NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF EDUCATION

COURSE CODE: EDU 808

COURSE TITLE: MATHEMATICS CURRICULUM AND INSTRUCTIONS IN SECONDARY SCHOOLS

COURSE GUIDE

EDU 808
MATHEMATICS CURRICULUM AND INSTRUCTIONS IN SECONDARY SCHOOLS

Course Developer	Dr. A. O Sule Al-Hikma University Adewole Housing Estate Ilorin
Course Writer	Dr. A. O Sule Al-Hikma University Adewole Housing Estate Ilorin
Course Editor	Dr. J. O Aiyedun National Open University of Nigeria Lagos
Programme Leaders	Dr. (Mrs.) M. E. Aina School of Education National Open University Of Nigeria Lagos.
	Dr. J. O Aiyedun National Open University of Nigeria Lagos

NATIONAL OPEN UNIVERSITY OF NIGERIA

National Open University of Nigeria
Headquarters
14/16 Ahmadu Bello Way
Victoria Island
Lagos

Abuja Annex
245 Samuel Adesujo Ademulegun Street
Central Business District
Opposite Arewa Suites
Abuja

e-mail: centralinfo@nou.edu.ng
URL: www.nou.edu.ng

National Open University of Nigeria 2006

First Printed 2006

ISBN: 978-058-086-7

Printed by ……………..
For
National Open University of Nigeria

Content

What you learn in this Course

Working through this Course

To complete this course, you are required to read the study units, read reference books, and read other materials provided by NOUN. Each unit contains Self Assessment Exercises (SAEs) and Tutor Marked Assignments (TMAs) and at points in the course, you are required to submit assignments for assessment purposes.

This is a 2 credit unit Post Graduate course consisting of 14 units.

Course Materials

Major components of the course are:

1. Course Guide
2. Table of Content
3. Assignment File
4. Presentation Schedule

Course Guide

The course is divided into 3 modules made up of units as follows:

Module One

Unit 1 The Nature and Usefulness of Mathematics
Unit 2 Goals and Objectives of Mathematics Instruction
Unit 3 Mathematics Curriculum Planning and Design
Unit 4 Curriculum Development in Mathematics in Nigeria

Module Two

Unit 1 Development of Mathematics Syllabuses and Scheme of Work
Unit 2 Development of Course Plan, Unit Plan, Weekly Plan and Daily Lesson Plan
Unit 3 Contributions of Some Psychologists to Mathematics Instruction
Unit 4 Piagets Works and Contributions to Mathematics Instruction

Module Three

Unit 1 Meaning and Purpose of Evaluation of Instruction in Mathematics
Unit 2 Continuous Assessment in Mathematics Instruction
Unit 3 Problems and Prospects of Mathematics Instruction in Secondary Schools in Nigeria
Unit 4 Briefs of Some past Mathematicians
Unit 5 Mathematics Education Strategies

Table of Content

The content of each unit is presented as follows:

3 Introduction
4 Objectives
5 Main Body
4.0 Conclusion
5.0 Summary
6.0 Tutor Marked Assignment
7.0 References/Further Readings

Assignment File

There are tow aspects to the assessment of this course. First is the Tutor Marked Assignments while the second is the end of semester written examination. You are expected to apply the information, knowledge and techniques gathered during the course.

There are 14 Tutor Marked Assignments in the course, your tutor will tell you which ones to submit and only three of them will be used to count toward 30%, while the end of the semester examination will contribute the remaining 70%.

Presentation Schedule

Your tutor will mark and comment on your assignments, keep a close watch on your progress and on any difficulties you may encounter and also provide assistance to you during the course. You must mail your tutor marked assignments to your tutor well before the due date.

How to Get the Most from this Course

Tutors and Tutorials

You should try your best to attend tutorials.

This is the only chance you have for a face to face contact with your tutor and for you to ask him questions. You will learn a lot from participating in active discussions with him and fellow students.

Summary

We wish you success with the course and hope you will find I interesting, useful and rewarding.

Course Code	EDU 808
Course Title	Mathematics Curriculum and Instructions in Secondary Schools
Course Developer	Dr. A. O Sule Al-Hikma University Adewole Housing Estate Ilorin
Course Writer	Dr. A. O Sule Al-Hikma University Adewole Housing Estate Ilorin
Course Editor	Dr. J. O Aiyedun National Open University of Nigeria Lagos
Programme Leaders	Dr. (Mrs.) M. E. Aina School of Education National Open University Of Nigeria Lagos.
	Dr. J. O Aiyedun National Open University of Nigeria Lagos

NATIONAL OPEN UNIVERSITY OF NIGERIA

National Open University of Nigeria
Headquarters
14/16 Ahmadu Bello Way
Victoria Island
Lagos

Abuja Annex
245 Samuel Adesujo Ademulegun Street
Central Business District
Opposite Arewa Suites
Abuja

e-mail: centralinfo@nou.edu.ng
URL: www.nou.edu.ng

National Open University of Nigeria 2006

First Printed 2006

ISBN: 978-058-086-7

Printed by ……………..
For
National Open University of Nigeria

Table of Content

MODULE1

UNIT 1 THE NATURE AND USEFULNESS OF MATHEMATICS

CONTENTS

1.0 INTRODUCTION

In this unit, you will be introduced to what mathematics is concerned with, the nature of mathematics and the Usefulness of mathematics to life and to other subjects.

The important knowledge of subject matter in teaching and learning of Mathematics has been shown by researchers of Mathematics education and psychology to be very important. The Mathematics teacher is expected to have acquired a sufficient amount of knowledge in Mathematics before he/she could really teach the contents of School Mathematics Curriculum. This is why the nature of Mathematics is extensively discussed in this unit. Since Mathematics is a complex discipline, attempts are made to analyse the complexities involved. Further, Mathematics is the bed rock of science and technology. It is a tool for engineers, architects, agriculturists, economists, geographers, sociologists, business administrators and computer scientists. Hence, Secondary School students should understand the interrelationships among Mathematics, Biology, Chemistry, Geography, Economics, Physics, Social Studies and other subjects. However, before Mathematics teacher can successfully conceptualise such interdependencies, he/she must have the knowledge of the nature of Mathematics.

In the same vein, the Mathematics teacher should understand the ways in which adolescents think. At times, the ways are at variance with those of adults. The variance affects the interaction between Mathematics teachers and their students. This may lead to poor performance of students in Mathematics tasks and the learning of its development of adolescents should adequately guide the teacher in the selection of curriculum and instructional materials for his/her pupils. This implies that the Mathematics teacher must be familiar with the Mathematics Curricula that are available in Nigeria and the problems of implementing them.

We shall be dealing with these issues in this unit.

2.0 OBJECTIVES

At the end of this unit you will be able to do the following:

1. explain and describe the structure of Mathematics and its implications for teaching and learning of Mathematics.
2. explain adolescent ways of thinking and how it influences the teaching and learning of Mathematics at the Secondary School level, and
3. interpret the contents of JSS and SS Mathematics curricula
4. give in your own words, correct explanations of at least five aspects of the Nature and Usefulness of Mathematics
5. explain at least one example of the application of each of the identified aspects

6. identify the use to mathematics in other areas of life and subjects.

3.0 MAIN BODY

3.1 Study Approach

To get the best of this unit, you are advised to follow closely the following suggestions:

1. Carry out the activities as and when requested to do so **without** reference to the suggested solutions first.
2. Read through a paragraph straight on first and then applying some reflective thinking the second time and then with a paper and pencil put down the central idea of the paragraph
3. Think and put down at least one other example to buttress the central idea in the paragraph.
4. At the end of this unit carry out the exercises and activities. Then check the suggested solutions where applicable and score your performance.
5. If your score is less than 60% please go through the unit again repeating all steps
6. Finally check through the objectives to see if all the objectives have been achieved.

3.2 Nature of Mathematics

Like other discipline, Mathematics has its own structure. There are two major aspects of this structure viz; (a) Substantive structure (The domain of Product) and (b) Syntactical Structure (the process). We shall look at the two aspects shortly.

Before that, ask yourself, "What is Mathematics" and what is it all about. Usually, there is an issue on the nature of Mathematics. The issue is whether Mathematics was "invented by Man" or Mathematics might guide your answer choice?

There are two worlds which contain Mathematics: (i) The world of Mathematics which is equivalent to the "idea world" and (ii) The Physical world. In each world, we have the product and process aspect. Remember that a set of another set of processes will lead to a product or products, products may serve as input into another set of processes, resulting into products of higher dimensions.

3.2.1 Products Aspects of Mathematics

Since you will be teaching structure in Algebra, Geometry and Statistics at the Secondary School level, it is relevant here that the nature of the products of Mathematics is made very clear to you. The products are concepts (define and undefined), postulates, theorems, rules and principles. These are in the world of Mathematics which is the idea world.

In the physical world, we have date, problems or events. Note the small number of products of Mathematics in the physical world. You might have come across some textbooks in which authors attempt to define every concept in Mathematics. Euclid did it but some of his definitions were not precise. They contain contradictions. For example, Euclid defined a point, a straight line, set and others. These are examples of undefined concepts. Can you find out more of such concepts in Mathematics?

However, let us examine some other products:

3.2.2 Mathematical Model

The final product of Mathematics is the mathematical model. A Mathematical model refers to a set of mathematical terms and statement which appear to be idealistic in the *physical world* but faithful reflection of data and event discrete product of Mathematics (such as a formula, a graph, a number system, or collection of point son a line) is called a Mathematical model of a kind. Much of mathematics taught in the junior – senior secondary school classes would be more meaningful to students if the concept of Mathematical model are introduced.

It is concerned with the computational model such as, to find the simple interest we use the model "I = PRT/100".

This is also related to algorithms model commonly employed in mathematics. For example to find the mean we use the model

$$M = \frac{\blacktriangleleft f(x)}{\blacktriangleleft f}$$

The consideration of Mathematical models lead us to the second aspect of Mathematics.

3.2.3 Process Aspects of Mathematics

Remember that, "Mathematics is also a verb". You can Mathematize. This is what Mathematics do. Deductive processes associated with actions such as assuming, computing, hypothesizing and proving are

Mathematic processes. Also, we have inductive processes implied by actions such as testing conjecturing, and generalizing. Furthermore, Mathematicians sometimes idealize physical situations by abstracting, and symbolizing both objects are relationships among objects. The language of Mathematics has been a powerful historic assest to developments in knowledge. Consider the Greek version of a2+b2 = (a +b)(a+b).

When symbols in the above statement had not been devised. The Greek could not easily handle problems involving the square of a binomial. Yet they were using this relationship by operating with the dissection and rearrangement of square and rectangular shape. Various problems in Mathematics are solved by observing, collecting date, and making inference form the date. These are the processes of Mathematics in the physical work.

3.3 Implications for Mathematics Instruction

Although, the concepts and assumptions which form the basis for every mathematical system are inventions of the human mind, they are not the thoughtless gibberish which might be assembled by a robot. There is a rationale behind each such product of Mathematics. Sometime that rationale takes the form of a motivating force, a need for a simpler way of handling a chore. Sometimes, the rationale has its root in history, in common usage, or in the etymology of a word. At other times, the rationale for teaching a particular concept at a time and in a particular way may be explained in terms of its sensible match to the previous learning of your student.

The history of Mathematics as well as the hierarchical nature of its products is essential to these connecting links, which illuminates the Mathematics that you teach. In some cases, careful analysis of concepts and principles will help provide you with ties which must be embedded in your teaching. In the use of inductive or deductive processes of Mathematics, care should be taken not to over-stress each of the processes. You should let your students see where induction will fail and where is the necessary and sufficient tool. The use of induction leading to generalization is good for the junior secondary school students but you should at a point at the JSS III notice the limit of the process. However, the choice of the processes you want to teach depends on the objectives you construct for a lesson.

3.3.1 The Usefulness of Mathematics in Secondary School Curriculum

Mathematics is the science of quantity and space. "It is a sytematised, organized and exact branch of science". It is a creation of the human mind, concerned primarily with ideas, processes and reasoning. So mathematics is much more than arithmetic – the science of number and computation; it is not enough with algebra - the language of symbols and relations; far more than geometry – the study of shape, size and analyses oscillation. It involves more than statistics – the science of interpreting data and graphs; more than calculus – study of change, infinity and limits.

Why should this subject be taught to everybody? Why should everybody learn mathematics? What are the advantages of devoting so much effort, time and money to teaching of mathematics? What is the importance to this subject in life and in school curriculum? What are the purposes and aims of teaching mathematics? How does it improve the worth and efficiency of the individual? What is the place of mathematics in our system of education? These questions pertain to the need for mathematics education in our schools.

A serious teacher of mathematics will be interested in finding answers to these questions. He must feel convinced about the usefulness of his subject, so that he will be able to convince his students to feel likewise. This will also develop in him a keen interest in the subject. His students in turn, will be affected by his interest. He should be able to explain to the learner why he needs this subject in hi curriculum – the planned experiences offered to the learner under the guidance of the school. Because the education of children and youths always takes place in a particular society, it is evident that any consideration of acceptance of any subject in school curriculum should take cognizance of the goals of education in that society.

The Nigerian National Curriculum Conference (1969) had observed that the socio-economic development of the previous decade indicated a general progress but secondary education must remain a terminal education for majority. Also, the National Policy on Education (1977) looks at secondary education as a terminal education for some. The two basic functions the secondary schools must serve, therefore, are

i) to prepare children for life, and
ii) to give those with the necessary background the opportunity to proceed to higher institutions (Nigeria, 1977:10)

The rationale behind the selection of these objectives was to strike a balance and to cater for the interests of the majority who may be unable to go on to the tertiary level, while the rest is adequately prepared to serve as a nucleus of Nigeria's future manpower needs. Considering the assessment methods of students learning mathematics in secondary education which are geared toward the cognitive achievements only, there is an apparent domination by the latter function of the secondary course over the one of preparation for like.

The purpose of this unit is to show how mathematics education will help in achieving this dual function.

The National policy on education (1977) hope s by "…. Inculcating the following values….. (2) faith in man's ability to make rational decisions….." (Nigeria, 1977:4), the national objective would be achieved. One, then, sees mathematics education as a worthwhile pursuit based on its intrinsic value. The ability of man to reason logically and think critically is one of the marks of an educated man. So, the study of mathematics could be an end in itself. But this stand will expose me to the questions raised by Robin Barrow in his book "Common Sense and the Curriculum".

"…..that pursuits have to be assessed as worth-while or worthless by intuition it follows, so long as intuition judgment as to what is worth-while continue to differ, that we have no good grounds for insisting that any particular curriculum is worthwhile in that it is not worthwhile, on the grounds that he cannot see the allegedly self-evident value fo the pursuit in question. If they cannot see that they are worthwhile, how can we claim that it is self evident that they are?" (R. Barrow; 1977,38) so you will read and find the arguments on those areas that seem to involve empirical facts in this unit.

3.3.2 Mathematics as a Science of Numbers and Measurement

The concept of number is as old as man himself, but numeric symbol developed when it became necessary for man to keep records of the numbers of this belongings, or of objects around him or to solve some daily problems.

Examples:

- The set of counting or natural numbers, N= {E, 2, 3, …} developed to enable the ancient Indians and the Arabs to count and keep records of their sheep and other objects.

- The set of whole numbers, W = {0,1,2,3,….} became necessary to enable the ancient Indians measure whole units
- The set of integers, Z ={… -3, -2, -1, 0, 1, 8, 3,…} developed to aid the Greeks solve problems of the form 1+ ☐
- the set of rational numbers (of the form p where q is non-zero) became necessary to enable mathematicians have a system in which division is always possible.

You will read the details of these and other number systems from the primary education studies on history of number and types of numerals.

Imagine a world without measurements. What a chaotic world that would be! There would be nothing like money, or time. The drugs we take will have no doses, speed will either be too much or too slow or not in existence at all. Historians will not be able to write or read history, magistrates cannot determine the numbers of years of imprisonments, lawyer cannot quote pages of law books, and administrators will not know the number of staff nor be able to plan for them. You cannot buy or sell. Life will come to a stand still".

3.3.3 Mathematics as Symbolic Universal Language

Mathematics makes use of symbols to represent concrete objects, words or expressions, other symbols, abstract ideas or concept. For example, the concept of twoness is represented by the symbols for addition and Δ represents a triangle.

The beauty of mathematics as a symbolic language lies in the simplicity and brevity of the symbolic forms the impersonal character of the symbols devoid of emotions.

Examples:

- The beauty of "Mr. Bex, the magistrate is on trial for arEDU robbery" we can write "p is on trial for arEDU robbery?
- Instead of "Mrs. May, the first lady is the defence witness number one", we can write, "y is DW_1"
- For the following long sentences: "A games master bought 5 rackets and 8 tennis balls from one shop for a total sum of ₦132. In the second shop he spent ₦156 to buy 7 rackets and 4 tennis balls. Find the unit prices of a racket and a tennis ball. We can simply write: Solve simultaneously 5r + 8b and 7r + 4d = 156 where r is the price per racket and b is the price per tennis ball.

From these examples, we see the mathematical symbols are freely used in law courts. Other examples are “PH/2” which stands for “prosecution witness number 2”. Of course in keeping records to cases, of prisoners, fines terms of imprisonment, dates and case references, mathematics concept are utilized.

In fact, in all science subjects; physics, chemistry, engineering, agriculture, biology, integrated science, geography and others mathematical symbols are freely used. For example H_2^O in chemistry is the chemical symbol for water, while *NaCl* is the formula for common salt. This mathematically shows that one molecule of sodium chemically combines with one molecule of chlorine. The mastery of the use of this symbolic language by Nigerian students will inculcate in them the habit of brevity, clarity and precision of expression and will bring them nearer to unity with other human beings in the world.

3.3.4 Mathematics Involves the Study of Geometry and Patterns

Mathematics looks at nature and classifies objects according to their shapes, sizes and spaces. One-faced solid – the moebus strip or belt is used as belts in garri-processing machines. Also two-faced solids are used as belts in automobiles. Mathematics differentiates plane figures from solid shapes. Among the plane shapes are Δ for triangle, □ for square and ⬠ for pentagonal shapes. Among the solid shapes are Δ, the tetrahedron, □ the cube and ■ the cuboid. Sizes, shapes and space are very much utilized in housing schemes, in building bridges, in Engineering construction, in surveying and in works of art.

Space perception is necessary for students for students to form proper sense of land utilization for agricultural and other purpose.

It look for order, common properties and constant occurrences in objects, numbers or shapes.

Example (i) certain number can be arranged to form squares such as:

□	□□	□□□	□□□□
	□□	□□□	□□□□
		□□□	□□□□
			□□□□
1	4	9	16 etc

These are Square numbers

Other numbers form triangular numbers such as:

□ □ □ □
□ □ □ □ □ □
□ □ □
□ □ □ □
1 3 6 12

Similarly, we have pentagonal and hexagonal numbers. Some other number patterns can be forEDU in the form of operation thus

10-1=9
100-1=99
1000-1=999
10000-1=9999

The geometric patterns are very much used in tessellations of surfaces, tiling and carpeting of floors and walls, and in fine and applied and in envelopes of surfaces and curves.

3.3.5 Mathematics as the Science of Generalisation

The concept of patterns in mathematics also leads to the concept of generalization from the common properties, patterns of common occurrences from particular instances. In this from, mathematics draws collusions, inferences, predictions by the application of **inductive** reasoning is the abstraction of generalizations by considerations of common properties of particular cases.

For Example:

$1=1^2$
$1 + 3+= 2^2$
$1 +3+5 = 3^2$
$1 + 3 + 5 + 7 = 4^2$

One can conjecture from this pattern that 'the sum of the first n odd numbers is n^2 or that "the sum of all odd numbers staring from I is always a square numbers".

Application: All statistical inference, or predictions about rainfall temperature ranges, demographic surveys, population growth forecasts, feasibility surveys, agricultural yields and educational projections, all planning's including budgeting are on mathematics statistical generating conclusions.

3.3.6 Mathematics as the Study of Relations between Two or More Quantities

In doing this, mathematics investigates the effects of various in one or more independent variables on the other parameters taking part in the relationship. This is used in industrial, economic and social projections.

Example

- In industries, one considers the effects of changing the model of production to the total income from the finished products; such as the effect of changing from Peugeot 404 of Peugeot 504 in the income
- In economics, the government considers the effect of increase in salary to the amount of inflation in the cost of commodities.
- In politics, the government considers the effect of creation of more states to the politics, stability of the country
- In education, the ministry considers the effects of University Free primary Education to the school primary population.

This functional or relational consideration is similar to the "rate of change" used in calculus in mathematics

Example: In History, the ages of rock fossils, and even the age of the earth can be determined using the rate decay or "carbon in model called"

> A lot of the past history of mankind has been determined using the aspect of mathematics which is the historian's imaginations. The world is in pairs. Considerations of the concept in mathematical models of base two numerations which made it possible for man to talk to and receive and receive information's from the computers.
>
> Even in languages, the rate of assimilation of vocabularies from one language into another, can be determined using the mathematical model of rate of change. For example, it has been established that the Swahili "oce", the Igbo "oche", the Yoruba "oche" all have the surtax origin and meaning of the English equivalent "chair".

4.0 CONCLUSION

In the words of the great educationist Herbert, "The real finisher of our education is philosophy", but it is the office of mathematics one is able to rely on sheer thinking. By eliminating irrationality, mathematical methods produced the realistic school of thought in philosophy owes a good deal to men like Pascal, Descartes and Leibnitz. They were all great mathematicians first. These aspirations in secondary school will go a long way in fulfilling aspirations of the National Policy on Education 1977 in preparation for High Education.

5.0 SUMMARY

The main central ideal in this unit are summarized below

- It is not easy to give one definition of mathematics, but we can explain aspects of mathematics by what it does.
- The recent emphasis on the National Policy on Education through the Mass Mobilization for self reliance, social justice and economic recovery (MAMSER) is that education shall make our students self reliant. One aspect of mathematics is creativity and creativity is the mother of self reliance and industrial and technological development, mathematics is concerned with

a. number and numeration
b. symbols
c. sizes, shapes and spaces
d. the study of patterns
e. generalizations
f. measurements
g. models
h. relations and functions

Finally, mathematics is utilized by all subjects including itself.

6.0 TUTOR MARKED ASSIGHMENT

Attempt the following questions in order to check your understanding of the content of the unit:

1. Define in your own words the mathematical model system
2. Explain the physical world of mathematics to a JSS 1 student
3. Explain the idea world of mathematics in your own words.
4. Explain the differences between induction and deduction in the model of mathematics.
5. Identify and explain five common errors usually made by teachers in communicating mathematics ideas to students.

6. Do you agree that physical models used in mathematics instruction do not convey the exact meanings of concepts to students? Give reasons for your position.
7. Make five sentences on "life without mathematics"
8. Go to the following professionals:-

   a. a headmaster of a primary school
   b. a housewife
   c. a motor mechanic
   d. an English language teacher

   Find out from each person, two aspects of mathematics that affect his/her profession or work.
9. Suggest and explain at least three ways mathematics can help us solve the problem of the high cost of garri in Nigeria.
10. Take any subject, write one way, mathematics ideas are used by that subject.

Self Assessment Exercises

Self Assessment Exercise 1

Write 3 sentences describing what you consider mathematics to be

Self Assessment Exercise 2

Give two other examples in which mathematics uses itself.

Self Assessment Exercise 3

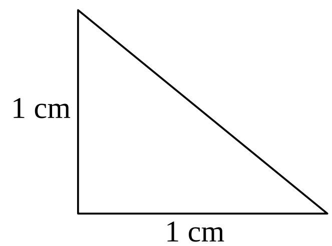
Think of and write down two effects, one on you and the other on the human race of having a world without numbers.

Self Assessment Exercise 4

1. Examine any textbook on physics find and write 2 examples of the use of mathematical symbols in physics.
2. Name and explain one way in which mass communication utilizes mathematical symbols.
3. Do you think that mathematical symbolism has any part to play in computer programming? Support your answer with one example.

Self Assessment Exercise 5

1. Identify and draw first four pentagonal numbers.
2. Show how to cover a rectangular surface 3 cm by 5 cm completely with shapes



7.0 REFERENCES/FURTHER READINGS

Farrell, M. A. & W. A. (1980) Systematic Instruction in Mathematics for the middle and high school years Reading Mass Addison Wesley Publishing company

Fakunde R.A. (1981) Teaching Arithmetic & Mathematics: In the Primary School. Ibadan University Press.

Krulick, S. & Weiss, J. B (1975) Teaching Secondary School Mathematics Philadelphia: W. B. Saunders Company.

Lass P. N & Paling F. A. (1983) Teaching Mathematics in Nigeria Primary Schools. Ibadan: Ibadan University Press.

Odili Godwin A. (1986) Teaching Mathematics in the Secondary School. Port Harcourt: Anachuna Education Books

UNIT 2 GOALS AND OBJECTIVES OF MATHEMATICS INSTRUCTION

CONTENTS

1.0 INTRODUCTION

Before ever one starts a new project he decides what he wishes to accomplish; a suitor does not choose a girl until he has defined the type of girl he wants; the roadside mechanic does not select a tool until he knows what operation he intends to perform; and in building a house one does not buy the materials without knowing the specifications in his plan. The owner of the house can only accept it from the contractor if and only if it satisfies the specifications established before work on it was begun. Similarly, you must design your mathematics programme to reflect what you wish to attain. So you must select mathematics programme to reflect what you wish to attain. So you must select mathematics textbooks and instructional aids or teaching methods appropriate for the programme designed. If the objective of mathematics instruction is not known, how can you evaluate a learner's performance? It is only when your destination is known that you can confidently select appropriate material, content, or instructional

methods. Again, you can evaluate the effect of our instruction when you have seen the effort we have made in attaining these goals.

What should be the appropriate goals for mathematics education? These must definitely agree with the general goals of Education. Scopes (1973) divides these goals under a number of sub-headings- Utilitarian, Social, Cultural and Personal goals. The question now is, how do you select your goals for mathematics instruction to attain these educational goals?

Here you may differentiate between the two words: goals and objectives, although the border between them is hazy. You shall take the word 'goal' to represent a larger, i.e, more general domain; and 'objective' to represent the more specific day-to-day aspects of the larger goals.

You have earlier identified how these goals can be selected to attain the national goals of Education. You may further want to discuss how these goals are broken down to operational objective. In this unit, you will be introduced to the factors to be considered in selection goals and objectives in mathematics. These factors shall be highlighted under the following headings.

(a) The need of the society-the Nation
(b) The need of the individual-the citizen
(c) What role can mathematics play in satisfying these needs?

2.0 OBJECTIVES

By the end of this unit, you should be able to:

1. enumerate factors that determine the selection of educational goals and objectives
2. explain what is meant by needs of the society;
3. explain the needs of individual, and
4. state and explain, with examples, what roles mathematics can play in achieving the selected goals and objectives.

3.0 MAIN BODY

3.1 Study Approach

1. To get the best out of this unit, you are expected to read the unit with personal commitment. Reflective thinking, comparing and contrasting with your past experience and happenings in the society are recommended.
2. You are expected to jot down and summarise the central idea of any paragraph you have read.

3.2 Goals of Teaching Mathematics

While some educators/teachers prefer to use the two terms interchangeably, it is useful to distinguish between goals and objectives with a view to facilitating effective teaching and learning in the Classroom.

Educational goals are the overall foreseen ends or general targets of education which are derived from the needs, values, problems and philosophy of the society concerned. In other words, aims communicate general educational outcomes which are long-range.

The goals of education in Nigeria, for example, are:

(1) the inculcation of national consciousness and national unity;

(2) the inculcation of the right type of value and attitudes for the survival of the individual and the Nigerian society;

(3) the training of the mind in the understanding of the world around; and

(4) the acquisition of appropriate skills, abilities and competencies both mental and physical as equipment for the individual to live in and contribute to the development of his society (FRN 1981, p.8).

A close observation of the enumerated aims of education in Nigeria suggests that goals are usually broad and abstract, and they are of little imEDUiate value to the Classroom teacher in planning a particular lesson in that they cannot assist him in his decision on content, teaching strategy and/or evaluation.

Thus Rae and Mcphillimy (1976) remarked that:

> goals are necessary start points, but, as they stand, they are not especially helpful to the classroom teacher because they do not assist her in deciding what approach to adopt in her teaching or what form of assessment to use (p.16).

However, the objectives of the teachers set for the student/pupils in the classroom should be consistent with the overall aims of education.

3.2.1 Objectives of Teaching Mathematics

Educational Objectives are statements of the change in the behaviour of the learner which will result from instruction, education treatment or stimulation. Thus objectives are much more specification treatment or stimulation. Thus objectives are much more specific.

3.2.2 Relationship Between Goals and Objectives in Mathematics

Accurate decision making on any programme is not easy. It requires a complete overview of the programme, through understanding of the parts that make up the programme, accurate forecasts, predictions from past records and projection for the future. It requires proper planning.

In business, or industrial projects, this is called feasibility surveys. Projections are made as to the success or failure probabilities of the business. "Trial and error" in business or other projects is never scientific. It leads to failure. In mathematics education, similar planning projections are made. These projections in education are sometimes called goals and sometimes called objectives of a country on education or on mathematics education.

They take "objective' to mean specific objective in a daily classroom lesson. In this unit the word goal will not be used often but when it is ever used, its meaning will be the same as "general objectives". For the word "objectives" we shall use the phrase "specific objective" to explain the specific behavioural objectives in any daily lesson plan. The meaning of these words and phrases will be clearly described later.

General Objective

Objective in education are broadly divided into two categories: general objectives and specific behavioural objectives. A general objective may be described as a part of the global objectives set out by a nation or an educational institution for the achievement of the national or institutional goals. For each subject or discipline, the government or the institution defines the general objectives in terms of that subject. Scopes (1973) divided general objectives under a number of subheading: utilitarian, social, cultural and personal goals. For mathematics curriculum, general objectives may be found from the following:

(a) From time to time government issues statements defining its policy on education. General objectives for any particular area of emphasis may be stated in the government policies or government gazzattes on education.

(b) Another source of general objectives may be the prefaces, forewords or philosophies behind the formulation of curriculum units. These are normally stated in the front pages of the curriculum.

For example in the national curriculum for Junior Secondary Schools volume I, 1985 p.3 part of the general objectives read:

"It is clear from the National Policy that Secondary Education is expected to be:

(i) Preparation for useful living within the society

(ii) Preparation for higher education

(iii) Again a general objective is found in front of 1978 Teacher Education Mathematics Curriculum as approved by the National Critique Workshop on Mathematics.

There it is stated that the objective of the teacher education programme is to produce teachers who will promote and achieve effective and efficient teaching and learning of mathematics in primary schools…"

(d) Another source of general objectives is the column of the curriculum itself which has the heading "Objectives".

In some curriculum, (such as the J.S.S Volume I, 1985 Science. Curriculum on Mathematics) the heading "Performance Objective" is used. What the planners of such curricula have in mind at this stage is general objectives. Performance objectives shall be explained later in this unit.

General objectives are useful to education because among other things, they provide general directives. When a person starts on a journey, he should have an idea of where he is going to. The general objective for each school subject points to the general objective of the nation or the institution. Again general objectives make for harmony among subjects and within subjects. They are necessary for the proper definition of the contents to be selected to meet the national or institutional objective.

Definitely the objectives stipulated in general objectives associated with learning units rather than with daily lessons. These expressions shall be explained in this later. To achieve general objectives we need more than one daily lesson and so general objectives can be broken down into several daily lesson objectives.

3.2.3 Factors that Determine Selection of Objectives

Mathematical objectives or goals are never selected or constructed in isolation of the needs of the country making use of the mathematics. Certain questions ask at this level may be:

(i) What are the pressing problems of the society at the material point in time? As we know societal problems change with time, people and government in power. Mathematics as we saw from discussion in the unit on history of mathematics, develop mainly to solve the problems of the day. A teacher wanting to develop educational goals for mathematics should ask questions:- such as, What are the needs of society? What is the government emphasizing at the moment?

(ii) Which of the needs of the society can mathematics directly affect?

(iii) What facilities are there for mathematics to tackle those needs?

The key factor or element in stating behaviour objectives is “action-verb”. Teachers are thus encouraged to select and use those verbs that most precisely specify the student behaviour that will be acceptable evidence that the objective of the lesson has been achieved.

Here are two examples of objectives written in behavioural or observable terms:

1. At the end of the lesson, the students should be able to identify and explain three political factors and two economic factors for the partition of West Africa, without a reference to outside material.

2. Using their own words, the students should be able to distinguish between cognitive and affective domains of education.

The merits or advantages of stating objectives in observable or measurable terms are obvious. In short, when objectives are stated in behaviour terms, they:

(a) assist the teacher to plan his teaching – for example, they provide sound basis for selecting appropriate content, instructional materials and methods of teaching;

(b) give the teacher a clear sense of direction in his work. That is, the objectives direct instructional activities in the classroom;

(c) provide a meaningful basis for evaluating student performance;

(d) provide the learners or students with the opportunity to ascertain their own progress. In other words the students can compare their performance against the specified or stated objectives;

(e) encourage students to progress at their own rate, channeling their efforts into relevant activities.

(f) alert the teacher to check whether his pupils have already covered the work he proposes to do.

Despite the merits of specifying objectives in behavioural terms, there are some objections to its use in the classroom. Some of the objections or criticisms include the following as observed by Nicholls and Nicholis (1978), and Rae and Mcphillimy(1976).

3.2.4 The Needs of the Individual

Societal needs are different from the needs of the individual.

For example at present, Nigeria societal needs are how to succeed as a nation politically, socially and economically.
Problems to be solved include continued military intervention, religious problems, socio-ethnocentric problems; corruption, bribery and nepotism. These problems affect the individual, but the imEDUiate needs of the individual are how to have shelter, food and drink in the society. The selector of mathematics goals must ask the following questions:

“In what ways can mathematics aid the individual to satisfy these needs?
By going through the mathematics curriculum in schools, can the individual make a successful living-useful to himself and useful to the nations?

3.3 The Reference of Mathematics Curriculum

Some of the questions the mathematician will ask are the following:

(i) What aspects of mathematics need to be utilized and emphasized in order to satisfy the needs of the society and the needs of the individual?

