3.0 PART ONE: ANALYSIS OF DIFFICULT QUESTIONS

3.1 MATHEMATICS ALT A (121)

In the year 2022 Mathematics Alternative A was tested in two papers. **Paper 1 (121/1)** and **Paper 2 (121/2)**. Each paper consisted of two sections: Section 1 (50 marks) consisting of 16 compulsory short answer questions of not more than four marks each and Section II (50 marks), with eight questions of 10 marks each where candidates answer any five.

Paper 1 (121/1) tests mainly Forms 1 and 2 work while Paper 2 (121/2) tests mainly forms 3 and 4 work of the syllabus.

This report is based on an analysis of performance of candidates who sat the year 2022 KCSE Mathematics Alt A.

3.1.1 Candidates' general performance

The table below shows the performance of both papers in the last five years.

Table 10: Candidates' performance in Mathematics Alt A for the last five years 2018 – 202

Year	Paper	Candidature	Maximum Score	Mean Score	Standard Deviation
2018	1		100	24.07	21.16
	2	658904	100	28.82	20.85
	Overall		200	**52.88**	**41.1**
2019	1	694445	100	31	24.037
	2	694347	100	23	20.904
	Overall		200	**55.08**	**43.91**
2020	1	742796	100	22.27	19.41
	2	742760	100	14.45	14.97
	Overall		200	**36.72**	**33.45**
2021	1	822376	100	23.66	19.87
	2	822242	100	16.39	15..27
	Overall		200	**40.04**	**33.98**
2022	1	877215	100	16.17	16.42
	2	877128	100	14.22	14.84
	Overall		200	**30.38**	**29.93**

From the table the following observations can be made:

(i) There was a drop of 7.495 marks in the mean mark of 121/1 compared to the previous year 2021. In 121/2, there was a drop of 0.22 marks in the mean mark compared to the previous year 2021.

(ii) The overall mean mark out of 200 dropped by 9.66 marks.

3.1.2 Individual question analysis

The following is a discussion of some of the questions in which the candidates had major weakness in. As a result, these questions were poorly performed. The discussion is based on the report from the chief examiners.

3.1.3 Mathematics Paper 1 (121/1)

In section 1 of 121/1, questions 2, 11 and 15 were popular among the candidates. In Section II, questions 19(b), 21(a) and 24 were the most popular among the candidates. Majority of the other remaining questions were said to have been relatively poorly performed. Questions 8, 11, 17, 18 and 19 will be discussed further in this section.

Question 8

A lorry left town A for town B and maintained an average speed of 50 km/h. A car left town A for town B 42 minutes later and maintained an average speed of 80 km/h. At the time the car arrived in town B, the lorry had 25 km to cover to town B.
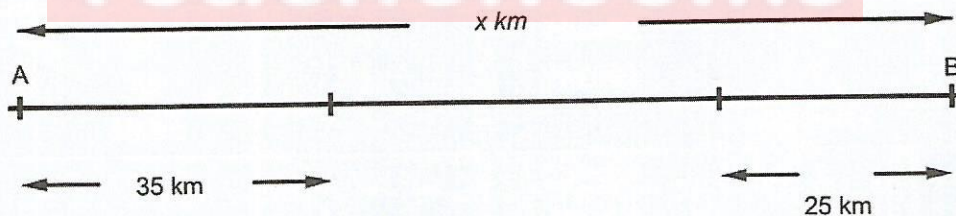Determine the distance between town A and B.   (3 marks)

The question tested on solving problems involving relative speed..

Weaknesses
- Failure to correctly interpret the question

Expected response

Distance moved by lorry in 42 min

$$= 50 \times \frac{42}{60}$$

$$= 35 \text{ km}$$

Let distance AB $= x$ km
The time taken by lorry to move a distance of $x - (35 + 25) = (x - 60)$ km is the same time taken by the car to travel a distance of x km

$$\frac{x}{80} = \frac{x - 60}{50}$$

$$80x - 4800 = 50x$$

$$30x = 4800$$

$$x = 160 \text{ km}$$

Alt

Let the time taken by lorry to travel from A to B = t hrs

Distance AB= $(50 \times t)$ km

Time it would take lorry to travel 25 km = $\dfrac{25}{50}$ hrs = 30 min

Excess time taken by lorry compared to car=$(42+30)$min=1.2 hours

Time taken by car to travel from A to B = $(t-1.2)$ hrs

$50t = 80(t - 1.2)$

$t = 3.2\,hrs$

$x = 3.2 \times 50$

$= 160\ km$

Advice to teachers

The concept and laws of relative speed are founded on ability to come up with equations based on time, distance and speed. It is important to show not just how to use the relationships but also the processes of derivation and principles behind their relationships.

