SOUTH EASTERN KENYA UNIVERSITY

UNIVERSITY EXAMINATION 2016/2017

FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN MATHEMATICS , BACHELOR OF EDUCTION ARTS AND SCIENCE , BCHELOR OF ECONOMICS AND BACHELOR OF ECONOMICS AND STATISTICS

SMA 322: NUMERICAL ANALYSIS 1

DATE: 6TH DECEMBER, 2016 **TIME: 10.30-12.30PM**

INSTRUCTIONS

ANSWER QUESTION ONE AND ANY OTHER TWO

QUESTION ONE (COMPULSORY) (30MARKS)

a. Highlight four systems of arithmetic which are often used in digital circuits. (4marks)
b. Using Double Dadd method, convert $(1101001\quad)_2$ in to its decimal equivalent. (4marks)
c. Evaluate and interpret the condition number for $f(x) = \left(x^2 - 1\right)^{\frac{1}{2}} - x$ for $x = 200$ (4marks)
d. Using $1's$ and $2's$ complemental subtraction, subtract $(1010\quad)_2$ from $(1101\quad)_2$ and compare the results. (4marks)
e. Construct a forward difference table for $f(x) = x^3 + 2x + 1$ for $x = 1,2,3,4,5$. (5marks)
f. Compute the number of digits is to be taken in computing $\sqrt{2}$ so that the error does not exceed 0.1%. (4marks)
g. Determine the maximum relative error for the function $F = 3x^2y^2 + 5y^2z^2 - 7x^2z^2 + 38$ for $x = y = z = 1$ and $\Delta x = -0.05, \Delta y = 0.001$ and $\Delta z = 0.02$. (5marks)

QUESTION TWO (20 MARKS)

a. Consider the trigonometric function $f(x) = \cos x$

i) Find the Taylor series expansion of $f(x)$ about 0. (3marks)

ii) Assuming Taylor series is truncated to $n = 6$ terms, determine the relative error at $x = \frac{\pi}{4}$ due to truncation. Express it as a percentage (4marks)

iii) Determine the upper bound on the magnitude of the relative error at $x = \frac{\pi}{4}$ expressed as a percentage. (3marks)

b. Using Romberg's method, compute $I = \int_0^{1.2} \frac{1}{1+x}\,dx$ correct to four decimal places.

(10marks)

QUESTION THREE (20MARKS)

a. Express $f(x) = 3x^3 + x^2 + x + 1$ in the factorial notation , interval of differencing being unity. (8marks)

b. Find the first and second derivatives of the function tabulated below at the point $x = 1.5$ (7marks)

x	1.5	2.0	2.5	3.0	3.5	4.0
$f(x)$	3.375	7.0	13.625	24.0	38.875	59.0

c. Given that $f(2) = 4$, $f(2.5) = 5.5$, find the linear interpolating polynomial using Aitken's iterated interpolation and hence find an approximate value of $f(2.2)$ (5marks)

QUESTION FOUR (20MARKS)

a. Given that $\sin(0.1) = 0.09983$ and $\sin(0.2) = 0.19867$,

i) Find an approximate value of $\sin(0.15)$ by Lagrange interpolation. (4marks)

ii) Obtain a bound on the truncation error. (3marks)

b. Perform three iterations of the Newton-Raphson method to solve the system of equations.

$x^2 + xy + y^2 = 7$

$x^3 + y^3 = 9$

Take the initial approximations as $x_0 = 1.5$, $y_0 = 0.5$ and the exact solution to be $x = 2$, $y = 1$ (8marks)

c. By constructing a difference table and taking the second order difference as constant, find the sixth term of the series $8, 12, 19, 29, 42, k$ (5marks)

QUESTION FIVE (20MARKS)

a. Consider the nonlinear function of the variable x given by $F(x) = x - \frac{1}{3}e^{x} = 0$ use $x^{(1)} = 1.2500$ to obtain the first four iterates of the root of the above equation. (6marks)

b. Evaluate the following :

i) $\Delta \log x$ (3marks)

ii) $\left(\frac{\Delta^2}{E}\right)x^2$ (4marks)

iii) $\nabla \sin x$ (3marks)

c. A second degree polynomial passes throughthe points $(1,-1),(2,-2),(3,-1)$ and $(4,2)$. Find the polynomial. (4marks)