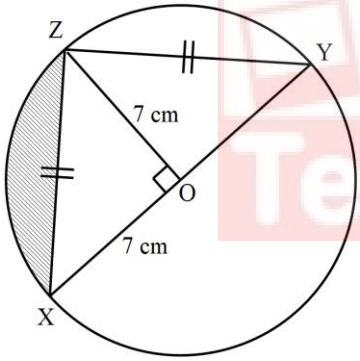
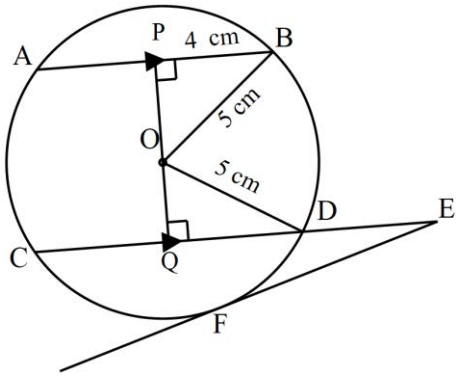
TERM 2 – 2023 (JULY)
MATHEMATICS PAPER 2 (121/2)
MARKING SCHEME

NO.	WORKING	MARKS	REMARKS
1.	Consider column vectors **PQ** and **QR** $\mathbf{PQ} = \mathbf{OQ} - \mathbf{OP} = (-\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k}) - (2\boldsymbol{i} + 3\boldsymbol{j} + 4\boldsymbol{k})$ $\mathbf{PQ} = -3\boldsymbol{i} - 5\boldsymbol{j} - \mathbf{3}\boldsymbol{k}$ Also $\mathbf{QR} = 5\boldsymbol{i} + 8\boldsymbol{j} + 7\boldsymbol{k} - (-\boldsymbol{i} - 2\boldsymbol{j} + \boldsymbol{k})$ $\mathbf{QR} = 6\boldsymbol{i} + 10\boldsymbol{j} + 6\boldsymbol{k}$ $\mathbf{QR} = 6\boldsymbol{i} + 10\boldsymbol{j} + 6\boldsymbol{k} = -2(-3\boldsymbol{i} - 5\boldsymbol{j} - \mathbf{3}\boldsymbol{k})$ $\Rightarrow \mathbf{QR} = -2\mathbf{PQ}$ hence **PQ//QR** Point Q is common to the PQ and QR Thus P, Q and R are collinear	B1 B1 B1	$\mathbf{PQ} = \begin{pmatrix}-1\\-2\\1\end{pmatrix} - \begin{pmatrix}2\\3\\4\end{pmatrix} = \begin{pmatrix}-3\\-5\\-3\end{pmatrix}$ $\mathbf{QR} = \begin{pmatrix}5\\8\\7\end{pmatrix} - \begin{pmatrix}-1\\-2\\1\end{pmatrix} = \begin{pmatrix}6\\10\\6\end{pmatrix}$ $\begin{pmatrix}6\\10\\6\end{pmatrix} = -2\begin{pmatrix}-3\\-5\\-3\end{pmatrix}$
	Total	**3**	
2.	$= 15(x^2)^2\left(\frac{1}{x}\right)^4$ $= 15\frac{x^4}{x^4}$ $= 15$	M1 A1	
	Total	**2**	
3.	 $A = \frac{90}{360} \times \frac{22}{7} \times 7^2 - \frac{1}{2} \times 7 \times 7$ $= 38.5 - 24.5$ $= 14.00$	M1 M1 A1	Area of sector OXZ – M0 if π = 3.142 used Area of ΔOXZ, A0 – if not 14.00
	Total	**3**	
4.	$m^2 = \frac{ht^2}{t^2 + 2w}$ $m^2t^2 + 2wm^2 = ht^2$ $2wm^2 = ht^2 - m^2t^2$ $2wm^2 = t^2(h - m^2)$ $t^2 = \frac{2wm^2}{h - m^2}$ $t = \pm\sqrt{\frac{2wm^2}{h-m^2}}$ or $t = \pm\sqrt{\frac{-2wm^2}{m^2 - h}}$	M1 M1 A1	Removal of square root Collecting terms in t together A0 – if $\pm$ is not committed
	Total	**3**	

