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University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATIONS FOR DEGREE OF BACHELOR OF SCIENCE IN COMPUTER SCIENCE/ BACHELOR OF SCIENCE IN STATISTICS/ BACHELOR OF SCIENCE IN MATHEMATICS AND COMPUTER SCIENCE/ BACHELOR OF SCIENCE/ BACHELOR OF SCIENCE IN ACTUARIAL SCIENCE

SMA 2100: DISCRETE MATHEMATICS

DATE: DECEMBER 2013 **TIME: 2 HOURS**

INSTRUCTIONS: *Answer question **one** and any other **two** questions*

QUESTION ONE – (30 MARKS)

(a) Rewrite each interval in set builder form:

(i) $A = [-3,5]$ (2 Marks)

(ii) $B = [3, 8]$ (2 Marks)

(b) Determine whether the function f is an onto function, where f is the function from $\{a,b,c,d\}$ to $\{1,2,3\}$ defined by $f(a)=3,\ f(b)=2,\ f(c)=1$ *and* $f(d)=3$. (3 Marks)

(c) Given that p is true, q is false and r is true, find the truth value for the proposition $p \wedge (q \rightarrow r)$. (3 Marks)

(d) Given that $A = \{1,2,3\}$, find $\wp(A)$. (3 Marks)

(e) Given that $f(x)$ and $g(x)$ are defined in the set of real numbers as $f(x) = \dfrac{2x+7}{4x-5}$ and $g(x) = \dfrac{x-2}{2x-7}$ determine $\dfrac{f}{g}(3)$. (3 Marks)

(f) Given that $ax^2 + bx + c = 0, \quad a \neq o,$ prove that $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. (3 Marks)

(g) Let $t_1 = 2,\ t_2 = 4$ and $t_3 = 8$ and $n \geq 1,\ \ t_{n+3} = t_{n+2} + t_{n+1} + 2t_n$. Find a pattern for t_n and prove your answer. (3 Marks)

(h) Let $u = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$
$A = \{1, 2, 3, 4, 5\}, \quad B = \{2, 4, 6, 8, 10\}\ and\ \ C = \{1, 3, 5, 7, 9\}$.
Find $A' \cap (B \cup C)$. (3 Marks)

(i) Use mathematical induction to prove that for any positive integer
$n, 1 + 3 + 5 + \cdots + (2n - 1) = n^2$ (5 Marks)

QUESTION TWO – (20 MARKS)

(a) Given that $f(x) = 3x + 2$ and $g(x) = x^2$. Find $f(x) - g(2x)$ (2 Marks)

(b) Given that $f: x \to 8x + 4\ and\ g: x \to x - 3$. Show that $(f \circ g)^{-1} = g^{-1} \circ f^{-1}$.
(8 Marks)

(c) Out of a group of 92 students, every student studies one, two or three languages. 50 students study French, 40 students study English and 35 study German. In addition 15 students study French and English, 10 students English and German, while 13 French and German. Find the number of students studying:

(i) At least two languages.
(ii) One language only
(iii)All the three languages. (10 Marks)

QUESTION THREE – (20 MARKS)

(a) Distinguish between a tautology and a contradiction. (3 Marks)
(b) Determine the truth values for the proposition $p \leftrightarrow q$, given that $p: 1 < 5\ and\ q: 2 < 8$.
(4 Marks)
(c) Given that proposition P is false, proposition q is true and proposition r is false, determine the truth value of the proposition.
$(p \vee \neg r) \wedge \neg\big((q \vee r) \vee \neg(r \vee p)\big)$. (6 Marks)

(d) Prove that:

(i) $p \leftrightarrow q \equiv (p \rightarrow q) \wedge (q \rightarrow p)$ (4 Marks)

(ii) $\neg(p \vee q) \equiv \neg p^{c} \wedge \neg q$ (3 Marks)

QUESTION FOUR – (20 MARKS)

(a) Use mathematical induction to prove the proposition that the sum of the first n positive integers is $1+2+3+\ldots+n=\frac{n(n+1)}{2}$ (6 Marks)

(b) Prove that for any integer x, x is odd if and only if x^2 is odd. (5 Marks)

(c) Prove that the sum of two odd numbers x and y is even. (5 Marks)

(d) Prove by contradiction that the sum of a rational number and an irrational number is irrational. (4 Marks)