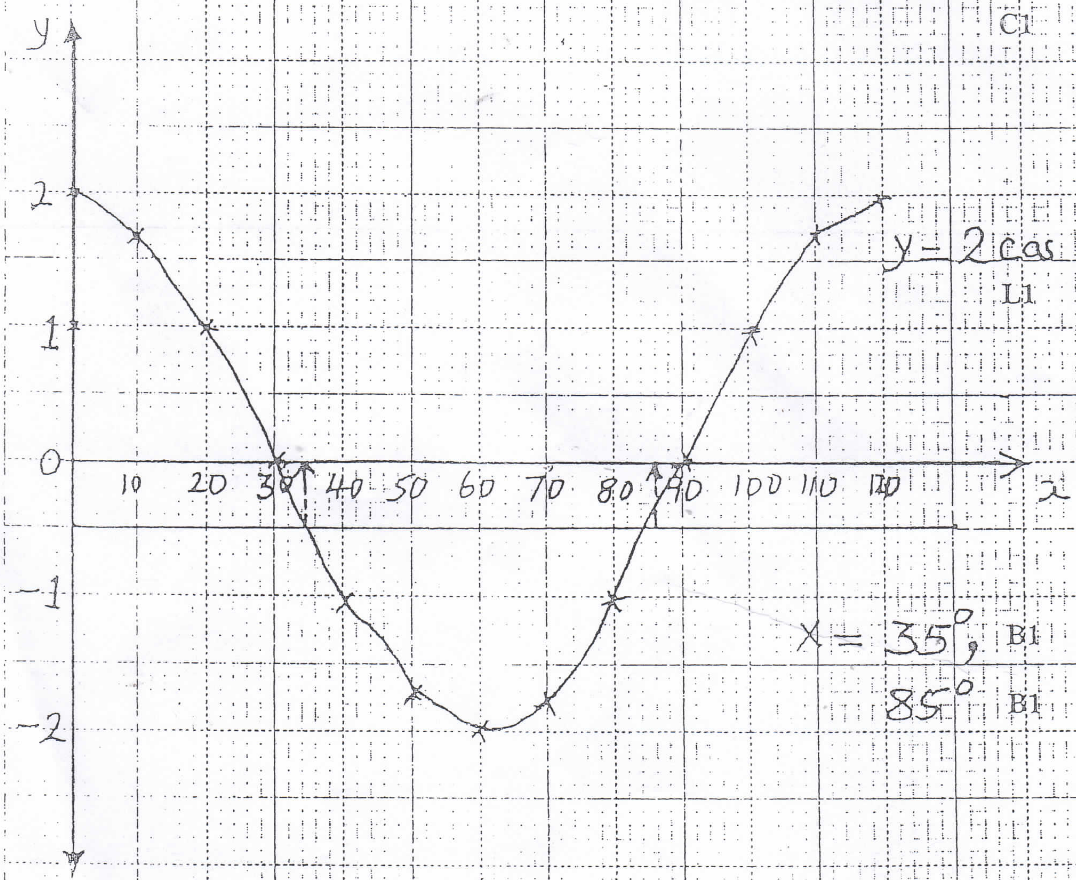
X	0	10	20	30	40	50	60	70	80	90	110	120
Cos 3x	0.8660		0.0000		-0.8660			0.5000	0.0000		0.8660	
y	1.73		0.00		-1.73			-1.00	0.00		1.73	