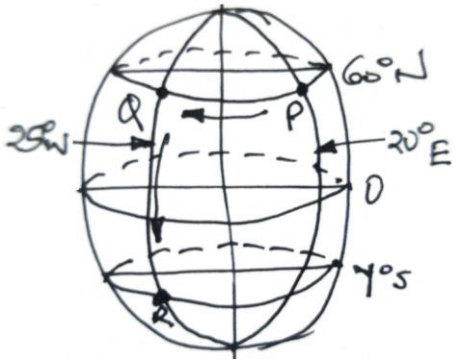
NO.	WORKING	MARKS	REMARKS
5.	(a) Graph (b) Value of c 1.0 ± 0.05	 S1 P1 L1 B1	
	Total	**4**	
6.	$\mathrm{M} = \frac{kr^2}{\sqrt{h}} \Rightarrow \mathrm{M}_1 = \frac{kr_1^2}{\sqrt{h_1}}$ $r_1 = \frac{100-25}{100} r = 0.75r$ $h_1 = \frac{100+21}{100} = 1.21h$ $\mathrm{M}_1 = \frac{k(0.75r)^2}{\sqrt{1.21h}} \Rightarrow \mathrm{M}_1 = \frac{kr^2}{\sqrt{h}}\left(\frac{0.75^2}{1.1}\right)$ $\mathrm{M}_1 = \left(\frac{0.75^2}{1.1}\right)\mathrm{M}$ $\%\text{ change} = \frac{\left(\frac{0.75^2}{1.1}\right)\mathrm{M} - \mathrm{M}}{\mathrm{M}} \times 100$ $= -48.86\%$ A decrease/reduction by 48.86%	M1 M1 A1	Expression for M_1 Percentage change A0 if left as -48.86%
	Total	**3**	
7.	After 1 year $\mathrm{A} = 48{,}000\left(1 - \frac{15}{100}\right)$ $= 40{,}800$ $\text{Half price} = \frac{48{,}000}{2} = 24{,}000$ $\text{Difference} = 40{,}800 - 24{,}000 = 16{,}800$ $16{,}800 = 40{,}800\left(1 - \frac{10}{100}\right)^n$ $\frac{16{,}800}{40{,}800} = 0.9^n \rightarrow \frac{7}{17} = 0.9^n \rightarrow \log 7 - \log 17 = n \log 0.9$ $n = \frac{\log 7 - \log 17}{\log 0.9} = 8.422 \rightarrow 8.422 + 1 = 9.422$	 M1 M1 A1	 Difference between cost at the end of year 1 and half the amount Taking logarithms
	Total	**3**	

NO.	WORKING	MARKS	REMARKS
8.	$20\pi = \pi(3x)^2 - \pi(x+2)^2$ $20\pi = \pi\{9x^2 - (x^2 + 4x + 4)\}$ $20 = 9x^2 - x^2 - 4x - 4 \Rightarrow 8x^2 - 4x - 24 = 0$ $\Rightarrow 2x^2 - x - 6 = 0$ $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4 \times 2 \times -6}}{2 \times 2}$ $x = \frac{1 \pm \sqrt{49}}{4} \Rightarrow x = \frac{1 \pm 7}{4}$ $x = \frac{1-7}{4} = -1.5$ (disccriminate) $x = \frac{1+7}{4} = 2$ Hence $x = 2$	M1 M1 A1	Forming quadratic equation in x When $\sqrt{49} = 7$ seen or factors
	Total	**3**	
9.	$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{\sin^2\theta + \cos^2\theta}{\cos\theta \sin\theta}$ $= \frac{1}{\sin\theta \cos\theta}$	M1 A1	
	Total	**2**	
10.	$\frac{\sqrt{48}}{2} + \frac{6(\sqrt{5}+\sqrt{3})}{(\sqrt{5}-\sqrt{3})(\sqrt{5}+\sqrt{3})}$ $\frac{4\sqrt{3}}{2} + \frac{6(\sqrt{5}+\sqrt{3})}{5-3}$ $\frac{4\sqrt{3}}{2} + \frac{6\sqrt{5}+6\sqrt{3}}{2}$ $\frac{4\sqrt{3}+6\sqrt{3}+6\sqrt{5}}{2}$ $\frac{10\sqrt{3}+6\sqrt{5}}{2} = 5\sqrt{3} + 3\sqrt{5}$ Hence $a = 3$ and $b = 5$	M1 M1 A1	Rationalizing the denominator Simplification
	Total	**3**	
11.	$a + 9d = 435$ $9000 = \frac{25}{2}\{2a + 24d\} \rightarrow a + 12d = 360$ $a + 9d = 435$ $\underline{a + 12d = 360}$ $-3d = 75 \rightarrow d = -25$ $a = 435 - (9 \times -25)$ $a = 660$ Hence $a = 660, d = 25$	M1 M1 A1	Forming two equations in a and d ✓ attempt to solve the simultaneous equations Both values a and d
	Total	**3**	
12.	(a) Period $= 540^0$ $540^0 = \frac{360^0}{k} \rightarrow k = \frac{360^0}{540^0} = \frac{2}{3}$ (b) Amplitude $= \frac{4}{3} = 1\frac{1}{3}$	B1 B1	 B0 if $\frac{4}{3}$
	Total	**2**	