Question 11

A Kenyan bank bought and sold United Arab Emirates (UAE) dirhams on two different dates as shown below.

		Buying (Ksh)	Selling (Ksh)
1st August 2021	1 UAE dirham	28.40	28.90
16th August 2021	1 UAE dirham	28.00	28.40

A Kenyan tourist who travelled to UAE on 1st August 2021 converted Ksh 130 050 to UAE dirhams.

During her stay in UAE, she spent 3 520 UAE dirhams. She arrived back to Kenya on 16th August 2021. On the same day she converted the remaining amount of money to Kenya shillings at the same bank.
Calculate the amount of money in Kenya shillings that she received from the bank.   (3 marks)

The question tested on conversion of currency from one form into another given the exchange rates.

Weaknesses
* Most of the candidates were unable to distinguish when to use buying and selling rates.

Expected response

On 1st August 2021, the **bank sells** 1 UAE dirham @ sh28.90

$$\text{Ksh } 130\,050 = \frac{130050}{28.90} = 4500 \text{ Dirhams}$$

Remaining amount of UAE dirhams $= 4500 - 3520 = 980$ Dirhams

On 16th August 2021, the **bank buys** 1 UAE dirham @ sh28.00

Amount received from the bank

980 Dirhams $=$ Ksh 980×28.00

$\qquad\qquad\qquad = $ Ksh 27 440

Advice to teachers

Teachers should emphasize to learners that banks are in the business of buying and selling **foreign currencies**. The banks make a profit from the business. Individuals are not the ones who sell or buy not sell or buy the currency they have. They convert the currency they have to another and they get this service from banks. Teachers also need to emphasize that exchange rates fluctuate. Organizing visits to financial institutions may also help learners to understand the concepts.

Question 17

A contractor hired Wema and Tatu to transport 144 tonnes of stones to building sites A and B. To transport 48 tonnes of stones for a distance of 28 km, the contractor paid Ksh 24 000.

(a) Wema transported 96 tonnes of stones to site A, a distance of 49 km.
   (i) Calculate the amount of money that was paid to Wema.   (2 marks)

   (ii) For every 8 tonnes of stones Wema transported to site A, he spent Ksh 3 000.
   Calculate the profit Wema made.   (3 marks)

(b) Tatu transported the remaining 48 tonnes of stones to site B, a distance of 84 km. If Tatu made 44% profit, calculate the amount of money Tatu spent to transport the stones.
                                                                               (3 marks)

(c) Determine the ratio of the profit made by Wema to that made by Tatu.   (2 marks)

The question tested on solving problems involving direct and inverse proportions

Weaknesses

- Majority of the candidates had challenges interpreting the question and in comprehending the information in the question.

Expected Response

(a) (i) Amount paid to Wema

$$= 24000 \times \frac{96}{48} \times \frac{49}{28}$$

$$= Ksh\ 84000$$

(ii) Amount spent by Wema

$$= 3000 \times \frac{96}{8}$$

$$= Sh.\ 36000$$

Profit (Wema)

$$= 84000 - 36000$$

$$= Ksh\ 48000$$

(b) Amount to Tatu

$$= 24000 \times \frac{48}{48} \times \frac{84}{28}$$

$$= Ksh\ 72000$$

Cost of transporting (Tatu)

$$= \frac{100}{144} \times 72000$$

$$= Ksh\ 50,000$$

(c) Profit (Tatu) $= 72000 - 50000$

$$= Sh.\ 22000$$

Ratio of profit

Wema : Tatu $= 48000 : 22000$

$$= 24 : 11$$

A variety of alternative methods was also used.

Advice to teachers
Teachers need to expose learners to application of linear inequalities in real life situations.

Question 23

A supermarket sold 530 packets of milk daily when the price was Ksh 50 per packet. Whenever the price per packet was increased by Ksh 4, the number of packets sold daily decreased by 20.