(ii) To what depths of mathematics content should we teach to reach the goals?

(iii) What teaching techniques and teaching aids are available for mathematics?

(iv) What human and material resources are available to handle the necessary aspects of mathematics necessary for the achievement of the goals?

(v) If these resources are not available, do we need some trainings or some fundamental foundation works to be done to make them available?

Now let us discuss how these questions are really important for goal development as far as mathematics is concerned.

Mathematics cannot be taught in isolation of the societal needs or the needs of the individuals. A teacher of mathematics must be convinced that what he is teaching will be useful to the students and to the society. With this conviction, he can then convince the students. In our modern society of attachment of high values to material living, every student will love mathematics more if he understands in advance what mathematics can do for him, for his future success as an individual and as a good citizen. This leads us to the utilitarian aspects of mathematics.

The history of mathematics in Nigeria shows that before the sixties, mathematics was taught in primary schools as arithmetic with emphasis of the four rules – addition, subtraction, multiplication and division. The new curriculum is now emphasizing the teaching of mathematics which should consist of activities – doing talking, writing, manipulating objects and experimenting with them. In the words of Johnson and Rising (1972, p.171).

1. We sort objects, events, or ideas into classes or categories.

2. We become aware of relationship within the classes or categories involved.

3. We find a pattern which suggests relationship or structure.

4. We formulate conclusion which seems to describe the pattern of events or ideas involved.

5. We establish the generalization by a deductive proof.

Thus, we see that students make the best use of what they see, hear and do. (Fakuade, 1981, p.11) summarises our goals for teaching mathematics in primary school thus:

(i) In view of the inherent utility values of mathematics processes in domestic, business progressional life of each individual, primary education will fail… if mathematics is not taught in primary school… so we say we teach primary mathematics in order to prepare every individual rightly for life.

(ii) Primary education gives the necessary preparation for secondary education… primary education teaching will lay the foundation of their future advanced studies and in other school subjects that require the 'services' of mathematics".

One cannot avoid the utility aspects of mathematics: counting, notation, addition, subtraction, multiplication, division, weighing, measuring, selling, buying, are some of the "need-to know" fundamental process of mathematics in secondary and primary mathematics.

Four main mathematics methods – scientific, intuitive, deductive and inventive are used to investigate, interpret and to make decisions.

The study of mathematics, therefore, will go a long way to "equip students to live effectively in our modern age of science and technology". (Nigeria 1977, p.10). As Odili (1986, p.3) points it, the four main mathematics methods "will help us to raise" generation of people who can think for themselves, respect the views and feeling of others…"

In planning or selecting goals for mathematics, we need to consider all aspect of the use of mathematics and mathematics methods in statistics, in science, in decision making, in logical reasoning, in fostering unity through common symbolism rather than the inhibitions of language and philosophical considerations.

For example, members of the Constituent Assembly who are conversant with the logical reasoning of mathematics will make use of mathematics persuasions, logical reasoning, rather than group interest, to convince others and give Nigeria an unbiased, long lasting, practicable constitution. An individual well grooEDU in the arts of mathematics will be very successful in life for he shall definitely apply the precision of actions, clarity or purpose, brevity of words and certainty of expressions which are attributes of mathematics to plan and reach his goals. He has foresight in making correct prediction and inferences.

The power of mathematics in character building is well known through the ages. Mathematics is a discipline and people who pass through its rigours are patient, disciplined and reasonable.

3.3.1 Reasons for Stating Objectives in Behavioural Terms

The two operating words in this phrase are “specific” and “behavioural”

An objective stated specifically during the planning for a daily lesson or a lesson to last one class period in our schools, is referred to as a specific behavioural objective. It stipulates exactly what change in behaviour is expected of a student by the end of the lesson. It is assuEDU that the student in question was not able to perform that behaviour before the lesson. A behaviour objectives is specific when it stipulate one behaviour clearly defined to be achieved by the student.

Classroom learning depends upon both the teacher and the students. The teacher presents or creates an environment conducive for learning in the classroom and guides the students to perform certain acts capable of changing his behaviour to the desired or mapped out objective. With the pre-stated objectives, the teacher should focus on what he wants the students to achieve.

Every well stated specific behavioural objectives has five parts:

(i) Under what condition will the change in behaviour take place? The condition, the classroom environment, the directions to be given to the students that will make him change the behaviour must be stated.

(ii) The **who** whose behaviour has to change must be mentioned. Usually in the classroom situation, it is the pupil who is the learner who is expected to change in the behaviour and not the teacher.

However, it is possible that the teachers objective is to test how effective his teaching methods is, then the objective is state in terms of the teacher. Objectives may be stated to test the effectiveness of the use of a certain text-book or teaching aid or how effective a programme is. In these cases, the objectives are stated in terms of the teacher or the person testing the instruments. But in most cases, in the classroom the **who** to be considered is the students.

(iii) What specific behaviour will the pupil exhibit? The specific behaviour is usually stated in operating verbs, Bloom Hasting and Madans (1971) stated that to define objectives so that they are not open to multiple interpretation we must translate the verbs into action verbs that require direct observation. Some action verbs are: Write, state, recite, identify, classify differentiate, solve, compare, contrast, list. Some verbs that are open to multiple interpretations are know, believe, enjoy, understand, and appreciate. These ambiguous verbs should not be used for specific behavioural objectives.

(iv) What result will the change in behaviour achieve? This is known as the performance product or outcome of change in behaviour. For example, when we say the student writes conditions for congruency: The underline words form the performance product. This is the result or outcome of the students action.

(v) **To What standard** is the learner expected to perform for this performance to be considered acceptable to the teacher? This requirement means that every well stated specific behaviour objectives must be measurable. Examples of well stated behavioural objective are:

Example: By the end of the lesson, the pupils will be able to list the first five multiples of any number from 1 to 5 inclusive. Let us schematically write this using the five parameters of a well stated specific behavioural objectives:

Under what condition	**Who the pupils**	**What behaviour will be able to list**	**What result multiple of any number multiples**	**To what level of performance the first five**
Example ii: By the end of the lesson	The pupils	Will be able to estimate	1 to 5 inclusive heights ofs in other student in centimeters	To within a difference of plus or minus 5 centimeter.

3.3.2 Types of Mathematical Specific Behavioural Objectives

(Blooms Taxonomy of Educational Objectives). The most useful classification of specific behavioural objectives is that done by B. S. Bloom (1956) and his associates. They called this classification "The taxonomy of educational objectives". This is an attempt to arrange instructional objectives in behavioural groups, from simple behaviours easy to achieve to highly complex groups of behaviours. Generally, three categories (Domains) of instructional objectives are identified:

(i) Cognitive Domain;

(ii) Affective Domain; and

(iii) Psychomotor Domain.

Let us explain these terms before going over to Blooms taxonomy of objective in details.

3.3.3 Cognitive Domain

The cognitive domain is recognized to be the thinking area of the students behaviour. Generally categorized the cognitive domain includes the following:

(i) Knowledge: which includes simple recall of knowledge of specific facts, terminological generalizations, theorems, structures or algorithms.

(ii) Comprehension; Which includes ability to translate, interpret, explain correctly and interpret or extrapolate.

(iii) Applications: i.e. use of ideas theories, principles or concepts learnt in other situations.

(iv) Analysis: which involves identification of relations and organizations or order in a concept.

(v) Synthesis: which includes organization of ideas into reports, plans or system.

(vi) Evaluation: which implies passing judgement on basis of internal or external evidences.

3.3.4 Affective Domain

The affective domain of behaviour relates to students feelings and biases. Affective categories are:

(i) Receiving or attending

(ii) Responding or participating

(iii) Valuing or believing in the worth of a thing

(iv) Organizing values into a system

(v) Characterization by value.

This domain is difficult to measure but the Federal Government has mapped out a system of continuous assessment for teachers to measure affective achievement in the classroom.

3.3.5 Psychomotor Domain

The psychomotor domain is easy to identify and measure. It involves the ability to use our locomotor sensory organs such as ability of a child to write a number 2, draw a straight line, make an arc of a circle or ability to use the protractor correctly to measure angles. At the first instance, a child is learning these skills, his behaviour can be stated in behavioural psychomotor terms.

Above examples certainly show that it is NOT true, as stated by Johnson and Rising 1972, p.50, and echoed by Odili G. A. (1986, p.11) that "a third domain, that of psychomotor or physical skills, is generally outside the province of mathematics education"

3.3.6 Merits and Demerits of Performance Objectives in Mathematics

Although several investigators have sought to develop a classification system of instructional objectives, the most systematic approaches have been developed by professor B.S. Boom and associates in Taxonomy of educational objective Here they attempted to arrange instructional objectives in a hierarchical order from simple to complex classes of thinking – each category is built upon and depends on its predecessors. They developed a threefold division of instructional objectives (1) Cognitive Domain (2) Affective Domain and (3) Psychomotor Domain. This division roughly corresponds to thinking, feeling and acting

The general categories of the cognitive domain are:

Knowledge – ranging from knowledge of specific facts and terminology to knowledge of generalization, theories and structures.

Comprehension – including translation, interpretation and extrapolation.

Application – use of ideas principle, or theories in concrete situations.

Analysis of elements, relationships and organization.

Synthesis or organization of ideas into a report, a plan or a system
Evaluation on the basis of internal or external evidence.

The corresponding outline for affective domain includes:

Receiving or attending
Responding or participating
Valuing or believing in the worth of something
Organizing values into a system
Characterization by value or a value complex

"A third domain, that of psychomotor or physical skills, is generally outside the province of mathematics education '' (Johnson and Rising, 1972).

Some working on this, have tried to simplify the classification. The international evaluation of achievement study (Torsten (ed) 1967; Vol. I, Ch 4) has five categories of objectives:

A. *Knowledge and information*; recall of definitions, notations, concepts.

B. *Techniques and Skills*: Computation, manipulation of symbols.

C. *Comprehension:* Capacity to understand problems, to translate symbolic forms to follow and extend reasoning.

D. *Application of Appropriate* concepts to unfamiliar mathematical situations.

E. *Inventiveness:* reasoning creatively in mathematics.

Performance objective are now making remarkable impact on the teaching of mathematics. They provide a strong foundation for a firm organization of a teaching programmes by specifying in detail what we want student performance to beat at the end of a teaching sequence. Carefully stated objectives help teachers and curriculum developers to evaluate their product. In particular, they provide a framework upon which accountability could be mounted. Thus, our educational programmes will yield good results to justify the millions of naira we invest in it yearly.

Critics of the use of performance objectives argue that by stating objectives in advance teachers are not able to take advantage of opportunities which occur unexpectedly in the classroom. This is not so, for teachers can use these unexpected opportunities by directing them towards objectives which are considered important and desirable. In this way, unexpected opportunities lead to progress in the teaching rather than to mere diversion. In fact, a teacher with no predetermined objectives might be accused of using such opportunities for entertainment purposes.

The fact that performance objectives lend themselves to assessment raises another criticism from some critics. Furthermore, they argue that there are other important outcomes of education besides those which can be assessed. The first is part of the strong feeling that is sometimes expressed against measurement of all kinds, and yet teachers are constantly making evaluative judgments of their pupils' abilities and performance. For the second we can only urge patience and experiment; lack of current satisfactory measurement tools does not imply lack of techniques in the future. We must either wait for someone else to develop measuring devices or, better still, try to devise some ourselves.

Another charge often made against performance objective is that they tend to use the language of assembly-line processing of products, not of patient observation of students. A characteristic of performance objective which some teachers find equally disturbing is the concept of 'the one best way', which seems to be implied. Not only does there appear to be an implicit belief that everything must begin with objectives, but it is also fairly obvious that there is also a best way of writing and displaying them.

The difficulty of writing objectives in certain areas, especially in higher conceptual activities, causes some critics to reject the whole idea of objectives. That it is difficult does not mean we should state objectives that lead to an overemphasis on trivial outcomes of learning with a corresponding neglect of important outcomes.

4.0 CONCLUSION

It should be made categorically clear in this unit that an objectives in one domain has a counterpart in the opposite domain. In other words, each domain is sometimes used as a means to the other, though the more common route is from the cognitive to the affective to the psychomotor.

The importance of the teacher's knowledge or awareness of the taxonomy of educational objectives cannot be overstressed for it enables the teacher to:

(a) select his range of objectives and test questions;

(b) ensure that appropriate balance or weight is given to the major categories of objectives and questions;

(c) structure his teaching so as to ensure that it covers the more important types of learning (e.g. thinking skills).

5.0 SUMMARY

This unit has explained to you the following:

- That Goal has the same meaning as General Objective which stipulate the general global aim of what Government wants to do.
- That there are two types of objectives: general objectives and specific objectives.
- The general objectives can be identified in the Government policy on Education, in the curriculum or in government gazette.
- That specific behavioural objectives have five parts.
- We have given examples of each of these objectives and identified their importance in educational system.
- Certain categories of specific objectives were also explained. The Cognitive, Affective and Psychomotor domains.
- These objectives are to be used in the next unit for drawing learning unit plan and the daily lesson plan.

- In this unit, you have been exposed to some questions you must ask in selecting mathematical goals for the country. The emphasis has been on considerations and questions relating to the needs of the society at present and needs of the individual. It has been emphasized that these needs change from time to time, hence the need to plan and change the curriculum from time to time.

- For example, Nigeria before independence needed people who could compute, buy, sell and communicate with the white men. So arithmetic became included in the curriculum with independence, these needs changed and new curricula in mathematics came into use.
- The mathematical considerations that need to be taken are highlighted in this unit. These mainly hinge around the utility aspects of mathematics as a discipline.

6.0 TUTOR MARKED ASSIGNMENT

1. (a) What is an 'educational aim'?

   (b) What is an 'educational objective'?

   (c) Distinguish between 'aim' and 'objective'.

2. What are the purposes of writing educational objectives as given by Davies (1981)?

3. (a) What do you understand by 'behavioural objective'?

   (b) Write *five* behavioural objectives in your teaching subject(s).

Self Assessment Exercises

Self Assessment Exercise 1

Give an example of an intended specific behavioural objectives which is not specific. Note that the new term for specific behavioural objectives is "performance objectives. But for the purpose of this unit we shall keep to the name specific behavioural objectives.

Self Assessment Exercise 2

Refer to the document prepared by the Federal Government on Continuous Assessment and find one technique for measuring affective domain.

Self Assessment Exercise 3

When a student in mathematics is solving a problem on multiplication of two digit number by two digit number, what domain of objectives is he using?

7.0 REFERENCES/FURTHER READINGS

Bloom B. S. (ed) 1956: *Taxonomy of Educational Objectives Handbook1 Cognitive* Domain, David Mckay Co. Inc.

Fakuade, R. A. (1981): *Teaching Arithmetic and Mathematics in the Primary School*, Ibadan: University Press.
Federal Ministry of Education 1985; *National Curriculum For Junior Secondary School*, Vol. 1, Ibadan: Heinemann Educational Books.

Federal Ministry of Education March 1978: *Teacher Education, Mathematics Curriculum*.

Johnson and Rising 1972, *Guideline for Teaching Mathematics,* Wadsword Publishing Co.

Nigeria: Federal Republic of Nigeria (1977): *National Policy on Education*: Federal Ministry of Education.

Odili G. A. (1986): *Teaching Mathematics for Secondary School*. Anachuna Educational Books.

Scopes P. G. (1973): *Mathematics in Secondary School*, *A Teaching Approach*, Cambridge University Press.

UNIT 3 MATHEMATICS CURRICULUM PLANNING AND DESIGN

CONTENTS

1.0 INTRODUCTION

The most pressing issue as far as the present state of mathematics education is concerned, is that of deciding what should constitute mathematics curriculum to be disseminated in schools and how it should be taught. In this unit, the various aspects of curricular issue will be discussed.

2.0 OBJECTIVES

At the end of this unit, you must have been exposed to the:

1. Basic Definition of Curriculum.
2. Determinants of A Viable Curriculum.
3. Process of Mathematics Curriculum
4. Patterns of Curriculum Organisation

3.0 MAIN BODY

3.1 Study Approach

Student are expected to:

1. read each section of the unit slowly and understand it.

2. attempt all the assignment and activities indicated at the end of the unit.

3.2 Basic Concepts of Curriculum

We are confronted by different views on what 'curriculum' is. In other words, there are many meanings attached to the word 'curriculum'. It is a term that is often used to denote "the range or list of subjects" on a school's programme; "a group" of school subjects meant to be taught to the learner"; the 'course of study' or a 'syllabus' which is only part of the total curriculum – all these are parts of what we mean by 'curriculum'. The modern concept of curriculum, therefore, emphasises not only the academic subjects but other activities which are planned and guided by the school. However, here are some of the definitions of the word 'curriculum':

1. A programme or course of activities which is explicitly organized as the means whereby pupils may attain the desired objectives (Hirst and Peters 1980, p.60).

2. A Curriculum is a plan for learning (Wiles and Bondi 1984, p. 19).

3. The planned experiences offered to the learner under the guidance of the school (Wheeler 1967, p.11).

4. By 'curriculum' is meant the sum total of all the experiences a pupil undergoes (Bishop 1985, p.1).

5. All that is taught in a school including the time-table subjects and all those aspects of its life that exercise an influence in the life of the children (Farrant 1980, p.24).
6. All the learning which is planned or guided by the school, whether it is carried on in groups or by individuals, inside or outside the school (Kerr 1968, p.16).

3.2.1 Determinants of Curriculum

In any curriculum planning process, we must not loose sight of the fact that it must go in the context of many pressures from both within and without the school itself. Therefore, by determinants of curriculum, we mean "those forces that influence and shape the content and organization of the curriculum (Zais 1976, p.15). In other words, the determinants of curriculum are those factors which affect or determine the curriculum (Bishop 1985, p.2). Determinants of curriculum are also referred to in the literature as the sources or the foundations of the curriculum (Zais 1976). Thus, the determinants of curriculum include the following:

3.2.2 Society and Culture

Society and its culture exert an enormous influence on the curriculum (Zais 1976). A society is a collection of individuals who have organized themselves into distinct group… the members of the group must perceive themselves as having things in "common" which enable them to "belong" (Zais 1976, p.157). On the other hand, culture is defined as "an accepted way of life: it includes a vast array of easily observed facets of living, such as material products, political and social organizations, characteristic vocations, modes of dress, foods, games, music, child-rearing practices, and religious and patriotic rituals (Zais 1976, p.15). Thus, Bishop (1985) asserted that a curriculum must be designed in the light of the major trends and development within society and it must also reflect the major social and cultural needs of society (p. 6). In essence, the educational objectives of any nation (like Nigeria) should reflect the cultural values of the society. For example, any curriculum designed for Nigerian School Children should reflect the five main national objectives of Nigeria, which are:

1. a free and democratic society.
2. a just and egalitarian society.
3. a united, strong and self-reliant nation.
4. a great and dynamic economy.

5. a land of bright and full opportunities for all citizens (FRN 1981, p.7).

It is, therefore, the responsibility of educational planners to obtain information about values predominant in the society for which any particular curriculum is being developed (Lewy 1977, p.249).

3.2.3 Philosophy of Education

Philosophy endeavours to define the nature of the good life or the good society. Bishop (1985) remarked that one must consider philosophical questions such as what knowledge is considered most worthwhile, what are the permanent human qualities one wants to transmit to the younger generation, what is the purpose of education? Different societies will differ in their answers to such questions (.55-6). The philosophy of education of a given nation like Nigeria will therefore, influence the content, objectives and strategies of her school curriculum either at primary, secondary or tertiary level. Nigeria's philosophy of education, for example is based on "the integration of he individual into a sound and effective citizen and equal educational opportunities for all citizens of the nation at the primary, secondary and tertiary levels, both inside and outside formal school system" (FRN 1981, p.7).

In essence, therefore, the educational philosophy of any nation would help or serve to:

(a) Clarify objectives and learning activities;

(b) Guide and select appropriate content;

(c) Guide the selection of resources (human and material);

(d) Guide the selection of instructional strategies;

(e) Suggest evaluation devices.

3.2.4 Nature of the Subject Matter

Subject matter refers to the content of what is learnt by the learner (Obanya 1980, p.21). Since there is knowledge explosion, there is need to attend to *three* basic questions:

(i) what content should be selected?
(ii) what content should be mandatory for all students?
(iii) what content should be considered as electives?

The answers to these questions would shape the content of the curriculum. The curriculum planners should, in the final analysis, select and incorporate in the curriculum the content that would meet the needs of the society and/or facilitate the attainment of educational aims of the nation. In essence, the content chosen or selected must:

(a) be relevant to the society in which the learner will live;
(b) be meaningful to the learner;
(c) be activity-based, with the child himself involved in the process of inquiry and creativity.

3.2.5 Psychology of the Learner (or the child)

Man is capable of learning only what his genes will allow him to learn (Zais 1976, p.15). It is, therefore, important to determine whether the content and tasks or activities the curriculum encompasses are within the developmental scope of the pupils who are to deal wit it. In other word, in planning a school curriculum we must take the stages of development of the learner into account. Piagets theory or stages of cognitive development, for example, has very important implications for the curriculum – Piaget identified four stages of cognitive development: the sensorimotor (up to two years of age approximately); the pre-operational stage (2-7); the concrete operational stage (7-11); and the formal operational stage (11-15/17). These stages connote the notion of *readiness* which is a very important factor in curriculum planning. Therefore, the intellectual development of the child/learner should be taken into consideration by:

(a) providing a flexible curriculum i.e. providing wide and varied learning opportunities to cater for individual differences;
(b) providing for different learning rates;
(c) providing variations in the methods of teaching and instructional materials.

3.2.6 The Learning Theory

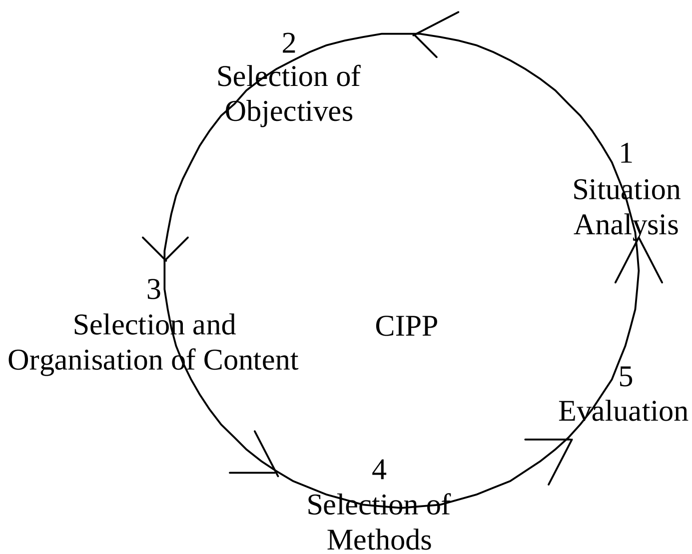
Notion about how human beings learn will affect the shape of curriculum (Zais 1976, p.16). Zais (1976) also remarked:

Whether conscious or sub-conscious, ideas about the nature of the learning process always influence the curriculum… Clearly, a sound and effective curriculum depends heavily on a well-founded theory of learning (p.244).

Therefore, theory of learning or psychology of learning will enable the curriculum planner to ascertain or determine the conditions requisite for learning of certain types of objectives; make proper placement of attainable objectives at the appropriate grade level.

3.2.7 The Process of Mathematics Curriculum Development

The process of planning learning opportunities intended to bring about certain changes in pupils/students and the evaluation of the extent to which these changes have taken place, is what is referred to in educational literature as 'curriculum development'. Thus, curriculum development is a continuous process which starts with situation analysis, continues with the formulation/selection of objectives, selection (and organization of content; selection of methods/learning activities; and then is put into action in the classroom (i.e. actual implementation) and then evaluation as contained in the diagram below.



3.2.8 Situation Analysis

It involves a review of the situation and an analysis of the external and internal factors constituting it. Factors to be considered include: changes and trends in society, expectations and requirements of parents, employers, the changing nature of the subject matter to be taught; education system requirement, pupils – their abilities, values and defined educational needs; material resources, teachers – their values, attitudes, skills, knowledge, experience, special strengths and weakness, their roles.

Essentially, situation analysis involves identifying tasks and problem, seeking possible solution, anticipating the difficulties and possible areas of resistance, planning the resources and organizational changes that will be needed (Bishop 1985, p.133).

3.3 Selection of Statement of Objectives

Having carried out a thorough analysis of the situation, one is now in a position to select the objectives to be realized. Objectives can be described as statements of the change in the behaviour of the learner which will result from instruction, educational treatment or stimulation.

Generally, educational objectives fall into three major categories or domains; the cognitive domain, the affective domain and the psychomotor domain.

If objectives are stated clearly, they offer considerable guidance in the selection of appropriate content and methods. Thus, the selection of an appropriate set of objectives is probably the most important stage in curriculum development.

3.3.1 Selection and Organisation of Content

Content can be considered as that which is presented to students or that which is made available to students for possible use. Therefore, content may be described as the knowledge, skills, concepts, principles, attitudes and values to be learned.

(a) Criteria for Selecting Content

Curriculum objectives operate s the final arbiters of content selection – in other words, the primary basis for content selection must always be the *stated objectives of the curriculum*.

However, a number of other criteria has been identified for selecting content – the criteria include: significance, validity, utility, human development, feasibility.

(i) **Significance:** refers to the *essentialness of the content to be learned*. Content to be learned is significant only to the degree to which it contributes to basic ideas, concept, principles, generalization etc. Significance also pertains to how the content contributes to the development of particular learning abilities, skills, processes and attitude formation.

(ii) **Validity:** refers to the *authenticity of the content to be selected.* In this time of information explosion, the content selected can quickly become obsolete, and even incorrect.

There is another aspect of validity – the content – selected should provide the learners with the experiences that would promote or effect the desired change in behaviour – the content must be related to objectives.

(iii) **Utility:** the criterion of utility attends to *content usefulness*. The content should be considered necessary for effective functioning in the society.

(iv) **Human Development:** content selected should be appropriate for the particular development level of the students; it should be within the capabilities of the students to process. In other words, content must be available in forms which are appropriate to the pupils' development level because "it is unrealistic to specify content calling for a high degree of abstractions of ideas when pupils are not developmentally ready for such cognitive processes" (Lewy 1977, p.68).

(v) **Feasibility:** addresses the question can the content selected be taught in the time allowed, with the resources available, with the expertise of the current staff?

(b) Criteria for Organisation of Content

Content should be organised in ways that will facilitate student learning – thus, the prime reason for organizing content is to render it comprehensible. Therefore, after the content to be included in a curriculum has been selected, it has to be organized in such a way as to produce major changes in the learners in the direction of stated educational objectives.

Content may be organised in two major ways: Vertical Organization and Horizontal Organization.

(i) Vertical Organisation

Vertical organization refers to arrangement of learning opportunities (or content) within the subject area so that what follows is slightly deeper and more complex than the previous task – vertical organization is deepening of knowledge – it is an arrangement according to stages – it deals with relationship of learning experiences in the subject area over time.

Sequence: is another aspect of vertical relationship in curriculum organization. Sequence often is referred to as the "vertical" organization of content to distinguish it from the "horizontal" organization, which concerns the arrangement of content at a given level of instruction (Zais 1976, p.340). Sequence is defined as 'the order in which curriculum content is presented'. In other words, sequences refers to the placement of learning in subject matter and processes in such a way that development can occur in a sensible order (Curhs and Bidmell 1977, p. 138). Four types of sequence have been identified:

(a) The simple-to-complex;

(b) Known to unknown;

(c) Concrete to abstract;

(d) Visible to invisible.

Here, sequence is defined as progression from simple subordinate components to complex structures.

(a) Prerequisite learning: This is common to subjects that depend for their exposition on laws and principles. In other words, one set of ideas or operations builds naturally upon preceding ones.

(b) Whole-to-part: The rationale here is that understanding of the whole, makes possible the understanding of partial phenomena.

(c) Chronology: Chronological sequence is utilized in a subject which has traditionally been perceived as a structure of chronologically recorded events. In other words, facts and ideas are arranged in a time sequence so that things are studied as they are related to one another in terms of time.

Whatever organizing principle one chooses in order to sequence content, the main concern should be that the topics in the content be sequenced to maximize learning by pupils (Lewy 1977, p.68).

(ii) **Horizontal Organisation**

Horizontal organization (sometimes referred to as scope and integration) is concerned with the side-by-side arrangement of curriculum components. It is an attempt to develop interrelationships between various subjects at the same grade level. In other words, horizontal organization is concerned with showing how ideas and skills in one subject are related to ideas and skills in other subject – for example, when what is learned in geography is related to what is learned in history in the same year, we say there is a horizontal relationship.

3.3.2 Selection of Teaching Methods

The method aspect of curriculum development implies ways/manners of presenting the content to the learners. Pupils do not learn best through the same methods. Therefore, a variety of methods is desirable with a view to increasing the possibilities of learning for the pupils. It should also be borne in mind that certain kind of objectives can best be achieved through the use of certain methods, that is, the objectives may influence the methods.

3.3.3 Evaluation Procedures

Curriculum evaluation is the final phase of development in which results are assessed and successes of both the learners and the programme are determined (Oliva 1982, p.25). Evaluation refers to the process of determining the degree to which the objectives of an educational activity or enterprise have been achieved (Bishop 1985, p223). In other words, evaluation is a process of finding out the extent to which the stated objectives of the curriculum have been attained. There are two types of curriculum evaluation – formative and summative.

Formative Evaluation: refers to the gathering of evidence during the process of forming or developing parts of the curriculum. Here, the task of evaluation is to determine what kinds of evidence can be used by members of the curriculum team to determine the adequacy of the materials in general as well as the kinds of evidence that can be used to determine in some detail what needs to be revised as well as why and where the revision is needed (Lewy 1977), p.85. In other words, formative evaluation is used for the improvement and development of an ongoing programme or curriculum.

Summative Evaluation: refers to the gathering of evidence or data that would be used to determine the overall effectiveness and quality of a new curriculum. The basic concern of summative evaluation, therefore, is the determination of the overall success of the entire programme including all its components.

3.3.4 Patterns of Curriculum Organisation

We have to organize the learning opportunities in order to facilitate the learning of students with a view to achieving the national educational objectives. Thus, the way and manner a curriculum is design, organized and developed for practical implementation is referred to as ‘pattern of curriculum organisation'. The main patterns or designs of curriculum organization include the following:

3.3.5 The Subject Centred Curriculum (Subject-Centred) Organisation

The subject design/pattern is the oldest and most widely employed form of curriculum organization. The subject design organizes the curriculum into a variable number of subjects, each of which purportedly represents a specialized and homogeneous body of knowledge – e.g. Physics, Chemistry, Biology, Geography, History and Economics (Zais 1976). The subjects themselves are sometimes subdivided into divisions.

Most teachers (especially at secondary and higher levels) are trained along subject lines and some teach only one or two subjects (Obanya 1980, p.28). Therefore, the subject centred curriculum is uniquely suited to take advantage of the teacher's subject matter expertise. Thus, teachers can easily plan, organize and teach in subject area. Another advantage of the subject-centred curriculum is that textbooks and other teaching materials generally are organized by subjects so that the material to be learned is clearly laid out. Furthermore, class scheduling can conveniently be compartmentalized to correspond to subject requirements: forty minutes each for English and Mathematics, eighty minutes for chemistry and home economics.

The major weaknesses attributed to the subject-centred curriculum include the following:

(a) It tends to fragment knowledge and the understanding of the pupils or students – no integration of knowledge;

(b) The subject-centred curriculum gives inadequate consideration to the needs, interests and experience of students;

(c) Its tendency to proliferate subjects;

(d) Narrows down knowledge to specified areas.

3.3.6 The Broad Fields Curriculum Organisation

This pattern of curriculum "represents an effort to overcome fragmentation and compartmentalization of subject-centred curriculum by combining two or more related subjects into a single broad field of study (Zais 1976, p.406). (In other words, it is an attempt at interdisciplincrity) – for example, Physics, Chemistry and Biology are combined as Integrated Science; and History, Geography, Economics, Government and Sociology combined as social studies, drama, music and art constitute creative arts.

The broad-field curriculum integrates separate subjects, thereby enabling learners to see relationships among various elements in the curriculum. In other words, it enables the learners to see the relationship between the subject and another. However, where teachers have not been specifically trained for the broad-fields curriculum, they tend to emphasise or stick to their respective areas.

3.3.7 The Core Curriculum Organisation

This pattern of curriculum is intended to provide common learning, or general education, for all students. That is, it constitutes the segment of the curriculum that teaches the common concepts, skills, values and attitudes needed by all individuals for effective functioning in the society. The core curriculum may be in form of:

(a) The Separate Subjects Core

It is a series of required individual subjects separately taught by subject matter specialists. In other words, the separate subjects core implies the separate subjects that all students are required to take, for instance, all the students at JSS level (in Nigeria) are expected to take English and Mathematics.

(b) The Fused Core

This is based on the total integration or 'fusion' of two or more separate subjects e.g. Integrated Science and social studies as practiced at JSS level in Nigeria.

3.3.8 The Learner-Centred Curriculum Organisation

The child-centred pattern of curriculum organization emphasises individuals development of the learner. That is, attention is given to individuals' development, their needs and their interests, needs and problems of the child.

In actual practice, a school's curriculum organization cannot be purely subject-centred, or child-centred, for example. Therefore, it is appropriate for the organization to be based upon some combination of the above patterns (Iyewarun, 1988:27).

4.0 CONCLUSION

To conclude this unit, the following questions should be reflected upon:

1. What is the scope of content for secondary school mathematics curriculum?
2. How is this content organized and sequenced?
3. What attention is given to conceptual development of mathematics processes?
4. Which alternative content should be included in the curriculum?

The consideration of those questions will affect mathematics teaching and learning during the 21st century.

5.0 SUMMARY

In this unit, you have been taught that the foundation or determinants of curriculum are:

1. The emerging needs/values of contemporary society.
2. The philosophy i.e. educational aims of the nation.
3. The nature of the subject matter.
4. The learners themselves.
5. The learning theory.