NO.	WORKING	MARKS	REMARKS
13.	$8x + 12y \geq 144$ or $2x + 3y \geq 36$ $x < 9$ $y \leq 2x$ $x + y \geq 10$	B1 B1 B1 B1	
	Total	**4**	
14.	 (a) Consider ΔPOB $PO = \sqrt{5^2 - 4^2}$ $PO = 3$ cm $OQ = 5 - 3 = 2$ cm $QD = \sqrt{5^2 - 2^2} = \sqrt{21}$ $CD = 2QD = 2\sqrt{21} = 9.2$ cm (1 decimal place) (b) FE $DE = \frac{3}{2} \times 9.2 = 13.8$ cm $FE^2 = 13.8 \times (13.8 + 9.2)$ $FE = \sqrt{13.8 \times 23}$ $FE = 17.8$ cm	 M1 A1 M1 A1	
	Total	**4**	
15.	Ali's before increase $= \frac{100 \times 5{,}040}{112} = 4{,}500$ Let Ben's before increase be Ksh. x $\frac{4{,}500}{x} = \frac{5}{3}$ $x = \frac{3 \times 4{,}500}{5}$ $x = 2{,}700$ Difference after Ben's increase $= 8{,}145 - 5{,}040$ $= 3{,}105$ $\%\ \text{change} = \frac{3{,}105 - 2{,}700}{2{,}700} \times 100$ $= 15\%$	M1 M1 M1 A1	
	Total	**4**	

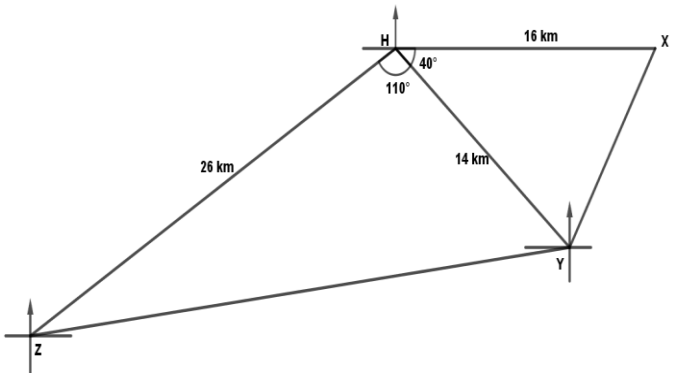
NO.	WORKING	MARKS	REMARKS
16.	$8.75 = \frac{1}{2} \times 5 \times h \rightarrow h = \frac{2 \times 8.75}{5} = 3.5$ cm	B1 B1 B1 B1	✓ height of ΔXPY Constant angle loci Parallel line to XY 3.5 cm above it Locus shaded
	Total	**4**	