If n represents the number of times the price was increased:

(a) write an expression in terms of n for:

    (i) the price of a packet of milk after the price was increased.   (1 mark)

    (ii) the number of packets of milk sold after the price was increased.   (1 mark)

    (iii) the total sales, in simplified expanded form, after the price of a packet of milk
          was increased.   (2 marks)

(b) Determine:

    (i) the number of times the price was increased to attain maximum sales.   (3 marks)

    (ii) the price of a packet of milk for maximum sales.   (1 mark)

    (iii) the maximum sales.   (2 marks)

The question tested on application of differentiation in finding maxima and minima of a function. Candidates were also expected to form the function from given information.

Weaknesses
- Most candidates could not form an equation for sales in terms of n.
- Most learners had difficulties interpreting the question

Expected Response

    (a) (i)   Price after increase
               If the price is increased once, new price = 50+4, If price is increased twice, new
               Price = 50+4×2 and so om.
               Therefore, new Price after n increases
               $= 50 + 4n$

       (ii)   No. of pks sold after price change
               $= 530 - 20n$

       (iii)   Total sales (s)
               $= (50 + 4n)(530 - 20n)$
               $= 26500 + 1120n - 80n^2$

(b) (i) For max sales

$$\frac{ds}{dn} = 1120 - 160n$$

$$1120 = 160n = 0$$

$$n = 7$$

(ii) Price after increases

$$= 50 + 4 \times 7$$
$$= \text{Ksh. } 78$$

(iii) Total sales (max)

$$= 78 \times (530 - 20 \times 7)$$
$$= 78 \times 390$$
$$= \text{Ksh } 30\ 420$$

Advice to teachers

Teachers need to expose learners to a wider variety of questions requiring application of maximizing and minimizing. Linking this area with business makes it more real life to learners

3.1 4 Mathematics Paper 2 (121/2)

In section 1 of 121/2, questions 3, 4, 5, 9, 12 and 13 were popular among the candidates. Questions 18 and 21 were popular among the candidates in Section II. Questions 1, 15 and 17 will be discussed further in this section.

Question 1

An investor took a loan from a bank that charged interest. The loan and the interest accrued were repaid in monthly instalments. The investor repaid Ksh 1 500 in the first month and in each subsequent month the instalments were reducing by Ksh 50 until the loan was fully repaid. Determine the maximum amount that may be paid for that loan. (3 marks)

The question tested on application of A.P formula in solving problems in real life situations.

Weaknesses
- Most candidates found it difficult to generate an AP with a negative common difference.
- Many candidates could not identify the sum of the series as the amount repaid for the loan.

Expected response

$$1500 + 1450 + \ldots + 0$$

$$0 = 1500 + (n-1) \times -50$$

$$n-1 = \frac{1500}{50}$$

$$n = 31$$

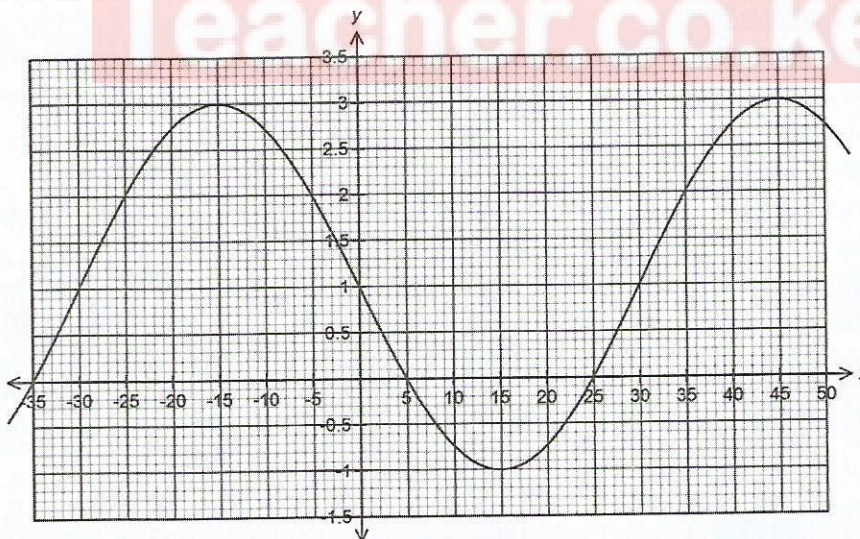$$S_{30} = \frac{31}{2}\{3000 + 30 \times -50\}$$

$$= 15.5 \times 1500$$

$$= 23250$$

Advice to teachers

Teachers need to expose learners to a wider variety of questions requiring application of AP and GP in real life.