B2 all
B1 at least 8

S1 Scale

P1

C1 Curve drawn



$y = 2\cos 3x$

L1 Line drawn

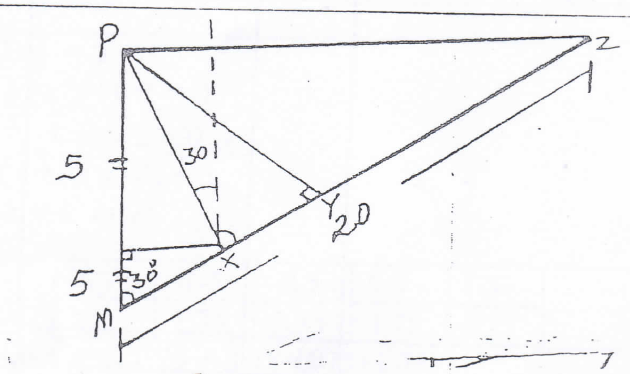
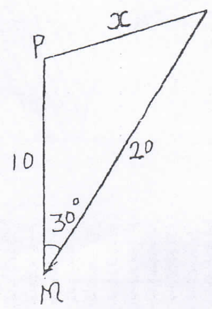
$x = 35^\circ$, B1

85° B1

Amplitude = 2 units B1

Period = 120^0 B1

10 Marks

	Taxable income = 293,232/= p.a = 293,232/= p.a = 24,436/ p.m. c). Allowance = $\frac{15.}{100}$ x 24,436 = 3635.40 Monthly basic salary = taxable – Allowance income p.m = 24436 – 3635. 40 = Shs. 20,800.60	M_1 A_1 M_1 A_1	Addition of incomes ✓ taxable income p.m
		10	
18	 a). Cos 30 = $\frac{5}{MX}$ mx = $\frac{5}{\cos 30}$ = 5.774 b) $360^0 - 30^0$ = 330^0 c) $\cos 30^0 = \frac{MY}{10}$ my = 10 cos 30^0 = 10 x 0.8660 = 8.66m d) By cosine rule $x^{2} = 10^2 + 20^2 - 2 \times 200\cos 30^0$ $= 500 - 400\cos 30^0$ $= 500 - 346.41$ $= 153.59$ $x = 12.39$ 	M_1 A_1 M_1 A_1 M_1 M_1 A_1 M_1 M_1 A_1	
		10	

19. The height of a number of orange trees in an orchard were measured to the nearest (cm) and recorded in the table below.

Height (cm)	Frequency	c.f
131 - 140	13	13
141 – 150	23	36
151 – 160	36	72
161 – 170	50	122
171 – 180	35	157
181 – 190	28	185
191 - 200	15	200

Using an assumed mean of 165.5, calculate

a) The mean height (3mks)

x	$x-A$	fd
135.5	−30	−390
145.5	−20	−460
155.5	−10	−360
165.5	0	0
175.5	10	350
185.5	20	560
195.5	30	450
		$\Sigma fd = 150$

$\bar{x} = A + \frac{\Sigma fd}{\Sigma f}$

$165.5 + \frac{150}{200}$

$165.5 + 0.75$

166.25

b) The standard deviation of the distribution (4mks)

d^2	fd^2
900	11700
400	9200
100	3600
0	0
100	3500
400	11200
900	13500
	$\Sigma = 52700$

$S = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$

$\sqrt{\frac{52700}{200} - (0.75)^2}$

$\sqrt{263.5 - 0.5625}$

$\sqrt{262.9375} = 16.215$

16.22

c) The quartile deviation (3mks)

$\frac{Q_3 - Q_1}{2}$

$Q_1 = \frac{1}{4} \times 200 = 50^{th}$

$150.5 + \left(\frac{50-36}{36}\right) \times 10 = 154.3888$

$170.5 + \left(\frac{150-122}{35}\right) 10 = 178.5$

$\frac{178.5 - 154.3888}{2} = 12.055$

20. The points A (5,-1) B (1,-2) and C (x ,y) of a triangle are mapped onto A[1] (1,5) B[1](2,1) and C[1] (4,2) by a matrix $\mathbf{N} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Find

a) Matrix **N** of the transformation. (4mks)

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 5 & 1 \\ -1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 2 \\ 5 & 1 \end{pmatrix}$$

$5a - b = 1$ $\quad$ $5c - d = 5$

$a - 2b = 2$ $\quad$ $c - 2d = 1$

$10a - 2b = 2$

$a - 2b = 2$

$9a = 0$

$a = 0$

$b = -1$

$10c - 2d = 10$

$c - 2d = 1$

$9c = 9$

$c = 1$

$d = 0$

$N = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

b) Co-ordinates of C (2mks)

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix} =$$

$x = 2$

$y = -4$

$C(2, -4)$

c) A" B" C" are the image of A' B'C' under a transformation represented by matrix $\mathbf{M} = \begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}$.