NO.	WORKING	MARKS	REMARKS
17.	(a) (i) Let $\mathbf{M} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$		
	$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} A & B \\ 2 & 5 \\ 3 & 3 \end{pmatrix} = \begin{pmatrix} A' & B' \\ -4 & -1 \\ 3 & 3 \end{pmatrix}$	M1	Matrix × Object = Image
	$\begin{pmatrix} 2a+3b & 5a+3b \\ 2c+3d & 5c+3d \end{pmatrix} = \begin{pmatrix} -4 & -1 \\ 3 & 3 \end{pmatrix}$		
	$2a+3b=-4$ $\quad$ $2c+3d=3$	M1	Correct attempt to solve for the unknowns
	$5a+3b=-1$ $\quad$ $5c+3d=3$		
	$\underline{-3a=-3 \rightarrow a=1}$ $\quad$ $\underline{-3c=0 \rightarrow c=0}$		
	$2+3b=-4 \rightarrow b=-2$ $\quad$ $0+3d=3 \rightarrow d=1$		
	$\mathbf{M} = \begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}$	A1	Matrix M
	(ii) Coordinates of C′		
	$\begin{pmatrix} 1 & -2 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} C \\ 4 \\ 1 \end{pmatrix} = \begin{pmatrix} C' \\ x \\ y \end{pmatrix}$	M1	Matrix × Object = Image
	$\begin{pmatrix} 4-2 \\ 0+1 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \end{pmatrix} \Rightarrow C'(2,1)$	A1	Coordinates of C′
	(b) Matrix for reflection along $y-x=0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$	B1	Identification of matrix for reflection along $y-x=0$
	$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} A' & B' & C' \\ -4 & -1 & 2 \\ 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} A'' & B'' & C'' \\ 3 & 3 & 1 \\ -4 & -1 & 2 \end{pmatrix}$	M1	Matrix × Object = Image
	$A''(3,-4)$, $B''(3,-1)$ and $C''(1,2)$	A1	Coordinates
	(c) Let the matrix be $\begin{pmatrix} p & q \\ r & s \end{pmatrix}$		
	$\begin{pmatrix} p & q \\ r & s \end{pmatrix}\begin{pmatrix} 2 & 5 & 4 \\ 3 & 3 & 1 \end{pmatrix} = \begin{pmatrix} 3 & 3 & 1 \\ -4 & -1 & 2 \end{pmatrix}$	M1	
	$\begin{pmatrix} 2p+3q & 5p+3q & 4p+q \\ 2r+3s & 5r+3s & 4r+s \end{pmatrix} = \begin{pmatrix} 3 & 3 & 1 \\ -4 & -1 & 2 \end{pmatrix}$		
	$2p+3q=3$ $\quad$ $2r+3s=-4$		
	$5p+3q=3$ $\quad$ $5r+3s=-1$		
	$-3p=0 \rightarrow p=0$ $\quad$ $-3r=-3 \rightarrow r=1$		
	$0+3q=3 \rightarrow q=1$ $\quad$ $3s=-6 \rightarrow s=-2$		
	Hence matrix= $\begin{pmatrix} 0 & 1 \\ 1 & -2 \end{pmatrix}$	A1	
	Total	**10**	

NO.	WORKING	MARKS	REMARKS
18.			
	(a) Let the change in latitude be θ $60 \times \theta = 5400$ $\theta = \frac{5400}{60} = 90^0$	M1	
	Let the latitude of R be Y^0 $60^0 + Y^0 = 90^0 \rightarrow Y = 90^0 - 60^0 = 30^0$ Hence R→ $(30^0S, 25^0W)$	A1	
	(b) Change in longitude= $20^0 + 25^0 = 45^0$	M1	
	$PQ = \frac{45}{360} \times 2 \times \frac{22}{7} \times 6370 \cos 60^0$	M1	
	PQ = km	A1	
	$QR = \frac{90}{360} \times 2 \times \frac{22}{7} \times 6370$ QR = 10,010 km Total =2502.5 + 10,010 = 12,512.5 km		
	(c) PQ in nautical miles	M1	
	$= 60 \times 45 \cos 60^0 = 1350$ nm	M1	
	Time $= \frac{5400 + 1360}{600}$ $= 11.25$ hours	A1	
	(d) Total time for travel 11 hr 15 minutes + 1 hr 40 min = 12 hr 55 minutes Sunday 0215 hrs − 12 hrs 55 minutes = Saturday 1415 hrs − 55 minutes + 3hrs	M1	
	$(add\ 3hrs\ time\ difference)$	A1	
	= 1620 hrs − Saturday		
	Total	**10**	

NO.	WORKING	MARKS	REMARKS
19.	(a) (i) Taxable Income in K£ per annum $= \frac{38,000 + 14,000 + 8,500 + 3,300}{20} \times 12$ $=$ K£ 38,280	M1 A1	
	(ii) Aggie's PAYE $6,000 \times 2 = 12,000$ $6,000 \times 3 = 18,000$ $6,000 \times 4 = 24,000$ $6,000 \times 5 = 30,000$ $6,000 \times 6 = 36,000$ $6,000 \times 7 = 42,000$ $2280 \times 8 = 18,240$	M1 M1	First six slabs all correct Last slab
	Gross Tax per annum $= 12,000 + 18,000 + 24,000 + 30,000 + 36,000 + 42,000 + 18,240$ $= 180,240$	M1	
	$\text{PAYE} = \frac{165,840 - 18,000}{12}$ $=$ Ksh. 13,520	M1 A1	
	(b) Net Pay per month Gross pay $=$ Ksh. 60,880 Deductions $= 1,000 + 6,250 + 500 + 1,000 + 3,000$ $= 11,750$	M1	Deductions
	Net Pay $= 63,800 - (13,520 + 11,750)$ $=$ Ksh. 38,530	M1 A1	Difference
	Total	**10**	