Question 11

The figure below represents the curve of the function $y = 1 - A \sin wx$ for the range $-35° \leq x \leq 50°$.

Determine the values of A and w.   (3 marks)
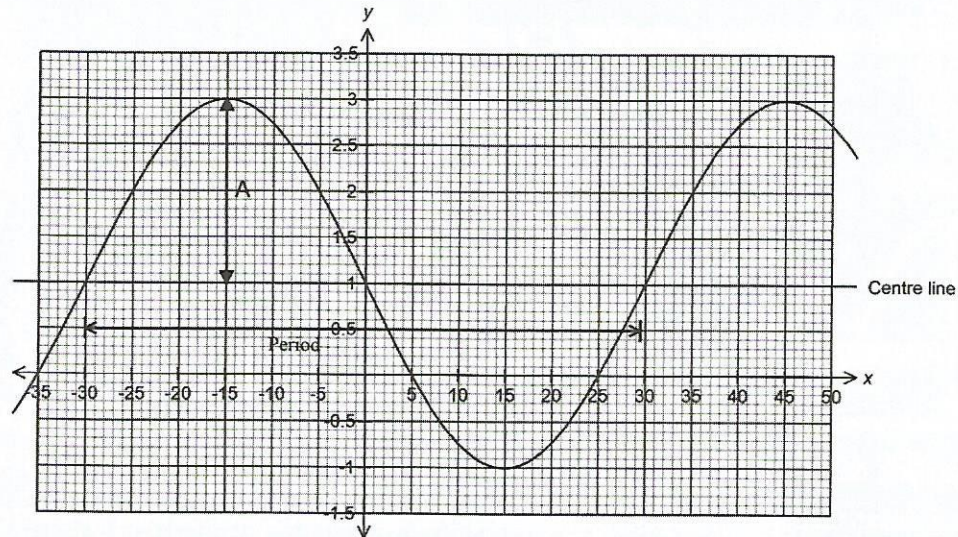
The question tested the candidates' ability to deduce from a trigonometric graph the amplitude, period, wavelength of a wave.

Weaknesses

Inability to identify centre line of the wave as $y = 1$ and not the x–axis.
Counting squares instead of units while getting amplitude

Expected response

Amplitude, A = 2

Period = 60°

$$w = \frac{360}{60}$$

$$= 6$$

Advice to teachers

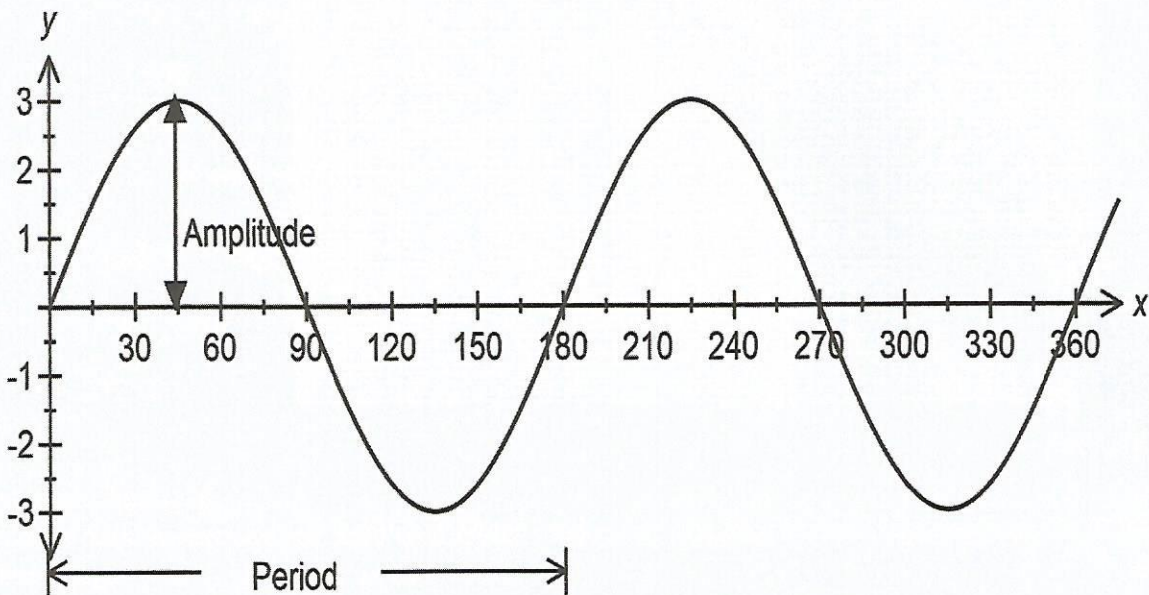**Requisite definitions and build-up of equisite knowledge**

