MERU UNIVERSITY OF SCIENCE AND TECHNOLOGY

P.O. Box 972-60200 – Meru-Kenya.

Tel: 020-2069349, 061-2309217. 064-30320 Cell phone: +254 712524293, +254 789151411

Fax: 064-30321

Website: www.must.ac.ke Email: info@mucst.ac.ke

University Examinations 2013/2014

FIRST YEAR, FIRST SEMESTER EXAMINATION FOR THE DEGREE OF MASTER OF SCIENCE IN APPLIED STATISTICS

STA 3106: TIME SERIES ANALYSIS

DATE: DECEMBER 2013 **TIME: 3HOURS**

INSTRUCTIONS: *Answer questions **one** and any other **two** questions*

QUESTION ONE - (30 MARKS)

a) Consider a white noise model

$x_t = e_t; \quad e_t \sim iidN(0, \sigma_e^2)$

i. Obtain $E(x_{t)}$. (2 Marks)

ii. Autocorrelation function s_j; $j = 0,1,2, \ldots$.. (5 Marks)

b) For each of the following ARMA processes check for stationarity and or invertibility properties;

i. $x_t = -1.3x_{t-1} + 0.4e_{t-2} + e_{t-1} + e_t$; (4 Marks)

ii. $x_t = 0.5x_{t-1} \pm 1.3e_{t-1} + 0.2e_{t-2} + e_t$; (4 Marks)

c) i) Define stationarity in the weak sense. (2 Marks)

ii) Using recursive substitution, express an AR (1) process interms of $MA(\infty)$ process. (3 Marks)

iii) Hence show that the AR(1) process is weakly stationary. (6 Marks)

d) obtain the second order differential data for Y_t and the differenced data on the same axis and comment on the role of differencing

Time t	1	2	3	4	5	6	7	8	9	10
Data Y_t	4.1	3.5	1.6	4.1	5.3	4.9	5.5	3.6	3.1	2.8

(4 Marks)

QUESTION TWO (20 MARKS)

a) The following data shows quarterly averages of sales of a given product.

Year	1992				1993				1994			
Quarter	Mar	Jun	Sep	Dec	Mar	Jun	Sep	Dec	Mar	Jun	Sep	Dec
Sales Y_t	9.9	9.5	8.3	8.7	9.9	8.8	7.0	7.9	9.3	7.5	6.9	6.9

i. Assuming an additive model, obtain the deseasonalized data (Z_t). Hence,

ii. Plot Y_t and Z_t on the same axis. Comment. (10 Marks)

b) Using the Lag Operator approach, transform an AR(1) process to an $MA(\infty)$ process and hence, show that;

$R(1) = cov(x_t, x_{t-1}) = \frac{\emptyset}{1-\emptyset^2}\sigma_e^2$

Assume that $e_t \sim iidN(0, \sigma_e^2)$. (10 Marks)

QUESTION THREE (20 MARKS)

a) Consider the time series process

$x_t = e_t + \beta e_{t-1}; e_t \sim iidN(0, \sigma_e^2)$

i. Obtain $E(x_t), var(x_t), cov\ (x_t, x_{t+h})$ for $h = 0,1,2$ lags.

ii. Hence obtain the correllogram plot of the time series process. (10 Marks)

b) i) Show that the $MA(\infty)$ process below is non-stationarity.

$x_t = e_t + C(e_{t-1} + e_{t-2} + ..; C = constant$

ii) show that $Y_t = \nabla x_t$ for the process in (i) above is however weakly stationary. Obain the autocorrelation function of Y_t. (10 Marks)

QUESTION FOUR (20 MARKS)

a) A time series process is given by

$m_t = \sum_{k=0}^{p} c_k t^k ; t = 0, \pm, \pm 2, ... c_k = constants$ show that the first order difference of m_t is a polynomial degree $(p-1)$ in t and hence that

$\nabla^{p+1} m_t = 0$ (6 Marks)

b) Define the difference operator $\nabla_{12} x_t = x_t - x_{t-12}$ Y_t is a stationary time series process, mean zero.

i. If $x_t = a + bt + s_t + Y_t$; s_t = seasonal component with period d= 12, show that $\nabla\nabla_{12} x_t = (1-B)(1-B^{12})x_t$ is stationary. (8 Marks)

ii. $x_t = (a + bt) + s_t + Y_{t;} s_t$ = seasonal component with period d = 12, show that $\nabla_{12}^2 \mathrm{x_t} = (1 - \mathrm{B}^{12})(1 - \mathrm{B}^{12})\mathrm{x_t}$ is stationary. (6 Marks)

QUESTION FIVE (20 MARKS)

a) Consider $\{x_t\}$, stationary time series process with $E(x_t) = 0$ and $R_x(h) = cov\,(x_t, x_{t+h})$. Define $f_x(\lambda)$ to be the power spectrum of x_t.

Show that $Y_t = \sum_{j=\infty}^{\infty} \alpha_j x_{t-j}\,;\alpha_j = constants$ has a power spectrum defined by

$f_y(\lambda) = f_x(\lambda)\left|\sum_{k=\infty}^{\infty} \alpha_k e^{e-i\lambda k}\right|^2$. (14 Marks)

b) Hence from (a), show that
$x_t = e_t + \beta e_{t-1}$ has a power spectrum defined by
$f_y(\lambda) = \frac{\sigma^2}{2\pi}\left(1 + \beta^2 + 2\beta\cos\lambda\right)$

Hint: the power spectrum of a white noise is given by $f_e(\lambda) = \frac{\sigma^2}{2\pi}$. (6 Marks)