CHUKA UNIVERSITY

UNIVERSITY EXAMINATIONS

FIRST YEAR EXAMINATION FOR THE DEGREE OF BACHELORS OF EDUCATION SCIENCE/ARTS, BACHELORS OF SCIENCE, BACHELORS OF ARTS(MATHS-ECONS), BACHELORS OF SCIENCE(COMPUTER SC, APPLIED COMP SC), BACHELORS OF SCIENCE(ECON STATS)

MATH 112: BASIC MATHEMATICS

STREAM: BED.SC, BED. ARTS, BA MATHS ECON, BSC.COMP SC, BSC.APP. COMP, BSC.ECON STAT

TIME: 2 HOURS

DAY/DATE: MONDAY 8/12/2014 **8.30A.M – 10.30 A.M**

INSTRUCTIONS:

- **Answer Question ONE and any other TWO questions**
- **Do not write on the question paper**

QUESTION ONE: (30 MARKS)

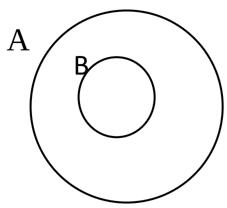
(a) Define the following terms as used in set theory

(i) The intersection of sets A and B

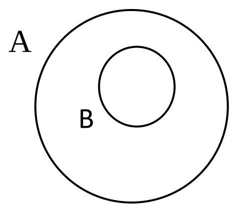
(ii) An empty set

(iii) The difference of sets A and B (3marks)

Hence shade the indicated operations in the given Venn diagrams



$A \cap B$



$A - B$ (2marks)

(b) Let the universal set $\varepsilon = \{x : x\ is\ an\ integer\ and\ 20 \leq x \leq 80\}$;

$M = \{x : x\ is\ a\ perfect\ square\}$; and $N = \{x : ones\ digit\ of\ x\ is\ 9\}$. Find :

(i) $P = \{x : x\ is\ divisible\ by\ 6\}$ (1mark)

(ii) $N \cup P$ (iii) $M \cap N$ (2marks)

(c) Let p be the proposition `` We are in the government'' and $q$ the proposition `` Our party is Jubilee'' is of the form: ``if p, then q''. Write this sentence in the form:

(i) `` p, only if q'' (1mark)

(ii) In contrapositive form i.e. $\sim q \rightarrow \sim p$ (1mark)

(iii) $p \rightarrow q$ (1mark)

(d) Let $X = \{1,\ 2,\ 3,\ 4\}\ and\ Y = \{1,\ 2,\ 3\}$. Define a relation R by the set

$R = \{(x, y)\ such\ that\ x \leq y\ \ \forall x \in X, y \in Y\}$. Find the set R. (3marks)

(e) Given two functions $f(x) = \frac{x+4}{5}$ and $g(x) = \frac{x-1}{2}$, find

(i) $(f\ 0\ g)(x)$ (3marks)

(ii) $(f\ 0\ g)^{-1}(2)$ (3marks)

(f) (i) Determine the 4 terms between 4 and 128 which together form a geometric sequence (3marks)

(ii) Evaluate $\sum_{n=1}^{10}(\frac{3}{4})^n$ (3marks)

(g) Verify the identity $\frac{1+sinx}{cosx} + \frac{cosx}{1+sin\theta} = 2secx$ (3marks)

(h) How many different four figure numbers can be formed from the digits 0,1,2,3,4,5,6 if repetition is not allowed. (2marks)

QUESTION TWO: (20 MARKS)

(a) Determine the domain and range in function $y = \sqrt{x^2 - 25}$ (3marks)

(b) Given two functions $f(x) = \frac{x+4}{5}$ and $g(x) = \frac{x-1}{2}$,

Show that $(f\ 0\ g)^{-1}(x) = (\ g^{-1} 0\ f^{-1})(x)$ (5marks)

(c) (i) The number of bacteria in a colony was originally 3million. The number doubled itself after every one hour. Calculate the number of bacteria generated in the colony during the 7th hour. (2marks)

(ii) Find the least number of terms of the G.P $2 + 6 + 18 + 54 + \cdots$.that must be taken in order that the sum exceeds 125,000. (4marks)

(d) (i) Solve for p in the equation $p_{C_2} = 28.$ (3marks)

(ii) Omondi has 5 teachers. In how many ways can he invite at most three of his teachers to his graduation party? (3marks)

QUESTION THREE: (20 MARKS)

(a) Use analytic method to prove that if ε is the universal set, A and B are sets in ε, with the notation $\varepsilon - A = A^C$ then

(i) $(A \cup B)^C = A^C \cap B^C$ (3marks)

(ii) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$ (6marks)

(b) Construct the truth table for the following propositions stating whether a fallacy, tautology or an inderterminate.

(i) $[(p \rightarrow q) \Lambda (q \rightarrow r)] \rightarrow (p \rightarrow r)$

(ii) $\sim[p \rightarrow (p \, v \sim q) \rightarrow q]$

(iii) $\sim[p \rightarrow (p \, v \, q)]$ (8marks)

(c) In an examinationn60 candidates sat for Mathematics, 80 sat for English and 50 sat for Chemistry. If 20 sat for Mathematics and English, 15 for English and Chemistry, 25 for Mathematics and Chemistry and 10 sat for all the three subjects. Illustrate this information on a venn diagram and hence determine the total number of candidates who sat for the examination. (3marks)

QUESTION FOUR: (20 MARKS)

(a) Show that $\sqrt{2}$ is not a rational number

(3marks)

(b) (i) Given the complex numbers $z_1 = 1 - i, \quad z_2 = 7 + i,$
Find $\frac{z_{1-}z_2}{z_1 z_2}$ in the form of $x + yi$ (5marks)

(ii) Express $z = 1 - i$ in the of $(r, \emptyset)$ (2marks)

(c) (i) Evaluate $(1 + i)^{11}$ giving your answer in the form of $a + bi$ (4marks)

(ii) Use De Moivre's Theorem to prove that

$(\cos\theta + i\sin\theta)^{k+1} = \cos(k\theta + \theta) + i\sin(k\theta + \theta)$ (3marks)

(iii) Given two complex numbers $z_1 = (6, 45^0), z_2 = (2, 30^0)$. Show that

$z_1 z_2 = (12, 75^0)$ (3marks)

QUESTION FIVE: (20 MARKS)

(a) (i) Simplify the expression $\frac{1}{\sqrt{x^2-a^2}}$ in terms of $tan\theta$, if $x = acosec\theta$ (3marks)

(ii) Prove that $\frac{1+tan^2\theta}{sec^2\theta} = cos2\theta$ (4marks)

(b) Solve $sin\theta + 5cos\theta = 2, 0 \leq \theta \leq 360$ (5marks)

(c) Use mathematical induction to prove that
$2^0 + 2^1 + 2^2 + 2^3 + 2^4 ... + 2^n = 2^{n+1} - 1$ (5marks)

(d) Determine the constant term in the binomial expansion $(2a - \frac{3}{a})^{18}$
(3marks)