Amplitude

The amplitude (A) of a wave is the distance from the centre line (or the still position) to the top of a crest or to the bottom of a trough.

Period

The distance or time taken for a wave's shape to repeats itself.

Example using the wave whose equation is

Example using the wave whose equation is $y = 3\sin(2x)$

From the wave of the function $y = 3\sin(2x)$ drawn above the amplitude is 3 and the period 180°.

The same results can be obtained by interpretation of the equation of wave.

Amplitude

The maximum value of the sine of an angle is at 90°, 450°, 810°,...... which is 1. With that in mind, the maximum possible value of $\sin(2x)$ in $y = 3\sin(2x)$ is 1 making the maximum possible value of y = 3. The Amplitude is 3.
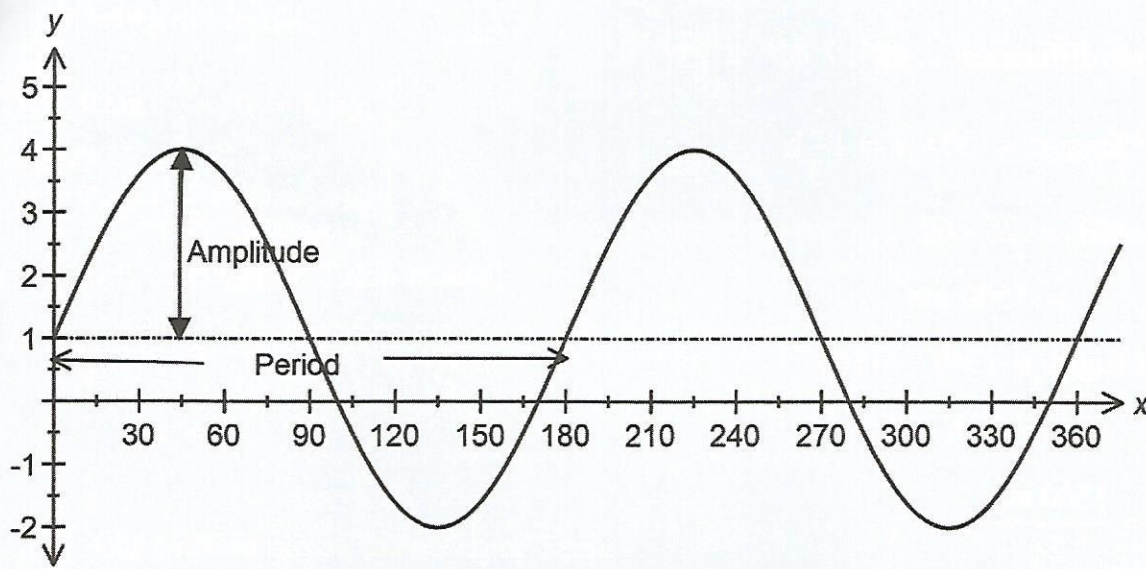
The period is got using the formula: $\text{Period} = \dfrac{360}{w}$ where w is the coefficient of x in $\sin(wx)$.

In the case of $y = 3\sin(2x)$, period $= \dfrac{360}{2} = 180°$.

Effect of adding a constant

Suppose we add a constant part the trigonometric part of $y = 3\sin(2x)$

Eg $y = 1 + 3\sin(2x)$

The effect of adding a constant to a is that the wave is translated by vector $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$. However, the amplitude and the period of the wave as measured from the new centre line remain unchanged.

Question 14

An aircraft took off from an airport A(0°, 40°W) at 1100 h local time. The aircraft landed at airport B (0°, 65°W) at 1200 h local time.
Determine the speed of the aircraft in knots. (4 marks)

The question tested on two objectives: Calculation of time in relation to Longitudes and calculation speed in knots and kilometres per hour.

