SOUTH EASTERN KENYA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION (ARTS), BACHELOR OF EDUCATION (SCIENCE), BACHELOR OF SCIENCE (ECONOMICS AND STATISTICS) BACHELOR OF ECONOMICS.

SMA 206: INTRODUCTION TO ANALYSIS

DATE: 8TH DECEMBER, 2016 **TIME: 4.00-6.00PM**

INSTRUCTIONS

Answer Question One and Any Other Two Questions

QUESTIONS ONE: 30 MARKS (COMPULSORY)

a) given the following typing sets;

: The set of Natural number

: The set of integers

: The set of Rational Numbers

: The set of Irrational Numbers

: The set of Real numbers

i. Starting with Real numbers construct a chart/ diagram to depict the relationships of set inclusion among the above sets. **(3 marks)**

ii. Identify with reasons which of the above sets have a field structure **(2 marks)**

b) Show that every open interval in R is an open set. **(4 marks)**

c) State the completeness axiom for , the set of real numbers. **(2 marks)**

d) By computing lim sup and lim inf determine the convergence or divergence of the sequence. **(4 marks)**

(e). Show that the $\lim_{x\to 0}\left(\frac{3x+2}{x}\right)$ is schizophrenic near $x = 0$ **(4marks)**

(f). State the squeeze theorem for sequences **(3 marks)**

(g). Explain what is meant by unbounded above and unbounded below of the extended reals and describe the order relationship between its two fictitious points and any real number. **(4 marks)**

(h). If n is a positive integer , then for all real values of x ,

prove that $\frac{d}{dx}[x^n] = nx^{n-1}$ **(4marks)**

QUESTION 2 (Optional) 20 MARKS

a) Give a general definition of neighbouhood of a point R (the set of real numbers). Using relevant examples. **(3 marks)**

b) Let and T be neighbourhood of a point . Prove that the intersection is also a neighbourhood of . **(7 marks)**

c) Using relevant examples define a limit point of a set. **(3 marks)**

d) Show with reasons that the set: is closed but not open. **(7marks)**

QUESTION 3 (Optional) 20 MARKS

(a). Use the numerical computation to determine the value of

$\lim_{x\to 4}\left[\frac{1}{(4x-2)}\right]$ **(6marks)**

(b). Let $f(x) = \lim_{x\to 2}\left[\frac{-3x}{x^2-4x+4}.\right]$, by illustration, find its limit. **(8 marks)**

(c). Prove that the image of a countable set under any map is countable. **(6 marks)**

QUESTION 4 (Optional) 20 MARKS

a) Distinguish clearly between a sequence and a series. **(4 marks)**

b) Identify the criterion for convergence of a sequence hence show that the sequence converges to 3 from first principles (by definition). **(5 marks)**

c) Give the definition of the derivative of a function . **(3 marks)**

d) Using (d) above, show that if $f(x)$ is differentiable at $x = c$, then is continous at $x = c$, i.e differentiability implies continuity. **(8 marks)**

QUESTION 5 (Optional) 20 MARKS

(a). Prove that the rational number system is not complete **(7 marks)**

(b). Prove that if r and s are real numbers with $r < s$, then there is a rational number $\frac{m}{n}$ such that $r < \frac{m}{n} < s$. **(8 marks)**

(c). Prove that every convergent sequence is cauchy **(5 marks)**