SOUTH EASTERN KENYA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION SCIENCE AND ARTS, BACHELOR OF SCIENCE MATHEMATICS

SMA 306: COMPLEX ANALYSIS 1

DATE: 7TH DECEMBER, 2016 **TIME: 10.30-12.30PM**

ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (30MARKS)

a. State the necessary and sufficient conditions for $f(z) = u(x, y) + iv(x, y)$ to be analytic in a region $\Re$. (4marks)

b. Find the value of

i) $(1 + i)^{99}$ (3marks)

ii) $(\sqrt{3} + 3i)^{\frac{1}{2}}$ (3marks)

c. Evaluate the following limit $\lim_{z \to 0} \frac{x + y - 1}{z}$ (4marks)

d. Show that $|\sin z|^2 = \sin^2 x + \sinh^2 y$ (4marks)

e. Express the following equation in terms of conjugate coordinates $2x - 3y = 5$ (4marks)

f. Express the complex number $3 + 3i$ in polar form and draw the vector associated to this number in complex plane. (4marks)

g. Determine the continuity of the function $\frac{z^2 + 4}{z(z - 2i)}$. (4marks)

QUESTION TWO (20MARKS)

a. Determine whether the following functions are analytic or not

i) $f(z) = 3z^2 + 7z$ (4marks)

ii) $f(z) = |z|^2$ (3marks)

b. Construct an analytic function whose real part $u(x, y) = x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$ (5marks)

c. Derive the Cauchy Riemann equations in polar form. (5marks)

d. Evaluate the integral $\frac{1}{2\pi i}\oint \frac{z^2+5}{z-5}dz$ (3marks)

QUESTION THREE (20MARKS)

a. If $z = x+iy$ and $w = u+iv$, prove that $\exp(z+w) = \exp z.\exp w$ (4marks)

b. Consider the function $f(z) = x^2 + y + i(2y - x)$. Determine the value of x that will make the function analytic and find $f'(z)$. (5marks)

c. Find the Laurent series about the indicated singularity for each of the following functions and give the region of convergence of each series

i) $\frac{e^{2z}}{(z-1)^3}; z = 1$ (6marks)

ii) $\frac{z - \sin z}{z^3}; z = 0$ (5marks)

QUESTION FOUR (20MARKS)

a. Suppose that $u(x,y) = x^2 - y^2$ and $v(x,y) = 2xy$, show that v is a harmonic conjugate of u in some domain and it is not generally true that u is a harmonic conjugate of v there (7marks).

b. Find the image of unit circle $|z| = 1$ under the mapping $w = z^2$ (6marks)

c. State the Cauchy's integral formula and hence evaluate $\frac{1}{2\pi i}\oint_c \frac{\cos \pi z}{z^2 - 1}$ where c is a rectangle with vertices $-i, -2-i, -2+i, i$ (7marks)

QUESTION FIVE (20MARKS)

a. Expand $f(z) = \frac{1}{z^2-1}$ as a Laurent series about $z = 1$ (4marks)

b. State the Cauchy'sresidue theorem and use it to evaluate $I = \frac{1}{2\pi i}\int_c \frac{z+2}{z(z+1)}dz$, wherec is the circle $|z| = 2$ (10marks)

c. Given that $w = \frac{1}{z} = \frac{x - iy}{x^2 + y^2}$, test for analyticity of the function and determine if the function is a harmonic or not. (6marks)