Weaknesses
- Most candidates could identify/Define knot as a measure of speed.
- Most candidates did not interpret local time
- Most candidates could not get the correct time of flight by factoring longitude difference between Airports A and B.

Expected response

Longitude difference $= 65° - 40° = 25°$

Time difference $= 20 \times 4 = 1\frac{2}{3}$ h

distance AB $= 25 \times 60$

$\qquad\qquad = 1500$ nm

Time of flight $= 1340 - 1100$

$\qquad\qquad = 2\frac{2}{3}$ h

Speed $= \dfrac{1500 \text{ nm}}{2\frac{2}{3}}$

$\qquad\quad = 562.5$ knots

Advice to teachers
The knot is a measure of speed. Teachers need to teach it alongside teaching of speed in km/h. The meaning of the local time at a place needs to be emphasized. Teachers need to expose learners to situations where there is need to calculate factor time difference due to longitude difference when calculating the duration of flight.

Question 17
A wholesaler stocks two types of rice: Refu and Tamu. The wholesale prices of 1 kg of Refu and 1 kg of Tamu are Ksh 80 and Ksh 140 respectively. The wholesaler also stocks blend A rice which is a mixture of Refu and Tamu rice mixed in the ratio 3 : 2.

(a) (i) A retailer bought 10 kg of blend A rice. To this blend, the retailer added some

Tamu rice to prepare a new mixture blend X. The ratio of Refu rice to Tamu rice

in blend X was 1 : 2.

Determine the amount of Tamu rice that was added. (3 marks)

   (ii) The retailer sold blend X rice making a profit of 20%. Determine the selling price

of 1 kg of blend X. (3 marks)

(b) The wholesaler prepared another mixture, blend B, by mixing x kg of blend A rice with

y kg of Tamu rice. Blend B has a wholesale price of Ksh 130 per kg.

Determine the ratio x : y. (4 marks)

The question tested on applications of Proportions to mixtures.

Weaknesses
- Many candidates were unable to correlate the new blend from the initial blend.

Expected response

(a) (i) In 10 kg of blend A:

$$\left. \begin{array}{l} \text{Refu}: \dfrac{3}{5} \times 10 = 6\,\text{kg} \\[2mm] \text{Tamu}: \dfrac{2}{5} \times 10 = 4\,\text{kg} \end{array} \right\}$$

Let x be kgs of Tamu added:

$$\frac{6}{4+x} = \frac{1}{2}$$

$$x = 8$$

(ii) 1 kg of Refu mixed with 2 kg of Tamu

$$\text{Wholesale price} = \frac{1 \times 80 + 2 \times 140}{3}$$

$$= 120$$

$$\text{Retailer's price} = \frac{120}{100} \times 120$$

$$= 144$$

(b) In x kg of Blend A

$$\text{Refu} = \frac{3}{5}x, \quad \text{Tamu} = \frac{2}{5}x$$

In blend B:

$$\text{Refu} = \frac{3}{5}x\,\text{kg}; \quad \text{Tamu} = \frac{2}{5}x + y\,\text{kg}$$

Cost of 1 kg of Blend B

$$\frac{80 \times \dfrac{3}{5}x + 140x\left(\dfrac{2}{5}x + y\right)}{x+y} = 130$$

$$48x + 56x + 140y = 130x + 130y$$

$$104x - 130x = 130y - 140y$$

$$-26x = -10y$$

$$\frac{x}{y} = \frac{5}{13}$$

$$\therefore \text{Ratio } x:y = 5:13$$

Advice to teachers

Teachers need to emphasize on construction of non-horizontal parallel lines. Learners should also be exposed to a wide variety of ways of constructing parallel lines.

Question 22

Fifty teachers in a sub county attended a workshop. The table below shows the distribution of the distances (d) in kilometres travelled by the teachers from their respective school to the training venue.

