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University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE, BACHELOR OF SCIENCE IN ACTUARIAL SCIECE, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN SCIENCE (GENERAL) AND SECOND YEAR, SECOND SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR SCIENCE IN INFORMATION TECHNOLOGY AND BACHELOR OF SCIENCE IN COMPUTER TECHNOLOGY

SMA 2101/STA 2104: CALCULUS I/CALCULUS FOR STATISTICS I

DATE: DECEMBER 2013 **TIME: 2HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

a) Evaluate

$$\lim_{x\to 0}\frac{\sqrt{3x+1}-1}{2x}$$ (3 Marks)

b) Find derivatives of the following functions

i. $y=(x^2+1)(x^3-5x)$ (3 Marks)

ii. $y=\left(5\sqrt{x}-x^4\right)^7$ (3 Marks)

iii. $y=\frac{x^2-1}{x^2+1}$ (3 Marks)

c) Find the values of the constant a so that the following function is continuous for all real x

$$f(x)=\begin{cases} ax+5; & x<1 \\ x^2-3x+4; & x\geq 1 \end{cases}$$ (3 Marks)

d) Using the definition of a derivative (from first principle), find $\frac{dy}{dx}$ for the function $y = x^2 + 3x - 2$. (4 Marks)

e) Given that $y^4 = (x+2)^3 + 9xy$, find $\frac{dy}{dx}$. (3 Marks)

f) Evaluate

$$\int_1^2 \left(\frac{4}{x^2} + 5 \right) dx$$ (3 Marks)

g) A body moves in a straight line according to the law of motion $s = t^3 - 4t^2 - 3t$. Find the value of acceleration at the instants velocity is zero. (5 Marks)

QUESTION TWO (20 MARKS)

a) Find the derivative of the following function

i. $y = 6\sqrt{x} + 5\cos x$ (1 Mark)

ii. $y = ln\left(\frac{3x^2-2}{3x^2-4}\right)$ (3 Marks)

b) Prove that if $y = \sqrt{x} + \frac{1}{4}\sin(2x^2)^2$, then $\frac{dy}{dx} = \frac{1}{2\sqrt{x}} + 2xcos(2x)^2$. (3 Marks)

c) Prove using the definition of derivative, that if $f(x) = \cos x$ then $f'(x) = -\sin x$. (5 Marks)

d) Find $\lim_{x\to\infty} \left(\frac{x+6}{x^3 + 2x + 1} \right)$ (3 Marks)

e) Air is being pumped into a spherical balloon at a rate of 4.5 cubic feet per minute. Find the rate of change of the radius when the radius is 2 feet. (5 Marks)

QUESTION THREE (20 MARKS)

a) Find the equations as the tangent line and the normal line to the curve $(x^2 + 2y)^3 = 2xy^2 + 64$ at the point (0,2). (7 Marks)

b) i) Find $\frac{dy}{dx}$ for the curve representing an involute of a circle whose parametric equations are;

$x = 2(\cos t + t\sin t)$

$y = 2(\sin t - \cos t)$ (4 Marks)

ii) Prove that a function represented parametrically by the equations

$x = 2t + 3t^2$

$y = t^2 + 2t^3$

satisfies the equation; $y = \left(\frac{dy}{dx}\right)^2 + 2\left(\frac{dy}{dx}\right)^3$. (4 Marks)

c) Use the first derivative to find all critical points of the curve $y = \frac{x^3}{6} + \frac{x^2}{4} - x + 2$ and identify each critical point as a local maximum, a local minimum or neither.

(5 Marks)

QUESTION FOUR (20 MARKS)

a) Differentiate the following with respect to x

$y = \frac{e^x + e^{-x}}{e^x - e^{-x}}$ (5 Marks)

b) The displacement of a particle moving along a straight line after t seconds is given by $s = t^3 - \frac{5t^2}{3} - \frac{16t}{3} + 4$

i. Find the initial acceleration. (3 Marks)

ii. Calculate the time the particle was momentary at rest. (3 Marks)

iii. Calculate the minimum velocity of the particle. (2 Marks)

iv. Calculate the distance moved by the particle between the 3rd and 4th seconds. (2 Marks)

c) Given that $y = \sin^{-1}\left\{\frac{1-x^2}{1+x^2}\right\}$, show that $\frac{dy}{dx} = \frac{-2}{1+x^2}$. (5 Marks)