6.0 TUTOR MARKED ASSIGNMENT

1. What do you understand by 'Curriculum Development"?
2. What is meant by 'situation analysis'?
3. Why is it important to have objectives in curriculum development?
4. (a) What is meant by 'evaluation' in curriculum development?
   (b) What are the functions of the following types of evaluation in curriculum development?
5. What do you understand by 'pattern of curriculum organisation'?
6. State and explain four designs (Patterns) of curriculum organization.
7. Compare and contrast 'the subject centred curriculum' and the 'broad fields curriculum' organizations.

SELF ASSESSMENT EXERCISES

Self Assessment Exercise 1

What do you understand by the word ‘curriculum?

Self Assessment Exercise 2

Give and explain clearly *five* determinants of curriculum.

7.0 REFERENCES/FURTHER READING

Audrey and Howard Nicholis (1972, 1978): *Developing a Curriculum: A Practical Guide,* George Allen and Unwin (Publishers) Limited.

Bloom B. S. (ed) *Taxonomy of Educational Objectives Handbook I Cognitive Domain* (New York: David Mckay Co. Inc. (1956) and David R. Krathwohl, B. S. Bloom and B. B. Masia *Taxonomy of Educational Objectives Handbook II: Affective Domain* (New York: David Mckay Co. (1964).

Johnson and Rising, (1972): *Guidelines for Teaching Mathematics*. Wadsworth Publishing Company, Inc. Belmont, California.

Mager R. F. (1962): *Preparing Objectives for ProgramEDU Instruction,* Federal Publishers, Belmont, California.

Scopes P. G. (1973): *Mathematics in Secondary School*; *A Teaching Approach*. Cambridge University Press, London, N. W. 12DB.

Tanner D. (1972): *Using Behavioural Objectives in the Classroom* (New York) The Macmillan Co.

Torsten (ed) (1967): *International Study of Achievement in Mathematics* Stockholm (Almquist and Wicksell) and New York (Wiley), Vol. 1, Ch. 4.

Wheeler D. K. (1967): *Curriculum Process*, Hodder and Stoughton Limited, Kent.

UNIT 4 CURRICULUM DEVELOPMENT IN MATHEMATICS IN NIGERIA

Contents

1.0 INTRODUCTION

This unit surveys the highlights of the development of school mathematics in Nigeria since the introduction of Western education. It exposes the achievements, trials and tribulations of secondary school mathematics with emphasis on the dynamics of curriculum change in mathematics education in Nigeria.

In this unit you will be exposed to various changes made in mathematics curriculum from the pre-independence era, through the first and second Republic and through the military regimes to present day Nigeria. Emphasis will be placed on the changes in Primary and Secondary mathematics curriculum and necessary references will be make to changes at the higher education level.

To bring our ideas to focus, we shall discuss mathematics curriculum developments under the following headings:-

1. Early pre-independence mathematics curriculum in Nigeria before 1960.

2. Mathematics curriculum in the first Nigeria Republic 1961–1969
3. Mathematics curriculum reforms in Nigeria during the oil boom period (1970 – 1976) – The modern mathematics controversy in Nigeria
4. The Benin (Mathematics) Conference of 1977
5. Mathematics curriculum developments from the end of “Modern” mathematics (1979) to the present time.

2.0 OBJECTIVES

By the end of this unit, you should be able to:

1. state the type of mathematics curriculum in operation in different parts of Nigeria in the pre-independence era and give at least two reasons for the use of such curriculum;
2. explain the need for a change of the pre-independence Nigeria after independence in 1960;
3. explain the various mathematics curriculum changes and the agents of these changes from 1961 to 1969 in Nigeria;
4. give at least two reasons for the introduction of modern mathematics in Nigeria in the early seventies;
5. analyse critically the problems associated with the introduction of modern mathematics in Nigeria;
6. narrate accurately the steps that led to the abrogation of modern mathematics in Nigeria in 1977; and
7. trace the efforts so far made in mathematics curriculum development from 1979 to now.

3.1 MAIN BODY

3.2 Study Approach

1. You are expected to study this unit with personal commitment, since the events of this study affected you directly in the past, are affecting you now and will affect you in the future.
2. Make notes of important milestones in dates and characters involved in the curriculum changes in Nigeria.
3. Write out clearly the mathematics curriculum differences between the pre-independence era and the second republic oil boom mathematics development efforts.
4. Refer to newspaper cuttings of the periods referred to in this unit and make notes of mathematics curriculum events not mentioned in this unit particularly the civil war effects on the curriculum.

3.2 Criteria for Selecting Curriculum Content

Curriculum objectives operate as the final arbiters of content selection -in other words, the primary basis for content selection must always be the stated objective of the curriculum.

However, a number of other criteria has been identified for selecting content – the criteria include:

(i) significance
(ii) validity
(iii) utility
(iv) human development
(v) feasibility

(i) Significance – refers to the essentialness of the content to be learned. Content to be learned is significant only to the degree to which it contributes to basic ideas, concepts, principles, generalizations, etc. Significance also pertains to how the content contributes to the development of particular learning abilities skills, processes and attitude formation.

(ii) Validity – refers to the authenticity of the content to be selected. In this time of information explosion, the content selected can quickly become obsolete, and even incorrect.

There is another aspect of validity – the content selected should provide the learners with the experiences that would promote or effect the desired change in behaviour – the content must be related to objectives.

(iii) Utility – the criterion of utility attends to content usefulness. The content should be considered necessary for effective functioning in the society.

(iv) Human development – content selected should be appropriate for the particular developmental level of the subjects; it should be within the capabilities of the students to process. In other words, content must be available in forms which are appropriate to the pupils' developmental level because "it is unrealistic to specify content calling for a high degree of abstractions of ideas when pupils are not developmentally ready for such cognitive processes" (Lewy 1977, p. 68).

(v) **Feasibility –** addresses the question can the content selected be taught in the time allowed, with the resources available, with the expertise of the current staff?

3.2.1 Methods of Organisation of Curriculum Content

Content should be organized in ways that will facilitate student learning- thus, the prime reason for organizing content is to render it comprehensible. Therefore, after the content to be included in a curriculum has been selected, it has to be organised in such a way as to produce major changes in the learners in the direction of stated educational objectives. content may be orgainsed in tow major ways: Vertical organization and Horizontal organization.

(i) Vertical Organisation

Vertical organization refers to arrangement of learning opportunities (or contnent) within one subject area so that what follows is slightly deeper and more complex than the previous task-vertical organization is depending of knowledge – it is an arrangement according to stages – it deals with relationship of learning experiences in the subject area over time.

Sequence is another aspect of vertical relationship in curriculum organization. Sequence often is referred to as the "vertical" organization of content to distinguish it from the "horizontal" organization, which concerns the arrangement of content at a given level of instruction (Zais 1976, p. 340). Sequence is defined as 'the order in which curriculum content is presented'.

In other words, sequence refers to the placement of learning in subject matter and processes in such a way that development can occur in a sensible order (Curhs and Bidmell 1977, p.138). Four types of sequence have been identified:

a. The simple-to-complex – here, sequence is defined as progression from simple subordinate components to complex structures.
b. Prerequisite learning – this is common to subjects that depend for their exposition on laws and principles. In other words, one set of ideas or operations builds naturally upon preceding ones.
c. Whole-to-part – the rationale here is that understanding of the whole, makes possible the understanding of partial phenomena.
d. Chronology – chronological sequence is utilized in a subject which has traditionally been perceived as a structure of chronologically recorded events. In other words, facts and ideas are arranged in a time

sequence so that things are studied as they are related to another in terms of time.

Whatever organizing principle one chooses in order to sequence content, the main concern should be that the topics in the content be sequenced to maximize learning by pupils (Lewy 1977, p.68).

(ii) Horizontal Organisation – Horizontal organization (sometimes referred to as scope and integration) is concerned with the side by side arrangement of curriculum components. It is an attempt to develop interrelationships between various subjects at the same grade level in other words, horizontal organization is concerned with showing how ideas and skills in other subject. For example, when what is learned in geography is related to what is learned in history in the same year, we say there is a horizontal relationship.

3.2.2 Pre-Independence Mathematics Curriculum in Nigeria before 1960

Before the advent of the foreign powers, Nigeria was not a political entity. Various ethnic groups enjoyed their traditional ways to transmitting knowledge – political, social and mathematical – from generation to generation.

With the amalgamation of Northern Nigeria and Southern Nigeria by Lord Luggard, and increase in contact between the first Europeans and Nigerians, some education of the natives to just enough language and mathematics to enable them act as escorts, messengers and interpreters in the white man's trade and expeditionary missions.

Europeans started penetrating Nigeria from the South in large numbers by 1842. They came not only as traders and men of adventures or explorers, but also as religious missionaries.

Then it became necessary to establish formal Western Oriented education as they moved from South to the North.

The white man needed the natives who could read, write and calculate. And so, Arithmetic was one of the three R's introduced in the early schools. Books used were all of foreign origin. Badmus (1977, p.15) stated that early mathematics books used were Efficiency Arithmetic, a shilling Arithmetic and Larcombe Arithmetic. Before 1960, the only subjects compulsory in Higher Elementary Colleges were English and Arithmetic (Lassa, 1972, p.2).

By early fifties, the concept of mathematics as subject in schools has developed in Christian Missions Schools in the South but the subject was done in three different sections as arithmetic, Algebra and Geometry. These had different periods in the timetable. The books commonly used were Durell and Channon and Smith each in the three volumes.

By 1956 mathematics became one single subject in the West African School Certificate Examinations. But arithmetic remained as a separate subject to mathematics in Teacher Training Colleges. The move to make mathematics one was supported by the argument that

> the integral mathematics emphasizes the important process and concepts in mathematics, rather than routine calculation. It provides alternative to solving the problem in more than one way, hence it reinforces insight into and the understanding of the problem.

Alternative B was for students commercially inclined and Additional mathematics which had topics from Pure Mathematics, Mechanics and Statistics, was for students who were mathematically inclined.

While mathematics was being emphasized in secondary schools, there was no effort to prepare the pupils for mathematics in the primary levels. Primary mathematics remained arithmetic.

Most of the teachers of mathematics in the Colleges were whites or a few Nigerian grade two or "pivotal" teachers (i.e those who did grade two after passing the West African School Certificate). After independence, there was need to train Nigerian technicians, engineers and teachers who will take over from the foreigners. There was population explosion in schools. The mathematics curriculum as existed was no longer suitable to satisfy these influences in the South. In the Northern part of Nigeria, the Arabic education emphasized supreme obedience to Allah and constituted authority, and so there was no need for western type of mathematics. Western education emphasized in the South freedom of thought and speech and logical reasoning through mathematics. After independence the imbalance in education between the North and the South became clear and there was need for change in the curriculum.

3.2.3 Mathematics Curriculum in the Period (1961-1969)

A call for curriculum changes during this period was made at the International Conference on Science in the Advancement of New States of Rehovoth, Israel in July 1960. But the call was for curriculum change throughout the developing African nations.

The African Education Programme was then initiated at the Endicott House Conference held at Delham Massachussets in 1961. Nigeria participated at this conference. There were also participants from the United States. The aim was to help African Countries to mount their reforEDU mathematics curricula.

The African Mathematics Group (AMP) then took off with workshops held in Entebbe and Mombassa. The most successful AMP project in Nigeria was the Lagos workshop which started in January 1964, with Professor Grace Alele Williams as Director. The Federal Government of Nigeria commissioned an organization called Nigeria Education Research Council (NERC) to write a new mathematics curriculum for the country in 1969. The modern mathematics curriculum was the result.

3.2.4 Mathematics Curriculum Changes from (1970-1976)

By January 1971, modern mathematics was started in all Primary schools in Lagos. The programme would not be introduced in the Eastern part of the country because of the war problems. However by 1974 modern mathematics has started almost in all states of the Federation.

The modern mathematics programme was far from being successful particularly in the Northern States which had not trained teachers to handle the curriculum.

Modern mathematics curriculum was also prepared for the secondary school with emphasis on making mathematics meaningful. In September 1973, a national workshop drew up a modern mathematics for the country. People started to talk of 6-Ms' of modern mathematics. Modern mathematics makes mathematics more meaningful. The curriculum of modern mathematics intended to use set theory to make mathematics concepts meaningful. But very few teachers could do this.

The efforts of NERC were augmented by the efforts of professional bodies like the Mathematical Association of Nigeria (MAN) and Institutes of Educations in Universities, and the West African Examinations Council (WAEC).

The WAEC included modern mathematics examination among the papers in the school certificate examinations in 1974. results were very poor.

The wind of change to modern mathematics also affected the Teacher Training curriculum. Money was not the problem. The present teacher training mathematics curriculum was produced in 1974 to take care of the newly introduced Universal Primary Education in that year. With the Universal Free Primary Education (UPE) there was population explosion in schools in the country. Adequate planning were however not made before embarking upon this noble venture. Politicians just used the UPE to win elections. To cope with the primary school population explosions, market women and drop-outs from secondary schools were give two years crash programmes at the Teacher Training Education to enable them teach in Primary Schools.

There were mass failures at all levels. "Expo" or student cheating at Examination became the order of the day.

By 1975 people started crying over the mass failure in mathematics. Accusing fingers were pointed at modern mathematics. Many called for the abolition of modern mathematics and for the re-emphasis of traditional mathematics. Before people knew it the huge controversy over the continued use of modern mathematics had started on pages of newspapers. Some Nigerian mathematicians including Professor Chike Obi – the father of mathematics in Nigeria – also joined the call to abolish modern mathematics. Their main reasons were:

1. that it was impossible to teach the abstract concepts of modern mathematics to primary school pupils.
2. that the "modern mathematics is a repressive campaign mounted by imperialist against African scientific and technological development" (Obi, Chike Feb. 21 1976)

A change of mathematics curriculum was therefore imminent.

3.1.5 The Benin Conference of January 1977

The Federal Government felt concerned not only with the failure rate but also with the modern mathematics controversy. In the midst of all these, the Federal Ministry of Education invited eminent mathematics educators from all over the country ot a conference at Benin on 6th and 7th January 1977.

The main purpose for the conference, as stated in the letters of invitation was for these experts on mathematics to advice the Government on the path to follow as regards mathematics in the country.

At the Conference, the Federal Minister for Education, Col. A. A Ali gave the participants and the nation the greatest EDUical shock that changed mathematics education in the country.

Rather than seek for advice, the minister in his address announced with military finality that with imEDUiate effect, the Federal Government has abolished "Modern" Mathematics and has decided to introduce traditional mathematics in schools. He supported this decision with the common arguments against modern mathematics on the pages of newspapers and coming from highly place mathematicians in the country. (Nigeria Herald, 1977)

There was confusion and shock among many participants at the conference. The problems created by this radical announcement will be discussed in the next part.

3.1.6 Recent Mathematics Curriculum Development from the End of "Modern" Mathematics to now (1979 – Date)

One of the good outcomes of the abolition of modern mathematics was the impetus, it gave a Task Force forEDU in 1976 by the NERC to "make concrete proposals for the "development of appropriate mathematics curriculum for the different levels including suggestions for implementation" (Akintola, 1977). This Task Force which did not meet in 1976 then found it necessary to meet at University of Ibadan February 1977. At this first session the Task Force resolved to introduce not just arithmetic but mathematics.

The Force agreed:

1. that there would have been no controversy in the first place if there had been adequate information and publicity about the rationale behind the modernization of mathematics
2. that the core of mathematics is the same in both the so-called "traditional" and "modern" mathematics
3. that the task to teaching mathematics had been further aggravated by the inadequacy of teachers and increased enrolment at the primary and the secondary levels of the system, especially due to the introduction of universal education .
4. that most of the arguments against the "new" mathematics have been based on sentiments, personal basis and isolated cases. (Odili G. 1986 p.45)

Meanwhile, the National Policy on Education (NPE) was first published in 1977 and secondary education was separated into three years of junior secondary and three years of senior secondary. The NERC produced a new curriculum for both junior and senior secondary mathematics. A national critique workshop on mathematics curriculum met at Bolingo Hotel, Onisha in March, 1978 under the distinguished chairmanship of the eminent mathematician professor Ezeilo, then Vice-Chancellor of the University of Nigeria.

The products of the Bolingo workshop were subsequently recommended by the Joint Consultative Committee on Education to the National Council of Education which approved them. They were then published in March 1978 in the form of primary school mathematics curriculum, primary teacher education mathematics curriculum, and secondary school mathematics curriculum by the Federal Government of Nigeria. The curriculum in further mathematics for science and technology students expected to run parallel with the senior secondary school mathematics curriculum was finalized in 1986.

4.0 CONCLUSION

Another independent effort, dealing with the problems of the school mathematics curricula, had been organized by the Comparative Education study and Adaptation Centre (CESAC) at the University of Lagos. Starting as early as August 1976, CESAC held a series of conferences and workshops aiEDU at developing a new syllabus for secondary-level mathematics and organizing a carefully planned timetable of text writing, trial testing of the material and teacher training.

In October 1977, the National Council of Education met to coordinate the various activities. The NERC was asked to draw up primary and teachers' college syllabuses while CESAC was to continue its work on a secondary syllabus. The works of these two groups were presented at a National critique workshop held in Onitsha in March 1978. the Onitsha Conference adopted the NERC primary syllabus with only minor modification. However, ti was observed that the secondary syllabus did not follow the new structure as contained in the National Policy on Education – a three-year junior secondary school followed by a three-year senior secondary school. The juniour secondary syllabus was to be a general one having a dual function – cater for those who would end their schooling after that level, and those who would continue. Consequently, two syllabus were worked out for the junior and senior level, drawing ideas from both the CESAC plan and the NERC recommendations.

The new syllabuses were approved by the Federal Ministry of Education in 1978. The contents cant be said to represent something of a compromise between the "traditional" and "modern" positions (if, indeed, any real dichotomy existed). Some of the "modern" content and terminology is dropped (e.g., set theory as such, is not taught till the fourth year of secondary school), but many of the "modern" ideas remain. Geometry is included throughout the primary programme and remains intuitive geometry in the junior secondary syllabus, with formal proofs given a broader treatment (to avoid memorization of certain "required" proofs given a broader treatment (to avoid memorization of certain "required" proofs) in the senior syllabus. Statistics is well covered from bar charts in Primary 4 to standard deviation and ideas of correlation in the senior secondary syllabus. This satisfies the aspiration of both "modern" adherents and those who want practical utility in mathematics. Computation skills remain important with pronounced discussion on estimation, approximation, and deciding if answers are reasonable.

With the approval of the new syllabuses by the Federal Government, the syllabus crisis of the 70s has been resolved. But much work remains to be done to bring these syllabuses to successful classroom use.

5.0 SUMMARY

1. This unit has documented the development of mathematics curriculum from the pre-independence era to the modern period.
2. In the beginning the mathematics curriculum in school was Arithmetic and books used were foreign. Between 1956 and 1960 mathematics was taught as one single subject in the country.
3. Between 1961 and 1969 major curriculum changes were initiated such as the Entebbe Mathematics Experiment and the School Mathematics Projects (SMP).
4. By 1974, these changes brought about the Modern mathematics era in Nigeria.
5. Modern mathematics gained momentum and because of oil boom, the Federal government introduced the Universal Primary Education. The explosion brought in by the UPE in school population, in untrained teachers and in expenditure, brought down the standard of performance of the students. But most people blaEDU the poor performance on the modern mathematics.
6. On January 7th 1977 the Federal Minister of Education Col. A. A Ali delivered an address at the Benin Conference which abolished modern mathematics. 1979 saw the birth of the present 6-3-3-4 mathematics curriculum an experiment whose results are yet to be tested.

6.0 TUTOR MARKED ASSIGNMENT

1. What was the nature of mathematics curriculum in use in Nigeria before independence in 1960?
2. Write and explain two facts about the Entebe mathematics?
3. 'Modern' mathematics brought with it a lot of controversies in Nigeria, write five facts about these controversies.
4. Write one sentence to explain the major objective of the "Modern" mathematics.
5. Name two important characters that gave the last blows that brought the downfall of "modern" mathematics
6. State 3 facts about the role NERC in mathematics curriculum changes in Nigeria.
7. What one contribution to the mathematics curriculum reforms in Nigeria would you attribute to CESAC?
8. In what year was the 6-3-3-4 mathematics curriculum launched in Nigeria?
9. Compare and contrast the 6-3-3-4 mathematics curriculum with the "modern" mathematics curriculum.
10. State and explain the objective of the 6-3-3-4 mathematics curriculum.

SELF ASSESSMENT EXERCISES

Self Assessment Exercise 1

Compare and contrast pre-independence Northern Nigeria and Southern Nigeria regarding mathematics curriculum in the elementary and secondary schools.

Self Assessment Exercise 2

Explain one good effect of the introduction of modern mathematics in schools in Nigeria.

7.0 REFERENCES/FURTHER READINGS

Akintola, J. M. "address to the National workshop on Primary and Teacher Education Mathematics Curriculum", Lagos, September 26, 1977.

Ali, A. A. (1977): Modern Mathematics, Not beneficial to the Nation. Nigeria Daily Times: January 14, 1977.

Ali, A. A.: "Modern Mathematics: Not Beneficial to the Nation" address to conference of mathematics educators, text in Daily Times, Lagos 7th January, 1977 p.29

Badmus, G. A. "An Evaluation of the Mathematics Education component of Nigeria Primary School Curricula between 1930 and 1960:. A paper presented to the 15th Annual Conference of the Mathematical Association of Nigeria 1-3 August 1977 at Ahmadu Bello University, Zaria.

Beeby, C. (1970) Curriculum Planning in Howson Developing a New Curriculum. Heinemann Education 1970.

Federal Ministry of Education (1077) A report of the NERC national Task Force on Mathematics: A study of the Problems and Issues of Mathematics Teaching in Nigeria. Published by the Nigerian Educational Research Council, Lagos. 1970.

Federal Ministry of Education: Primary Mathematics curriculum, Junior Secondary Mathematics curriculum, Secondary Mathematics Curriculum, 1979

Lassa, P.N "A Survey of Mathematics Training of Grade II Teachers" – presented at National Seminar on Primary and Teacher Education Mathematics Syllabus, Lagos 26-30 1977.

Obi, Chike, Daily Sketch (Nigeria) No. 3739, February 2, 1976

Odili, G. A. Teaching Mathematics in the Secondary School: Anachuna Educational Books 1986

Ohyche, R.O. "Change in Mathematics Education since the late 1950s – Ideas and Realisation, Nigeria Educational Studies in Mathematics 9 (1978) (271-281) D. Reide Publishing Company, Dordrecht, Holand.

Skemp, R. R "Not what you teach but how you teach it: Modern vs Traditional Mathematics" Times Educational Supplement, No. 3073, April, 1974

Taiwo, C. O. "Yoruba Mathmatics, M.Sc. Dissertation (Institute of Education, London University, London 1986) pp. 8-10

Ukeje, B. O "The Entebbe Mathematics" West African Journal of Education. Vol. 9, No. 1, February 1965

West African Examination Council (1953 – 1966) Annual Reports on Examinations Published by the Examinations Council, Accra, Ghana

Williams, G. A "Dynamics of Curriculum change in Mathematics" Lagos State Modern Mathematics Project. West African Journal of Education, Vol. XVIII, No. 2, June 1974.

MODULE 2

UNIT 1 DEVELOPMENT OF A MATHEMATICS SYLLABUS AND A SCHEME OF WORK

CONTENTS

1.0 INTRODUCTION

Success in any activity is determined by how far the planning was done. The pastor spends more than three hours preparing a thirty-minute sermon; the lawyer spends the whole of his evening in his chambers, the engineer spends more time on his drawing board planning than he spends at the site executing his plans. Similarly, the mathematics teacher needs to have time to prepare his work, time to arrange his work in advance, so that the translation of theory into practice is successfully carried out in the classroom. The teacher who uses the same lesson plan every year without preplanning is unprogressive. An experienced teacher must have some time to reflect on the successes or failures of lessons he has taught several times. So the satisfactory teaching in the classroom should not be left for each day to look after itself. Planning involves consideration of the whole school curriculum and breaking it into an operational plan. Planning includes preview of a year's work, units that may be taught in a period of a week to a month and daily lesson. In this unit therefore, you will be exposed to:

(i) the basic meaning of a syllabus
(ii) the meaning of scheme of work
(iii) professional roles of mathematics teacher on syllabus and scheme of work
(iv) how the mathematics teacher can draw scheme of work

2.0 OBJECTIVES

By the time you finish studying this units you should be able to:

(i) define and explain the meaning of a syllabus and a scheme of work
(ii) identify the roles of mathematics teachers in drawing a syllabus and a scheme of work

3.0 MAIN BODY

3.1 Study Approach

You are expected to get hold of a secondary school mathematics curriculum and study the arrangements of its contents. Try to observe the main topics, contents, activities and materials contained in it.

3.2 SECONDARY SCHOOL MATHEMATICS SYLLABUS IN NIGERIA

A *syllabus* is defined as:

(i) a condensed outline or statement of the main points of a course of study, springing up from the broad curriculum of the school (Bello 1981, p.20).
(ii) a written summary in which information about all the main points from all the subjects in the curriculum is presented (Robinson 1980, p.42).
(iii) a concise written outline of a course of study; a list of items which have been written down to be taught and learnt from year to year, a blueprint enabling teachers to carry out a particular part usually a formal part of the curriculum (Nwagu 1978, p.26).

The teacher has certain roles to play in the implementation of the proposals contained in the syllabus. The professional roles/duties include the following:

1. the teacher should make a thorough study of the syllabus so as to have an adequate knowledge of what is to be taught.
2. the teacher should acquaint himself or herself with what the students have done in the previous class and the work the students will do when they move on toe the next class. This step s necessary because "in order for the current syllabus to be well understood, the teacher must build upon the foundation of knowledge gained in the previous year and work towards preparing a good foundation for the students' work in the next class (Bello 1981, p.22).
3. the instructional and learning materials that would be needed/required to achieve the objectives of a given syllabus should be sought for by the teacher and used at the appropriate time.
4. there should be modification of the syllabus whenever occasion calls for such e.g. adjustment of the syllabus to community resources. In other words, the syllabus should not be regarded as a rigid handout to be studied and obeyed indiscriminately.
5. the syllabus should be broken into scheme of work. That is, the teacher should make a clear and logical breakdown of the syllabus yearly, termly and/or weekly topics.
6. evaluation of each stage of the syllabus is the responsibility of the teacher in order to ascertain two basic elements:

   (a) Achievement of the stated objectives; and
   (b) The effectiveness of the teacher's instruction.

3.2.1 Scheme of Work in Mathematics

A given syllabus is only a indication of content, and does not prescribe specifically the order in which topics should be taught. It is from the syllabus that the teacher will work a detailed plan indicating what he will do in each term, and then break it down even further and show what ground he will cover in each week of the term. A scheme of work is therefore described/defined as follows:

1. it is a list of sub-topics or units drawn from the broad topics in the syllabus (Robinson 1980, p.46).
2. in a scheme of work, the broad outlines of the syllabus are developed in much greater detail to show the topics the teacher intends to deal with and in what order, what work will be given to

the pupils in the form of exercises or other assignments, and what books and equipment will be used (Nwagu 1970, p.27).

3. it involves listing a series of topics to be taught in the course of a term in the order in which they are to be taught (Obanya 1980, p. 30).

Here, the content that is to be covered during the year is outlined. Since in mathematics each new step depends on the thorough understanding of what he has done before, the content should be arranged in a logical sequence. We also consider what teaching techniques are to be needed so that teachers' and pupil's notes, workbooks, are acquired and rooms and timetables are well arranged. The way the programme may be evaluated is put into consideration. Whether there will be weekly or monthly test is determined. In some schools, the scheme of work is prepared by the Head of Department himself or at a meeting of all members of mathematics department. The latter is likely to yield better results.

In essence, a scheme of work is a clear statement of the work the teacher proposes to do in a given period of time. However, the teacher should not be afraid to deviate from the scheme of work should this become necessary. In other words, the scheme of work should be flexible so that there could be room for adding, deleting or rearranging the topic if there is need for such.

As far as possible, an attempt should be made in the scheme of work to indicate:

(a) the objectives of each topic;
(b) the instructional materials and methods/techniques to be used for the teaching of different topics;
(c) the evaluation devices or techniques to be employed/used in finding out the attainment of the stated or stipulated objectives.

There are certain guidelines to be considered while drawing up a scheme of work – the guidelines include the following:

1. the teacher must undertake a careful study of the prescribed syllabus so that he does not deviate from laid-down subject requirements – the teacher should avoid the temptation either to digress too far or overemphasise certain aspects of the syllabus. The pupils' previous experience should be taken into consideration.
2. the teacher should consider how long the term is and the number of periods devoted to the subject. (i.e. time allocation).

3. the different periods or the year during which certain topics are to be taught should be considered – for example, adjustment or modification of the syllabus to community environment may be necessary, taking into consideration available resources.
4. there should be taken into consideration the sequence of the topic e.g. the simple to complex, prerequisite learning and whole-to-part, taking into consideration pupils' ability or intellectual level(s).

3.2.2 Reasons Why Teachers of Mathematics Need to Draw Scheme of Work

A scheme of work is desirable because:

1. it serves as a guide for the teacher to know how much he or she is expected to cover and then strive to cover it – that is, it serves as a useful record of a teacher's forecast of the work to be covered in each week.
2. it helps the teacher to know when a topic is coming up and when to gather the necessary instructional materials that would facilitate effective teaching of the topics.
3. it helps to guarantee some measure of continuity in pupils' learning since each topic is suppose to relate to the learning of previous ones.
4. it serves as a basis of evaluation – this is, the teacher will base his evaluation of the pupils on the topics outlined in the scheme of work.
5. it affords the principal/head teacher and the school supervisors an idea of what the class has been doing.
6. in a case of change of teacher, the successor knows exactly how far the work has been covered and where he should start from.

4.0 CONCLUSION

In this unit basic concept of mathematics syllabus and a scheme of work have been discussed. The students have been exposed to the techniques of drawing up a syllabus and a scheme of work with specific references to secondary school mathematics. The unit was concluded with different reasons why teachers of mathematics at the secondary schools should be knowledgeable about the syllabus and scheme of work operational components.

5.0 SUMMARY

In this unit, you have leant the meanings of a syllabus and scheme of work. You have also been exposed to the methods of drawing up a mathematics syllabus and a scheme or work. With these relevant experience you are expected to be able to guide secondary school teachers in the rudiments of mathematics syllabus and scheme or work.

6.0 TUTOR MARKED ASSIGNMENT

1.(a) What do you understand by the word 'syllabus'?
(b) Discuss *five* professional duties a secondary school teacher should carry out to ensure that a given 'syllabus' is used to the best advantage of the learners.
2.(a) What is a 'scheme of work'?
(b) Discuss *five* reasons to justify the need to draw up a scheme of work in your teaching subject(s).

SELF ASSESSMENT EXERCISES

1. Take a secondary school mathematics textbook and break the content into its relevant syllabuses.
2. Proceed further to break the mathematics syllabuses into appropriate scheme of work.

7.0 REFERENCES/FURTHER READINGS

Association of Teacher of Mathematics: *Notes on Mathematics in Primary School*; London: Cambridge University Press.

Flectcher, T. J. (ed) *Some Lessons in Mathematics, A Handbook on The Teaching of Modern Mathematics:* London: Cambridge University Press.

Green, Thomas F. *The Activities of Teaching:* New York, McGraw-Hill 1971.

Hyman Ron, (1970): *Ways of Teaching:* Philadephia: J.B. Lippincott.

Johnson, D. A. and Rising, G.R. (1969): *Guideline for Teaching Mathematics:* Belmont, California: Wasworth Publishing Company.

Laing R. A. and Peterson, (1973): *"Assignment: Yesterday, Today and Tomorrow – Today"* Mathematics Teacher, October.

Scopes, P.G. *Mathematics in Secondary School – A Teaching Approach:* London: Cambridge University Press.

UNIT 2

DEVELOPMENT OF MATHEMATICS COURSE, UNIT, WEEKLY AND DAILY LESSON PLANS

CONTENTS

1.0 INTRODUCTION

Educational planning is of many types depending on the duration of plan and nature of objectives.

One such plan is the course plan. Once the curriculum has been constructed, the classroom teachers first task is to break the curriculum into learning units, organize the learning units into termly plans. Here it is assuEDU that the curriculum had already mapped out the content into years, in what is called yearly plan. Actually this aspect of work is that of the curriculum planners.

In a course plan certain questions must be asked and answered. How many terms are there in the academic year? How many working weeks are available for the academic term? Are the weeks different from term to term? Do we anticipate holidays, strikes, closure of school? How many lesson periods are available in schools per week?

The teacher must reasonably predict answers to these questions. In general, it is safer for you as the teacher of mathematics to understand Course plan, Unit plan, Weekly plan and Daily lesson planning.

In the contents of this unit therefore, you will be exposed to

(i) the importance of educations planning
(ii) how to draw course plan
(iii) the meaning of unit plan;
(iv) the meaning and examples of a daily lesson plan;
(v) features and format of a lesson note

2.0 OBJECTIVES

By the end of this unit, you should be able to:

(i) define and explain the meaning of course plan, unit plan, weekly and daily lesson plans;
(ii) list the features of a lesson note; and to study how you can
(iii) write a standard lesson note.

3.0 MAIN BODY

3.1 Study Approach

Get hold of a primary school or junior secondary school curriculum and study it. Try to observe the vertical and horizontal structures of any page of the curriculum.

Note: that the vertical structures are the broad topics while the horizontal structures are the main topics, objectives, content, activity/materials and remarks.

3.2 The Importance of Educational Planning

The saying "well planned, half finished summarises the importance of planning in any endeavour a person undertakes. A teacher is a leader in the class. As a leader, he needs poise and confidence to be a role model for the students. He needs to have purpose, direction, knowledge of the topic on hand and an understanding of the students. All these characteristics, the teacher acquires in planning any lesson, with these characteristics, the teacher is easily loved, obeyed and given attention by student. Gagne (1970) suggests that informing students of what is expected of them serves as direction that can facilitate students achievement. Planning helps the teacher to streamline the content of his teaching, making it not too much or too little for the time available.

