

## 2. Binomial expansion

<p><b>1</b></p> <p>a)</p> $\left(2 - \frac{1}{x}\right)^8 = 2^8 - 8 \cdot 2^7 \left(\frac{1}{x}\right) + 28 \cdot 2^6 \left(\frac{1}{x}\right)^2 - 56 \cdot 2^5 \left(\frac{1}{x}\right)^3 + 70 \cdot 2^4 \left(\frac{1}{x}\right)^4 + \dots$ $256 - \frac{1024}{x} + \frac{1792}{x^2} - \frac{1792}{x^3} + \frac{1120}{x^4}$ <p>b)</p> $(1.75)^8 = 256 - \frac{1024}{4} + \frac{1792}{16} - \frac{1792}{64} + \frac{1120}{256}$ $= 256 - 256 + 112 - 28 + 4.375$ $= 88.375$	<p>M<sub>1</sub></p> <p>A<sub>1</sub></p> <p>M<sub>1</sub></p> <p>A<sub>1</sub></p>	$2 - \frac{1}{x} = 1.75$ $0.25 = \frac{1}{x} \therefore x = 4$
		4
<p><b>2.</b></p> <p>(2+x)<sup>5</sup></p> $25 + 5(2)4x + 10(2)3x^2 + 10(2)2x^3 + \dots$ $= 32 + 80x + 80x^2 + 40x^3 + \dots$ <p>(1.970)<sup>5</sup> = (2 - 0.03)<sup>5</sup></p> <p>x = -0.03</p> $(1.97)^5 = 32 + 80(-0.03) + 80(-0.03)^2 + 40(-0.03)^3$ $= 32 - 2.4 + 0.072 - 0.00108$ $= 29.67092$ $= 29.67$	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p>	
		04

1.
  - a)  $I^5 + 5(-3x)^1 + 10(-3x)^2 + 10(-3x)^3 + 5(-3x)^4 + (3x)^5$   
 $1 - 15x + 90x^2 - 270x^3 + 405x^4 - 243x^5$   
 $1 - 15x + 90x^2 - 270x^3 + 405x^4 - 243x^5$
  - b)  $3x = 1 - 0.997$   
 $x = 0.001$   
 $= 1 - 15(0.001) + 90(0.001)^2 - 270(0.001)^3 + 405(0.001)^4$   
 $= 1 - 0.015 + 0.00009 - 0.00000027 + \dots$   
 $= 1 + 0.00009 - 0.015 - 0.00000027$   
 $= 1.00009 - 0.01500027 = 0.98508973$   
 $= -0.9851 \text{ (4 d.p)}$
2.
  - (i)  $5 + \frac{x}{2}^6 = 15625 + \frac{3125}{3}X + \frac{9375}{4}X^2 + \frac{625}{2}X^3 + \dots$
  - (ii)  $X = 1$   
 $\left(\frac{11}{2}\right)^6 = 15625 + \frac{3125}{3} + \frac{9375}{4} + \frac{625}{2}$

$$= 15625 + 1041.667 + 2343.75 + 312.5$$

3.

$$(\sqrt{3} + 2x)^6 = (\sqrt{3})^6 + \sqrt{6}(\sqrt{3})^5 \cdot 2x + 15(3)^4(2x)^2 + 20(\sqrt{3})^3(2x)^3$$

$$= 27 + 108x\sqrt{3} + 270x^2 + 480x^3\sqrt{3}$$

$$\sqrt{3} + 2x = 3\sqrt{3}$$

$$\sqrt{2x + 2} = 3$$

$$x = \sqrt{3}$$

$$27 + 108\sqrt{3}\sqrt{3} + 270\sqrt{3}^2 + 480\sqrt{3}(3)^3$$

$$= 27 + 324 + 810 + 4320 = 5481$$

4.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

$$P(\text{Sum odd}) = \frac{18}{36} = \frac{1}{2}$$

5.  $\angle PQR = 180 - (35^\circ + 75^\circ)$   
 $= 70^\circ$

$$PR^2 = 12^2 + 8.4^2 - 2(12)(8.4) \cos 70^\circ$$

$$PR = 145.61 = 12.07$$

6. (a) Terms;  $2^5, 2^4(\frac{3}{x}), 2^3(\frac{3}{x})^2, 2^2(\frac{3}{x})^3, 2^1(\frac{3}{x})^4$

$$\text{Co eff } 1, 5, 10, 10, 5$$

$$(2 + \frac{3}{x})^5 = 2^5 + 5(2)^4(\frac{3}{x}) + (2)^3(\frac{3}{x})^2 + 10(2)^2(\frac{3}{x})^3 + 5(2)(\frac{3}{x})^4$$

$$= 32 + 2140x^{-1} + 720x^{-2} + 1080x^{-3} + 820x^{-4}$$

(b)  $9.5 = 2 + \frac{3}{x}$

$$\frac{3}{x} = 7.5$$

$$x = \frac{3}{7.5} = 0.4$$

$$(9.5)^5 = 32 + \frac{240}{0.4} + \frac{720}{(0.4)^2} + \frac{1086}{(0.4)^3} + \frac{810}{(0.4)^4}$$

$$= 53647.625(3d.p)$$

7.  $X^5 - 5x^4(0.2) + 10x^3(0.2)^2 - 10x^2(0.2)^3 + 5x(0.2)^4 - (0.2)^5$

$$X^5 - 5x^4(\frac{2}{10}) + 10x^3(\frac{2}{10})^2 - 10x^2(\frac{2}{10})^3 + 5x(\frac{2}{10})^4 - (\frac{2}{10})^5 + x^5 - (\frac{4}{10})x^3 - (\frac{8}{100})x^2 + 5x \cdot 16 - \frac{2^5}{10^5}$$

$$X^5 - x^4x^3 - \frac{8}{100}x^2 + 80x - \frac{2^5}{10^5}$$

$$90, 392, 079$$

8.  $\log(x+24) = \log(x(9-2x))$

$$X + 24 = 81 - 18x$$

$$X = 3$$

9.  $\frac{1+x}{12} = \frac{1}{2} + \frac{x}{48} + \frac{5x^2}{432} + \frac{5x^3}{432}$

$$\left(1 + \frac{x}{12}\right)^6 = 1 \frac{1}{4}$$

12

$$\frac{x}{12} = \frac{1}{4}$$
$$x = 3$$

$$\left(\frac{5}{4}\right)^6 = 1 + \frac{3}{2} + \frac{9}{48} + \frac{27}{432}$$

$$= 2.7500$$

10. (a)  $(1 + \frac{1}{2})^8 = 1 + 8(\frac{1}{2}) + 28(\frac{1}{2}x)^2 + 56(\frac{1}{2}x)^3 + 70(\frac{1}{2}x)^4 + 567(\frac{1}{2}x)^5 + 2(\frac{1}{2}x)^6 + 8(\frac{1}{2}x)^7 + (\frac{1}{2}x)^8$   
 $= 1 + 4x + 7x^2 + 7x^3 + 4.375x^4 + 1.75x^5 + 0.4375x^6 + 0.0625x^7 + \frac{1}{256}x^8$

(b)  $(1.05)^8 = 1 + 4(0.1) + 7(0.1)^2 + 7(0.1)^3$   
 $= 1 + 0.4 + 0.07 + 0.0074...$   
 $= 1.48$

11.  $81 + 27x + 9x^2 + 3x^3 + x^4$   
 $81 + 108x + 54x^2 + x^4$   
 $81 + 108(0.02) + 54(0.02)^2$   
 $= 83.182$