MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

SECOND YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

SPH 203: MATHEMATICAL METHODS FOR PHYSICS I

Date: 5th December, 2016 Time: 12.00 - 3.00 pm

INSTRUCTIONS:

- Answer question ONE (Compulsory) and any TWO questions in SECTION B.

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SPH 203: MATHEMATICAL METHODS FOR PHYSICS I

SEPTEMBER-DECEMBER, 2016 SESSION

INSTRUCTION

Answer question *1 (Compulsory)* and any other ***TWO*** questions in **section B.**

SECTION A

QUESTION 1 (30 MARKS)

a) Define the following terms.

(i) Sequence **(1 mark)**

(ii) Series **(1 mark)**

(iii) Asymptote **(1 mark)**

(iv) Interval of convergence of a power series **(1 mark)**

b) Determine whether the sequence whose general term is given by $a_n = \frac{3n(-1)^n}{n+1}$ converges to a limit or diverges. **(2 marks)**

c) Show that the series $\sum_{k=1}^{\infty} \frac{1}{(k+4)(k+5)}$ is convergent. **(3 marks)**

d) State the ratio test for convergence of a series. **(2 marks)**

e) Apply the linearity, product and chain rules to differentiate the function

$f(x) = (9x^3+1)^2 \sin 5x + \ln x^3 - e^{4x}$ **(3 marks)**

f) Use the method of integration by parts to evaluate the definite integral

$\int_1^N \ln x dx$ **(3 marks)**

g) Represent $f(x) = \sin x$ by a Taylor series at $x = \frac{\pi}{3}$. **(3 marks)**

h) Calculate $\tanh x$ given that $x = 1$. **(2 marks)**

i) A force $\bar{F} = 20\hat{i} + 10\hat{j} + 6\hat{k}$ acts on a wheel at a point $\bar{r} = 2\hat{i} + 3\hat{j} + \hat{k}$ from the centre of the wheel. Calculate the torque on the wheel. **(2 marks)**

j) A physical system in motion traces a path defined by the parametric equations $x = 2t^3 + 1$ and $y = t^2 \cos t$. Determine the gradient function of the path traced. **(3 marks)**

k) Use de L'Hopital's rule to evaluate $\lim_{x \to 0} \frac{\sin x}{e^x - e^{-x}}$ **(2 marks)**

l) State TWO main axioms of a vector space. **(1 mark)**

SECTION B

Answer any TWO questions in this section.

QUESTION 2 (20 MARKS)

a) (i) Define a linear map. **(3 marks)**

(ii)Let a map T be defined as $T\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} x \\ -y \end{pmatrix}$. Show that T is linear. **(7 marks)**

b) Apply the concept of integration of rational functions to evaluate

$$\int \frac{x^2 + 2x + 4}{(x+1)^3} dx$$ **(6 marks)**

c) Calculate the direction cosines and direction angles of the vector $\bar{a} = 2\hat{i} + 5\hat{j} + 4\hat{k}$. **(4 marks)**

QUESTION 3 (20 MARKS)

a) A ladder $20\ m$ leans against a vertical wall. The foot of t he ladder is being drawn away from the wall at the rate of $2\ m/s$. How fast is the top of the ladder sliding down at the instant when the foot is $10\ m$ from the wall? **(10 marks)**

b) Given $\vec{A} = \hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{B} = 3\hat{i} + 2\hat{j} + \hat{k}$. Find:

(i) $\vec{A} X \vec{B}$ **(3 marks)**

(ii) $\vec{A}.\vec{B}$ **(2 marks)**

(iii) Angle between $\vec{A}$ and $\vec{B}$ **(2 marks)**

(iv) Unit vector perpendicular to both $\vec{A}$ and $\vec{B}$. **(3 marks)**

QUESTION 4 (20 MARKS)

a) State the root test for convergence of a series. **(3 marks)**

b) By using the answer in 4(a), test for the convergence of the series $\sum_{k=1}^{\infty}\left(\frac{5}{k}\right)^{k}$ **(2 marks)**

c) Evaluate $\int \frac{x}{(4x^2+3)^6}dx$ **(5 marks)**

d) A ball is dropped from x metres above a flat surface. Each time the ball hits after falling a distance h, it rebounds a distance, rh where r is a positive number less than 1. Find the total distance the ball travels up and down. **(10 marks)**

QUESTION 5 (20 MARKS)

a) The unitless equivalents of Pauli spin matrices are given by $\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, $\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ and $\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ where $i = \sqrt{-1}$. Show that the commutation brackets $[\sigma_x, \sigma_y] = 2i\sigma_z$ and $[\sigma_y, \sigma_z] = 2i\sigma_x$ **(8 marks)**

b) (i) Find the eigenvalues of the matrix $A = \begin{pmatrix} 4 & -2 \\ 1 & 1 \end{pmatrix}$ **(5 marks)**

c) Water runs into a conical tank at the rate of 2 m^3 per minute. Determine the speed at which the water level is rising, given that the smallest face of the tank is on the ground and its depth is twice its radius. **(7 marks)**