Planning helps the teacher to identify the sequence of thought, sequence of activities and content development of the topic. Planning helps teacher prepare ready- made questions that help him direct the pupils to the right objectives.

3.2.1 Basic Concept of a Course (Termly) Plan

After breaking the syllabus in to course termly plan the next stage is for the teacher to further identify learning units within the termly content. A leaning unit is a broad unit of a base concept capable of being broken into more than one daily lesson content. An example of a learning unit from the primary curriculum is "Addition of Fractions". Certainly this is a concept which cannot be taught effectively in one lesson.

It is best if one learning unit fits into the number of lessons in one week. In this case the learning unit can then be called the course plan.

The phrase **scheme of work** seems misunderstood by teachers. It is not synonymous with the weekly lesson plan. The scheme of work entails the entire labour of breaking the contents of a curriculum into yearly plan, termly plan, and learning unit plan and even subdividing the learning unit lesson into its component daily lesson plans.

3.2.2 Weekly Plan

It may be better to plan lessons in learning unit than in weekly plans. This is so because if a learning unit plan is not exhausted in one week, it can be carried unto the next week. This maintains continuity of coverage. However, with proper planning, it is possible to make every learning unit plan coincide with the weekly plan for supervision purposes.

3.2.3 Unit Plan

A unit plan is usually prepared for a number of lessons to cover a particular topic or set of related ideas. Its main purpose is to put the daily lessons into broader perspective and at the same time force a deeper analysis of the content to be taught. It allows the teacher to ask for special teaching aids. Such aids may require special funds which may take some time to get from the bursary department.

The following questions are considered when writing unit plans:

(a) Why is this unit important? What are the objectives of the unit? What does it have that will appeal to the student? Which are the keys to future progress?

(b) What are the central ideas and unifying concepts around which activities may be organized? What should be stressed most? How should the class time allocated to this unit be subdivided?

(c) What teaching strategies are appropriate? Is this the first time students are meeting these ideas or is it to give wider perspective to ideas previously introduced? Can students develop the ideas themselves? What materials are available to provide a varied attack on the unit?

(d) What previous concept, skills and experiences are needed for this unit? How can the content be modified for students of varying ability? What extra practice can be provided for weak students? What special teaching techniques may be used with them? What can other students do whenever students receive special attention? What enrichment topics should be included for all students? What activity should be assigned to bright students?

(e) What teaching techniques will best suit this class? What are the difficult spots that require special attention? How did I teach this material previously? Should I change my approach or techniques? What lessons require practical work?

(f) What teaching aids wil this unit require? What supplementary books or pamphlets would be helpful for students? What models are appropriate? What should the bulleting board display? Are any excursions suitable? Who might be a suitable outside speaker or class participant?

(g) What kind of evaluation should I use? What ways are best suited to the content of this unit and this class?

(h) What kinds of assignments should the students prepare? Are long-term assignments appropriate? Can the students learn part of the material independently?

These questions help the teacher to escape from the textbook. They provide the basis for developing a careful plan of attack that may be put into operation through the daily lesson plans. A unit plan should contain these elements:

(i) Statement of the objectives;

(ii) Description of the skills and the previous knowledge of the students;

(iii) Content outline to be taught and the basic skills and important ideas selected for mastery;

(iv) Selection of possible learning activities and stating briefly the teaching procedures and techniques;

(v) A list of materials to be used;
(vi) Description of the assignments and evaluation instruments.

3.2.4 Daily Lesson Plan

The lesson plan may vary in format but, in general, it should answer in greater detail the same question as asked in the unit plan. Similarly, the lesson plan has the element as the unit plan except that it is focused on the objectives and content of a single lesson. No doubt educationists. Psychologists and mathematics teachers agree that well-planned lessons give confidence to the teacher which is shown on the performance of the class. The lesson plan should answer questions such as the following:

a. What are you going to teach?
b. Why is it important for students to learn these ideas?
c. How are you going to introduce the lesson?
d. What activities are you going to make pupils do?

   How are you going to get students to discover the important ideas of the lesson
e. What materials are you going to use?
f. How are you going to evaluate pupils learning gain?
g. How are you going to end the lesson? What assignments would you provide for further practice and study?

Alternatively, these questions can be compressed into three basic questions What, ‘Why’ and ‘How’.

The lesson plan may have any convenient format. Ordinarily it is arranged in the order in which activities are to take place. One suggested format is provided to guide you in this unit as follows:

3.2.5 Format for Lesson Plan

Every unit plan (or weekly plan) should contain the following pieces of information:

i. The topic to be planned for
ii. The Class for which the topic is being planned
iii. General objectives. These can be identified from the curriculum. They are necessary to focus the attention of the teacher to the stages and parts of development in the contents development.
iv. Entering behaviour: This is a set of past experiences of the students necessary for the new concept or unit to be learnt. The entering behaviour helps the teacher to plan for how to address the students, and what new content to introduce and give the

teacher the point of entry into the student. This means that the teacher utilizes the entering and starts from it to introduce the new concept.

v. The daily lessons involved in the learning unit. This is the most important aspect of this format. To be able to break the learning unit into a suitable number of daily lessons enough to cover the unit is an art every teacher must acquire. The teacher must be able to judge what quantity of contents is suitable for one lesson period. One general criterion that aids the teacher is this: "Teach one concept at a time". In other words, let one specific objective do depending on the relative intelligence of the class. But in most case one objective is enough for a lesson of about 40 minutes (in most case 35 minutes) in primary school.

vi. Some suggested teaching aids may be identified at the learning unit plan level. This makes the planning of the daily lesson plan easy.

Above are the essential six parts of a learning unit plan. Please note that evaluation and teaching techniques are not necessary at this planning level, but these are necessary in the daily lesson plan level.

From above format we see that the main purpose of a learning unit plan is to identify number of daily lesson to cover a particular learning unit or a set of related ideas. It then help us in putting the identified daily lessons into broader perspective and at the same time forces a deeper analysis of the daily content to be taught.

3.2.6 An Example of Lesson Plan

i. Topic: Addition of fractions

ii. Class: 5

iii. General objectives: By the end of this unit. The students should be able to:

a. add fractions with common denominators accurately .

b. explain why fractions with different denominators cannot be added directly – there is need to change them to a common denominator

c reduce fractions with different denominators to a common denominator

eg. $\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10}$

d add correctly fractions with different denominators: and

e add correctly all types of fractions – mixed with proper fraction and mixed with mixed fraction.

iv **Entering Behaviors : Students have dealt with:**

a equivalent fractions
b least common multiples or LCM
c addition of whole numbers
d representation of fraction using charts.

v. **Daily Lesson Involved**

a. Addition of fraction with common denominators
b. Reducing fractions with different denominator to a common denominator
c. Addition of fractions with different denominators of the type

$$\frac{1}{2} + \frac{2}{5}$$

d Addition of all type of fractions

Example

1 mixed with proper fractions such as 11/2 +2/3
2 mixed with mixed fractions such as 11/2 + 2/3/5

vi **Teaching Aids**

a. Fraction boards showing equivalent fractions
b Fraction charts for addition.

3.2.7 Features of a Lesson Plan

What do you mean by a Note of lesson? Note of lessons may mean different things to different authors, For the purpose of this unit, a note of lesson is taken to mean the same things as a daily lesson plan.

A daily lesson plan is a write- up that spells out clearly, the specific objectives of a lesson, the entering behavior, the teaching aids, the activities involved, the evaluation techniques, the teaching strategies and the closure of the lesson.

The main areas of interest are the objectives, the entering behavior and the content development of the lesson. The teacher and the student performance acts help to focus attention to what exactly is being done by the teacher and the student to achieve the objectives.

3.3 Needs for Writing Note of Lessons

The reasons we gave for the need of writing learning unit plans also apply in the writing of notes of lesson. Specifically a note of lesson or a daily lesson plan helps the teacher to:

(i) Focus his attention to the realization of the specific objectives stated;
(ii) identify the teaching aids necessary for teaching the lesson;
(iii) identify the entering behavior- those experiences and past knowledge of the pupils which will help the teaher introduce the new concept;
(iv) define the sequence of thought and actions necessary to develop the content and to direct the pupils to achieve the objective; and
(v) anticipate questions both from himself and from the student and get ready for the answers

3.3.1 A SAMPLE LESSON PLAN FOR MATHEMATICS

Topic: Parallelogram

Objective: Given a parallelogram, the student should be able to identify its properties in ten minutes.

AssuEDU Knowledge: The definition of a quadrilateral. The angles, sides and diagonals of quadrilateral. The types of quadrilaterals: generally, rectangle, square and parallelogram.

Learning Activities

1. **Introduction:** Name some shapes which are parallelograms. From what you know about parallel lines cut by a transversal, what angle relationships do you know?

2. **Content**

(a) The sides which are opposite to each other are equal in length.
(b) The angles which are opposite to each other are equal.
(c) The diagonals bisect each other.
(d) The diagonals bisect the parallelogram so that two congruent triangles are forEDU.

3. **Method and Procedure:** Measure the angles and sides of several parallelograms. How do these measures compare? What generalizations can you state based on these measurements? (Illustrate these relationships with the Geo-board)

4. **Assessment:** The students are given many different figures on a work card and asked to identify those that are parallelograms.

5. **Materials:** Geo-boards with elastic thread, work-cards and mathematics set.

Assignment: Write the proof of this theorem: A diagonal of a parallelogram divides the parallelogram into two congruent triangles.

3.3.2 Different Formats For Daily Lesson Plans

Generally as Odili (1986p. 50) puts it, the daily lesson plan should answer the following questions:-

(a) what are you going to teach?
(b) why is it important for students to learn these ideas?
(c) how are you going to introduce the lesson?
(d) what activities are you going to make the pupils do? How are you going to get the students to discover the important ideas of the lesson?
(e) what materials are you going to use?
(f) how are you going to evaluate pupils learning gain?
(g) how are you going to end the lesson? What assignment would you provide for further practice and study?"

There are many formats now available. There are however general guideline steps which every format must include:

(i) The content itself
(ii) The class must be stated
(iii) The specific behavioural objectives
(iv) The entering behaviour or behaviours
(v) The teaching aids, if any
(vi) The development of the content which must include the sequential development of the teaching strategies or skills used
(vii) The evaluation questions
(viii) The closure or the ending of the lesson

Explanation of Format

For effective daily lesson plan you should put into consideration the following three words that start with letter C-the Content, the Child and the Curriculum.

(a) Content

The teacher must be conversant with the content of what he is to teach. No teacher teaches effectively what he does not know. In order to understand the content fully the teacher must consult himself: How much of the content does he know? Can he explain the meaning of the concept in relation to other concepts or expressions in mathematics?

.Can he explain the meaning in relationship to the referent or referents i.e objects or ideas for which the mathematics concept which may be different from what others may have about the concept? The teacher should be able to give honest answer to these questions.

(b) The Child

Knowledge of the child also identifies his class. If the teacher knows the child, he can then plan for the type of teaching aids that child will love and appreciate, that will not distract or harm the child. The teacher also psychologically prepares himself to handle the child. He even identifies the depth of content the child can understand because knowing the child enables him to speculate on the maturity of the child.

(c) The Curriculum

Many teachers rarely consult the curriculum. But the 6-3-3-4 mathematics curriculum at any level contains a wealth of information for the effective teaching of any concept. The column for objectives identifies the necessary general objectives that must be achieved. The teacher can then write his own specific objectives that will lead to the general objective; the content column gives other topics relating to the concept under treatment and makes for relational awareness on the part of the teacher; the column for materials activities suggest the method to be used, activities to be carried out in class, the teaching aids to be used and experiments to carry out. The column for remarks spells out the depth to be reached for any concept, the areas for emphasis and areas not to be treated and special teaching strategies or aids to be used.

It is recommended that every teacher should think about the three C's before looking at the textbooks.

3.3.3 Textbooks and Teaching Aids

The teacher should be conversant with the use of textbooks. From the foregoing, it is clear that the teacher should not be a slave to the textbook. He should use the text-book that has the most content, arranged in order and depth suggested by the curriculum and with exercises and activities suitable in quantity and quality for the class identified.

The teacher needs not follow the text-book page by page blindly. He should feel free to consult other texts that treats a topic better in his own judgement than the prescribed text.

It is not necessary that every concept in mathematics should have cognitive, affective and the psychomotor domain of learning objectives unless these three are there in the lesson. Mostly in all lessons in mathematics we have cognitive domain of specific objectives. The affective is now becoming necessary because of the continuous assessment requirements. But the psychomotor may not be necessary unless the child is learning for the first time how to write numbers, make strokes, draw curves, or shapes or learn how to bisect or construct shapes.

Almost every concept in the primary school mathematics requires teaching aids. This is necessary because at this level the pupils are operating in what is known as concrete level. The top primary section may use semi concrete teaching ids, ie. Diagrams of real objects but where real objects are available it may be best to use them. Every aid must be suitable for the class and topic being treated.

The teacher should set out clearly step by step and sequential treatment of the concept. What exactly the teacher does at every step should be spelt out as well as what the students do. Actions or activities for the students are emphasized and not passive listening. In short the note will be dynamic enough to show that every student participates in the lesson. Also to be indicated in this section are the teaching strategies and skills to be utilized by the teacher to bring the students to the stated objectives.

3.3.4 Importance of Entry Behaviour (Previous Knowledge)

This is a part of the past experience (previous knowledge) of the child which are necessary for the introduction or understanding of the new concept.

Every entering behaviour identified must be utilized in the new lesson.

3.3.5 Evaluation and Feedback

The teacher should include in his plan diagnostic exercises and formative exercises to identify areas of need, areas of weakness on the part of students, the programme or the teaching methods depending upon the stated objectives.

These exercises should be similar to the ones used as example by the teacher, and should be graded from simplest to the hardest. There should be sufficient exercises to keep everybody including the fast performers, busy.

4.0 CONCLUSION

The teacher should think out during the planning the best way to end a lesson. It might be the use of more practice exercises for take home, a summary of the lesson or the working of the evaluation exercises. This closure varies from teacher to teacher but it is best especially in the spirit of the 6-3-3-4 of education to give assignments that will require application of the concept learnt in the lesson.

A Sample Lesson Plan on Addition of Fraction

Topic: A first lesson on addition of fraction with different denominators.

$$\frac{1}{2} + \frac{2}{5}$$

Class: 5

Specific Behavioural Objectives

By the end of this lesson the students should be able to add correctly two fractions with different denominators.

Entering Behaviour

i. The students have students have studied equivalent fractions and how to find them.
ii. They can add fractions with common denominators.
iii. Pupils can change improper fractions to proper fractions.
iv. They can also write fractions in their lowest term.

Teaching Aids

i. Fractions charts
ii. Equivalent fraction board

Content Development and Learning Activities

Step I

Introduction or set Introduction: Quickly find out if the pupils can add fractions with common denominator. Thus write on chalkboard

$$\frac{5}{10} + \frac{4}{10} = \boxed{}$$

Let each child write the answer. Go round and check their answers.

Step II

Write on chalk board the problem $\frac{1}{2} + \frac{2}{5} = \square$ and demand an answer from the pupils. Pause a little for them to think. Expect to hear from some of the pupils that this cannot be done straight on. And ask the question why? The pupils should reply it is because the fractions have different denominators.

Step III

Open up the equivalent fraction chart

Teacher: Write out at least five equivalent fractions for each of the fraction"

Pupils:

$\frac{1}{2} = \frac{2}{4} = \frac{3}{6} = \frac{4}{8} = \frac{5}{10}$

$\frac{2}{5} = \frac{4}{10} = \frac{6}{15} = \frac{8}{20} = \frac{10}{25}$

Teacher goes round to supervise and help those that could not do this.

Teacher: Identify the equivalent fractions with common denominators in the sets.

Pupils: $\frac{5}{10}$ and $\frac{4}{10} = \square$

Teacher arranges on the chalkboard the new problem $\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \square$

Teacher: Now write down the required sum.

Pupils copy in their notes $\frac{1}{2} + \frac{2}{5} = \frac{5}{10} + \frac{4}{10} = \square$

Teacher goes round to check answers. Another example is then given

$\frac{1}{3} + \frac{3}{4} =$

Teacher asks students to follow the same method as above to arrive at the answer. Evaluation: Add the following fractions:

1. $\frac{1}{2} + \frac{1}{3} =$
2. $3 + 1 =$

5 4

3. $\frac{1}{2} + \frac{2}{3} =$

4. $\frac{3}{4} + \frac{5}{6} =$

5.0 SUMMARY

As the pupils are working teacher goes round to mark each individuals,
Closure: Solve this problem at home with your parents:
"If I have $\frac{1}{3}$ of my school fees from my father and $\frac{1}{5}$ of it from my mother.
What total fraction of my fees have I now got?

We shall look at your work when you report to school tomorrow.

6.0 TUTOR MARKED ASSIGNMENT

1. Write one main difference between unit plan and a lesson plan.
2. Explain the term learning unit.
3. Write 2 main characteristics of an entering behaviour.
4. Identify a learning unit and state one reason why it is a learning unit.

SELF ASSESSMENT EXERCISES

Self Assessment Exercise 1

Suppose you want to teach a lesson from your years of experience without planning. State two problems you may face in class.

Self Assessment Exercise 2

Carry out the following experiment: pay observation visit to four classrooms in the primary schools in your area. Observe the goings on in the classroom on lessons on mathematics. Take notes and fill this chart:

School Class/Teacher	Action by the teacher	Action by the student
1		
2		
3		
4		

Answer this question for each teacher; Who is more active, the teacher or the students?

7.0 REFERENCES/FURTHER READINGS

Gagne Report M, (1970): *The Conditions of Learning* Hold Rinihat and Winston, New York.

Odili, G. A. (1986): *Teaching Mathematics in the Secondary Schools*, Anachnna Educational Books.

Fakude, R. A. (1981): *Teaching Arithmetic and Mathematics in the Primary School*, Ibadan: University Press, Ibadan.

UNIT 3 PSYCHOLOGICAL BASES FOR MATHEMATICS INSTRUCTION IN SECONDARY SCHOOLS IN NIGERIA

CONTENTS

1.0 INTRODUCTION

Educational psychology is generally concerned with the study of human behaviour. For class teachers, educational psychology will enable them cope with problem of how children learn and under what conditions maximum learning can take place. In this unit you will see how educational psychology can be applied to gainfullly teaching and learning mathematics.

In this unit and also in unit 2 of this module you will study the cognitive aspect of learning mathematics and the contributions of three psychologists-Piaget, Bruner and Gagne to the learning of mathematics. They are, not the only contributors but they represent a selection whose choice for further reason will become apparent as you read the unit.

2.0 OBJECTIVES

By the end of this unit you should be able to:

(1) name at least five areas of psychological studies on mathematics education;
(2) watch questions, and problems on mathematics learning and teaching with the appropriate field of educational psychology research;
(3) give three stages according to Burner, when a child attains cognitive learning;
(4) associate a given strategy for solving a problem with the appropriate stage;
(5) identify Gagne's contribution to the study of mathematics; and
(6) distinguish between Bruner's and Gagne's contributions t the learning of mathematics.

3.0 MAIN BODY

3.1 STUDY APPROACH

(1) Read each of this section slowly to understand it.
(2) Do an activity as you come across it. Do not refer back to the text when doing the activity .
(3) Check the answers as soon as you finish the activity. If you do not get them right re-read the text and then go through the activity again
(4) You will probably need a good dictionary to help in understanding new words.

3.2 PSYCHOLOGICAL CONSIDERATIONS FOR MATHEMATICS INSTRUCTION

With educational psychology always in search for a generally acceptable instructional paradigm and with the history of mathematics education having divergent views on the development of mathematics its curriculum and instructional strategies the discussions on the bases of educational psychologists in this unit entails an uphill task for the mathematics teachers. In order to allow you to understand the content of this unit clearly .The contribution of this educational psychologists will be taken in turns.

3.2.1 The Contribution of Jerome S. Bruner

Jerome Burner worked on the process of thought in general. Later he applied this to the process of learning mathematics. He devised experiments to help him observe how mathematical thinking in children develop. The investigation concerned the individual strategies by which a child tries to discover a given logical relationship.

The procedure in most of the experiments was to present a number of cards to the child. Each card has its diagrams of triangle, circle or square separately or a combination of these. Each card was red or green or blue. So there were three variable- number, shape and colour-each with three values. A concept such as red triangles was thought of by the experimenter and the subject chose cards to which the experimenter answered either Yes or No: if the card was red and had triangles on it. and No if not. Subjects were asked to find the concept, which the experimenter had in mind in the least number of trials. Sometimes more variables were used, sometimes the numbers of choice were restricted.

From this single procedure, Bruner was able to claim that learning in general depended on four factors.

(i) the structure of the concept that is to be learnt:
(ii) the nature of the learner's intuition:
(iii) the desire of the learner to learn:
(iv) the readiness for learning- (biological readiness).

Thus Bruner considered adequacy of both the subject matter and the learner himself necessary for the leaning of Mathematics. By this he meant that the learner must be intuitively ready to learn and the materials to be learnt must be presented in a form (or structure) that matches the learners "readiness stage" This led to his controversial, but yet popular, assertion that" Any concept can be taught effectively in some honest form to any child at any age provided such a concept is introduced at the child's language level".

This sort of reasoning let him to attempt a classification of these levels or stages. The following are the three stages through which Brunner says a child goes through in cognitive learning.

i. **The Enactive Stage:** At this stage the child thinks only in terms of action. This stage is characterised by the mode of representing past events through motor responses. The child enjoys touching and manipulating objects as teaching proceeds. Specifically no serious learning occurs at this stage. Topics can however be introduced to a child at this stage using concrete materials. The child's methods of solving problems are limited because he cannot "act the solution" – he cannot solve problems.

ii. **The Iconic Stage:** This is the stage of manipulation of images. Here he builds up mental images of things already experienced. Generally, such images are composite being forEDU from a number of experiences of similar situations. Learning at this stage is usually in the form or in terms of seeing and picturing in the mind any object which transforms learning. The child uses thinking thereby making transfer of learning considerably easy. Bruner emphasised that before any image is forEDU to represent a sequence of acts, certain amount of motor skills and practice have to take place.

iii **The Symbolic Stage:** At this stage child possesses the ability to evaluate learning. Logic, language and mathematical symbols are used to discuss what has been learnt. Acquired experiences are translated into symbolic form The three stages can be illustrated this way, using the concept of addition of positive whole numbers. Consider the problem: 3+2=5. First the child must work with block, marbles, counter or other real objects. Take the three first, take another two mix them up and then count the mixture (or union) At the second stage he will be able to work with worksheets containing pictures of objects(images). Instead of the physical objects he is now able to recognise their image and can solve something like it while not necessarily requiring the production of the ducks physically. At the final stage he can solve the real' problem 3 + 2 =5 using symbols 3 and 2 and 5. Bruner's three stages, in brief of the sequence

Action

Image

Word

and correspond approximately to Piaget,s

sensor motor

Perceptual

Abstract

modes of cognitive functioning. Also Burner, like Piaget, believe that all mathematics could be learnt by discovery approach provided that search is started early enough in the life of the child by presenting to him

concrete materials relevant to the concept we want him to learn at a higher stage. For example properties of a triangle could be taught by making sure children play with triangular shape object at the pre-school (Kindergarten) stage, draw enough image of triangles at the at the primary school and by the junior secondary school they would have been sufficiently equipped with various terms to discover for themselves some, if not all the properties of a triangle.

3.2.2 The Contributions of Robert Gagne

As a behaviourist psychologist. Gagne devoted his time to study the conditions of learning. He believes that learning occurs as a result of interaction between the learner and the environment. Learning is known to have taken place when we notice (observe) Gagne maintains that the stages described by Piaget are not necessarily the inevitable result of an inborn "timetable" but are instead a consequence of children having learned sets of rules that are progressively more complex. How do children acquire these sets of rules? According to Gagne children are "taught " the rules by their physical and social environment.

Notice the differences in what Piaget and Burner are claiming on one hand and what Gagne is saying. If we follow Piagets (and Burner) assertion we will assume that children will develop complex concepts. Understanding and problem solving skills when they are ready. That is when their nervous systems have matured sufficiently and they had enough experiences with simpler more elementary problems.

Mathematics teachers who see learning as a process of discovery are likely to borrow heavily from Piaget and Burner. Others who will see learning as produced primarily by children environment are likely to take their cues from Gagne. The following illustrates Gagne contribution to the learning of mathematics.

We have already mentioned that he emphasised the idea of pre-requisite knowledge in learning mathematics. That is the idea that one cannot master complex concepts without mastering the fundamental concepts necessary for such complex concepts. For instance. a child can not successfully add fractions without the knowledge of finding common denominator of fraction. That is,

A child cannot do $\frac{1}{3} + \frac{2}{5}$ unless he has been

Lead to learn that $\frac{1}{3} = \frac{5}{15}$ and $\frac{2}{5} = \frac{6}{15}$

therefore $\frac{1}{3} + \frac{2}{5} = \frac{2}{15} + \frac{6}{15} = \frac{11}{15}$

Besides there is an interEDUiate step which we have ignored in this sequence: That is the fact that fractions with same denominator could be added as like terms.

That is $\frac{n}{A} + \frac{m}{A} = \frac{m + n}{A}$ for A's Which are real numbers for instance;

$$\frac{2}{3} + \frac{5}{3} = \frac{2 + 5}{3}$$

3.2.3 Gagne on ProgramEDU Learning and Learning Sets

Gagne is also noted to have applied the method of programEDU learning to the analysis of mathematical learning. He extended the idea of "learning set" to mathematics. This is the idea that learning a series of similar but not identical tasks brings more than the acquisition of so many discrete skills. The learner acquires in addition a generalized readiness to learn similar and even different but related materials. We say if this happens, that the learning has generalized to produce a "set" for learning related and more general task.

We will use one of Gagne's example to illustrate this

We start by analysing the steps required to solve a simple algebraic equation in one unknown e.g. 4x + 5 = x + 11

Solution $4x + 5 = x + 11$

$4x – x = 11 – 5$

$3x = 6$

$\frac{3x}{3} = \frac{6}{3}$

$x = 2$

We now trace back from the final step to the beginning for the process, asking the question, what would the individual have to know in order to be able to perform the new task, being given only instruction?

x =2 Basic Concept

$\frac{3x}{3} = \frac{6}{3}$ …….. Division by coefficient of $\overline{x \quad 2}$ a real number

3x = 6 ……… Addition (Subtraction) of similar terms

4x – x = 11- 5……… Collection of like terms

……….. Transpositions in an equation

………… Directed numbers

Thus at some stage in solving linear equations, the learner would have to

(i) Collect like terms
(ii) Add (subtract) similar terms (Learning sets)
(iii) Transpose
(iv) Divide by a real number

Gagne built up a programmne in which each of the sets in the hierarchy leading to final solution was taught and he then ask himself the question how far learning a lower-order set made for improved learning of a higher-order one. His hypothesis were confirEDU, that the lower supporting set was required to be mastered before the next one would be achieved.

3.2.4 Gagne on Teaching Method

The problem of learning by discovering methods has been mentioned the earlier that Piaget and Burner supported it. Some other psychologists hold a different view. Gagne held a compromising view which he referred to as guided discovery. Guided discovery has been shown to be best in the field of elementary mathematics

3.2.5 Robert Gagne's Theory of Learning

The research of the psychologist M. Gagne into the phases of a learning sequence and the types of learning is particularly relevant for teaching mathematics Professor Gagne has used mathematics as a EDUium for testing and applying his theories about learning and has collaborated with the University of Mary land Mathematics project in studies of mathematics learning curriculum development.

The Objects of Mathematics Learning

Before examining Gagne four phases of a learning sequence and eight types of learning, it is appropriate to discuss the objects of mathematics learning, which are considered in his theory. These objects of mathematics. Learning are those direct and indirect things which we want students to learn in mathematics. The direct objects of mathematics learning are facts skills, concepts, and principles: some of the many indirect objects are transfer of learning, inquiry ability problem solving ability self-discipline. and appreciation for the structure of mathematics. The direct objects of mathematics learning –facts skills concepts. and principles-are the four categories in to which mathematics content can be separated.

Mathematical facts are those arbitrary conventions in mathematics such as the symbols of mathematics. It is a fact that 2 is the symbol for the word two. that + is the symbol of the operation of addition. And the sine is the name given to a special function in trigonometry.

Facts are learned through various techniques of rote learning such as memorization drill. Practice tiEDU tests, games and contest people are considered to have learned a fact when they can state the fact and make appropriate use of it in a number of different situations.
Mathematics skills are those operations and procedures which students and mathematicians are expected to carry out with speed and accuracy Many skills can be specified by sets of rules and instruction or by ordered sequences of specific procedures called algorithms. Among the mathematics skills which most people are expected to master in school are long division addition of fractions and multiplication of decimal fractions, constructing right angles, bisecting angles, and finding unions or intersections of set objects and events. These are examples of other useful mathematical skills which are learned through demonstrations and various types of drill and practice such as worksheets, work at the chalkboard, group activities and games Students have mastered a skill when they can correctly demonstrate the skill by solving different types of problems requiring the skill or by applying the skill in various situations.

A concept in mathematics is an abstract idea which enables people to classify objects or events and to specify whether the objects and events are examples or non-examples of the abstract idea. Sets, subsets, equality, inequality, triangle, cube, radius and exponent are all examples of concepts. A person who has learned the concept of triangle is able to classify sets of figures into subsets of triangles and non-triangles. Concepts can be learned either through definitions or by direct observation. By direct observation and experimentation young children learn to classify plane objects into sets of triangles, circles, or square, however few young children would be able to define the concept of a triangle. A concept is learned by hearing, seeing, handling, discussing, or thinking about a variety of examples and non-examples of the concept and by contrasting the examples and non examples. Younger children who are in Piaget's stage of concrete operations usually need to see or handle physical representations of a concept to learn it: whereas older formal operational people may be able to learn concepts through discussion and contemplation. A person has learned a concept when he or she is able to separate examples of the concept from non-examples.

Principles are the most complex of the mathematical objects. Principles are sequences of concepts together with relationships among these concepts. The statements, "two triangles are congruent if two sides and the included angle of one triangle are equal to two sides and the included angle of the other and, the square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides are examples of principles. To understand the principle about congruent triangles one must know the concepts triangle angle, and side. According to Gagne (1966) in a chapter appearing in the book "Analyses of concept Learning" edited by Herbert, Klaus, Meier and Chester W Harris:

It would appear then that principles can be distinguished from what have previously been called concept in two ways. First the performance required to demonstrate that a concept has been learned is simply an identification. That is a choice from a number of alternatives: a principle in contrast must be demonstrated by means of performances that identify its component concepts and the operation relating them to one another. Second, this means that the inference to be made about EDUiating processes is different in the two cases. A concept is a single EDUiator that represents a class of stimuli (or objects), whereas a principle is a sequence of EDUiators, each one of which is itself a concept.

Principles can be learned through processes of scientific inquiry, guided discovery lessons, group discussions, the use of problem solving strategies and demonstrations. A student has learned a principle when he or she can identify the concepts included in the principle, put the concepts in their correct relation to one another, and apply the principle to a particular situation.

It probably would not be a very precise or useful activity to classify all the objects of secondary school mathematics into the four object categories-facts, skills, concepts and principles. Even the experts in mathematics and learning theory would disagree about the proper category for many mathematical objects.

In general, the objects progress in order of complexity from simple facts, to skills and concepts, through complex principles. Also the classification of many (may be even most) mathematical objects is relative to the observer's own view-point, which is an important fact (or is that a principle!) for every mathematics teacher to know. A student who merely memorizes the quadratic formula knows a fact. A student who can plug numbers into the quadratic formula and come up with two answers has learned a skill. A student who can classify 5,3, and 4 as constants and x as a variable for the quadratic equation 5x2 + 3x+4=0 is demonstrating acquisition of a concept. And a person who can derive (or prove) the quadratic formula and explain his derivation to someone else has mastered a principle. Consequently, the quadratic formula which is a principle may be regarded as either a fact a skill or a concept by a student whose viewpoint of the quadratic formula is not as sophisticated as that of a mathematician

As a mathematic teacher you should develop testing and observation techniques to assist you in recognizing students view points of the concepts and principle which you are teaching All of us have at time memorized the proofs of theorems, with no understanding of the concepts and principle involved in the proof in order to pass tests. While this subterfuge is a form of learning, it is not what teachers hope to have student learn by proving theorems. The point to recognize here is that many times when teachers are teaching what they view as mathematics principles, students are internalising as fact or skills the information which is being presented.

The Phases of a Learning Sequence

Gagne has identified eight sets of conditions that distinguish eight learning types, which he calls Signal learning. Stimulus response learning. Chaining verbal association. Discrimination learning, Concept learning, Rule learning and Problem solving Gagne believes that each one of these eight learning types occurs in the learning in four sequential phases. He calls these phases the apprehending phase, the acquisition phase, the storage phase and the retrieval phase.

The first phase of learning the apprehending phase is the learners awareness of a stimulus or a set of stimuli which are present in the learning situation Awareness or attention will lead the learner to perceive characteristics of the set of stimulus what the learner perceives will be uniquely coded by each individual and will be registered in his or her mind This idiosyncratic way in which each learner apprehending a given stimulus result in a common problem in teaching and learning. When a teacher presents a lesson (stimuli) he or she may perceive different characteristics of the content of the lesson than are perceived by students and each student may have a somewhat different perception than every other student. This is to say, that learning is a unique process within each student and as a consequence each student is responsible for his or her own learning situation. The uniqueness of individual perceptions explains why students will interprete facts, skills, concepts, and principles differently from the way a teaching and learning some what imprecise and unpredictable. It does have many advantages for society. Each person is able to apply his or her unique perceptions of a problem and its solution to a group discussion of the problem which results in more appropriate solutions of problem in our society.

The next phase in learning is the acquisition phase. Is attaining or possessing the fact skill, concept, or principle which is to be learned, acquisition of mathematical knowledge? Can it be determined by observing or measuring the fact that a person does not possess the required knowledge or behaviour before an appropriate stimulus is presented, and that, he or she attained the required knowledge or behaviour imEDUiately after presentation of the stimulus.