Write down the co-ordinates of A" B" C" (2mks)

$$\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}\begin{bmatrix} 1 & 2 & 4 \\ 5 & 1 & 2 \end{bmatrix} = \begin{bmatrix} -3 & 3 & 6 \\ 0 & 0 & 0 \end{bmatrix}$$

A" (-3, 0)

B" (3, 0)

C" (6, 0)

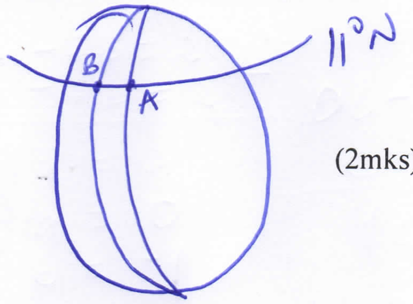
d) A transformation **N** followed by **M** can be represented by a single transformation **K**. Determine K (2mks)

MN

$$\begin{pmatrix} 2 & -1 \\ 0 & 0 \end{pmatrix}\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} -1 & -2 \\ 0 & 0 \end{pmatrix}$$

M1 $\quad$ A1

21. A ship left port A (11^0 N, 32^0 W) and sailed due west to another port B. The journey took 160 hours at an average speed of 28 knots. Given that radius of the earth is 6370km and $\pi = {}^{22}/_{7}$



a) Calculate the distance between A and B

i) In nautical miles (2mks)

$D = S \times T$

$28 \times 160 = 4480 nm$ M1 A1

ii) In km (2mks)

$4480 \times 1.853 = 8301.44$ km M1 A1

b) Calculate the average speed of the ship in km/h correct to 2d.p (2mks)

$S = \frac{D}{T} = \frac{8301.44}{160}$ M1 $= 51.88$ km/h A1

c) Calculate to the nearest whole number the longitude of port B and hence state its position (4mks)

$4480 = \theta \times 60 \times \cos 11$ M1

$\theta = \frac{4480}{60 \times 0.9816}$

$= 76.06$

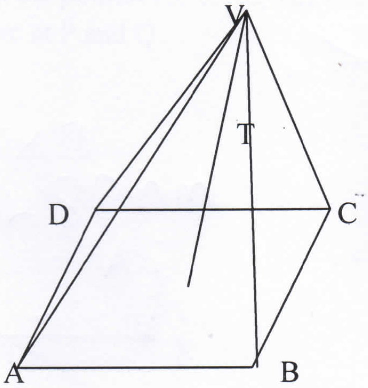
$= 76^0$ A1

Longitude diff $= x - 32 = 76$ M1

$x = 108^0$

$(11^0N, 108^0W)$ A1

22.



The right pyramid above (not drawn to scale) has AB = 12 cm and BC = 16 cm. O is the centre of the base with OV = 15 cm.

Calculate, giving your answer to four significant figures.

a) The length of the slant edge (2mks)

$AC = \sqrt{12^2+16^2}$

$= 20\text{cm}$

$AV = \sqrt{10^2+15^2}$

$\sqrt{325}$

$= \underline{18.03\text{ cm}}$

b) The angle between the lines VA and VC (2mks)

$\tan\theta = \frac{10}{15} = 0.6667$

$\theta = \tan^{-1} 0.6667$

$= \underline{33.69°}$

c) The angle between the plane ABV and the base ABCD (3mks)

