MASENO UNIVERSITY

UNIVERSITY EXAMINATIONS 2016/2017

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION SCIENCE WITH INFORMATION TECHNOLOGY

MAIN CAMPUS

SPH 402: STATISTICAL MECHANICS

Date: 3rd December, 2016 Time: 12.00 - 3.00 pm

INSTRUCTIONS:

- Answer question ONE (Compulsory) and any TWO questions in SECTION B.

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UNIVERSITY EXAMINATIONS FOR 2016/2017 ACADEMIC YEAR

FOURTH YEAR FIRST SEMESTER EXAMINATIONS FOR THE DEGREE OF BACHELOR OF SCIENCE AND BACHELOR OF EDUCATION SCIENCE WITH INFORMATION TECHNOLOGY (MAIN CAMPUS)

SPH 402: STATISTICAL MECHANICS

SEPTEMBER-DECEMBER, 2016 SESSION

INSTRUCTION

Answer question *1(Compulsory)* and any other ***TWO*** questions in **section B.**

SECTION A

QUESTION 1 (30 MARKS)

(a) Define the following terms as used in Statistical Mechanics

(i) Phase space **(1 mark)**

(ii) Density of states **(1 mark)**

(iii) Statistical weight **(1 mark)**

(iv) Degree of freedom **(1 mark)**

(b) The Hamiltonian of a physical system in a phase space is defined by

$H = \frac{P^2}{2m} + \frac{1}{2} m\omega^2 q^2$, where the symbols have their usual meanings. Determine the Hamilton's canonical equation of motion. **(3 marks)**

(c) State TWO properties of particles obeying Bose-Einstein statistics, hence derive the equation for determining the total number of microstates within this distribution. **(4 marks)**

(d) Calculate the statistical weight of a macrostate consisting of 5 particles obeying Maxwell-Boltzmann distribution and two energy levels 1 and 2 such that level 1 has 2 particles and a degeneracy of 2 while level 2 has a degeneracy of 3.

(3 marks)

(e) Distinguish between a microstate and a macrostate. **(2 marks)**

(f) Starting with Gibb's free energy, $G = F + PV$, derive expressions for entropy, volume and chemical potential. **(3 marks)**

(g) Show that the conditions for equilibrium of a system in a heat bath can be derived from the inequality $dU_A - T_R dS_A + P_R dV_A - \mu_R dN_A \leq 0$ where the symbols have their usual meanings. **(3 marks)**

(h) Derive the equation for the partition function of a spin-$\frac{1}{2}$ paramagnetic system.

(3 marks)

(i) Black body radiation in a box of volume V and at temperature T has internal energy $U = \sigma V T^4$ and pressure $P = \frac{1}{3}\sigma T^4$ where σ is the Stefan-Boltzmann constant. Determine the Helmholtz free energy in terms of U. **(3 marks)**

(j) Explain the formation of Bose-Einstein condensate. **(2 marks)**

SECTION B

Answer any TWO questions in this section

QUESTION 2 (20 MARKS)

(a) Starting with an example of a Fermi sphere, derive the equation for density of states. **(10 marks)**

(b) Derive Hamilton's equations of motion in a phase space described by generalized position (q) and momentum (p) coordinates. **(10 marks)**

QUESTION 3 (20 MARKS)

(a) (i) Distinguish between a canonical ensemble and grand canonical ensemble.

(2 marks)

(ii) Show that the average internal energy in a canonical ensemble is given by

$\overline{E} = \frac{1}{Z}\sum_i \varepsilon_i e^{-\beta\varepsilon_i}$ where the symbols have their usual meanings. **(3 marks)**

(b) Show that the Stirling's approximation for a very large number of particles, N, is given by $\ln N! = N\ln N - N$. **(5 marks)**

(c) Determine the conditions under which a composite isolated system C resulting from interaction of two isolated systems A and B attains thermodynamic equilibrium. **(10marks)**

QUESTION 4 (20 MARKS)

(a) Show that the Fermi-Dirac distribution function is given by $f_{FD} = \left(e^{\left(\frac{\varepsilon_i - \mu}{k_B T}\right)} + 1\right)^{-1}$.

(10 marks)

(b) Show that the entropy, S, of a thermodynamic system is given in terms of the Boltzmann constant, k_B and statistical weight, Ω, in the form $S = k_B \ln\Omega$.

(10 marks)

QUESTION 5 (20 MARKS)

(a) Derive the equation of the energy density of electromagnetic radiation from a black body in the form

$$u(\upsilon,T)d\upsilon = \frac{8\pi h\upsilon^3}{c^3\left(e^{\left(\frac{h\upsilon}{k_B T}\right)} - 1\right)}d\upsilon$$ where the symbols have usual meanings.

(12 marks)

(b) Show that the value of wavelength, λ, for which $u(\lambda,T)$ obtained in 4 (a) above is maximum is given by $\lambda_m T = 2.9 mmK$. Use the solution $x = 5$ for $1 - e^{-x} = \frac{x}{5}$. **(8 marks)**