After a person has acquired a new capability, it must be retained or remembered This is the storage phase of learning. The human storage facility is the memory, and research indicates that there are two types of memory short-term memory has a limited capacity for information and lasts for a short period of time. Most people can retain seven or eight distinct pieces of information in their short-term memories for up to thirty seconds. An example of how short term memory operates is our ability to look up a seven digit telephone number, remember it for a few seconds while we are dialling, and forget the numbers as soon as someone answers our call. Long term memory is our ability to remember information for a longer period of time than thirty seconds and much of what we learn is stored permanently in our minds.

The fourth phase of learning, the retrieval phase is the ability to call out the information that has been acquired and stored in the memory. The process of information retrieval is very imprecise, disorganized, and even mystical At times desired information such as a name can not be retrieved from memory upon demand but will pop up later when one is thinking about something that appears to be completely unrelated to the moment when the name was wanted. Other information is stored so deeply in memory that special techniques such as electrical stimulation of the brain or hypnosis are required to initiate retrieval

These four phases of human learning apprehending acquisition storage retrieval have been incorporated into the design of computer systems though in a much less complex form than they appear in human beings. A computer apprehends electronic stimuli from the computer user. Acquires these stimuli in its control processing unit, and stores the information upon demand. The infinitely more complex learning process in people is illustrated every day in mathematics classrooms if students are to learn a procedure for finding an approximation to the square root of any number which is not a perfect square. They must apprehend the method acquire the method, store it the in memory and retrieve the square root algorithm when it is needed to aid student in progressing through these four stages in learning the square root algorithm. The teacher evokes apprehension by working through an example on the chalkboard facilitates acquisition by having each student work an example by following by step, evokes retrieval by giving a quiz the next day

Types of Learning

The eight types of learning which Gagne has identified and studied (signal learning, stimulus response, learning chaining, verbal association, discrimination learning, concept learning, rule learning and problem solving) will be presented and explained, below. Some of the conditions appropriate for facilitating each learning type will be found under the referenced materials.

3.2.6 J.P. Guilford Structure of Intellectual Model

While Jean and other have studied the stages of intellectual development, J.P Guilford and his colleagues have developed a three dimensional model containing 120 distinct types of intellectual factors. These 120 intellectual factors appear to encompass most of the human mental ability which can be specific and measured in formulating this model. Guilford and his associates have attempted to define and structure general intelligence into a variety of very specific mental aptitudes Their findings verify what many perceptive teachers have observed: even very intelligent students may have difficulty carrying out certain mental tasks: whereas other students who have attained low scores on general intelligence tests may do surprisingly well at some types of mental activities it is quite important for teachers to understand that individual student may possess a variety of specific mental strengths and weaknesses. Tests have been designed to measure many of these factors of intelligence and it is possible to select appropriate tasks to assist people in strengthening their specific cognitive inadequacies.

When a teacher finds that a student seems to be unable to attain even a minimal level of mastery of certain skills the school psychologist may be able to determine which intellectual ability are poorly developed in that student and may suggest activities to improve those abilities. Even a teacher who works in a school where the services of a psychologist are unavailable or are available only for students with several intellectual or emotional handicaps, can recognise certain inadequately developed mental skills in some students and can assist them in developing those skills Teachers can have a significant positive influence upon the formation of each student self image and every teacher should recognise and encourage those unique talents which each individual possesses Teachers can also negatively affect students some teachers indicate through covert and overt actions that students who are not particular proficient and interested in the teacher specialty have little prospect of leading a useful and happy life. Every mathematics teacher should appreciate the value of mathematics, however each teacher should be objective enough to understand that mathematic is only small, and in some cases unimportant, concern in the lives of many successful people

3.2.7 Guildford Intellectual Variables

Guildford model of intellectual aptitudes, which is called: *The Structure of Intellect Model*, was developed at the University of California using a statistical procedure called factor analysis to identify and classify various mental abilities. The model was substantiated by testing people varying in age from two years through adulthood. The Structure of Intellectual Model, which has been used as a tool by researchers studying the variables in intelligence, characterizes learning and intellectual development as being composed of three variables. The first of these variables, *operations,* is the set of mental processes used in learning. The second variable, *content,* categorises the nature of the material being learned. *Products,* the third variable in intelligence, refers to the manner in which information is organised in the mind.

Operations of the Mind

Guilford has identified five types of mental operations which he calls *memory, cognition, evaluation, convergent production,* and *divergent production. Memory* is the ability to store information in the mind and to call out stored information in response to certain stimuli. *Cognition* is the ability to recognise various forms of information and to understand information. *Evaluation* is the ability to process information in order to make judgments, draw conclusion, and arrive at decisions. *Convergent production* is the ability to view given information in a new way so that unique and unexpected conclusions are the consequence. A student who imEDUiately answers ½ when asked to give the sine of 30_0 is using his or her memory. A child who can separate a mixed pile of square and triangles into separate piles of squares and triangles is exercising a degree of cognition. When a member of jury sits through a trial, deliberates in a closed session with other jury members, and concludes that the defendant is guilty as charged, that person has used his or her mental ability of evaluation. An algebra student who finds the correct solution to a set of three linear equations in three unknowns has used his or her convergent production ability. A mathematician who discovers and proves a new and important mathematical theorem is exhibiting considerable ability in divergent production.

Contents of Learning

Guilford, in his Structure of Intellect Model, identifies four types of content involved in learning. He calls the things that are learned *figural, symbolic, semantic,* and *behavioral contents. Figural contents* are shapes and forms such as triangle, cubes, parabolas, etc. *Symbols contents* are symbols or codes representing concrete objects or abstract concepts. ♀ is a symbolic representation for a woman, and and + is the mathematical symbol for the operation of addition *Semantic contents* of learning are those words and ideas which evoke a mental image when they are presented as stimuli. Tree, dog, sun, war, fear, and red are words which evoke images in people's minds when they hear or read them. The *behavioral contents* of learning are the manifestations of stimuli and responses in people: that is, the way people behave as a consequence of their own desires and the actions of other people. The concrete shapes and forms (figures), the character representations (symbols), the spoken and written words (semantics). And the actions of people (behaviors) combine to make up the information that we discern in our environment.

Products of Learning

In Guilford's Model, the six products of learning (the information is identified and organized in the mind) are units, classes, relations, systems, transformations, and implications. A unit is a single symbol, figure, word , object, or idea sets of units are called classes, and one mental ability is that of classifying units Relations are connections among units and classes. In our minds we organize units and classes into interrelated structure so that we are aware of the relationship among these two product of learning. A system is a composition of units, classes, and relationship into large and more meaningful structure. Transformation is the process of modifying, reinterpreting, and restructuring existing information into new information. The transformation ability is usually thought to be a characteristic of creative people. An implication is a prediction or a conjecture about the consequences of interactions among units, classes, relations, systems, and transformation. The way in which the real number system is structured, illustrates how the mind organizes information into the six products of learning. Each real number can be considered as a *unit*, and the entire set of real numbers is a *class*. Equality and inequality are *relations* in the set of real numbers. The set of real numbers together with the operations of addition, subtraction, multiplication, and division and the algebraic properties of these operations is a mathematical *system*. Functions defined on the real number system are *transformations*, and each theorem about functions on the real numbers is an *implication.*

The 120 (5x4x6) distinct intellectual abilities defined in Guilford's Structure of Intellect Model result from taking all possible combinations of the five operations, four contents, and six products. For instance, the intellectual aptitude, memory for figural units, is the ability of a person to remember figural objects which he or she has seen. An example of this aptitude in mathematics is a student's ability to reproduce a geometric figure after he or she has been shown an example of that particular figure. The following list of operations, contents, and products indicates how the 120 intellectual aptitude can be forEDU by combining any operation, with any content, with any product, to form an ordered triple.

Guilford's Factors of Intellectual Ability

Operations		*Contents*		*Products*
1. memory	→	1. figural	→	1. units
2. cognition		2. symbolic		2. classes
3. evaluation		3 semantic		3. relations
4. convergent production		4. behavioral		4. systems
5. divergent production				5. transformations
				6. implications

Although this model of human intelligence is useful in identifying varables in learning and helps to explain various learning aptitudes and abilities, one limitation of the Structure of Intellect Model should be noted. Any attempt to structure and categorize complex human abilities into a model must result in an oversimplification of reality. Most of the facts, skill, principles, and concepts which teachers teach and students learn require complex combinations of intellectual abilities. When a student is unable to construct proofs in plane geometry, it may be quite difficult to determine which mental aptitude (or set of aptitudes) is causing this learning problem. Proving theorems in plane geometry may require a unique combination of a large subset of the 120 intellectual abilities, and most mathematics teachers have neither the skills nor resources to identify and measure these specific mental variable in each student. Even though the services of a trained psychologist may be required to determine precisely the intellectual deficiencies in a particular student and prescribe reEDUial activities, every teacher should learn to recognize certain general learning insufficiencies and assists students in overcoming some of their learning problems. The first step in dealing with these natural human intellectual variations is to recognize that every student's intellect is comprised of many different factors which may be present in varying degrees in each student. The next step is to observe each student's individual performance in specified areas of mathematics and attempt to identify his or her distinct strengths and weaknesses. The third step is to provide individualized work (as students' needs require and time permits) for students so that they can both apply their stronger intellectual abilities in learning mathematics and improve their weaker intellectual aptitudes. This step suggests that there are two approaches to overcoming learning handicaps. One approach is for the learner to bypass his or her weaknesses and apply his or her intellectual strengths to each task. Another approach is to attempt to strengthen intellectual deficiencies. Both methods of attacking intellectual shortcoming are useful and both can be employed simultaneously in the classroom. Finally, every teacher should strive to learn more about the nature of intelligence and learning by reading professional journals and participating in in-service workshop, college course, and post baccalaureate programs.

3.2.8 Dienes on Learning Theory for Mathematics

Zoltan. P. Dienes, who was educated in Hungary, France and England , has used his interest and experience in mathematics education and learning psychology to develop a system for teaching mathematics. His system, which is based in part upon the learning psychology of Jean Piaget, was developed in an attempt to make mathematics more interesting and easier to learn. In his book *Building* up *Mathematics*, Professor Dienes summarized his view of mathematics educations as follows:

At the present time there can hardly be a single member of the teaching profession concerned with the teaching of mathematics at any stage, from infants upwards, who can honestly say to himself that all is well with the teaching of mathematics. There are far too many children who dislike mathematics, more so as they get older, and many who find great difficulty with what is very simple, let us face it: the majority of children never succeed in understanding the real meanings of mathematical concepts. At best they become deft technicians in the art of manipulating complicated sets of symbols, at worst they are baffled by the impossible situations into which the present mathematical requirements in schools tend to place them. Not all too common attitude is 'get the examination over', after which no further thought is given to mathematics. With relatively few exceptions, this situation is quite general and has come to be taken for granted. Mathematics is generally regarded as difficult and tricky, except in a few isolated cases where enthusiastic teachers have infused life into the subject, making it exciting and so less difficult.(p.1).

Mathematical Concepts

Dienes regards mathematics as the study of structures, the classification of structures, sorting out relationships within structures, and categorizing relationships among structures. He believes that each mathematical concept (or principle) can be properly understood only if it is first presented to students through a variety of concrete, physical representations. Dienes uses the term concept to mean mathematical structure, which is a much broader definition of concept than Gagne's definition. According to Dienes there're two types of mathematics concepts-pure mathematical concepts, and applied concepts.

Pure mathematical concepts deal with classifications of numbers and relationships among numbers, and are completely independent of the way in which the numbers are represented. For instance, six, 8, xii, 1110 (base two), and ∆∆ ∆∆ are all examples of the concept of even number; however each is a different way of representation of a particular even number.

Notational concepts are those properties of numbers which are a direct consequence of the manner in which numbers are represented. The fact that in base ten, 275 means 2 hundreds, plus 7 tens, plus 5 units is a consequence of our positional notation for representing numbers based upon a powers-of –ten system. The selection of an appropriate notational system for various branches of mathematics is an important factor in the subsequent development and extension of mathematics. The fact that arithmetic developed so slowly is due in large part to the cumbersome way in which the ancients represented numbers. We have already mentioned the problems which occurred in the development of mathematical analysis in England as a consequence of the English mathematicians' insistence upon using Newton's cumbersome notational system for calculus, rather than the more efficient system of Leibniz.

Applied concepts are the applications of pure and notational mathematical concepts to problem solving in mathematics and related fields. Length, area and volume are applied mathematical concepts. Applied concepts should be taught to students after they have learned the prerequisite pure and notional mathematical concepts. Pure concepts should be learned by students before notional concepts re presented, otherwise students will merely memorize patterns for manipulating symbols without understanding the underlying pure mathematical concept. Students who make symbol manipulation errors such as $3x+2=4$ implies $x+2=4-3$, $\underline{x+2}=x$, $a^2 \cdot a^3 = a^6$, and x^2+5 2 are attempting to apply pure and notational concepts which they have not adequately learned.

Dienes regards concept learning as a creative art which can not be explained by any stimulus-response theory such as Gagne's stages of learning. Dienes believes that all abstractions are based upon intuition and concrete experiences; consequently his system for teaching mathematics emphasizes mathematics laboratories., manipulative objects, and mathematical games. He thinks that in order to learn mathematics (that is, to be able to classify structure and identify relationships) students must learn to

(1) analyze mathematical structures and their logical relationships

(2) abstract a common property from a number of different structures or events classify the structures or events as belonging together,

(3) generalize previously learned classes of mathematical structures by enlarging them to broader classes which have properties similar to those found in the more narrowly defined classes, and

(4) use previously learned abstractions to construct more complex. higher order abstractions.

3.3 DIENE'S STAGES OF LEARNING MATHEMATICAL CONCEPTS

Dienes believes that mathematical concepts are learned in progressive stages which are, somewhat analogous to Piaget's stages of intellectual development He postulates six stages in teaching and learning mathematical concepts(1) free play, (2) games,(3) searching for communalities, (4) representation,(5) symbolization, and (6) formulization.

Stage 1: Free Play

The free play stage of concepts learning consists of unstructured and undirected activities which permit students to experiment with and manipulate physical and abstract representations of some of the elements of the concept to be learned. This stage of concept learning should be made as free and unstructured as possible: however, the teacher should provide a rich variety of materials for students to manipulate. Even though this unregulated period of free play may appear to be of little value from the point of view of a teacher, it is an important stage in concept learning. Here students first experience many of the components of a new concept through interracting with a learning environment, which contains concrete representations of the concept. In this stage students form mental structures and attitudes which prepare them to understand the mathematical concept.

Stage 2: Games

After a period of free play with representations of a concept, students will begin to observe patterns and regularities which are embodied in the concept. They will notice that certain rules govern events. That some things are possible and that other things are impossible. Once students have found the rules and properties which determine events, they are ready to play games, experiment with altering the rules of teacher made games and make up their own games. Games permit students to experiment with the parameters and variables within the concept and to begin analyzing the mathematical structure of the concept. Various games with different representations of the concept will help students discover the logical and mathematical elements of the concept.

Stage 3: Searching for Communalities

Even after playing several games using different physical representations of a concept, students may not discover the mathematical structure which is common in all representations of that concept. Until students become aware of the common properties in the representations, they will not be able to classify examples and non-examples of the concept. Dienes suggests that teachers can help students see the communality of structure in the examples of the concept by showing them how each example can be translated into every other example without altering the abstract properties which are common to all the examples. This amounts to pointing out the common properties found in each example by considering several examples at the same time.

Stage 4: Representation

After students have observed the common elements in each example of the concept, they need to develop, or receive from the teacher, a single representation of the concept, which embodies all the common elements found in each of the examples. This representation could be either diagrammatic representation of the concept, a verbal representation or an inclusive example. Students need a representation in order to sort out the common elements which are present in all examples of the concept. A representation of the concept will usually be more abstract than the examples and will bring students closer to understanding the abstract mathematical structure underlying the concept.

Stage 5: Symbolisation

In this stage the student needs to formulate appropriate verbal and mathematical symbols to describe his or her representation of each concept, however, for the sake of consistency with the textbook, teachers probably should intervene in students' selections of symbol system. It may be well to permit students to first make up their own symbolic representations, and then have them compare their symbolizations with those in the textbook. Students should be shown the value of good symbol systems in solving problems, proving theorems, and explaining concepts. For example, the Pythagorean theorem may be easier to remember and use when it is represented symbolically as a $a^2 + b_2 = c$, rather than verbally as "for a right triangle, the square of the hypotenuse is equal to the sum of the square of the other two sides".
One difficulty caused by some symbolic representations of rules,

formulas, and theorem is that the conditions under which each rule, formula, or theorem can be used are not always apparent from the symbolism. Our symbolic statement of the Pythagorean theorem does not state the conditions under which the theorem can be used; however, the verbal statement does specify that the theorem applies to right triangles. Many students who are quite good at remembering rules have no trouble matching the appropriate rule to each specific problem-solving situation.

Stage 6: Formulization

After students have learned a concept and the related mathematical structure, they must order the properties of the concept and consider the consequences. The fundamental properties in a mathematical structure are the axioms of the system. Derived properties are the theorems, and the procedures for going from axioms to theorems are the mathematical proofs. In this stage, students examine the consequences of the concept and use the concept to solve pure and applied mathematics problems.

Games

Dienes believes that games are useful vehicles for learning mathematical concepts throughout the six stages of concept development. He calls the games played in the undirected play stage, where students are doing things for their own enjoyment, *preliminary games*. Preliminary games are usually informal and unstructured and may be made up by students and played individually or in groups. In the middle stages of concept learning, where students are sorting out the elements of the concept, *structured games* are useful. Structured games are designed for specific learning objectives and may be developed by the teacher or purchased from companies which produce mathematics curriculum materials. In the final stages of concept development, when students are solidifying and applying the concept, *practice games* are useful. Practice games can be used as drill and practice exercises, for reviewing concepts, or as ways to develop applications of concepts.

Principles of Concept Learning

Dienes (1971), in his book *Building up Mathematics,* summarizes his system of teaching mathematics in four general principles for teaching concepts. His six stages in concept learning are refinements of these four principles:

1. Dynamic Principle. Preliminary, structured and practice and/or reflective type of games must be provided as necessary experiences from which mathematical concepts can eventually be

built, so long as each type of game is introduced at the appropriate time. We shall see that this break-up can be further refined.

Although, while children are young, these games must be played with concrete material, mental games can gradually be introduced to give a taste of that most fascinating of all games, mathematical research.

2. Constructive Principle. In the structuring of the games, construction should always precede analysis, which is almost altogether absent from children's learning until the age of 12.

3. Mathematical Variability Principle. Concepts involving variables should be learned by experiences involving the largest possible number of variables.

4. Perceptual Variability Principle or Multiple Embodiment Principle. To allow as much scope as possible for individual variations in concept-formation, as well as to induce children to gather the mathematical essence of an abstraction, the same conceptual structure should be presented in the form of as many perceptual equivalents as possible. (pp. 30-31)

3.3.1 Applying Dienes' Theory in a Mathematics Lesson

In applying Dienes' six stages for concept learning to planning a mathematics lesson, you may find that one stage (possibly the free play stage) is not appropriate for your student or that activities for two or three stages could be combined into single activity. It may be necessary to plan unique learning activities for each stage when teaching younger elementary school students; however older secondary school students may be able to omit certain stages in learning some concepts. Dienes' model for teaching mathematics should serve as a guide, and not a set of regulations to be followed slavishly.

The concept of multiplying negative will be discussed here as an example of how Dienes' stages can be used as a guide in planning teaching/learning activities. Since nearly all students learn to add, subtract, multiply and divide natural numbers, and to add and subtract integers before learning to multiply integers, we will assume that these concepts and skills have been mastered by our hypothetical students.

For students who are in sixth or seventh grade, one could begin the free play session by informally discussing the arithmetic operations on the natural numbers and the algebraic properties of natural numbers. The teacher might also discuss adding and subtracting integers and the commutative and associative properties for addition. He or she may even choose to substitute an informal review for free play. Or the free play and game stages could be combined into several games such as the following simple play and game stages. The teacher should prepare enough decks of standard playing cards with the face cards removed so that there is one deck for every five students in the class. Students playing in groups of five would each be dealt four cards. Each student would group his or her cards into pairs, then take the product of the numbers showing on the cards in each pair of his or her cards to obtain add the two product-sum is the winner of that hand in his or her group. The numbers on black cards (clubs and spades) are considered to be positive numbers, and numbers on red cards (hearts and diamonds) are negative numbers. Consequently students would imEDUiately be confronted with the problem of how to group negative cards to get large positive products and sums. Various groups may agree upon different rules for handing the product of two negative numbers. For instance, a black 2 and 4 and a red 7 and 5 could be used to make (2x4) + (-7x-5) = 43, if the correct rule that the product of two negative integers is a positive integer is formulated. If not, then negative numbers would be of no help in organizing a winning hand. Some students will certainly ask each other or the teacher about how to score negative integers.

To decide how to handle the product of two negative numbers, the teacher could present a series of problems involving a search for communities. For instance, these problems could be discussed in class:

1. Assume that bad people are negative and good people are positive. Also assume that moving into community is a positive act and leaving community is a negative act. What is the net effect of five bad people leaving two different communities? The class should decide that these events constitute ten positive happenings.
2. What is the effect on your cash balance of subtracting four, three dollar debts from your newspaper route account book? Several students should imEDUiately observe that the effect is positive twelve dollars.
3. Finish this table:

   - 3 X 3 = - 9
   - 3 X 2 = -6
   -3 X 1 = -3
   - 3 X 0 = 0
   -3 X –1 = ?
   -3 X –2 =- ?
   -3 X –3 =?
4. -3 X (7 + -2) = (-3 X 7) + (-3 X –2) = -21 + ?

but
-3 X (7+-2) = -3 X 5 = -15
so
what number is ? ?

As a mathematics teacher, you may be able to construct other examples showing that the product of two negative integers is a positive integer.

In the representation stage of forming the concept of multiplying negative integers, students should be able to observe a diagram representing the concept and describe the general property of multiplication of two negative integers. The following diagram, shown in figure 3.3, is one way to represent that the product of two negative integers is a positive integer.

In the symbolization stage, each student should be able to explain the diagram in figure 3.3 and use it to show examples of the concept. Each student should also explain that the diagram shows that the product of two negative integers must be a positive integer in order for the distributive property to be true for multiplication and addition of integers. Finally, the class should adopt the symbol system that for any natural numbers a and b, (-a) (-b) = +ab; and for any integers x, y, z, x(y +z) = xy +xz.

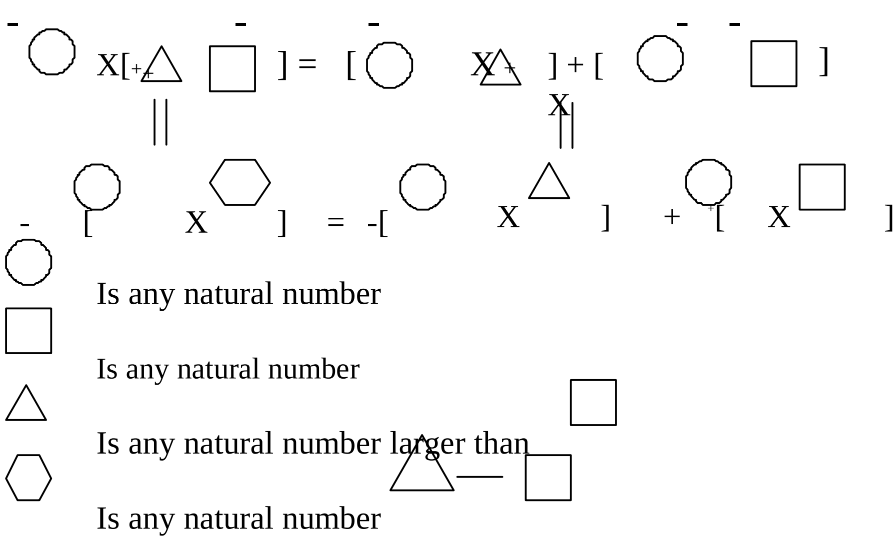


Figure 3.3 A representation of the concept that the product of two negative integers is a positive integer.

This concept can be formalized by recognizing that the statement, “the product of two negative integers is a positive integer,” is an axiom. Theorems such as y X z = z X y and x(y+z) = xy +xz can also be stated and proved.

Diene's approach to teaching and learning mathematics can be summarized in the following list of subprinciples, which are inherent in his four principles for concept learning.

1. All of mathematics is based upon experience and students learn mathematics by abstracting mathematical concepts and structures out of real experiences.
2. There is a fixed natural process that students must carry out in order to learn mathematical concepts. The process must include:
   (a) A play and experimental period involving concrete materials and abstract ideas.
   (b) An ordering of experiences into a meaningful whole.
   (c) A flash of insight and understanding when the student suddenly comprehends the concept.
   (d) A practice stage to anchor the new concept so that the student can apply it and use it in new mathematical learning experiences.
3. Mathematics is a creative art and it must be taught and learned as an art.
4. New mathematics concepts must be related to previously learned concepts and structures so that there is transfer of old learning to new learning.
5. In order to learn mathematics, students must be able to translate a concrete situation or event into an abstract symbolic formulation.

3.3.2 Ausubel's Theory of Meaningful Verbal Learning

During the nineteen fifties many mathematics educators came to believe that the prevailing lecture method for teaching mathematics was resulting in rote learning which was not meaningful to students. As new mathematics programs with an emphasis upon understanding of concept were developed and implemented in schools during the nineteen sixties, verbal expository teaching began to fall into disrepute. Many people felt that expository teaching resulted in role learning. and models such as discovery learning, inquiry, and mathematics laboratories were thought to be more appropriate method for fostering meaningful learning. However there were people who still believed that since the lecture method of teaching had worked reasonable well in the past, it should not be discarded as a bad teaching strategy. Throughout this period, the learning theorist, David P. Ausubel argued that expository teaching was the only efficient way to transmit the accumulated discoveries of countless generations to each succeeding generation. And that many of the recently popular methods were not only inefficient. But were also ineffective in promoting meaningful learning Ausubel's theory of meaningful verbal learning contains a procedure for effective expository teaching resulting in meaningful learning. To Ausubal, the lecture or expository method is a very effective teaching strategy, and he believes that educators should devote more effort toward developing effective expository teaching techniques.

Now that studies of mathematics skills in children and young adults (for example, studies conducted by the National Assessment of Educational Progress, NAEP) indicate that all is not well in applying arithmetic skills, many people are beginning to question the new mathematics programs and the new teaching methods. A study completed in 1975 by the NAEP and reported indicate that young adults between the ages of 26-35 could not solve simple consumer arithmetic problems. An earlier NAEP study found that people in these same age groups were reasonably proficient in solving textbook-type arithmetic problems, so there appears to be a problem in teaching meaningful, real-world applications of arithmetic. This unfortunate dilemma for mathematics education may lend some support to Ausubel's contention that the popular non-expository teaching methods do not necessarily result in the learning of meaningful problem-solving procedures.

4.0 CONCLUSION

Gagne's division of learning into eight types from the simplest(signal learning) through the progressively more complex types (stimulus response learning chaining vernal association, discrimination learning and concept learning) to the higher order types (rule learning and problem- solving)is a useful and valid way to view learning However , learning does not usually progress I a sequence of easily definable and identifiable steps, and the various learning types do not occur in chronological sequence as do Piaget of intellectual development All of these eight learning types can, and so occur nearly simultaneously in all but a few people through most of their lives As a teacher you should understand Gagne's different types of learning and select teaching strategies and classroom activities which promote each learning type when that particular type seems to be appropriate for learning the mathematics topic that you are teaching most teaching /learning sequences will require several of these eight types of learning which may interact in very complex ways

5.0 SUMMARY

The seven theories which are presented and discussed in this chapter are attempt by their developer to structure and explain the very complex processes of in structure and learning. No single theory provide a complete model of either teaching, in spite of the limitation of these theories, each has applications for teaching ad learning secondary school mathematics.

Piaget and Skinner have formulated two very different models of human learning. Piaget has developed a theory of intellectual maturation and development, whereas Skinner has studied the conditions under which human behaviors take place. Although they are different approaches to the study of learning and behavior, these two theories do complement each other; each has many applications in teaching mathematics.

Guilford has determined what he believes to be the 120 mental abilities which comprise general intelligence, and his findings can be of considerable use to teachers in identifying and dealing with specific learning problems in individual students.
Bruner's theory of instruction is useful to teachers in helping them formulate general approaches to teaching, and much of his work has been shown to be directly applicable to teaching mathematics.
Based, in part, upon Piaget's theory of intellectual development, Denies has developed a theory of teaching mathematics which contains a sequence of strategies for teaching mathematics concepts. He has also described how specific topics from secondary school mathematics can be approached by using his six stages in concept development as a general model for teaching and learning mathematics.

Gagne and Ausubel, while concerned with refining theories of learning and instruction, have developed techniques and strategies for classroom teaching. Both of the men have formulated models for structuring the content of a discipline such as mathematics. Gagne has taken a bottom to top approach to structuring content into learning hierarchies which build upon simpler, prerequisite facts skills, and concepts to learn more complex skills, concepts and principles. Ausubel has developed a theory of meaningful verbal learning which can be used by teachers when presenting material in a lecture or expository mode to students. Since a large proportion of mathematics teaching is carried out in a lecture mode, Ausubel's procedures for structuring information so that it can be learned in an efficient and meaningful way can be very useful to secondary school mathematics teachers.

The various theories of teaching and learning can be used as a basis for designing and presenting mathematics lessons and also provide a rich background of information which teachers can use in developing and improving the effectiveness of their classroom strategies for teaching mathematics to students in secondary schools.

6.0 TUTOR MARKED ASSIGNMENT

Who holds what view? Write B for Burner and A for Gagne against the following statements.

1. Learning depends on the structure of the concept. Nature of intuition aid desire of learner of learner and readiness for learning.
2. The three stages Enactive, Iconic and Symbolic consists of the sequence action, image and words respectively.
3. How far a child learns depends on the set of progressively complex rules he has learnt.
4. All mathematics could be learnt through discovery.
5. Children learn complex sets of rules by interaction with their environment.
6. At the enactive stage learner represents events through motor response only.
7. Specifically no learning occurs in the first stage of development.
8. Learning lower order sets is a pre-requisite for improved learning of a higher order one.

SELF ASSESSMENT EXERCISES

Self Assessment Exercise 1

According to Bruner, a theory of instruction should be both prescriptive and normative. Explain what he means by these two terms and why it is necessary for instructional theories to be both prescriptive and normative. Discuss the four major features which prescribe the nature of the instructional process, and which Bruner believes should be contained in any theory of instruction.

Self Assessment Exercise 2

Define each of the eight types of learning which Robert Gagne has identified, and give an example from mathematics education of each learning type. Suggest some teaching strategies which would be appropriate for promoting each one of eight learning types.

Self Assessment Exercise 3

J.P. Guilford has identified five operations of learning, four contents of learning, and six products of learning. Define and give an example for secondary school mathematics of each of these fifteen characteristics of intelligence.

Self Assessment Exercise 4

Zoltan Dienes has categorized three types of mathematics concept pure concepts, notational concepts, and applied concepts. Define and give several examples of each type of concept, and suggest teaching/ learning activities which would be appropriate for each type.

Self Assessment Exercise 5

Discuss Ausubel's two preconditions for meaningful reception learning and explain how they can be applied to meaningful teaching and learning.

7.0 REFERENCES /FURTHER READING

Dienes Z.P (1971) Building up mathematics 4th Ed. Hutchinson Ed. Ltd London.

Lindgren H.C (1980) Educational Psychology in the Classroom 6th Ed. O.U.P. Oxford.

Gagne, R.N (1925) Essentials of Leaming for Instruction (Expanded Ed). Dryden Press USA

Servais W. & Varga T. (1971) Teaching School Mathematics. A Unesco Source book UNESCO.

UNIT 4 PIAGET'S WORKS AND THEIR IMPLICATIONS FOR MATHEMATICS INSTRUCTION

CONTENTS

1.0 INTRODUCTION

Jean Piaget (born in 1896 and died in 1980) was a precocious adolescent who, at 15, published a research article on mollusks. He later changed his interest from shell creatures to interest in the development of knowledge in humans. After his doctorate degree from the University of Lausanne, Piaget worked in Alfred Binet's laboratory School in Paris. There, Piaget became interested in what lies behind children's answers, particularly their incorrect answer. He began to use a semi clinical interview to uncover the basis for children's behaviour.

To appreciate Piaget's works, it is necessary to understand his position about knowledge. To Piaget, knowledge is the transformation of experience by the individual, not just the accumulation of pieces of information. With that view about knowledge, Piaget sought evidence of the nature of that transformation in the behaviour of infants when an object is hidden from them and in the behaviour of adolescents faced with a complex solutions task.

One aspect of Piaget's theory designated as the stage-dependent theory resulted from the appearance of qualitatively different intellectual abilities in sequence of stages related to age characteristically. For example, the ability to engage in logical classification was observed in the performance of children beginning at age seven or older. Piaget's analysis of a host of responses to tasks resulted in the identification of four major stages which form an invariant sequence. These four stages in order of appearance are: the sensory motor, the preoperational, the concrete operational and the formal operational stages. The second aspect of Piaget's theory designated the stage-independent theory, includes Piaget's explanation of the development of intellectual structures and his views on the nature of knowledge and the nature of knowing.

Educational psychology is generally concerned with the study of human behaviour. For class teachers, educational psychology will enable them cope with problem of how children learn and under what conditions maximum learning can take place. In this unit you will see how educational psychology can be applied to gainful teaching and learning of mathematics.

In this unit and also in Unit 2 of this module you will study the cognitive aspect of learning mathematics and the contributions of three psychologists-Piaget, Bruner and Gagne to the learning of mathematics. They are not the only contributors but they represent a selection whose reason for further choice will become apparent as you read the unit.

2.0 OBJECTIVES

By the end of this unit, you should be able to:

1. name at least five areas of psychological studies on mathematics education.
2. watch questions on problems on mathematics learning and teaching with the appropriate field of educational psychology research.
3. recount at least one study by Jean Piaget to test either the concept of conservation of number or the concept of invariance.
4. watch various terms used in the unit with their correct meaning as used by Piaget. And
5. select by the aid of a checklist. Mathematics topics for a suitable stage in Piagets stages of development.