$VX = \sqrt{18.03^2 - 6^2}$

$= 17.00$

$\cos\beta = \frac{8}{17.00}$

$\beta = \cos^{-1} \frac{8}{17.00}$

$\beta = \cos^{-1} 0.4706$

$\beta = \underline{61.93°}$

d) The pyramid is chopped at point T to form a frustum such T divides OV in the ration 2:1. Find the volume of the frustum. (3mks)

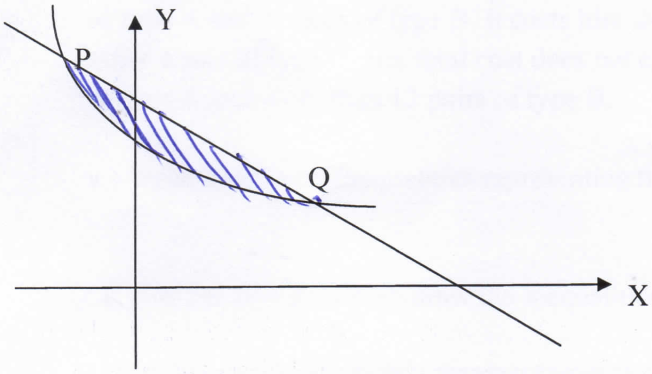
$\frac{1}{3}AH = \frac{1}{3} \times 16 \times 12 \times 15 = 960\text{ cm}^3$

$\frac{1}{3}Ah = \frac{1}{3} \times 4 \times \frac{16}{3} \times 5 = 35.56\text{cm}^3$

V of frustum = $960 - 35.56$

$= \underline{924.44\text{ cm}^3}$

23. The figure below shows the sketch of the curve y = $x^2 - 2x + 4$and a straight line PQ which cuts the x-axis and the y-axis at the points (10, 0) (0, 10) respectively.
The line also intersects the curve at P and Q



Find.

a i.) The equation of the straight line in the form y = m x + c (2mks)

$y = /x^2 + 2x + 4/$

$g = \frac{\Delta y}{\Delta x}$

$g = \frac{10-0}{0-10}$

$g = -1$ M1

$\frac{-1}{1} = \frac{y-10}{x-0}$

$y - 10 = -x$

$y = -x + 10$ A1

ii.) The co-ordinates of P and Q (4mks)

$y = x^2 - 2x + 4$ — (1)

$y = -x + 10$

$-x + 10 = x^2 - 2x + 4$ M1

$x^2 - 2x + x - 6 = 0$

$x^2 - x - 6 = 0$

$x^2 - x - 6 = 0$ M1

sum = −1, prod = −6 } 2, −3

$x^2 + 2x - 3x - 6 = 0$

$x(x+2) - 3(x+2) = 0$

$x_1 = 3$

$x_2 = -2$ A1

$P = (-2, 12)$ B1

$Q = (3, 7)$

B Use integration to find the area of the shaded part. (4mks)

A. of Trapezium = $\int_{-2}^{3} (-x + 10)\,dx$

$\left[-\frac{x^2}{2} + 10x\right]_{-2}^{3}$

$\left(-\frac{(9)}{2} + 30\right] - \left[-\frac{4}{2} - 20\right]$ M1

$25.5 + 22$

$= 47.5$ square units

Area enclosed by curve

$= \int_{-2}^{3} (x^2 - 2x + 4)\,dx$

$\left[\frac{x^3}{3} - \frac{2x^2}{2} + 4x\right]_{-2}^{3}$ M1

$(9 - 9 + 12) - \left[-\frac{8}{3} - 4 - 8\right]$

$12 + 14\frac{2}{3} = 26\frac{2}{3}$ s. units

A. of shaded region

$47.5 - 26\frac{2}{3} = \cancel{20.8}\ 30\frac{1}{6}$ square units A1

M1

24)

