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University Examinations 2014/2015

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF BACHELOR OF EDUCATION, BACHELOR OF SCIENCE, BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER, BACHELOR OF SCIENCE IN STATISTICS, BACHELOR OF SCIENCE IN ACTURIAL SCIENCE, BACHELOR OF SCIENCE IN COMPUTER SCIENCE,BACHELOR OF SCIENCE IN INFORMATION TECHNOLOGY, BACHELOR OF SCIENCE IN COMPUTER TECHNOLOGY, BACHELOR OF SCIENCE IN COMPUTER SCIENCE AND FORENSICS

SMA 3111: DISCRETE MATHS

DATE: DECEMBER 2014 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question* ***one*** *and any other* ***two*** *questions*

QUESTION ONE (30 MARKS)

a) Given that proposition p is false, proposition q is true and proposition r is false, determine whether each proposition is true or false

(i) $p \vee q$ (1 mark)

(ii) $\neg p \vee \neg(q \wedge r)$ (2 marks)

b) Given that u= $\{1,2,3,4,5,6,7,8,9\}$ is a universal set, and A= $\{1,4,7\}$, B= $\{2,4,5,7\}$ and C= $\{3,4,7\}$ are set, list the members of the following sets;

(i) $A \cup B$ (1 mark)

(ii) $(A \cap B)^c$ (2 marks)

c) Given that A= $\{a,b\}$, B= $\{2,3\}$ and C= $\{3,4\}$, determine Ax(B $\cup$ C) (2 marks)

d) Use truth tables to prove that the propositions $(p \wedge q) \vee (\neg p \wedge \neg q)$ and $p \leftrightarrow q$ are logically equivalent (5 marks)

e) Given that $f(x) = \frac{1}{2}x+4$, $g(x) = x-5$, find $(f \circ g)(x)$ (2 marks)

f) Use mathematical induction to show that $1+2+.......+n = \frac{n(n+1)}{2}$ for all integer $n \geq 1$

(5 marks)

g) In each of the following venn diagrams shade the indicated sets:

$A \cap (B \cup C)$ (3 marks) $(A \cap B) \cup (A \cap C) \cup (B \cap C)$ (3marks)

h) Given that f is a function from $\{a, b, c, d\}$ to $\{1, 2, 3\}$ defined by f(a)=3, f(b)=2 f(c) =1 and f(d)=.....Determine whether f is onto (4 marks)

QUESTION TWO (20 MARKS)

a) let $t_1=2$, $t_2=4$, $t_3=8$ and for ≥ 1, $t_{n+3}=t_{n+2}+t_{n+1}+2t_n$. Find a pattern for t_n and prove your answer. (4 marks)

b) Prove that for any integer x, x is odd if and only if x^2 is odd. (5 marks)

c) Prove by contradiction that the sum of a rational number and an irrational number in irrational (6 marks)

d) Prove that $\sqrt{2}+1$ is an irrational number (5 marks)

QUESTION THREE (20 MARKS)

Out of a group of 120 students, the following information is given

65 students study French

45 students study German

42 students study Russian

20 students study French and German

25 students study French and Russian

15 students study German and Russian

8 students study all the 3 languages

a) Draw a venn diagram for the above information (10 marks)
b) Find the number of students studying exactly one language (3 marks)
c) Determine the number of students who take none of the languages (4 marks)
d) Find the number of students studying exactly two languages (3 marks)

QUESTION THREE (20 MARKS)

a) By use of truth tables, establish whether the statements given below are a tantology or contradiction
   (i) $(p \wedge q) \vee (\neg p \vee (p \wedge \neg q)$ (5 marks)
   (ii) $(p \wedge \neg q) \wedge (\neg p \vee q)$ (5 marks)
b) Use a truth table to establish whether the argument below is valid.

   $p \wedge q \rightarrow \neg r$

   $p \vee \neg q$

   $\neg q \rightarrow p$

   $\therefore \neg r$ (6 marks)

c) Show that the statements $\neg(p \wedge q)$ and $\neg p \vee \neg q$ are logically equivalent. (4 marks)

QUESTION FOUR (20 MARKS)

a) Given that f(x) =3x+2 and g(x) = x^2. Find f(x) – g (2x)

b) Given that f(x) = $\frac{1}{2}x + 4$, and g(x) =x-5, show that $(fog)^{-1}(x) = (g^{-1} of^{-1})x$. (8 marks)

c) Show that the exponential function f:$\mathbb{R} \rightarrow \mathbb{R}^+$, f(x) =$e^x$ is an injective function. (5 marks)

d) If f: $\mathbb{R} \rightarrow \mathbb{R}$ is the function defined by the rule f(x) =4x-1 for all real numbers of x. Show that f is onto (4 marks)

QUESTION FIVE (20 MARKS)

a) Given that A $\{1,2,3\}$, find p(A) (3 marks)

b) Given that A= $\{1,2,\}$ and B=$\{3,4,5\}$. Prove that AxB $\neq$ BxA (6 marks)

c) Given the sets A=$\{a,b\}$, B=$\{2,3\}$ and c=$\{3,4\}$

Find (i) Ax(B $\cup$ C) (3 marks)

(ii)(AxB) $\cup$ (AxC (8 marks