Distance d (km)	0 – 4	5 – 9	10 – 14	15 – 19	20 – 24	25 – 29
No. of teachers	4	7	11	14	9	5

(a) On the grid provided, draw a cumulative frequency graph to represent the information above. (4 marks)
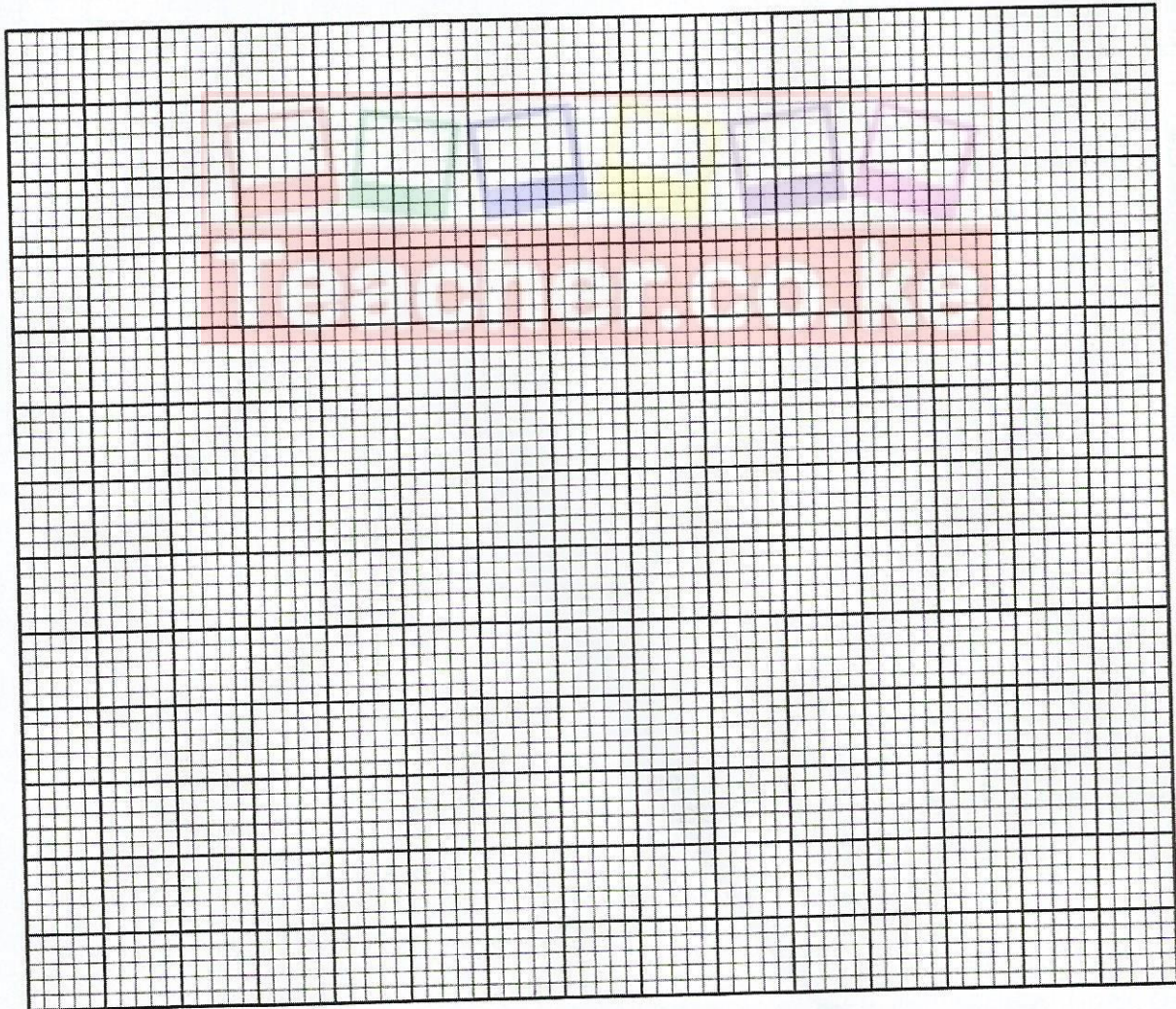
(b) Use the graph to estimate:

(i) the median distance (1 mark)

(ii) the number of teachers who travelled a distance d km where $15 \leq d \leq 23$

(3 marks)

(c) Each of the 75% of all the teachers who travelled a distance d km where $d \leq 10$ km, used a motor bike and each was charged Ksh 50.

Determine the total amount of money raised by the motor bike operators. (2 marks)

The question tested ability of candidates to make cumulative frequency table, estimate the median and the quartiles by ogive and interpret measures of dispersion.