3.0 MAIN BODY

3.1 STUDY APPROACH

1. Read each of this section slowly to understand it.
2. Do an activity as you come across it. Do not refer back to the text while doing the activity.

3. Check the answers as soon as you finish the activity. If you do not get them right.
4. You will probably need a good dictionary to help in understanding new words.

3.2 PSYCHOLOGICAL BASIS FOR MATHEMATICS INSTRUCTION

You can consider the problems of mathematics education through such questions as:

i. why is pupil A learning mathematics faster and better than pupil B even though they are of the same age and in the same class?
ii. Why is a pupil not producing comparatively good result as he did when he is younger even though he worked as hard?
iii. Why is a child not learning mathematics while, his/her brother is doing very well? And so on.

In other words we can analyse intellectual and cognitive activities in a systematic way be considering differences as follows: difference between individuals at a given age or time; and differences in the same individual between different times based on learning, developing and thinking.

Differences between individuals at a given time lead us to study mathematics ability by use of such tools as mental and scholastic tests (e.g. aptitude test, prognostic tests, intelligence test) referred to generally as the tests of special ability. Although these tests and their results are important: studies on the differences in the same individual between different times are considered very significant for the teacher of mathematics.

It was rationale of two kinds of difference that led psychologists to direct their research in mathematics education to the following five major fields:

(a) Intellectual ability and individual differences in mathematics aptitude and attainments.
(b) Experimental study on the growth of thinking, particularly with reference to number, spatial relations, geometry, classification, the development of mathematical concepts and operations and the nature of mathematical experience.
(c) The nature of concept, productive thinking and problem solving
(d) Mathematics learning with reference to methods, programEDU learning, learning sets (learning to learn) and the spread of learning.

(e) Curriculum studies aiEDU at revising and improving the mathematics curriculum.

Piaget, Burner and others who were concerned with studying the development of thinking of the individual along a span of time are referred to as developmental psychologists. Gagne represents the group which drew conclusions about how children think by observing their behaviour (responses) to simple tasks (stimulus) and are referred to as the behaviour (S-R) psychologists. S-R stands for **stimulus response**.

Piaget, Burner and Gagne are representatives of their schools of thought who have contributed in one way or the other to mathematics education. You can now see why we have chosen to discuss them in this module on psychological basis of mathematics education.

3.2.1 Contributions of Piaget to the Learning of Mathematics

Jean Piaget was a French-Swiss psychologist who was originally trained as a biologist. For more than fifty years he studied and analysed the growth and development of children's thinking. His school in Geneva is noted for the study of psychological problems underlying the learning of mathematics. His work has the greatest significance for teachers of mathematics especially at the primary level.

This applies to his study of children's general intellectual development and specifically to the development of mathematical concepts. You will now study this in more detail.

Piaget view cognitive development in terms of well-defined sequential stages in which a child's ability to succeed is determined partly by his biological readiness for the stage and partly by his experiences with activity and problems in earlier stages. The age 0-2 years is knows as the **Sensorimotor** stage where the child relates to its environmental through its senses only.

Towards the end of the second year of life, children 'have rudimentary understanding of space and are aware that objects have an existence apart from their imEDUiate experience of them.

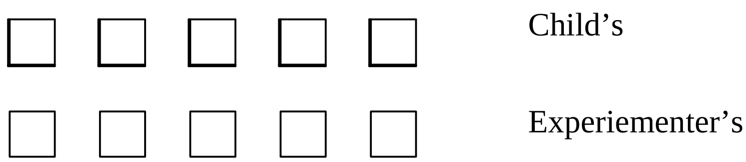
The pre-operational stage, (2-7 years) which generally cover the cognitive development of children during the pre-school (KINDERGARTEN) years, is marked by the ability to deal with reality in symbolic ways.

The thought processes of children in this stage are, however, limited by **Centering** (inability to consider more than one characteristics of an

object at a time). Children at this stage also have difficulty with reversibility (The ability to think back to the causes of events).

Because of these deficiencies, they cannot **conserve** (retain) important characteristics of objects and event, and cannot engage in logical thinking in any concrete sense. The child is said not to posses the concept of conservation of number, volume, quality or space.

Piaget demonstrated the lack of conservation in two experiments
The following is an account of the experimental procedure:
The child was presented with two rows of five plastic squares each row arranged (as show below) in one-one correspondence. All the squares are equal in size.

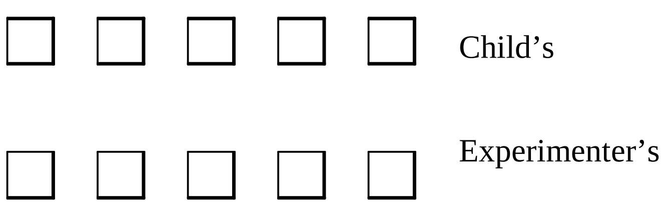


The experimenter then says to the child. The first row is yours and the second row in mine.

Question: Have we both the same number of squares

Child's response: Yes (Both the same)

The experimenter later arranges the squares as shown below by spreading out one row (the child's).



He then repeats the statement and question to the child. The child responded by saying he (the child) has more-that is they are no longer both the same.

The child at the pre-operational stage is influenced by the perceptual features of the stimulus (i.e plastic squares). He lacks the ability to see that movement has not altered the plastic squares. Conservation of numbers is a fundamental requirement in the understanding of numbers, for without it a child cannot match sets by pairing to establish equivalent sets. For instance, for the child who is learning a number sentence such as:

3 + 2 = 5
000 + 00 = 00000

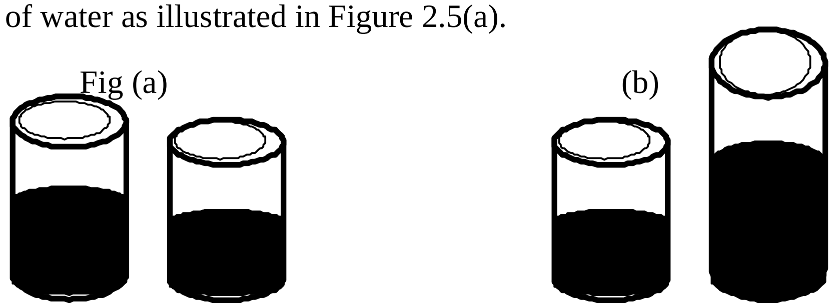
and forms a union of disjoint set of three counters and another set two counters, the physical movement of the counters into a new pattern may mean: they change in their number characteristics; and though they may recite 3 plus 2 equals 5, he lacks understanding.

The implication for the mathematics teacher is that it is a waste of time (and probably harmful to children) to try to tell children things that cannot be experienced through their senses, that is, through seeing, feelings as well as hearing. Abstract mathematical ideas should therefore not be introduced at this stage. Children at this stage must be permitted to manipulate objects and symbols, so as to be able to appreciate reality. Mathematically oriented recreational facilities (e.g games, play blocks, counters, marbles etc) are important tools for learning mathematics at this stage.

The period of concrete-operational stage (7-12 yrs) is particularly important to the primary school teacher because most primary school children are in this stage of development. This stage marks the beginning of what is known as **logico-mathematical** aspect of experience. It is illustrated by the case of the child who learn that counting a set of objects leads to the same result whether he counts from front to back, back to front, or whatever configuration in which the objects are arranged. Also logico-mathematical experience underlies the physical act of grouping and classifying in what is known as the algebra of sets.

Piaget studies the concrete operational stage using the concept of conservation of invariance, which is a basic characteristics in this stage. For example a child is shown two identical glasses containing the same amount of water as illustrated in Figure 2.5(a).

Fig (a) (b)



The water in one glass is then poured into a taller glass with smaller diameter as shown in Figure 2.5(b).

If the child understands the amount of water is still the same no matter the nature of the glass container and rejects what perception tells him (one look like it contains more water), he is using logic and has arrived at the concrete operational thought level for his concept. The amount of water is conserved and remains invariant (unchanged) after the transfer into another container. This is referred to as the **concept of conservation of invariance**. The child realizes that the process can be reversed- that if the liquid is poured back into the initial container, the amount should remain the same.

Another example is the process of adding one number to another number. In thought, this would be: if I want to add 4 to 3, I would take a collection of three marbles, physically and four more and then determine the result. But in practice I would represent the process internally. Reversibility would imply that if I recognize that 3+4 = 7 then I would recognize that 7-4 = 3.

The psychological condition for a reversible operation is that of conservation, i.e + A is reversed by –A.

The concrete-operational stage is therefore important for mathematics learning because many of the operations a child is able to carry out at this stage are mathematical in nature. For example the operation of classification, ordering, construction of the idea of nature, spatial and temporary operation. This includes all the fundamental operations of elementary logic of classes and relations of elementary mathematics, geometry and even physics.

There is however one limitation at this stage. Children have difficulty from hypothetical assumptions. Care must be taken in the type of materials that is included in their mathematical curriculum.

Children can reason abstractly if they are not affected by the limitations of the concrete-operational stage. They can proceed to the fourth stage of Piaget's process of cognitive developments. This is known as the formal-operational stage, from about 12 years. Only one-fourth of adolescents and one-third of adults, however, ever fully functions at the formal operational level as shown by results of research.

At this level, the child now reasons or hypothesizes with symbols or ideas rather than needing objects in the physical world as a basis for his thinking. He can use the procedures of the logician or scientist, a hypothetic-deduction procedure that no longer ties his thoughts to existing reality. He has attained new mental structures and constructed new operations.

3.2.2 The Stage-Independent Theory of Piaget On Interaction

Piaget holds that individual intelligence develops through the person's interaction with his or her environment. Piaget insists that knowledge is active, for her/him to know an idea or an object requires that the student manipulates physically or mentally and thereby transforms it. At different developmental stages the activities of transforming knowledge takes on different forms.

The construct of interaction may be the most important of Piaget's contributions. According to this concept, when you want to solve a problem, you will spontaneously and actively interact with those characteristics of the real situation that you perceives as relevant to your problem.

3.2.3 The Stage Independent Theory of Adoptation

Piaget provides us with other construct that explains the functioning of interaction and suggests the parameters of teaching behaviours most likely to result in interaction. Just consider your own mental activity as you read these paragraphs. Each of you comes to the reading with different experiences, differing degrees of understanding of Piaget, and different retention of past school learning. You read the same words. Some of the words are familiar, but may be used in a slightly novel way; the same ideas might present a totally new notion. In essence, you are trying to get a cognitive understanding of the ideas in the paragraph. In so doing, you are carrying on a matching patching task. You may try to match the inferred ideas (new ideas) to the ones you possess that seem to be similar or you may patch previous cognitive knowledge on the basis of new ones.

Piaget has specific labels for the match patch function described above. He labels the process where an individual interacts with an experience (a real object, a situation, inferred ideas through reading listening or seeing) adaptation. Piaget describes adaptation in terms of two concurrent functions assimilation and accommodation. In the illustration above, the reader assimilates (matches) the ideas inferred from the paragraphs as he or she simultaneously accommodates (patches) prior cognitive structures to the new input.

Notice that when adaptation (matching- patching) occurs, the individual is changed (in a cognitive sense) while he or she changes the experience.

3.3 The Stage-Dependent Theory of Cognitive Development

The last two of the Piagetian Stage are designated as Concrete Operational and Formal Operational. These are the stages that span the age range from 8 to 15 (16) an age range that includes your mathematics students.

3.3.1 Characteristics of the Concrete Operational Stage

Operations are mental actions that are reversible. The first indicator of the stage `is the ability of the child to conserve number i.e. to realize that the number of objects is unchanged regardless of their arrangement. However, the same child may assume that the length of a pipe cleaner is changed when it bent or that the amount of clay in a round ball is altered if the clay is made into a long thin pipe. In other words, ability to conserve numbers, length, mass, volume etc. are not all achieved at the same time even though, all the schemes require the ability to consider several perceptions simultaneous and the ability to reverse a mental action.

By the end of the concrete operational stage, the preadolescent is able conserve all of the preceeding relationships. If you are a physics minor, you all realize that volume can be a fairly complex concept, because it includes the ideas of interior volume, liquid volume, and displacement volume. Underlying all the above major advances of the stage of concrete operations are the ability of the child to classify and the ability of the child to work with relationships which order makes a difference.

The ability to classify is perhaps the most powerful of our thinking tools. In and out of school, we are asked to learn hierarchies of classification systems addition, much learning depends upon the ability to perform multiple classification i.e. the task we give a child when we ask him to select an isosceles right triangle from a collection of shapes.

Piaget’s data show that these abilities are developed gradually throughout other concrete operation stage as a result of interaction with sufficient experiences that require multiple classification. Ordering relationships are inseparable parts of learning to classify. For example, the full meaning of statements such as “6 is greater than 4 which is greater than 3” is not grasped by the pre-conceptual operation youngster. But the concrete operational child correctly conclude that “6 is therefore, greater than 3”and later, is able to reverse these relationships and handies “less than” statements.

However, all of the above abilities are developed during the stage of concrete operation by interacting with “Concrete” content. What is essential about the "concrete content" is that the experiences be real to the child and that such experiences reflect in as tangible a way as possible the concept or rule being developed. An enormous concrete base of activities, verbal responses, visual responses, and visual experiences is required for initial learning of an idea which are the instructional modes suitable for teaching concrete operational students?

3.3.2 Characteristics of Formal Operational Stage

The mental structures that develop by the end of the concrete operational stage are still available for use by the adolescent or preadolescent. They must be used to solve many real-life problems. In some cultures the concrete operational structures are the most complex that can be identified.

At the stage of formal operations, the adolescent can deal with the "form" of the situation and need not resort entirely to the concrete aspects of the problem. Piaget has identified four characteristics of the former operational stage, all of which depend upon one/another. They are(1) the treatment of the real as a subset of the possible, (2) hypothetic deductive reasoning, (3) Combinatorial analysis and (4) prepositional thinking.

At the formal level, reality is considered as a subset of the possible, with the result that hypotheses may proceed from non observed and non-experienced phenomena. This characteristics of the formal stage, i.e. the ability to imagine the possible as containing the real, the formal thinker from the restrictions of his or her sense. Further formal operations are characterized by prepositional thinking. The elements manipulated by the formal thinker are logical propositions, statements containing raw data rather than just the raw data itself.

This is where you use conjunction, disjunction, implication, negation and equivalence. This type of thinking is what Piaget called second degree thinking operation, that result in statements.

The four characteristics of formal operations, outline the manner in which the intellectually mature adolescent thinks. Presented with a new situation, the adolescent begins by classifying and ordering the concrete elements of the situation. The results of these concrete operations are divested of their intimate ties with reality and become simply propositions that the adolescence may combine in various ways. Using combinatorial analysis, the student regards the totality of combinations as hypotheses that need to be verified and rejected or accepted.

3.3.3 Implications of Piaget's Theories for Mathematics Instructions

Stage Independent Theory

There are some clear signals to the teacher in the functioning of adaptation
First, if assimilation- accommodation is to occur the gap between the new experience and past knowledge cannot be too large. If you analyse the nature of the content and search for prerequisites, some prior needs will be identified.
Then you can informally diagnose through home work assignments or a shared question/answer period or any number of teaching methods.
If adaptation is such an individual matter and the resulting knowledge is more heterogeneous than homogeneous, then diagnosis must be able to identify differences in individual understanding.
Diagnostic questions or tasks can be constructed to assess recall, comprehension or the ability to use concepts in a novel way.

The Stage-Dependent Theory

The fact the cognitive structures are nested and that one never loses the mental abilities of earlier stages is significant; if a task is truly one that requires formal operations, the student cannot operate on it meaningfully unless he or she has had sufficient concrete experience on which to draw. The teacher must analyse the content and its concrete prerequisites. Must diagnose the students intuitive background and must plan instruction to close the gap between the intuitive and the abstract.

4.0 CONCLUSION

Although it may be difficult or undesirable to attempt to hurry the stages of development, just described, teachers are important in providing appropriate readiness activities and in asking appropriate questions. Otherwise, the children may be delayed in achieving the various stages of development. Two of the following four basic factors affecting mental development-experience and social transmission are strongly affected by teachers.

These four factor affecting mental development as the child proceeds through the stages of development just described are organic growth, (maturation), experience, social interaction or transmission, and equilibration.

5.0 SUMMARY

1. Five fields of research in educational psychology were identified
2. Of the first four fields, the contributions from those working on problems of the development of thinking is by far the most significant for mathematics education. The contribution of Piaget in this area were discussed.
3. Piaget maintained that children's cognitive development follows a well-defined sequence of stages where they acquire an organized group of perceptions, ideas and understanding (called structures) that enable them to deal with the world. In the pre-operational stage, children begin to deal with their environment symbolically, but have difficulty with reversibility (the ability to think back to the cause of events).
4. Reversibility is acquired in the concrete operational stage. At these stage children learn to think logically and to engage in fruitful interactions with others.
5. During the formal operations stage, children are able to develop and test hypotheses. They can also think and act scientifically.

6.0 TUTOR MARKED ASSIGNMENT

Now that you have completed the unit, this assignment will help you to determine how much you have learnt from the unit.

What follows is a list of statements some of which arises from our discussion in unit I and are therefore TURE (T); others, are wild statements that have no psychological bases with respect to what you have learnt and are therefore FALSE(F).

Read each of these statements carefully. Write T for those you consider TRUE and F for those you consider FALSE.

1. child at the pre-operational stage have difficulty with reversibility.
2. The concept of conservation is present at the concrete operational stage.
3. Abstract thinking can never be achieved for school learners in Nigeria.
4. The psychological condition for a reversible operation is that conservation.
5. Piaget viewed cognitive development in terms of well defined sequential stages determined by biological readiness and experiences.
6. A child's chances of reaching Piaget's formal stage depends on the size of his brain.

7. The second stage of Piaget's development is marked by the ability to deal with reality in symbolic ways.
8. The thought process of a child in the pre-operational stage in limited by "centering".
9. Absence of conversation of number or volume is the result of deficiency of centering reversibility.
10. Logical thinking exist at the pre-operational stage since children can deal with reality in symbolic way at that stage.
11. Conservation of numbers is a fundamental requirement in the understanding of number.
12. Without the conservation of numbers a child cannot establish the idea of equivalent sets.

SELF ASSESSMENT EXERCISES (SAE)

1. The list A are five major fields of research in educational psychology as applied to mathematics education. In list B are questions about learning mathematics. Match the correct field with the correct question.

List A

(a) Intellectual ability with special reference to mathematics.
(b) Study of growth of pupil's thinking.
(c) Concept formation, productive thinking & problem solving.
(d) Learning mathematics.

List B

(i) What should be introduced?
(ii) How should new mathematics be taught?
(iii) How do the mathematics idea of children develop?
(iv) Are there mental factors at certain ages which can be associated with mathematics abilities?
(v) What are the qualities of productive thought in mathematics?
(vi) How can mathematical task be broken up to improve learning of mathematical ideas.

2. Explain one experiment used by Piaget to test the presence of the concept of conservation or invariance in a child.
3. List A contains more new terms you have encountered in this unit. List B are some definitions.
   For each term in A, choose a definition in B which is nearest to its meaning as used in this text.

List A

(i)	Concept	(iv)	Stimulus
(ii)	Operation	(v)	Responses
(iii)	Perception	(vi)	Invariance

List B

(a) An action that can be carried out in thought as well as physically.
(b) Any term that can be reorganized as a recurrent feature in an individual's thinking to represent a class of experiences.
(c) Our reaction (observable behaviour) as a result of a task.
(d) The ability to observe through our sense and interprete meaningful understanding of objects, events and situation that constitute our world.
(e) A task for which a reaction is expected.

That which is carried out in a hospital theatre

7.0 REFERENCES/FURTHER READING

Lindgen, H. C (1980), *Educationl psychology in the classroom (Sixth Ed).* Oxford University Press: New York.

Servais W. & Varga T. (1971), *Teaching School Mathematics* A UNESCO Source Book;

Clyde H. C (1970), *Mathemtics psychology An Elementary Introduction* Prentice Hall Inc Englewood Cliffs: New Jersey,

Furth H. (1969) Piaget, and Knowledge Englewood Cliffs. N.J Prentice Hall

Barbel, I & Piaget J. (1958) The growth of logical thinking from child to adolescent New York; Basic Books, Inc. Publishers.

Gallagher, J.M. & Reid D.K (1981). The learning theory of Piaget and Inhelder; Monterey, California: Brook/Cole Publishing Company.

MODULE 3

UNIT 1 MEANING AND PURPOSE OF EVALUATION OF INTERACTION IN MATHEMATICS

CONTENTS

1.0 INTRODUCTION

Evaluation can be regarded as an integral part of an instructional programme of activity regardless of the length of time or period the instruction lasted. Evaluation is an on going process throughout the period of instruction. It is also the last activity that completes the teaching exercise whether the teaching lasted for a year, a term or whether it is a teaching unit or single period of instruction.

In this unit you will be equipped with some knowledge and understanding of this important educational process, evaluation, as well as be made conversant with its indispensable purposes for educational thought and practices.

As a teacher at any level, this knowledge is basic for you to be able to assess your students' performance effectively.

2.0 OBJECTIVES

By the end of this unit you should be able to

1. define or explain in your own words the term 'evaluation'.
2. state or describe the purposes of evaluation

3.0 MAIN BODY

3.1 Study Approach

Being the first unit of this module, you need to

1. study it with diligence;
2. attempt the questions in the activities and the assignments with all seriousness.
3. go through it as many times as possible for effective recall.

3.2 Meaning of Evaluation

Evaluation is an educational process that helps in gathering relevant and adequate information or data about the attainment or otherwise of dimensions of behaviour associated with the educational objectives specified by either the classroom teachers or the curriculum designer for the purpose of instruction.

A breakdown of this lengthy stipulative definition has much to convey to you as a teacher.

In the first instance, one may be tempted to ask for the thing that is being evaluated. Is it students achievement (attainment or otherwise)?

Secondly, the fact that achievement involves dimensions of behaviour, shows that evaluation goes beyond measurement which is assignment of numbers of things (situations) according to certain rules. It may therefore be said simply that evaluation involves value judgements. It is a broad process involving a host of evidence-gathering techniques to help in decision-making about the quality of an individual's or a group's performance or the success of an instructional programme in relation to stated objectives.

A student's abilities or progress are being evaluated when his scores are interpreted in terms of standards of his won class; in terms of his educational or vocational plans and aspirations; or some other bases for making value judgement.

Some teachers mistakenly feel that the only way of evaluating is by setting cognitively based questions for the students to solve. Because of this anomaly, the Federal Government of Nigeria introduced the idea of continuous assessment in her school system which helps in getting information of the child or student in variety of ways. Here, not only the cognitive dimension is tested, but also the affective and psychomotor domains.

Nevertheless classroom tests are more often used for evaluation than other methods. Because of the importance of evaluation, some educationists have described it, as the quality control of the educational programmes.

Evaluation, involves a bulk of activities, ranging from preparing test item's, scoring students scripts, recording scores, charting, interpreting and measuring the progress the students have made in the desired directions.

It is true that it is a time-consuming activity, frequent and tedious, yet it is an indispensable and recurring task which becomes increasingly important when you commit yourself to the noble consciousness of having your students achieve their best academically.

For any kind of evaluation to be purposeful, you should have in mind very clearly the instructional objectives of the lesson or the teaching programme. Hence the emphasis laid on making instructional objectives to be specific, clear and measurable. When objectives are stated in behavioural and measurable terms, it makes evaluation smoother and more meaningful.

3.2.1 Types of Evaluation Techniques

There are diagnostic, formative and summative types of evaluation.

3.2.2 Diagnostic Evaluation: involves the assessment of learners' previous knowledge before any unit of institution. It is diagnostic as its name implies because it helps to identify learners' areas of weakness on the previous unit of instruction so that the teacher becomes aware of where to start a new unit of instruction and how to guide the students in the process.

3.2.3 Formative Evaluation: Formative evaluations involve the assessment of learners' achievement after a brief unit of instruction. It is a diagnostic tool for learners weakness; to obtain evidence of students' mastery of the important concepts and ideas of a lesson; thus providing feedback to the teachers about his teaching method and the outcome of instruction.

3.2.4 Summative Evaluation: Summative evaluation aims at much more general or large scale assessment of the extent or degree to which the larger stated aims of instruction have been attained over the entire course or some substantial aspect of it. Sessional examinations are examples of summative types. On the spot evaluation is of the formative type. Good formative test results obtained at regular intervals leads to good performance at the summative level which covers wide range of knowledge and skills.

This very important educational activity cannot be without a purpose. Hence the next section will concentrate on pointing to you the useful purpose of evaluation.

3.3 Purpose of Evaluation in Mathematics

5.3.1 Decision Making

Evaluation helps in Decision Making: At the end of a lesson, or course, unit, the teachers, school administrators and other school personnel make many decision about the students progress and in addition lead the students' to make many decisions for themselves.

But it is necessary to realize that decisions are not made on scanty or irrelevant information about a students' programme rather it is on the basis of a good deal of information. Hence the school should keep or have cumulative and considerable record of information about each student.

The relevance of the information is also very significant, in that, the data that will be helpful to place a student in senior secondary technical are not identical with the data that will be most relevant to place him in ordinary senior secondary school. In the same secondary school too, the data for placing a student for Further Mathematics will not be identical with the data for placing him for Agricultural Science.

Note also that before decisions are made they usually involve prediction but before the latter, available and accurate data should be at hand as a result evaluation.

5.3.2 Predictive Measurement

Moreover, evaluation helps for the purpose of prediction. For example questions like: "In what mathematics class is this student likely to make the best progress". Is she likely to be able to offer the courses she wants?

These questions demand predictions. Yes he is likely to do well in such a class, he will not or from the records available, he is not likely to do well in that class. He will not cope with the demand. These predictions now serve as prerequisites for individual or institutional decisions. But it should be emphasized that the accuracy of the judgements and inferences you make will only b increased by relevant data.

5.3.3 Placement and Promotions

Another important purpose of evaluation is that it helps for placement and promotion. So, evaluation tests the readiness of a student or child to be placed in a new school or promoted to a new class as the case may be. Common Entrance Examination and JAMB examination are all evaluative strategies for placement of pupils or students to new and higher institutions relative to the former. Sessional examinations in schools are evaluative strategies for the purpose of promotion of students from one class to a higher class. But better decisions of placement and promotion can only be made with cumulative considerable and relevant data and not just by one or two single achievement tests.

5.3.4 **Guidance and Counseling**

Evaluation also helps in guidance and counseling and career choice. Choosing a career is a type of decision-making which a student has to make by himself and or by parents, based on the predictions they can make or the school helps him to make by making available his cumulative records.

5.3.5 Assessment of Instructional Strategies

Evaluation helps in the assessment of the teaching methods and materials on how effective they were in the course of an instruction. Therefore, it helps in ReEDUiation purposes. This implies that if the feedback obtained from evaluation is unsatisfactory there will be that decision of making up(reEDUying the situation) what was lost. ReEDUiation will therefore involve re-teaching and re-planning on the part of the teacher of the teacher as well as relearning on the part of the learner (the student).

As a classroom teacher you can or must have discovered other purposes evaluation serves.

4.0 CONCLUSION

Evaluation has been defined in this unit and its purposes outlined. Its components have also been given, namely diagnostic, formative and summative.

5.0 SUMMARY

In this unit you have studies

1. the meaning of evaluation as a purposeful educational process which helps in gathering relevant and adequate data about learners' achievement based on the educational objectives specified by either the teacher or the curriculum designer.
2. five major purposes of evaluation were highlighted: decision-making, prediction, promotion/placement, career guidance and reEDUiation.
3. data collected should be relevant and adequate;
4. learners achievement should be on various dimensions rather than on only cognitive testing.
5. objectives to be evaluated should be clearly specified and well stated.

6.0 TUTOR MARKED ASSIGNMENT

1. Using your own words give a definition of 'evaluation'
2. How does evaluation help in decision making? Give three examples of such possible decisions.
3. Clear and well defined objectives is necessary in defining evaluation. Discuss.

SELF ASSESSMENT EXERCISES

1. From the definition of evaluation, point out some key aspects that evaluation must involve.
2. "To evaluate" sounds like "to measure" comment.

7.0 REFERENCES/FURHTER READINGS

Fakuade, R. A. (1981) Teaching Arithmetic and Mathematics in the Primary School. Ibadan University Press.

Kalejaiye, A. O (1985) Teaching Primary Mathematics Lagos, Longman Nigeria Limited

UNIT 2 CONTINUOUS ASSESSMENT IN MATHEMATICS INSTRUCTION

CONTENTS

1.0 INTRODUCTION

Ohuche (1981) defines assessment as involving the determination of the value and worth of a thing and implies making decisions. Assessment in mathematics therefore implies determining the value and worth of learning outcomes, instructional situations, programme effectiveness and related products of learning mathematics. On the other hand, continuous assessment points to periodically or occasionally evaluating the learner all through his school career and ensuring that all decisions made contribute to the final decision on his work. The Federal Ministry of Education (1980) defines continuous assessment as a mechanism whereby the final grading of a student in the cognitive, affective and psychomotor domains of behaviour takes account, in a systematic way, of all his performances during a given period of schooling. Such an assessment involves the use of a great variety of modes of evaluation for the purpose of guiding and improving the student.

It is usual in any educational system to use tests and examinations for the purpose of assessment. But these are not the only instruments which can be used. There are other methods of assessment which can be employed to provide a valid evidence needed to improve upon students' learning and teaching. For instance, there are instruments which have been developed to obtain information on students' attitude, opinion or interest in mathematics. Such instruments include projects, assignments, observation, questionnaires and interviews. They are to be employed and the data collected must include the three domains – cognitive, psychomotor and affective domains. Every decision takes account of all the previous decisions. As a result continuous assessment requires some cumulative records on each student.

Classroom assessment is known by its very nature to be an approximation having its observed and true components. The algebraic difference between these two yields an error. Experience has shown that decisions based on one short examination have on several occasions ruined the career of good students who fell sick during the only examination. On this note, the application of continuous assessment in mathematics education will give a more meaningful result.

The greatest problem facing the implementation of continuous assessment in mathematics is that the preaching of its gospel has not yet got to the grassroots of mathematics learning - the primary and secondary mathematics teachers. Neither do parents know what it is all about. As a result of this non-filtering of innovational educational ideas to the grassroots, our children have been exposed to a state of continuous testing in the name of continuous assessment (Obanya, 1984). Some state ministries instruct teachers to give 'X' number of tests a term. Students are told in advance when the tests are coming up. They prepare for the test by the traditional swotting method, drugging themselves with coffee and kola nuts and devising examination success tricks. Parents on their own part are given results in two columns, labeled "Test work" and "exam work". Obanya (1984) observes that these results say very little about the affective traits of the child and the way they are developing. The results of the continuous tests are not used for guidance or for improving teaching and learning. What we need therefore, is massive programme of in-service training for practicing mathematics teachers on the techniques of continuous assessment. This will aim at improving the attitude of teachers to record keeping, since continuous assessment depends on keeping cumulative records. Also definite orientation on the effect of cheating and biased standards on the effectiveness of the programme must be carried out.

Since the directive that continuous assessment should be used at all our educational levels for the evaluation of our students, it thus becomes imperative that every Nigerian teacher should closely understand it's usefulness and practice its workability.

This unit will therefore give you a great insight into the following. The meaning and rationale of continuous assessment, its importance, problems of its implementation, how these problems can be overcome and the skills required by every teacher to ensure effective use of continuous assessment .

2.0 OBJECTIVES

By the end of this unit, you should be able to:

1. Define continuous assessment and give the rationale for using it in schools.
2. State the problems of implementing continuous assessment and how to overcome them: and
3. Spell out the various skills that a teacher requires for ensuring effective use of continuous assessment.

3.0 MAIN BOY

3.1 STUDY APPROACH

Being the second unit of this module with a new concept or idea, you will need to:

1. Take a careful study of this concept to see how it relates to the various tests that have been learnt and studied with their uses.
2. Avoid rushing to assimilate the content of this unit.
3. Attempt carefully all the exercises that appear under the activities and assignments.

3.2 MEANING OF CONTINUOUS ASSESSMENT

In general term continuous assessment is viewed as that method of finding out the extent of what the pupils have gained from learning activities in terms of thinking and reasoning abilities, character or moral development, knowledge and skill acquisition. To stress this further, continuous assessment can also be said to be the final assemblage of the pupils grades while taking good cognizance of the cognitive, affective and psychomotor domains of their behaviour throughout the period of their stay in the school. It should also be noted that when such assessments are given, a great variety of modes of evaluation are also employed for the purpose of giving appropriate guidance and improvement to aid the pupils learning activities and performance in schools.

3.2.1 Rationale for Continuous Assessment

As a teacher who will continuously make use of continuous assessment in teaching pupils in schools, there is the need to really know why the continuous assessment is necessary for use in our schools. Some of these are discussed below:

1. With the view that an assessment is an important part of the teaching process, teachers are expected to also take active part in the final assessment of the pupils they teach.
2. Contrary to the old single examination system practiced in the past, continuous assessment is expected to give the final grades of all thepupil's performance throughout the period in the school. Such an assessment will give a more realistic picture of the pupils' ability than a single examination can do.
3. As teachers, the continuous assessment will provide you with useful tool to asses your own teaching performances and how to improve on them.

3.2.2 Problems of Implementing the Continuous Assessment System in Schools

Although the continuous assessment practice is now fully part of our educational system, it is important to give due consideration to certain significant problems that go along side with its implementation and at the same time propose solutions to them. These problems are directly for the educational system in general and for teachers in particular.

The first of these numerous problems has been identified as the problem of comparing the quality of the various tests and instruments of assessment used by different teachers in the various schools. In this respect it has been noted that majority of our classroom teacher have not received adequate and formal training in the construction of test and the use of various educational measuring instruments. If reports are made available in the form of written comments, these will bring in the difficulty of ensuring any kind of comparability between comments made by individual teachers even in the same school and parents on the other hand.

Another problem that require consideration is that of record keeping and continuous keeping of such record since such practice demands proper and accurate keeping of records on every pupil in the school.

