SOUTH EASTERN KENYA UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS, BACHELOR OF EDUCATION SCIENCE AND ARTS

SMA 320: METHODS 1

DATE: 15TH DECEMBER, 2016 **TIME: 4.00-6.00PM**

ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (COMPULSORY) (30MARKS)

a. Define the following terms
   i) Ordinary points (2marks)
   ii) Regular singular point (2marks)
   iii) Irregular singular point (2marks)
b. Identify any four special functions (4marks)
c. Identify the ordinary points and singular points if any for the ordinary differential equations
   i) $(a^2 - x^2)\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + n(n+1)y = 0$ (2marks)
   ii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - 4)y = 0$ (2marks)
d. Use the power series method to solve $y'' + y = 0$ (5marks)
e. Given that $\Gamma(x+1) = x\Gamma(x)$, determine $\Gamma(4)$ and $\Gamma(x+2)$ (4marks)
f. Find the Laplace transform for $-3t^4e^{-0.5t}$ (3marks)
g. Determine the Legendre polynomial $P_3(x)$ through $P_5(x)$ using Rodrigue's formular (4marks)

QUESTION TWO (20MARKS)

a. Use the method of Frobenius to solve the differential equation $3xy'' + y' - y = 0$ (10marks)
b. Using the Laplace transform or otherwise determine the solution ofthe Bessel equation $x^2 y'' + xy' + (x^2 - 4)y = 0$ (10marks)

QUESTION THREE (20MARKS)

a. Solve the problem $U_t = kU_{\theta\theta}$, $U(\theta, 0) = f(\theta)$ with separation of variables. (10marks)
b. Find the Fourier series of the function $f(x) = x^2$ for $0 \le x \le 2\pi$ (10marks)

QUESTION FOUR (20MARKS)

a. Solve $y'' + 2y' + 3y = \sin t$ using Laplace transform given $y(0) = y'(0) = 0$.(10marks)
b. Find a solution of $(a^2 - x^2)y'' - 2xy' + n(n+1)y = 0$, given that $a \neq 0$ by reduction to the Legendre equation. (10marks)

QUESTION FIVE (20MARKS)

a. Given the Laplace's equation $\nabla^2 \phi = 0$, solve it using the method of separation of variables. (10marks)
b. Find the inverse Laplace transform for

i) $\dfrac{\pi}{(s+\pi)^2}$ (4marks)

ii) $7(s-1)^{-3}$ (3marks)

iii) $\dfrac{1}{s} + \dfrac{2}{s^2}$ (3marks)