NO.	WORKING	MARKS	REMARKS
20.	(a) (i) $\text{P(Power)} = 1 - \text{P(No power)}$ $= 1 - \left(\frac{1}{5} \times \frac{1}{4} \times \frac{1}{3}\right)$ $= \frac{59}{60}$	M1 A1	Or otherwise all summed apart from $\text{P}(K'S'G')$
	(ii) Only two sources providing power at any time $= \text{P}(KGS')$ or $\text{P}(KG'S)$ or $\text{P}(K'GS)$ $= \left(\frac{4}{5} \times \frac{3}{4} \times \frac{1}{3}\right) + \left(\frac{4}{5} \times \frac{1}{4} \times \frac{2}{3}\right) + \left(\frac{1}{5} \times \frac{3}{4} \times \frac{2}{3}\right)$ $= \frac{1}{5} + \frac{2}{15} + \frac{1}{10}$ $= \frac{13}{30}$	M1 M1 A1	Identifying all the three possible situations Sum of the three above
	(b) (i) Total number of students $= 40 + 56 + 45 + 60 = 201$ $\text{P(Green or Yellow)} = \frac{40}{201} + \frac{60}{201}$ $= \frac{100}{201}$	 M1 A1	
	(ii) From Green or Pink and does not take foreign languages $= \text{P}(GF')\ \text{P}(PF')$ $= \left(\frac{56}{201} \times \frac{75}{100}\right) + \left(\frac{45}{201} \times \frac{80}{100}\right)$ $= \frac{14}{67} + \frac{12}{67}$ $= \frac{26}{67}$	 M1 M1 A1	
	Total	**10**	

NO.	WORKING	MARKS	REMARKS
21.	(a) Sketch  	M1 M1	Correct sketch Correct labelling
	(b) Distance YZ $h^2 = 14^2 + 26^2 - 2 \times 14 \times 26 \cos 110^0$ $h = \sqrt{1120.990664}$ $h = 33.48$ km	M1 M1 A1	Cosine rule applied Taking $\sqrt{h^2}$
	(c) Bearing of X and Y $h^2 = 14^2 + 16^2 - 2 \times 14 \times 16 \cos 40^0$ $h = 10.43$ km $\frac{10.43}{\sin 40^0} = \frac{16}{\sin Y}$ $Y = \sin^{-1}\left(\frac{16 \sin 40}{10.43}\right)$ $Y = 80.42^0$ Bearing $= 80.42^0 - 50^0 = 30.42^0$ Hence bearing 030^0	M1 A1 B1	Sine rule applied Value of Y Accept N30^{0}E/A0 if 30^0
	(d) P is the circumcentre of ΔHZY $\frac{33.48}{\sin 110^0} = 2R$ $R = \frac{33.48}{2 \sin 110^0}$ $R = 19.33$ km	M1 A1	
	Total	**10**	

NO.	WORKING	MARKS	REMARKS
22.	(a) Mean mark		

Marks	f	x	d	fd	cf
1 – 10	3	5.5	-50	-150	3
11 – 20	5	15.5	-40	-200	8
21 – 30	7	25.5	-30	-210	15
31 – 40	9	35.5	-20	-180	24
41 – 50	11	45.5	-10	-110	35
51 – 60	15	55.5	0	0	50
61 – 70	14	65.5	10	140	64
71 – 80	10	75.5	20	200	74
81 – 90	6	85.5	30	180	80
	$\Sigma f = 80$			$\Sigma fd = -330$	

	B1	All fd correct
$\text{Mean} = 55.5 + \frac{-330}{80}$	M1	Mean
$= 51.375$	A1	
(b) Ogive		
	S1	
	P1	
	C1	Smooth ogive drawn
(c) Students who scored 60% and above	M1	
$= 80 - 48 + 1$	A1	
$= 33$		
(d) Interquartile range		
$Q_3 = \frac{3}{4} \times 80 \rightarrow 60^{\text{th}} = 68.5, Q_1 = \frac{1}{4} \times 80 \rightarrow 20^{\text{th}} = 36.5$	M1	
$\text{IQR} = 68.5 - 36.5$	A1	
$= 32$		
Total	**10**	