Let no. of type A dresses be x

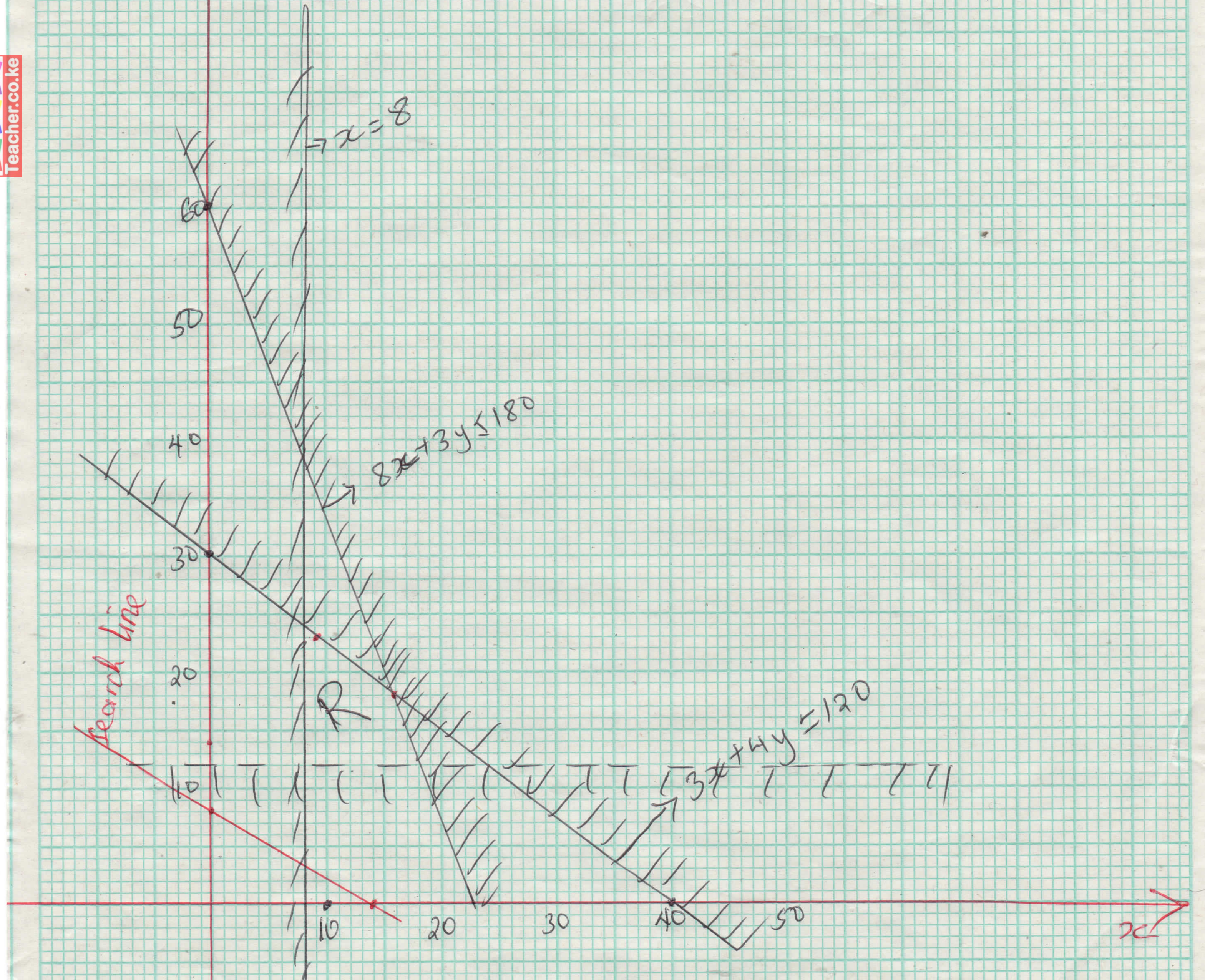
B y

$3x + 4y \leq 120$

$400x + 150y \leq 9000$; $8x + 3y \leq 180$

$x \geq 8$

$y \geq 12$.



$40x + 70y = k$

$40x + 70y = 560$

9 type A dresses &

23 type B dresses.