3.2.3 Solutions to the Problems of Implementation of Continuous Assessment

The following are some useful solutions for the proper implementation of the continuous assessment system

1. A consistent system of record keeping should be maintained by every school to ensure uniformity of standard.
2. All teachers should adopt a fairly uniform syllabi for each level of education for their pupils. There should also be uniformity in the setting of tests and various assessment instruments
3. There is the need for a coordinating committee to monitor all the activities involved in the system to ensure uniformity in the maintenance of approved standards in relation to administration standardized achievement tests and the use of various assessment instruments in all schools from time to time to ensure a relative high level of performance by pupils.

3.2.4 Teachers Roles In Implementing continuous Assessment in School

All classroom teachers are expected to live up to their expectations in the discharge of their duties to meet up good planning with a comprehensive view of their functions, rather than have a narrow perspective view of what they are supposed to teach. ‘Teachers are expected to be professionally qualified, knowledgeable, dedicated, honest and hardworking for them to smoothly and properly practice effectively the continuous assessment system in our schools.

In general, all teachers involved in the continuous assessment practice have to consistently plan and teach continuously, keep close observation of all the attributes that each pupil may manifest, make a systematic and comprehensive record on pupils academic progress and learning outcomes. The teacher is expected to diagnose and find solutions to each pupil’s difficulties and then help them as a unique individual by using the results of their assessment instruments to always counsel and guide the pupils always in the classroom learning situation.

3.3 Application of Continuous Assessment in Mathematics Cognitive Outcomes

There are many types of test items applied in learning achievement. Since mathematics achievement is not a unitary trait, we therefore devote this section in considering different common types of test items in mathematics and how to construct them. There are generally two types of test items-objective and essay types. The objective test items are those requiring the student to supply the answer or to select the answer from a given number of alternatives. The scoring of objective test is quick, easy and more accurate. However, we cannot use objective tests to measure such abilities like selection, organization and integration of ideas. To measure these we must depend on the essay question. On the other hand, the marking of essay type test is subjective. Different markers usually give different marks to the same solutions even when the same marking guide is used. Therefore in order to effectively assess the cognitive learning outcomes both the essay type and objective type test should be used.

f specification for Junior Secondary School Mathematics Behaviours

	A. COMPUTATION					B. COMPREHENSION				C. APPLICATION				
	Knowledge of specific facts	Knowledge of Terminology	Ability to perform the basic operations	Ability to changes No. from one mode to another	Ability to carry out algorithm	Knowledge of concepts	Knowledge of principle and rules	Ability to recognize pat-terms	Ability to transfer problem from one mode to another	Ability to read and interpret e problem	Ability to solve routine problems	Ability to solve over-all problems	Ability to make comparison	Total number of items for contents
	A_1	A_2	A_3	A_4	A_5	B_1	B_2	B_3	B_4	B_5	C_1	C_2	C_3	
														4
			2		1				1					
						1								3
			1	1			1							4
			2	1								1		
						1								1
				-							1	1		
o														3
			2			1						2		
						1							1	3
on													1	
						1				1				1
					1									2
						2	1					1		
				1				1						1
				1							1	1		
														1
	1					6							1	3
							1							4
d		1												4
	1								1			1	3	
	4									1				3
												2		
										1				3

l ar d c l rs an .	- 2	1 2	- 7	4	2		1 4	1 2	2	3	4	9		3 3 50

3.3.1 Objective Type Test

(a) **True/False Items**

In a True/False item, a statement is made and the testee is expected to indicate whether the statement is true or false, or question is asked and the respondent should answer Yes or No.

Example. Circle 'True' if this statement is correct and 'False' if not.

1. A Rhombus has the interior angles 90^0 True/False
2. The constant $\frac{\text{Circumference}}{\text{Diameter}}$ is a rational number, True/False
3. The arithmetic sum of deviations from the mean is zero, True/False

Write 'Yes' or 'No' against the question.

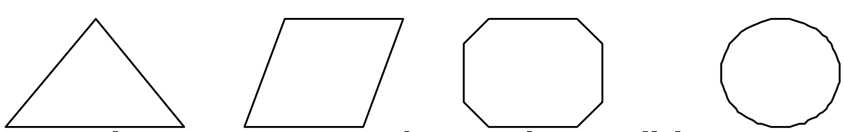
4. Is (x-3) a factor of $x^2-7x+12$?
5. Does an equilateral triangle have equal sides?

You will observe from these examples that True/False test items are often limited to testing facts. Also they do not stimulate much thinking since the testee is faced with only two choices and may, therefore, be encouraged to guess.

(b) Matching Items

In order to construct a 'matching' test item, two sets of alternatives are provided, for examples, the drawings of several shapes and their names may constitute two lists. The tested is then expected to match one item from a list to another item in the second list. In general, the number of alternatives in one list should be at least one more than the number in the second list, so that someone who knows the answer to all but one of the items will not automatically get the last answer correct. This type of test items is usually marked by giving one mark to each pair of items which are matched correctly and no mark for the pair which is wrongly matched.

Examples (1) Match each shape with its name



Rectangle, Hexagon, Cube, Circle, Parallelogram, Triangle

(2) Match the proper fraction and the decimal fraction which have the same value

3 4 8 2
4 5 10 5

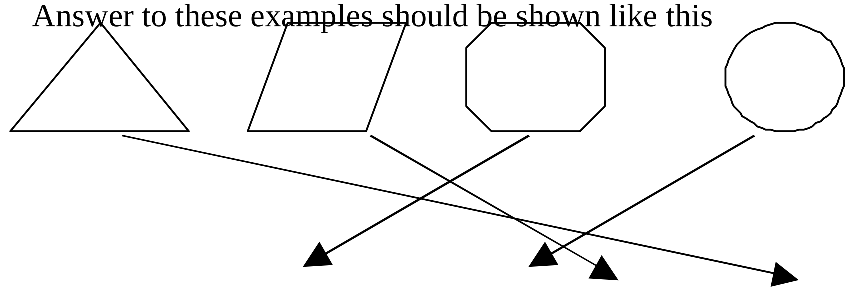
0.4 0.5 0.6 0.75 0.8

(3) Match the shape and the number of flat faces which it has

Cube Cylinder Tetrahedron
10 8 6 4 2

Answer to these examples should be shown like this

1.



Rectangle, Hexagon, Cube, Circle, Parallelogram, Triangle

2. 3/4 4/5 8/10 2/5

0.4 0.5 0.6 0.75 0.8

3. Cube Cylinder Tetrahedron

10 8 6 4 2

Matching items test the knowledge of facts. They also economize space because the questions which are asked in one item may constitute three or four 'True/False' or multiple choice items. For these reasons it is recommended that matching test items be used during classroom teaching.

(c) Multiple Choice Items

A typical multiple-choice test items has a stem which may be a statement or a question and four or five possible answers of which only one is correct. The options are usually selected so that they can attract pupils who do not know the correct answers. Usually, pupils are asked to indicate the correct answer by circling, underlying, writing the letter in front of their chosen answer or where computer sheets are used to shade the appropriate space.

Examples Circle the letter in front of the correct answer.

1. A cup 'A', half full of water, was emptied into and empty cup 'B' which then became one third full. If the process had been

reversed, i.e., 'B' had been filled half full and poured into 'A', what fraction of 'A' would have been filled.

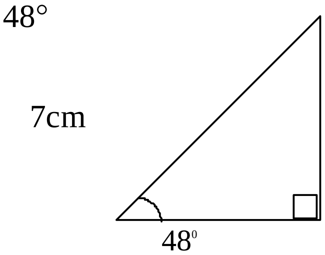
i.e. 'B' had been filled half full and poured into 'A', what fraction of 'A' would have been filled. (a) $\frac{1}{3}$ (b) $\frac{2}{5}$ (c) $\frac{3}{5}$ (d) $\frac{2}{3}$ (e) $\frac{3}{4}$

2. If $p = \sqrt{\frac{k}{x}}$ then x is equal to (a) $\frac{P^2}{k}$ $\frac{2\ 2}{k}$ $\frac{2}{k^2}$ $\frac{}{y2}$ (e) $\frac{P^2\ 2}{k^2}$

3. In the triangle ABC the length of AC is

A. 7 Cos 48^0 B. 7 Sin 48^0 C. $\frac{7}{\text{Sin}}$ D. 7 tan 48^0 E. $\frac{7}{\text{Cos } 48^0}$

48°



4. The mode of this set of numbers 3 3 4 4 5 6 4 7 7 8 9 3 4 4 9 7 6 is A. 3 B. 4 C. 7 D. 5 E. 9

5. Which of the following statements is false?
A. All rhombuses are parallelograms
B. All squares are rectangles
C. All squares are rhombuses
D. All rectangles are parallelograms
E. All quadrilaterals are parallelograms

Multiple-choice tests are usually scored by giving one mark to each correct answer. However, students may be penalized for incorrect answers by using the formula.

$$\text{Score} = \text{Right} - \frac{\text{Wrong}}{n - 1}$$ where n is the numbers of options.

That is, deducting one quarter of the total number of items got wrong from the total number of items got right, where the multiple choice items have five options. Where there are four options for the answer, one third of the number of items got wrong is deducted from the number of items got right. Most teachers seldom apply any penalty in marking multiple choice questions.

Multiple-Choice tests are commonly used in public examinations because of the ease and objectivity of scoring and because they provide a more adequate coverage of the syllabus than essay-type tests.

Precautions In Writing Multiple Choice-Test

i) The problem should be clearly and concisely stated in the stem and not in the options.
ii) One problem only should be included in an item.
iii) As much as possible the options of an item should be of the same structure and of about equal length.
iv) One and only one correct answer should be included in the normal multiple-choice type.
v) Each option, especially where expressions are involved, should be listed in a line.
vi) The correct options should be placed in random manner.
vii) Grammatical cues should not be allowed to give away the right altemative.
viii) The negative should be used only when very necessary in the stem. Double negatives should be avoided unless ability to handle double negatives is being tested.
ix) Such phrases as "all of these," None of the above," etc. should be used with caution.

Item Analysis

Item Analysis is a procedure used in appraising the effectiveness of the test Item and building up a filed of high quality items for future use.

We shall illustrate this by considering 38 test papers which have been scored.

First, we rank the papers in order from the highest to the lowest score. Select the 13 papers in the upper group (approximately one third) and the 13 papers in the lower group.

For each test item, for example, consider Question 2 of the examples on multiple choices, tabulate the number in the high group and low group who selected each alternative.
Information for item Analysis

Options	A	B	C	D	E	Omitted
Upper 13 papers	0	0	13	0	0	0

Lower 13 papers	2	1	5	2	3	0

From the table we compute the following:

i) ***Item Difficulty:*** This is done by calculation the percentage of pupils who got item right.

i.e. Item Difficulty $= \frac{\text{Number who got the item right}}{\text{Total number who tried the item}}$

Applying the formula to the above information
I.D =18

This allows the teacher to confine the class discussion of the test result to those items that caused pupils the greatest difficulty.

ii) ***Item Discriminating Power*** Here we determine the degree to which it discriminates between pupils with high and low achievement by using the formula:

Discriminating Power =

$$\frac{\text{Upper group who got item right-lower group who got it}}{\text{½Total number include in the item analysis}}$$

Applying the formula to the above table

$$D = \frac{13-5}{13} = \frac{8}{13} = 0.616$$

When this value is 1.00 then all the pupils in upper group got the item right while all those in the lower group failed it. We say that there is a maximum positive discriminating power. But when D = 0, where equal number in both upper and lower groups got the item right, we say there is no discriminating power.

Finding out how effective each distracter is can be done by inspection. When an item contains distracter that is not selected at all or by very few pupils then a revision is needed.

Generally, item analysis calls for reEDUial work on those areas and helps in curriculum revisions. It increases the skills in test construction.

(d) Multi-Facet Items

In a multi-facet question, a pupil is presented with a situation and is required to answer several independent questions using the given data.

Example 1: This table shows the number of chairs produced by a Furniture Company for six weeks.

	1st week	2nd week	3rd week	4th week	5th week	6th week
No. of Chairs	68	72	88	98	78	76

(ii) Find the mean number chairs
(iii) Draw a histogram to show the data

Example 2: A solid rectangular block has length 8cm, breath 5cm and height 3cm
(ii) Draw the block;
(iii) Find the surface area;
(iv) Find its volume.

Example 3: Five men can do a piece of work in 20 days.
(ii) How many days will it take 10 men?;
(iii) How many days will it take 1 man?

Example 4: Here are two fractions 1¾ and 1/n
(i) Find the sum;
(ii) Subtract the smaller one from the larger;
(iii) Multiply the two fractions;
(iv) Divide the larger fraction by the smaller one.

Example 5: A lorry travels on a road from kilometer 97 to kilometer 25 at a speed of 80 km/h.
(i) Find the distance it travels;
(ii) Find the time taken by the lorry to travel a distance of 40km.

In multi-facet items, marks are awarded for correct answers only and there is usually no penalty for an incorrect answer. The marks which are given to the separate questions should depend on their difficulty levels.

(e) Completion Item

In a completion item, a pupil is expected to supply the answer by filling in the correct response.

Examples:

1. One prime factor of 27 is ________________________
2. A Hexagon has ___________________________sides
3. A tetrahedron has __________________________faces
4. Petrol is measured in ________________________
5. Expressing 3 + 4 as a single fraction gives _______

$\overline{}$ $\overline{5}$x 5x

In classroom teaching, it is advisable to use a variety of test items in presenting mathematics questions to students. The variety in format will stimulate their interest and ensure that they do not get used to a particular format. Students who are used to test format may not do well on test items which use other formats.

The multiple-choice and multi-facet test items are more suitable for evaluating achievement at the end of a series of lessons. The true/false, matching and completion items are easy to construct and may be used during classroom teaching to test factual details of the lesson.

3.3.2 Essay-Type Test

As already mentioned, only essay-type test can be used to measure complex mathematical achievement. In addition, essay-type test gives room for proper integration and applications of mathematical concept to original problem-solving situation. The student is not restricted to a particular method but only required to provide a complete solution showing various steps in the solution. In objective tests, the student is not given any opportunity to score any mark for an answer narrowly missed. This is where essay-type tests tend to give a more accurate assessment of level of mathematical achievement which the objective tests cannot give. For example, a boy solved for

x in $x^2 - 7x + 12 = 0$

$x_2 - 7x + 12 = 0$

$(x - 3)(x-4) = 0$

either $(x - 3) = 0$ or $(x - 4) = 0$

$x = -3$ and $x = -4$

In an objective test, this student will score zero. But in an essay-type test, he may score three out of five marks. In marking essay-type items, the teacher should credit the important stages in the solution of problems so that pupils will be motivated and encouraged to continue to learn mathematics. It will also make the students to put down their reasoning in a logical manner and so assist the teacher to identify their mistakes. Five example of essay-test items are given below:

Problems:

1. (a) Simplify: i) $\frac{15}{6} \times 1\frac{1}{3}$

ii) $3\frac{1}{2} - \frac{1}{4}\left(\frac{15}{6} + 2\frac{2}{3}\right)$

(b) In a town of 5672 inhabitants, there were 87 births during 1980. Find the percentage birth rate.

2. (a) Divide 246 in the ratios 1½: 2:3⅓

(b) The length L of a rectangle is given as 6cm and the area A as 20cm, each correct to 1.s.f. between what limits does the breath B of the rectangle lie?

3. The average height of six trees is 7m 8cm.

(a) Use directed numbers to show how much each tree is above or below average when their heights are 7m 3cm, 7m 5cm, 8m, 7m 8cm, 7m 7cm.

(b) (i) What is the sum of the difference for the tree 7m 3cm high and the tree 7m 5cm high?

(ii) Find also the product of the differences in (i)

4. (a) Perform the following binary operation:
11110 + 110

(b) A number is written as 25 to the base y. Thrice the number is written as 77 to the base y. Find y.

5. (a) 12kg of tea N3.50 per kg was mixed with 18kg of another brand of tea at N4.20 per kg. If the mixture was sold at N4.00 per kg, find the percentage gain or loss.

(b) A piece of work could be done by 6 men working for 8 days at the rate of 10 hours a day. A contractor hires two groups of men to do the job. The first group consists of two men who worked for 12 hours a day for 4 days. The second group consists of 9 men who also worked for 4 days. If the job was completed, for how many hours a day were the men of the second group working.

In setting essay-tests it is essential that the teacher should be precise in his choice of words and in the problem he sets. Many a time the language in which the mathematics is set is more difficult than the mathematics itself. This should be avoided.

In marking essay-type test items, the teacher should first write down all the possible solutions stating all the steps that are important. The marking guide follows with marks specified for each question and how they will be awarded. For example, solutions to the five problems above are written below. Question 1 will carry eight marks, Question 2 eight marks, Question 3 ten marks, Question 4 ten marks and Question 5 fourteen marks. The marking guide is also provided.

To prepare a marking guide we use certain types of marks, these include method marks and accuracy marks. The method marks are given for student's ability to use a correct method at any stage of the solutions. The method mark is denoted by the symbol M, so that M_1 means one mark for the method at that stage. M_2 at another stage will mean two marks for the method at the appropriate stage.

Accuracy marks, denoted by the symbol A, are given for accurate work following a correct method. A_1 means that one mark is given for accurate work at that stage, while A_3 at another stage shows that three marks are to be awarded for accuracy at the stage. At times, due to an earlier error in the solution, a pupil works part of a problem correctly but gets an answer different from the correct one. Marks may be awarded to this answer if it follows from his own working. Accuracy marks are awarded only when the associated method marks are scored. A mark should not be awarded if the associated M mark is not gained. If a student loses the preceding M mark or A mark, he may still gain the latter M marks and A marks.

There are other types of marks different from the M and A marks. These are the independent mark, which are given for a particular answer or statement. They are not given when the answer or statement is different from the required one. These marks are usually denoted by B, so that B1½ means that one and half marks are given for a particular answer or statement and no other one.

The solutions of the five problems and their marking scheme are provided below. Study the scheme and use it to mark the sample; script of one student A shown imEDUiately after the marking scheme.

Solutions to Worked Examples

1. (a) i) $\frac{5}{6} \times 1\frac{1}{3}$

$$= \frac{11}{6} \times \frac{4}{3} = \frac{22}{3} = 3\overline{)22}^{\,7.3}$$

```
  21
  10
   9
   1 = 7.33
```

ii) $3\frac{1}{2} - \frac{1}{4}\left(15 + 2\frac{2}{3}\right)$

$$= 3\frac{1}{2} - \frac{1}{4}\left(\frac{33}{6}\right)$$

$$= 3\frac{1}{2} - \frac{1}{4}\left(\frac{21}{6}\right)$$

$$= 3\frac{1}{2} - \frac{21}{24}$$

$$= \frac{7 \times 21}{2 \times 24} - \frac{82 - 21}{24} = \frac{61}{24}$$

(b) Inhabitants 5672

Births 87

Years 1980

% Birth rate = $\frac{87}{5672} \times 100 = \frac{8700}{5672} = 1.53\%$

2. (a) 246 is to be divided in the ratio 1½: 2: 3⅓

sum of ratio is $\frac{3}{2} + 2 + \frac{10}{3}$

$= \frac{9 + 12 + 20}{6}$

$= \frac{2 + 29}{6}$

$= 2\frac{5}{6} = 6\frac{5}{6}$

+ 4

First share =1½ x 246

$= \frac{3}{2} \times \frac{6}{41} \times \frac{246}{1} = \frac{2 \times 6}{1} \times 246 = 9 \times 6 = 54$

Second share $= 2 \times 246 = \frac{2}{41} \times \frac{6}{1} \times \frac{246}{1}$

$= 2 \times 36 = 72$

Third share $= \frac{10}{3} \times \frac{6}{46} \times 246 = 23 \sqrt{2460} = 106\frac{22}{22}$

23

160

138

22

2. (b) Length of rectangle = 6cm

Area of rectangle = $25cm^2$

Breadth = 25 ÷ 6

= $\frac{26}{6} = 4\frac{1}{6}$

:. Breath b = 4cm

3. (a) …………………………………………………………………

Height of tree	Average	Diff. (4 - A)
7m 3cm	7m 8cm	- 5cm
7m 11cm	7m 8cm	3cm
7m 5cm	7m 8cm	- 3cm
8m	7m 8cm	+2cm
8m 8cm	7m 8cm	0 cm
7m 7cm		

……………………………………………………………………………………

(b) i) -5 + -3 = -8

-5 x –3 = 15

4. (a) 11110 ÷ 110   110√ 111 /11
110
110   :. 1110 ÷ 110 = 11

(b) 25y
(2y + 5)3 = 77y
6y + 15 = 7y + 7
7y - 6y = 15 - 7
y = 8

5. (a) 12kg of tea was brought at ₦3.50 per kg.
:. Total expenditure for the 12 is 12 x 3.50
i.e. 7 x 126 = ₦42.00
2 1
18kg of another brand was bought at ₦4.20 per kg.
:. Total expenditure is 18 x 4.20 = ₦75.60
:. Total expenditure for the two is (42.00 + 75.60) = ₦117.60
Total income for the expenditure is (₦4.00 x (18 + 12)
= 30 x ₦4.00 = ₦20.00
:. Profit is ₦(120.00 – 117.60) = ₦4.40
% profit is 240 x 100 = 240 = 2400 = 2 2
117.60 117.60 1176 49%

(b) 6 men worked 8 days at 10 hr/day
1 man will work 8 days at 10 x 6 hr/day
1 man will work 1 day at (10 x 6 x 8) hr/day
i.e. 480 hrs.
Now for 2 men working 4 day at 12 hrs/day
1 man will work 1 day at (12 x 2 x 4) = 96hrs.
Also let x hrs be for the 9n men
9 men working 4 days at x hr/day
1 man working in at 9 x hr/day
1 man working 1 day at 36 x hr/day.
Thus 96 + 36x = 480 hrs.
36 x = 480 – 96
= 384
x = 384 = 1⅔
36

Student should score a total of 34 marks – $M_1 A_0$ in Question 1 (a) (i), $M_2 A_0$ in 1 (a) (ii), B_1, M_1, A_1, in (b), $M_1 A_1 A_1, A_0$ in 2 (a) $B_0 M_0 A_0$ in 2(b), B_5 in 3(a), $M_0 A_0$ 3(b) (i) $M_1 A_1$ in 3 (b) (ii) M_0
A_0 in 4(a), $B_1 B_1 M_2 A_2$ in 4(b), $B_2 B_1 M_1 A_1 M_1 A_1$ in 5(a), $B_1 B_1$
$B_1 M_2 A_0$ in 5(b).

Note that although the student wrote down wrong answer for the subtraction (120-117.60) in Question 5(a), yet he scores the accuracy mark for the next step shows that 'he has recovered'.

You will notice that different teachers score differently in essay questions and even the same teachers differently at different times. This is mainly experienced where evaluation of answers is not guided by clearly defined outcomes, it tends to be based on less stable intuitive types of judgment. Although, the subjective nature of scoring essay questions will always involve some uncontrollable variations, the scoring reliability can be greatly increased by the approach discussed above.

Another limitation of essay questions is the amount of time required for scoring the answers. In fact, where classes are large, conscientious scoring becomes practically impossible. The likely practical solution is to reserve the use of essay questions for measuring complex mathematical achievement, which cannot be measured by other means.

5.3.6 Use of Test Results

Marks scored in mathematics test serve a variety of functions in the school. These include the following:

(i) Assessing the degree of mastery of some specific in order to permit a decision on what to teach or study next.
(ii) Rendering reports to the students, parents and the school on how well the student has been progressing in acquiring the broad range of skills, knowledge and understanding that represents the objectives of the course.
(iii) Helping the students make educational and vocational decisions. Guidance counselors and teachers keep and utilize these test results to know a student's past achievements, understand better his present strength and weaknesses, thus being in a better position to predict the areas in which he is likely to be successful in the future.
(iv) Modifying the test items in order to yield more valid information in future use.

3.3.4 Criteria for Desirable Tests

In test construction, there are certain criteria to be satisfied. The most essential of these can be classified under the heading of validity, reliability and usability. We now examine each of these criteria a little more closely.

(a) Validity

The perennial problem of predicting future performance of a student entering secondary school from entrance examination result can only be solved if the mathematics test scores predict success in mathematics class. Does the test measure a representative sample of the entry behaviour expected of a Form One student? This leads us to the idea of validity- that a test should really measure what it sets out to measure. It follows that mathematics test in the entrance examination, being a prognostic test, should reveal potentiality. A formative evaluation, used as first aid treatment for simple learning problem, should reveal weaknesses that can be put right. Sometimes we use only school certificate results (which is basically an achievement test) as a basis for selection procedures either for further education or employment. In fact, these are not valid uses of these examination scores.

(b) Reliability

If mathematical tests were 100% reliable then a student would score the same if he took the test today as if he took it yesterday or tomorrow or next week. But this is not practically possible. The best is to reduce the variation in scores by avoiding the factors that spoil the perfect testing situation. In our mock examination our students score highly, yet the same students about four weeks later score low marks in mathematics in school certificate examination. So any score should not be regarded as absolute, but a measurement which is accurate to within a given margin of error.

(c) Usability

In addition to mathematics test being valid and reliable, it is also important to consider how easy it is to administer, time required, how easy it is to score and how easy it is to interpret and apply. For easy administration the instructions should be clear and simple. Otherwise, we may have errors that will have adverse effect on the validity and reliability of the scores. Many subjects are competing in the secondary school as a result we shall always favour the shorter time test, this will help us utilize well the small time period for the large content we are given in mathematics. Nobody likes to wait very long for the result of any test, so teachers should give prompt attention to scoring of test.

Furthermore, the results of a test may show that a wrong answer is obtained by a majority of pupils, especially in a multiple-choice test. The teacher should find out why most pupils selected a wrong answer. This can be done by discussing the test item with the class and going through the question and the various options in order to check their corrections or otherwise. Occasionally, it is found that the students are right or the choice of a wrong option in a multiple choice is not attracting any pupil or is selected more by pupils who are strong in mathematics, then such an option should be dropped and replaced by a more attractive one.

4.0 CONCLUSION

In this unit, you have come to understand the meaning, rationales, problems and solutions to issues of continuous assessment in secondary schools. The unit also discussed teachers' roles in the implementation of continuous assessments. The unit is concluded with types of tests and criteria for desirable tests.

5.0 SUMMARY

1. Continuous Assessment is the overall assessment of pupil's learning activities in terms of his intellectual, social, physical and moral development. It is aiEDU at assessing pupil's academic progress and as well give him appropriate guidance and improvement in all his learning activities in schools.
2. The goal of continuous assessment practice is to help the assessors conduct all appropriate assessment that will be the basis of good planning of all educational programmes for all school-age pupils.
3. The comprehensiveness of continuous assessment practice show that it takes adequate care of all area of human learning. Some of these are school attendance, behaviour, health, habits, participation in school activities, self-concept values etc. All these are to be measured with the academic progress of pupils.
4. Continuous assessment practice require the use of many assessment instruments to determine the performance of individual pupils in the schools. Some of these assessment instruments are assignments, test, project, etc.
5. Under the system, all teachers are expected to make systematic and comprehensive records of assessment of pupils' learning experiences, diagnose and find the solutions to pupils' difficulties and individually help them to improve their academic, moral and physical standards.

6.0 TUTOR MARKED ASSIGNMENT

1. Mention two defects of using single examination or a paper-pencil test to determine academic achievement.
2. Explain the use of three of the assessment instruments which will help to determine the performance of your pupils in the school.
3. Clearly explain two of your expectations as a practicing teacher towards the use of continuous assessment practice in schools.

Continuous assessment in the mathematics cognitive domain raises three major questions:

(i) At what intervals shall the tests be given?
(ii) What kinds of test should be used?
(iii) What will be the weighting of the results obtained for each year of the J.S.S. programme?

Test could be given on bi-weekly basis. Each test should include both objective and essay type questions. Assignment should be given at the end of each instructional situation while class assignment takes 35-40%, it is suggested that bi-weekly tests take 60%. A comprehensive examination is conducted at the end of each term. Again the overall result for the term is shared in the proportion, 35-40% of class assignment and bi-weekly tests, and 60-65% of the end of term assessment. Finally, the results of students' achievement at the end of the year should be the average of the three terms work.

SELF ASSESSMENT EXERCISE

Self Assessment Exercise 1

As a practicing teacher, give two other reasons why Continuous Assessment should form an integral part of our educational system.

Self Assessment Exercise 2

1. Mention two other important abilities that the Continuous Assessment practice require of you as a practicing teacher that will aid your teaching.
2. Mention four (4) good qualities of an effective teacher.

7.0 REFERENCES/FURTHER READINGS

Federal Ministry of Education (1977): *National Policy on Education*, Lagos, Nigeria.

E. Ayotunde Yoloye (1984): *Continuous Assessment: A Simple Guide for Teachers.*

Airasian P.W. and G.F. Madaus, "*Functional Types of Student Evaluation*", Measurement and Evaluation in Guidance.

Anastasi A. (1976): *Psychological Testing*, 4th ed. New York. Macmillan Publishing Company Inc.

Bloom B.S. (Ed), (1956): *Taxonomy of Educational Objectives Handbook* I, Cognitive Domain, New York, David Mckay Co. In.

Bloom B. S., J.T., Hasting and G.F. Madaus, (1971): *Handbook on Formative and Summative Evaluation of Student Learning,* New York, MacGraw-Hill Book Company.

Cronback L.J. (1970): *Essentials of Psychological Testing,* New York, Harper and Row.

Gronlund N.E., (1976): *Measurement and Evaluation in Teaching* (3rd Ed.) New York, Macmillan Publishing Co. Inc.

Gronlund N.E., (1973): *Preparing Criterion – Referenced Tests for Classroom Instruction,* New York, Macmillan.

Mager R.F., (1962): *Preparing Instructional Objectives,* San Francisco, Fearon.

Stanley J.C. and K.D. Hopkins, (1972): *Educational Land Psychological Measurement and Evaluation*, Englewood Cliffs, N.J. Prentice-Hall Inc., Chapter 10.

UNIT 3 PROBLEMS CONFRONTING MATHEMATICS INSTRUCTION IN SCHOOLS

CONTENTS

1.0 INTRODUCTION

No teacher in Nigeria concerned with the teaching of mathematics at any stage, from infants upwards and, particularly at secondary level, can honestly say to himself that all is well with the teaching of the subject. There are far too many students today who leave the secondary school wit a dislike for mathematics. Those who teach physics, chemistry or other subjects requiring the application of mathematics complain of the odious task they face. My student teachers come back to the College from teaching practice with reports of their experiences which point at the same awareness. There exist many problems in the teaching of mathematics.

Great anxiety has been expressed by government, employers of labour, parents and teachers about the fact that large numbers of students, after a secondary school course, are unable to perform many of the simple arithmetical and mathematical operations needed in their everyday life and work. One reason we can advance is that a majority of students never get thorough understanding of the real meaning of mathematical concepts. In most cases, they are only clever in the art of manipulating complicated sets of symbols, or they are dazed by the unpleasant situations into which the present mathematical requirements in schools tend to place them. Eventually, they develop a common attitude – 'get the examination over', after which forget about mathematics. This is the position in our country now. Mathematics is generally regarded as a very difficult subject everywhere in the world. But the serious problem in Nigeria is that very little is being done to alter this state of affairs.

There has lately been however, a certain feeling of discomfort among those concerned with the teaching of mathematics. The mumblings of discontent have been getting louder and louder, yet they have remained only on newspaper pages without any co-ordinated plan of attack. Similarly, there are different viewpoints about the necessity of different types of reforms in its teaching. Some take delight only in criticism for the sake of criticism. It is the joint responsibility of those concerned to bring about necessary improvement and changes. The subject has to be popularised at all costs. With the 'setback' by the then Federal Commissioner for Education at the Benin Conference (1977) and also with the newly-approved syllabus introduced, we now hope that the Mathematics Association of Nigeria will awake to the challenges of the day. Encouragement should be given to those interested in finding solutions to these problems at all times in terms of funds for workshops, seminars, conferences and useful researches. Various governments have been expressing their intentions to establish quality education. I believe that a thorough analysis of these problems and possible reEDUies will place them on the correct line.

In doing so, you will be expected to consider the situations as they exist in our schools today under the headings: historical, political, socio-cultural, economic, academic mathematical, pedagogical and psychological.

In this unit, you will be exposed to some problems facing the teaching and learning of Mathematics not only in the Primary and Secondary level but also in all levels of mathematics education in Nigeria. Some possible suggestions as to the solutions of these problems shall be included.

These problems shall be examined form the historical, political, socio-cultural, economic and academic perspectives. It should be stressed that these problems are not mutually exclusive.

2.0 OBJECTIVES

By the end of this unit, you should be able to:

1. identify at least one historic problem facing the teaching of mathematics in Nigeria.
2. identify at least one political problem associated with the teaching/learning of mathematics.
3. state and explain at least one problem facing the teaching of mathematics arising from the social or cultural stratification of the country.
4. state and explain at least two problems facing the teaching of mathematics emanating from the economic situations of the country.
5. identify and explain at least six problems of academic perspective facing the teaching/learning of mathematics; and
6. suggest at least one solution to each of the identified problems.

3.0 MAIN BODY

3.1 STUDY APPROACH

For you to get the best out of this unit, the following procedures are recommended:

1. Read through a suggested problem once, using your reflective thinking to associate this problem with your past experiences. Jot down other problems associated with this recorded problems in this unit. At the end of the unit, send a list of such unrecorded problems to NTI and demand their comments and/or solution to them.
2. Make a list of all problems about mathematics published in our dailies which you come across. Compare them with the problems set out in this unit.
3. Note that the problem of teaching and learning mathematics in Nigeria is a dynamic one. It grows and changes from time to time and as such not all of its ramifications will be written down in this unit.