Weaknesses

- Generating an appropriate linear scale on the x – axis
- In cases of intervals like (b) (ii) and (c), incorrect computations. (see advise to teachers)

Expected Response

To be able to **anchor ogive at 0**, the horizontal axis need to have a scale that can accommodate 0.5 units. A scale of 2 cm to represent 5 units is able to achieve this.

(a)

U.C.B	4.5	9.5	14.5	19.5	24.5	29.5
C.F	4	11	22	36	45	50
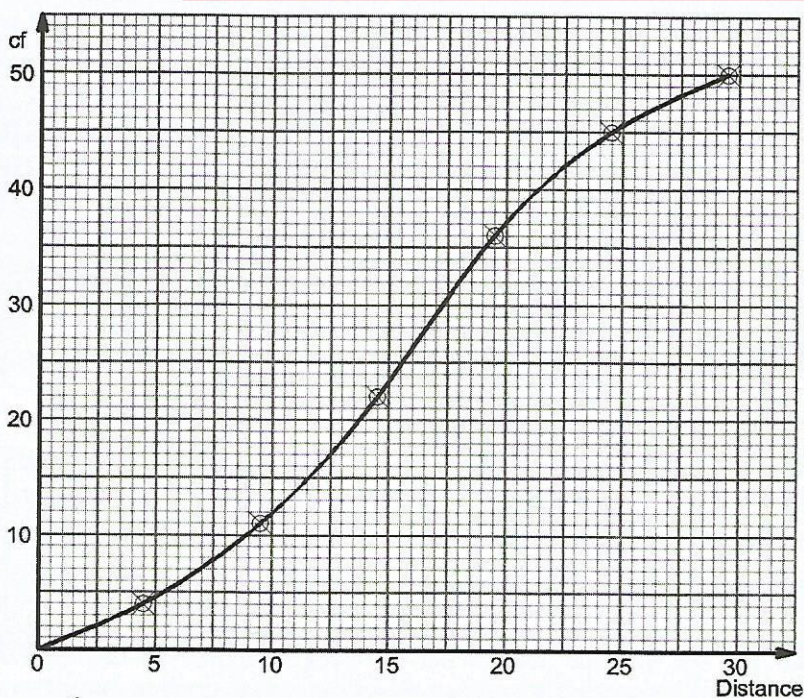
(b) (i) Median distance = distance covered by 25th teacher
$$= 15.5 \pm 0.1 \text{ km}$$

(ii) No of teachers in range $15 \leq d \leq 23$

No. of teachers $(d \leq 23) = 43$
No. of teachers $(d \geq 15) = 23$
No. of teachers $= (43 - 23) + 1$
$$= 21$$

(c) No. of teachers $(d \leq 10) = 12$

No of teachers $(d \leq 10)$ who used bike

$$= \frac{75}{100} \times 12$$

$$= 9$$

Amount raised $= 9 \times 50$

$$= \text{Ksh } 450$$

Advice to teachers

(a) Anchoring of ogive on the x axis
An ogive need to be anchored on the x- axis. A suspended ogive is said to be incomplete.
We need to teach students on the importance of anchoring their ogives

(b) Intervals
Consider this scenario from the cumulative distribution table.
There were 50 teachers who commuted. With the distances commuted arranged in ascending order, the 45th teacher travelled a distance of 24.5 km. How many teachers travelled a distance $d \geq 24.5$ km?

Its easy to say $(50 - 45) = 5$ teachers, but by hand count, it's the $45^{th}, 46^{th}, 47^{th}, 48^{th}, 49^{th}$

and 50th teachers which gives 6 teachers. When boundary is included, get the difference and then add 1.

Conclusion
Major weaknesses have been observed in most areas of the syllabus for Mathematics Alt A. These areas include Numbers (sequences and series, mixtures), Geometry (General construction, Loci, Longitudes and Latitudes), Commercial Arithmetic (Appreciation and depreciation), Measurements (mass, density and weight), Statistics (Ogive), Trigonometry (Amplitude and period of a wave), Transformations (Transformation matrices) and Differentiation (maxima and minima - application to real life).

Application of learned concepts to real life situations was observed to be a challenge to many candidates. Contexts introducing an item are supposed to clarify issues and not to make items wordy. To help learners understand the concepts, it is necessary to have more applications relating to real life situations in the course of the teaching and learning. There is also need to introduce learners to questions with contexts. The contexts relate mathematics to real life and give meaning to Mathematics.