NO.	WORKING	MARKS	REMARKS
23.	(a) Capacity filled by A and B in 1 hour $\text{Tap A} \rightarrow \frac{22}{7} \times 10^2 \times 2.1 \times 100 \times \frac{1}{1000} = 66 \text{ litres}$ $\text{Tap B} \rightarrow \frac{22}{7} \times 12^2 \times 1.4 \times 100 \times \frac{1}{1000}$ $= 63.36 \text{ litres}$ $\text{Total} = 66 + 63.36 = 129.36 \times 3600$ $= 456{,}696 \text{ litres}$	M1 M1 A1	Expressions for both A and B Sum of A and B
	(b) Time taken for ¾ full $\frac{3}{4} \times \left(\frac{22}{7} \times 14^2 \times 15\right)$ $= 6{,}930 \text{ m}^3$ $\frac{6{,}930 \times 1000}{456{,}696}$ $= 15.1742$ $\approx 16 \text{ hours}$	M1 M1 A1	Volume or capacity at¾ full Time 16 hours seen
	(c) Time for 75% full $\frac{22}{7} \times 14^2 \times 15 \times 1000 \times 0.75$ 48510000 litres $\left(\frac{22}{7} \times 0.15^2 \times 1.05\right) \times 1000$ $= 519.75 \text{litres}$ $\text{Time} = \frac{48510000}{519.75 \times 3600}$ $= 25.93 \text{ hours}$	M1 M1 M1 A1	Capacity drained in 1 hour by Capacity filled by A, B and C in 1 hour Capacity to be filled to reach 25% in height
	Total	**10**	

NO.	WORKING	MARKS	REMARKS
24.	(a) Number of trapezia $= 7 - 1 = 6$ $h = \frac{5-(-1)}{6} = 1$		

x	-1	0	1	2	3	4	5
y	$-\frac{8}{3}$	-3	$-\frac{8}{3}$	$-\frac{5}{3}$	0	$\frac{7}{3}$	$\frac{16}{3}$

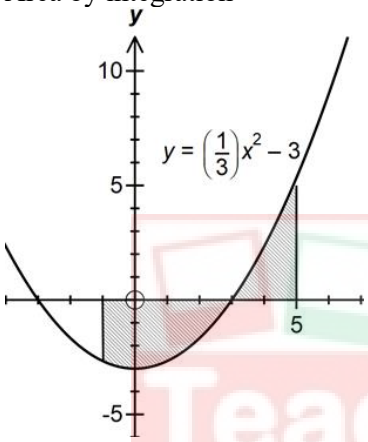
B1

$\text{A} = \frac{1}{2} \times 1\left\{\left(\frac{8}{3} + \frac{16}{3}\right) + 2\left(3 + \frac{8}{3} + \frac{5}{3} + 0 + \frac{7}{3}\right)\right\}$ M1

$\text{A} = \frac{1}{2}\left\{\left(\frac{24}{3}\right) + 2\left(\frac{29}{3}\right)\right\}$ M1

$\text{A} = \frac{1}{2}\left\{8 + \frac{58}{3}\right\} = 13\frac{2}{3}$ A1

(b) Area by integration



$\text{A} = \int_{-1}^{3}\left(\frac{1}{3}x^2 - 3\right)dx \rightarrow \text{A} = \left[\frac{1}{9}x^3 - 3x + c\right]_{-1}^{3}$ M1

$\text{A} = \left[\frac{1}{9} \times 3^3 - 3 \times 3 + c\right] - \left[\frac{1}{9} \times -1^3 - 3 \times -1 + c\right]$

$\text{A} = [-6 + c] - \left[\frac{26}{9} + c\right] = -\frac{80}{9} = \left|-8\frac{8}{9}\right| = 8\frac{8}{9}$

Also

$\text{A} = \int_{3}^{5}\left(\frac{1}{3}x^2 - 3\right)dx \rightarrow \text{A} = \left[\frac{1}{9}x^3 - 3x + c\right]_{3}^{5}$

$\text{A} = \left[\frac{1}{9} \times 5^3 - 3 \times 5 + c\right] - \left[\frac{1}{9} \times 3^3 - 3 \times 3 + c\right]$ M1

$\text{A} = \left[-\frac{10}{9} + c\right] - [-6 + c] = -\frac{10}{9} + 6 = 4\frac{8}{9}$ M1, A1

$\text{Total Area} = 8\frac{8}{9} + 4\frac{8}{9} = 13\frac{7}{9}$

(c) Percentage Error

$= \frac{13\frac{7}{9} - 13\frac{2}{3}}{13\frac{7}{9}} \times 100$ M1

$= 4\frac{1}{31}\%$ or 4.032258065% A1

	Total	**10**	