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**JARAMOGI OGINGA ODINGA UNIVERSITY OF SCIENCE AND TECHNOLOGY**

**SCHOOL OF MATHEMATICS AND ACTUARIAL SCIENCE**

**UNIVERSITY EXAMINATION FOR DEGREE OF BACHELOR OF EDUCATION AND ACTUARIAL SCIENCE.**

**3rd YEAR 1st SEMESTER 2015/2016 ACADEMIC YEAR**

**REGULAR (MAIN)**

**COURSE CODE: SMA 300**

**COURSE TITLE: REAL ANALYSIS**

**EXAM: STREAM: (BEd.,and Actuarial Sc.)**

**DATE: EXAM SESSION:**

**TIME: 2.00 HOURS**

**Instructions:**

1. **Answer question 1 (Compulsory) and ANY other 2 questions**
2. **Candidates are advised not to write on the question paper.**
3. **Candidates must hand in their answer booklets to the invigilator while in the examination room.**

**QUESTIONONE**

1. Let  and be sets and be a function. Define

i) an injective function . (2mks)

ii) asurjective function . (2mks)

1. Write down the first 5 terms of the following sequences and state whether it is monotonically increasing or decreasing.
2. . (3mks)
3. . (3mks)
4. Let  be a real-valued function which is bounded on [a, b]. Define the upper integral of  on [a, b]. (3mks)
5. Show that$\lim\_{x\to 2}x^{2}=4$. (5mks)
6. Show that  is uniformly continuous on [-1, 1]. (4mks)
7. Give the formal definition of a metric space. (3mks)
8. Let  be a function defined by

$f=\left\{\begin{array}{c}1, if x\in Q\\2, if x, otherwise\end{array}\right.$.

 Determine whether  is Riemannintegrable on the interval

[a, b]. (5mks)

**QUESTIONTWO**

1. State and prove the squeeze theorem for sequences. (6mks)
2. Use the squeeze theorem to show that for any, with,

.

 (10mks)

1. Show that every convergent sequence is Cauchy. (4mks)

**QUESTIONTHREE**

1. Let  be a non-empty set. Define the following

(i) a complete metric space on the set. (2mks)

(ii)a sequentially compact subset of a metric space. (2mks)

1. Let, where  is a non-empty set and  is a metric on. Show that a Cauchy sequence in  which has a convergent subsequence is convergent. (6mks)
2. If and , then show that . (10mks)

**QUESTIONFOUR**

1. Let  be a sequence of real numbers. Show that the limit of  is unique if it exists. (6mks)
2. Show that. (8mks)
3. Let, show that the sequence  diverges.(6mks)

**QUESTIONFIVE**

1. Let  be a real-valued function which is bounded on. Then show that  is integrableon if and only if, for any, there exists a partition on  such that. (10mks)
2. If  and  is integrableon both and, then  is integrableonand. (10mks)