3.2 SOURCES AND SOLUTIONS TO SOME INSTRUCTIONAL PROBLEMS IN MATHEMATICS IN NIGERIA

The problem of teaching/learning mathematics in Nigeria has many dimensions. These dimension are not necessarily mutually exclusive. Each problem identified has effect in and is affected by many other problems. For clarity of thought, however, let us consider the problems using the following headings:

3.2.1 Historical Problems

(j) The imbalance in development of mathematics education in Nigeria was brought about by the introduction no Western Education in the North. This differential development of education in the country created many problems in the country. The problem of uniform planning, the problem of uniform curriculum, the problem of over production of teachers of mathematics in the South, the quota system of admission into Colleges, all stem from the differential curriculum development in the country.

The following suggestions are made. The Federal Government should make available money for provision of teachers and necessary facilities in those states that are backward in mathematics.

Another suggestion is the inter-state cooperation in employment of over produced staff. The mathematics teachers excessively produced in the South, should be encouraged to take up employment in the North.

(ii) Another historical contribution to our problems is the fact that the Colonial masters started a curriculum of Arithmetic in the country. Human nature abhors changes. And so, when 'new' or modern mathematics was introduced many people, including the products of Arithmetic were vehemently opposed to it. We have seen the controversies arising from the introduction and subsequent abolition of modern mathematics in the seventies. Nigeria is still groping with the problem.

3.2.2 Political Problem

One of the worst problems that militate against educational growth in Nigeria is political. Nigeria is running a Federal system of government but the bulk of the revenue is centrally controlled. But it should not be since education is State controlled in many areas. Radical political decisions militate against the growth of mathematics education. We all know the effects of the Federal Ministry of Education's decision on January 7th, 1977 to abolish over night the modern mathematics from schools. The effects of this one radical political decision is well summarized by Odili (1986, p.44) thus.

There was confusion as mathematics educators, State Ministries of education, textbook publishers, and especially serious minded classroom teachers were caught by this bombshell of an announcement. Some reluctantly continued with whatever programmes they were using, while others scrambled to find twenty-five years old "traditional textbooks".

One other radical political decision, which rocked educational foundation in Nigeria, is the introduction of the Universal Primary Education (UPE) without proper planning in the early seventies.

This brought with it the population explosion in the primary schools without corresponding expansion in infrastructure, teachers, book and money in the secondary level to take care of or absorb the primary outputs. Standard of education was brought down and students resorted to cheating in examinations or what was popularly know as "Expo".

One of the suggested solutions to this hush-hush programmes in education is proper planning and consultation at all levels. Fundamental issues must be developed and proper spade-work done before the launching of the new programme.

Another solution is to apply the method of piece-wise implementation of the programme. This method is being applied now in the philosophy of pilot schools being organized by the Nigeria Educational Research Development Council (NERDC).

Political interference in Universities is robbing the Universities the innovative leadership as an instrument of change. The Government should allow the Universities a free hand to continue their research undertakings.

Military intervention in governments and constant changes in the government and functions of arms of the government are set backs to growth of education/programmes. For example, in two years in one State, there were two changes in military governors, two changes in commissioners. Each commissioner comes with his/her new educational plans. There is no continuity. Textbooks are changed at will.

3.2.3 Socio-Cultural Problems

The diversity in ethnicity in the country creates educational problems. Values and priorities vary from ethnic group to ethnic group. One group may prefer Arabic, nomadic or cultural education while another group would introduce formal classroom western education. The outputs of these systems would definitely be different and this militates against the much desired unity and uniform development of the country.

Some cultural differences bring about student unrest. Student unrests are also propagated by religious differences. The consequences of the students unrests are:

(i) closure of schools.
(ii) loss of property, loss of life, loss of academic year and loss of manpower.

Some other social problems are created by the get-rich vales by all means in our country. This permeates all our societies. Students want to cheat in examinations for best grades and best jobs or places in the University. Parents give heavy sums of money to get their children admitted into Universities or obtain good grades in examinations.

Some suggested solutions may be for the Federal Government to ban religious politics and ethnic politics on campuses.

One of the social problems facing mathematics education in Nigeria is the type of job a student who studies it in Nigeria can do to earn a good living. The only job open to a good mathematics graduate (1st class Honours) is lectureship, Civil Service, industrial work or banking. In all these, he would depend upon his salary only for a living and this salary is impossible to give him a decent living. His business counterpart is swimming in money. He cannot employ himself or he cannot practice self employment like the engineer, EDUical doctor, or a pharmacist. Thus, many students feel (genuinely) that as mathematicians they have no future.

A solution to this is for the government to make the study of mathematics very lucrative, give scholarship to mathematics students and make the conditions of service for mathematicians as lucrative as those of EDUical doctors. Science allowance should be restored for mathematics teachers. However, in the long run there is no substitute for self-sufficiency and self-actualization. In that sense MAMSER was the answer.

3.2.4 Economic Problems

Economic problems are so real that in some states, for example, Imo State, the enrolment in schools is less than one third of what it was in 1976. With the introduction of schools fees under the guise of many names – education levy, equipment levy, education rate, Parent-Teacher Association levy – many parents were forced under the then economic crunch to withdraw their children from schools.

In the sixties, education benefited from large government budget allocation and foreign aids and oil revenues, and so money was not the problem. For the present the government is even poorer than some individuals and because of instability in the country, foreign aids and foreign investments are not coming as quickly and as largely as we would like. And even the emphasis is not on foreign aids but on self-reliance by all Nigerians and Nigeria. This is the message of Structural Adjustment Programme (SAP).

What we need to do now is keep costs of our educational planning low but the qualities high. To do this effectively, government must intervene and make conditions of service (not salaries) of workers correspond to the essential food price indicts.

3.2.5 Academic Problems

There is a group of problems, which we shall call academic problems. They include:

Poor student quality;
Student indifference to mathematics;
Poor teacher quality;
Teacher dis-satisfaction and indifference;
Unavailability, expense or inappropriateness of textbooks;
Non-application of appropriate curricular;
Poor leadership on the part of headmaster/headmistresses and principals;
Non-appreciation of mathematics teachers by the Nigeria society

These shall be discussed briefly. Many Nigeria students are defeated in mathematics from their first year in primary school. We can blame this on poor handling of the subject by many primary school teachers. After their first three years those teachers who inherit these students receive mostly students of poor quality in mathematics. The best of these teachers are able to do some reEDUial work. Others despair.

In any case most students become indifferent to the subject by the time they reach the junior secondary school. You should listen to those teachers who show some dedication to teaching mathematics at this level tell their tales of woe.

Student indifference is compounded by poor teacher quality; especially at the primary school. The beginning point in teaching subject well is knowing the subject. Unfortunately, many primary school teachers and some NCE and degree holders in our secondary schools do not know enough school mathematics. Some who know mathematics are not able to teach it. They frighten students through calling them idiots, fools and all kinds of names.

Then there is teacher dis-satisfaction arising from both the low status of teachers in the society and the poor treatment of teachers by those responsible for their welfare. For this and other reasons some of which are economic many teachers have become indifferent to the jobs and carry out their teaching functions half-heartedly.

Moreover, the unhealthy work environment of teachers has to be pointed out as a problem for all teachers in general and mathematics in particular. The classroom are unexciting and uninspiring. Some basic equipment which may be used to foster some mathematics concepts and process are not available.

The textbook problem has many dimensions. First, a few of the more popular primary and secondary school books now on the scene were written ahead of the curriculum and do not completely reflect the said curriculum. Second, the general economic condition makes some parents less able to purchase books at a time when prices seem to increase daily. Third, some books are not available when needed. It is also the case that in many schools if copies of the various curricula are available they will be stuck away in the headmaster, or principals office. Many who teach mathematics would neither have seen or used the appropriate curriculum. In a study of some states, copies of the curriculum were in the Ministries of Education but not in the schools.

And then our schools are poorly led. Principals / headmaster waste much time on routine administrative matters and moving between their schools and ministries of education / school boards. They do not pay enough attention to providing academics leadership in their schools. In particular, mathematics and science teachers have complained of lack of financial and other support of their classroom and club activities.

The final problem which we may discus under this heading is that both mathematicians and mathematics teachers are not appreciated in the Nigerian society, indeed, the society does not pay due regard to teachers. In the case of mathematics and science where teachers are few this lack of appreciation has had serious adverse effects. Teachers who have the opportunity quit at short notice for 'greener pastures;.'

Many of these problems have obvious solutions. For the quality of the students and the ultimate improvement of the primary school teacher, mathematics should be taught as a specialist subject in the primary school. To increase teachers satisfaction and motivate them more, greater attention has to be paid to improving the environment of work of the teachers-classrooms and working conditions.

Government is already looking into the issue of textbooks. A solution lies not in the production of books by government but in regulating the activities of publishing companies and providing necessary subsidies on raw materials.

3.2.6 Mathematical Problems

The mathematical situation can be subdivided into three distinct point of view (i) the acquisition of techniques (ii)the understanding of ideas, and (iii)the contents

3.2.7 Acquisition of Techniques

One may feel that a great deal has been done or that people are working hard to improve the efficiency of the acquisition of techniques. Also, many studies have been conducted by educational psychologists on alternative methods of teaching arithmetical process, and many textbook writers have this in mind. But one unfortunate thing is that all these have been happening in developed countries and we lack work on this based on local materials and culture. The textbooks we use are written by foreign authors who base their work on results from studies carried out in their own environment. These textbooks have their materials presented in a ready-made form which goes against thinking, discovery and originality. We need to develop teaching methods that will agree with our own setup. Here we have larger classes and lack equipment. Our approach must be to improvise and adapt methods.

3.2.8 Understanding Ideas

The situation in the field of understanding of ideas can be described as desperate. We talk of the need for the understanding of mathematics. Unfortunately, the present system of teaching mathematical information fails to involve this in most cases. Consider this statement made by an elementary five lady teacher-one of the products of the one-year UPE course in a reply to one of my question: "I carried out the teacher's instructions but the why and therefore was hardly ever clear." Again, most students in JSS Three could solve two simultaneous equations when written in this form:

$$2x + 3y = 45; \qquad 5x + 4y = 74$$

But they could not solve the same problem put this way: Find two numbers such that twice the first added to three times the second is equal to 45 and also such that five times the first added to four times the second is equal to 74.

Again, a student may be conversant with all the technicalities of linear equations without having much idea what sort of thing a linear equation is. That is, a student may feel that he understands mathematics when in fact he does not. Such a student will not appreciate the interconnection of the various processes he knows. The teacher may also have the impression that a student understands something when in actual fact he does not, as students have a quick way of giving standard answers to standard questions and this gives a false impression of knowing a concept. A simpler question reveals a different state of affairs. Although the question of the measure of understanding of mathematical concepts is mainly an expression of opinion since, at the moment, understanding cannot be tested or measured or the facts about it placed on a statistically valid scale, it is my impression that most pupils do not understand the mathematics they are taught.

3.3 CONTENT OF MATHEMATICS

Many people feel that the mathematics syllabus is defective because it is heavy and lengthy. This could be true of the old syllabus especially when we were using the compartmentalized system, but the newly approved secondary school syllabi cannot be called heavy. The greater shortcoming of the syllabus is that it does not provide hints and instructions for teachers' guidance. So without such details the teacher is placed in a difficult situation in trying to use the syllabus very effectively. It is necessary for the curriculum experts who develop these syllabi to suggest the centers of concentration for different topics, the use of aids and devices, the connected practical work, project work, etc. Any such syllabus has to be comprehensive, so that the teachers all over the country can get some help from it. The statement of curriculum should not only be a collection of topics, but should define the actual procedures to be adopted for their effective teaching. The application and utility of mathematical content in actual life should also be mentioned side by side.

The confusion caused by the use of the terms "modern mathematics" and "traditional mathematics" during the late 1960's are both

unfortunate and misleading. For most schools in Nigeria were neither running the traditional mathematics syllabus nor the modern mathematics syllabus. Some work on set theory and number bases only were taken as modern mathematics while the remaining content was still being introduced in the traditional way. Yet these students were expected to sit for an examination based on the modern mathematics syllabus. Nevertheless, the new syllabus for the whole Federation had been prepared to take care of this misconception.

3.3.1 Pedagogical Problem

Considering the pedagogical problems one can see that in almost all schools in spite of the much talked-about modern approach, the traditional classroom approach prevails. In all the classes in schools around, much of the teaching is being directed at an average ability or rather at a level slightly above average: too fast a pace for the slower student and not enough scope is offered to the brilliant child. In an attempt to solve this problem, I suggest a new approach with emphasis on group and individual work. For it is clear that an essential factor in any learning process is the child's own readiness at any particular stage. Furthermore, the section on teaching method gives a variety of approaches that will help in overcoming the present problems.

3.3.2 Teaching Aids

There is a serious lack of mathematical teaching aids in our schools. According to the fingers of Piaget, up to the age of about twelve or thirteen (in our own culture up to fifteen) most children need the help of concrete materials to accompany their thinking. Actual objects have an appeal for the young student. He must be provided with things to see, touch and handle. There should be an adequate provision of concrete materials in the classroom. Without it, the subject will become very abstract. The establishment of a mathematical laboratory will remove this defect. Although it is mainly the job of the Management and higher authorities to provide all this, the teachers' enthusiasm towards its establishment is also wanting.

3.3.3 Classroom Organisation

A number of factors complicate the problem of classroom organization, such as, the number of children, the size of the room, the furniture, the availability of space near by and facilities.

In all our schools, no class has fewer than fifty students, one wonders how a single teacher could take care of fifty students at a time. In most cases the rooms are too small and poorly ventilated. It becomes difficult for the teacher to establish any close individual contacts with the students. Two major problems emanate from the above: how to arrange the children into suitable learning groups and how to provide them with materials with which to explore.

It may be advisable to limit the number of children exploring mathematics to half the class, while the other half is occupied at one side of the room with some written or other quiet work. This has the double advantage of creating more floor space, and having fewer students to supervise. A flexible timetable is required for this arrangement as, of course, the two halves of the class will change places later during the day. Better still is the need for providing modern tables or fixed work benches along one or more walls. The children's books can be accommodated in locker trays. This arrangement has the outstanding advantage that the student's desk is no longer considered a personal possession or fixed forms and benches: he does not need to work at a particular desk but can do so at any surface which best suit a teacher's arrangement. Tables should be light, easy to move and without shelves or drawers. Easy access to all equipment is an important factor in classroom organisation.

3.3.4 Teacher's Competence

It is a common defect in our educational setup, that, most of the subject teachers are not adequately qualified in the subject concerned. It is worst in the teaching of mathematics. Without proper qualifications and proper training, they fail to do justice to the subject. A teacher may be able to show good examination results in spite of his weak standing in the subject generally, but even that is not a sufficient criterion to allow him to continue with the teaching of mathematics. An adequate, high qualification developes self – confidence in the teacher and serves as a source of inspiration to his students.

The teacher's burden is usually great and, as a result, he may tend to follow the way of least line of resistance. He cannot adopt and prepare for effective methods as he has very little time available for him. His teaching load should be lightened to enable him to show his originality and initiative. The economic position of the teacher is not good enough. He is always worried about his financial position and a worried teacher cannot give of his best to the learners. He is tempted to engage in part-time activities to supplement his income. Consequently teaching suffers. He is a frustrated, discontented and half-hearted worker.

About ninety-five percent of the practicing teachers I chat with during my teaching practice supervision show no interest in the teaching of mathematics. Some said that they were forced by circumstances to take up teaching. They were always on the lookout for a better job and ready to leave the profession as soon as they got an opportunity to do so. As a result, they were not at all enthusiastic about their work and are not willing to consider new idea or question past theories or develop any personal approach of their own. How then can such teachers arouse interest in the pupils?

I, therefore, suggest that the Federal and State Government's should address themselves seriously to the question of mathematics teachers. A careful analysis of the statistics as they relate to students in secondary school who should be taught mathematics and the teacher that are available to teach them in a meaningful way will, I hope ,convince anyone that there is need to tackle the problems in the following consideration . First ,there should be a crash programme for training large numbers of mathematics teachers for secondary school. However, this should not be like the UPE teacher training programme. It is good to note that the National Teachers Institute (NTI) has taken up this work creditably.

Secondly, there should be regular weekend vacation and longer-term courses for teachers to update their knowledge and teaching methods. These courses should, wherever possible, lead to certificates or diplomas and there should be some means of rewarding deserving teachers either on obtaining certificate or diploma or after an inspection of their work or both

Thirdly, teachers who are qualified to teach mathematics and who have proved their worth should be retained as "classroom teachers" but given whatever promotion is due to them.

Finally, the professional training for the mathematics teachers as of now has its own shortcoming. One would expect our colleges of education and universities training mathematics teachers to spend more time in curriculum development and methodology of their subject than with courses in educational foundations. But the contrary exists now. According to S.T Bajah, "At the University of Ibadan Nigeria, physical science pre-service teachers spend what amounts to nearly four-fifths of their professional training as foundation courses" (Bajah, 1975). A lot of improvement is required in this area, so that our products will be able to tackle the multiple problems arising from the teaching of mathematics.

3.3.5 Psychological Problems

We shall now examine the psychology situation in the whole process of teaching mathematics. The classroom teacher will notice that the solution to most of the problem he encounters in the class could be found from researches conducted in the field of psychology. It then follows that the teacher should know something about psychology as it applies to mathematics education.

3.3.6 Individual Differences

The growing awareness of the presence of students' individual differences in learning potential and performance through the years has been, and may continue to be, a major curriculum problem secondary school programme in mathematics education. It is clear that this problem intensifies on arrival at the secondary school and becomes more marked as they progress. Here these differences are found in mental ability, the ability to reason or think reflectively, and to solve problems. This problem classified the students into three major divisions-slow learner, the average and the academically-talented students. Besides the so-called 1.Q., it is known that there exist some specific mathematical factors-the ability to use symbols, to do logical reasoning, to compute with accuracy and speed and to appreciate spatial relationships.

The question now is how can every student be given the opportunity to display his abilities as fully as possible, be he a quick or slow learner and what mathematics programme is the most significant for each of the three groups? Is there a close similarity or a wide differences in the content of the programme for each group? Both psychologists and educational experts tend to agree that basic content must be the same for all three groups. But they must vary in amount, method of organization and presentation, and attention. This requirement is not specific to mathematics teaching alone, but it is of great importance as the effect is more marked here.

Here are some ways of providing for individual differences

i. Students should work individually or in small groups on more or less different tasks. Each student should be allowed to choose what to do and given enough freedom to do it the way he wishes. The role of the teacher is to control and correct his errors. This encourages the students to be independent, autonomous and self-reliant at work. Their morale is increased because they are given room to make their choice. They are more likely to progress at their own pace.
   Alternatively, students of similar ability are grouped together. At times, groups are mixed and one child in each group is expected to act as a leader. Friends can be grouped together since the gifted is rarely patient enough to help his slower learner unless they are friends.
ii. The class may be divided by the teacher into two, three or four groups, based on their ability or achievement. While the teacher

is busy with one group the rest of the class are engaged in individual or group work. The division is made flexible to enable adjustments from time to time.

iii. The streaming of pupils into special classes according to ability has a similar aim. In some schools the best students are placed in form A while the second best group in Form B and so on. But some schools prefer to use interest. For example, the science-inclined students are placed in one class while Arts students are in another.

3.3.7 Motivational Variables

The problems of motivation arise from the attempt to create and maintain interest in the teaching of mathematics in secondary School. Motivational factors can be divided into those that are intrinsic or self-imposed and those that are extrinsic or externally-imposed. Some mathematics educators have described the externally-imposed motivation as exerting an unfavourable influence on the natural process of mathematics learning. Unfortunately, the present situation attaches importance to rewards and punishment. We are advised not" … to use a teacher-imposed external reinforcement schedule to determine what a child thinks, how he answers a question or how he attacks a problem". (Davis, 1964).

The most powerful and enduring source of motivation, therefore, lies within the intellectual curiosity of the student. Curiosity leads him to discover answers to questions. The internal urge to discover mathematical relations, principles and facts arise from intellectual curiosity about them. Discovery, in turn, arouses further curiosity. Two forms of reinforcement emerge from this (i) intrinsic rewards derived from solving a problem, and (ii) the reward derived from being able to tell your classmates or your teacher about what you have just accomplished. But the discovery method has some limitations for not all mathematics can be taught in this way. So we have to try other sources of motivation.

According to Polya: "The interest of the material to be learned should be the best stimulus to learning and the pleasure of intensive mental activity should be the best reward for such activity. Yet, where we cannot obtain the best we should try to get the second best, or the third best…" (Polya, 1965, p.103). So we may consider interest as having its source both within and outside mathematics. Students could derive interest in mathematics as a career. Before 1940, there were not much business or industrial job opportunities for the professional mathematicians. But presently the demand for professionally-trained mathematicians in business and industry is greater than the supply. So a variety of rewarding careers that are now available in mathematics will arouse interest in secondary school students.

Furthermore, the teacher can stimulate interest in mathematics by pointing out its applications in many fields of work. As mentioned in above, student must be made to understand the role mathematics will play to influence opportunities and requirements in different fields. Mathematical recreations can help to relieve boredom. This can take many forms: puzzles mosaics and tessellations, dissections and paper folding, etc. Mathematics clubs provide good environment for stimulating student interest. The programmes of the clubs activities are planned to satisfy the student's interest.

Teachers of mathematics, unlike their counterparts in English, Geography, Science, History and the Social Studies, have been slower in utilizing the 'eye and ear' appeal as a means of stimulating interest. The chapter on teaching aids discusses fully the use and contributions of multi-sensory aids to mathematical learning. However, from the standpoint of motivation alone, its importance is so great that it must not be overlooked.

Finally, we have agreed that the most difficult problems the teacher encounters is on motivation. We have also discussed the means and devices which will be of help in tackling these problems. But the most important factor in solving these problems is a sympathetic, well-inforEDU, competent and inspiring teacher.

3.3.8 Backwardness

These lead us to two types of backwardness – (i) General Backwardness and (ii) Particular Backwardness. General Backwardness- This means all-round backwardness. This is as a result of the so-called low I.Q. which may be between 50-70. supporters of this, claim that it could be raised by about 20% (Durojaiye). The removal of this is a combined responsibility of all the teachers in the school. But the mathematics teacher will be expected to pay special attention to his subject. The Russian psychologists who have rejected the idea of I.Q. have a better approach to this problem. They consider teaching to be a direct attack on teachability. Students of low teachability attain the required level of mastery of school material if they are given supplementary exercises.

Particular Backwardness-Here the student is backward in mathematics but fairly intelligent in other subjects. The removal of particular backwardness is easier than that of general backwardness. The student's performance in other subjects is a source of confidence and inspiration for him. This backwardness is the sole concern of the mathematics teacher. More serious cases may have to be treated in separate classes whereas borderline cases may be tackled in the classroom itself.

It becomes necessary, therefore, that teachers should have good knowledge about the causes and possible reEDUies. The backward student may have some physical deficiencies, like poor eyesight, defect in the hearing, headache or any other physical ailment which invariably does not allow the student to concentrate on his studies. Mathematics requires special concentration. The solution to these causes lies with a doctor or physician but some sort of physical exercise could be of help to the student. It is the responsibility of the teacher to refer the student to the hospital, depending on the nature of the case. In case of a serious physical deficiency, the teacher should try to persuade the parent to get their ward admitted to the school for the handicapped pupils.

Another possible cause of backwardness often mentioned may be mental. These causes are either inherited or influenced by environmental factors. The students may be of low teachability, or have some mental ailment,.psychological dissatisfaction, domestic problem, mental conflict, sense of insecurity, inferiority complex lack of interest in mathematics or the interest is dominated by something else. When the mental problem is simple, the teacher can tackle it with some probability of success. But serious complex cases should be referred to a psychologist or a psychoanalyst for thorough diagnosis for the pupil will go a long way in helping to treat such cases.

Furthermore, a students that has a distaste for the subject is likely to be backward. This could be acquired as a result of a teacher's attitude. If he fails to establish a rapport between his student and himself, the student tends to have a distaste for the subject. The quantity of work and how it is presented to the students may also tend to produce this distaste. A good taste for the subjects could be developed through the teacher's patience and persistence. We should avoid hasty conclusions about student's performance. At times the influence of the home may also create this distaste. Some parents would sometimes say that they never liked mathematics or they never wanted to study it or that failure in mathematics has been a tradition of their family, thereby giving negative suggestions to their children. Parents should avoid this and should be made conscious of their duty in this matter by the teacher.

The way the teacher maintains discipline in the class may cause backwardness. If he is too lenient, then some clever and mischievous ones may take undue advantage of it by not paying attention, the resultant effect is backwardness. Where he is very strict, the feeble-minded students may get unhappy and discouraged. They may develop a negative affection for the teacher and consequently for the subject.

Again, the teacher should not hesitate to explain the fundamentals of the subject over and over if required. If not the student may develop some doubts about the fundamentals which may lead to backwardness. The mathematics teacher's personality in the school and neighbourhood may also affect some of the students adversely.

A change of school by the student as a result of parents'/guardians/transfer or even a change in subject teacher may affect the student's performance. Whenever the change of school is unavoidable, the parents and the teachers must keep the student under close watch for the adjustment period. School administrators should avoid changing the teacher in the middle of a session as far as possible.

Enough attention must be given to the slow learners so that they will not feel they are being disowned or being left behind. At times it may require a teacher devoting extra time and attention to such students. The teacher should not overlook their need.

A student may develop some backwardness if he is continuously irregular or absent for a long time. Provision could be made for extra coaching for some time to give such students the opportunity to fill up the gaps and come up with other students of the class. Such temporary backwardness should not be allowed to last very long. Where the students involved are many, extra classes could be arranged for them. This has yielded positive results in most of the schools where the practice is to organize extra classes during the holidays or close to the examination.

4.0 CONCLUSION

In this unit, you have learnt some of the problems confronting mathematics instruction in secondary schools. Some solutions to these problems were also suggested to guide the students and the teachers
.

5.0 SUMMARY

In this unit, you have been exposed to some of the problems facing mathematics education in the country. Some suggestions have been made as to their solutions

- A few problems identified in this unit are:

(a) The inmbalance in rate of mathematics education growth between the North and South. Solution suggested: Allow the states sufficient fund to develop at their own rate.

(b) Political problems included constant changes in the governments among the governors and in the commissioners. This breeds discontinuity, waste of resources and corruption.

(c) Socio-cultural problems include, enthnicity and its attendant nepotism, quota system and distrust, and lack of lucrative jobs for mathematics graduates in the country. The society looks down on mathematics graduates because "they are poor"

(d) Economic problems include inability of parents to buy textbooks, pay school fees and feed the children well. Poor economic situation is also affecting the production of textbooks and teaching aids.

(e) Academic problems include poor, insufficient teachers, poor infrastructure poor teaching/learning environment, nature of mathematics as a language and poor method of teaching in schools.

6.0 TUTOR MARKED ASSIGNMENT (TMAs)

1. State and explain one problem facing mathematics before independence in Nigerian.

2. Effects of "modern" mathematics in Nigeria is NOT as adverse as the introduction of Universal Free Primary Education. Make two sentences to support or oppose this statement.

3. Name and explain one political problem in Nigeria

4. Suggest one solution to the problem of text books in Nigeria

5. State one problem posed by the nature of mathematics itself

6. Suggest two ways we can employ MAMSER to solve some of our educational problems in mathematics

7. Suggest one problem facing the teaching of mathematics left out in this unit.

SELF ASSESSMENT EXERCISES (SAE)

Self Assessment Exercises 1

Suggest one solution to the problem of imbalance in mathematics education brought about by introduction of Western education in the Southern and Arabic education in Northern part of the country different from the one suggested in this unit.

Self Assessment Exercise 2

Suggest one adverse effect it might have if the Federal Government allows total freedom to the institutions of higher learning in the country.

7.0 REFERENCES/FURTHER READINGS

Odili G.A *Teaching Mathematics in the Secondary Schoo,* Anachuna Educational Books 1986.

Bajah S.T, - "*Preparation of Secondary School Teachers of the Physical Sciences for the African Environment":* West African Journal of Education: Vol. XIX, No. 1, 1975, p.93)

Butter, Wren and Banics-"*The Teaching of Secondary School Mathematics:* McGraw-Hill Book Company, New York, 1970.

Davis, R.B.-"*Discovery in Mathematics, Addison Wesley,* 1964.

Johnson and Rising –"*Guidelines for Teaching Mathematics*- Wadsworth Publishing Company, Belmont, Califonia, 1972.

Peters, R.S. *"Ethics and Education"* George Allen and Unwin Ltd. 1966.

Piaget, J. (1) *The Child's Conception of Number*
(2) The Child's conception of Space

Polya, G. – "On Hearing, Teaching and Learning Teaching": American Mathematical Monthly, Vol. 70, p. 605.

Scopes, P.G? *Mathematics in Secondary Schools – A teaching Approach*-Cambridge at the University Press. 1973.

Servais, W and Varga T-*Teaching School Mathematcs" a* UNESCO sources books Pengium, 1971.

Wheeler, D.K,: "Curriculum Process": Hodder and Stoughton Ltd, Kent, 1967

UNIT 4 BRIEFS ON SOME PAST REKNOWNED MATHEMATICS

CONTENTS

1.0 INTRODUCTION

In this write-up, it may not be possible to write about the contributions of all or even most of the great mathematics in history.

You will, however, be exposed to the contributions of eminent mathematiccians whose work greatly affected the mathematical concept being commonly used in our secondary, primary and teacher training schools today.

You will be introduced to the great mathematician, Thales of Miletus and his deductive geometry; Pythagoras of Alexandra and his geometric theorems; Euclid and his postulational geometry; ArchiEDUes of Syracus and his calculus; John Napier and his invention of Logarithm: Fermat and Paschal and their modern theory of probability.

2.0 OBJECTIVES

By the end of this unit, you should be able to:

1. write and explain at least one important contribution to mathematics of each of the following; Eucid, Thales, Pythagoras, Napier, Fermat and Pascal; and

2. describe accurately the historical origin and development of the following mathematics concept: number and numeration, postulational and axiomatic geometry, logarithm, and calculus, and theory of probability.

3.0 MAIN BODY

3.1 STUDY APPROACH

In learning this unit the following point may be closely followed:

1. Read through the paragraph straight on jotting down new words unfamiliar to you.
2. Read through the paragraph once more.
3. Close the page and write at least one central idea about the paragraph.
4. Go back and check your central idea. If correct go to the next paragraph.
5. After going through the whole unit, then answer the questions at the end and check the suggested answers and score yourself.
6. If the score is too low (lower than 60%) go through the unit once more.

3.2 DISCOVERIES ON THE BASES OF MATHEMATICAL DISCIPLINES

The study of the history of mathematics is very important. It is not an overstatement to say that the history of mathematics is the history of civilization. Its study can teach us about history, about mathematics, about modern civilization and about man. But one sad thing is that history of mathematics has not got its rightful place in the curriculum, just because no time is given to it for there are many heavy courses to be covered in mathematics.

I believe that once introduced, it will become a source of fascination to the learners. To study previous mathematics demonstrates that modern mathematics did not suddenly appear in textbook form, but has evolved through a series of inspiration, errors, sidetracks and practical need. It shows that all the branches of mathematics were developed in relation to one another. So it makes the learner to avoid compartmentalization. Also in mathematics we can learn about the evolution of ideas, the unity and the status of mathematics. It creates the awareness of the interplay between mathematics and society. Mathematics was influenced by the changing needs of society but recently this has been reversed and mathematics dictates the pace of civilization.

Teachers will present mathematics as a dynamic and progressive subject, full of human interest. It does not only remind us of what we have, but will teach us how to increase our store of knowledge. It also warns the learner against hasty conclusions. Some mathematics topics are better introduced in the class by discussing their history. Students will be made not to attack an unsolved problem by the same method which has led other mathematicians to failure.

3.2.1 Thales of Miletus (624-548BC)

Thales is popularly referred to as the first of the seven wise men. He had advantage of travelling to the centers of ancient civilization. In Egypt he learned geometry, in Babylon, Thales came in contact with astronomical tables. The success of Thales' prediction of the eclipse of 585BC brought him to fame. He was unusually regarded as a clever man and the first philosopher.

Thales made proposal which is now known as the Theorem of Thales – that an angle inscribed in a semi circle is a right angle. He was the originator of deductive organization of geometry. Thales proved many theorems among which are the following:

1. A circle is bisected by a diameter.
2. The base angles of an isosceles triangle are equal.
3. The pairs of vertical angels forEDU by two intersecting lines are equal.
4. If two triangles are such that two angles and a side of one are equal respectively to two angles and a side of the other then the triangles are congruent.

There is no document from antiquity to certify that Thales did all these, but tradition attributed them to him.

Evidence of his wisdom is recorded by Aristotle who reported that Thales made fortune by "cornering" the olive presses in a year in which the olive crop promised to be abundant. He was said to have calculated the distance of a ship at sea through the proportionality of sides of similar triangles. Thales was the founder of the logic schools.

3.2.2 Pythagoras of Samos (580-500 BC) Pythagoras was the "Father of Greek Geometry"

Pythagoras traveled to Egypt and Babylon and to India and absorbed not only mathematics but also astronomical information. He settled at Croton and established a secret society in which he was a prophet and a mystic. He established the Pythagorean School with strict order of secrecy code of conduct. In this school, Pythagoras went in pursuit of philosophical and mathematical studies. The very words "philosophy" and "mathematics" are believed to have been coined by Pythagoras himself to describe his intellectual activities. He is said to have given two types of lectures. One for members of the school or order only and the other for those in larger community or outsiders. It was to the inner Pythagoreans, he revealed most of his mathematical discoveries.

Pythagoras was many things to his admirers- the philosopher, the astronomer, the mathematician, the abhorrer of beans, the saint, the prophet, the performer of miracles, the magician, the purifier of souls.

The motto of the Pythagoreans is that "All is number" and their emblem is a figure of a five pointed star. Some of the number works by the Pythagoreans are:- the classification of numbers into odd and even, prime and composite, perfect and deficient numbers whose sum of their divisors are equal to the numbers themselves. Example. 6 is a perfect number because its aliquot divisors are1, 2, and 3 and 1+2+3=6.
Note that 6 can divide 6 but it is not an aliquot divisor of 6.
28 is a perfect number because 28 = 1 + 2 +4 +7 + 14. Two numbers are amicable if the sum of aliquot parts of one gives the other number. Example: 220 and 280 are amicable number. These amicable numbers are built into talismans and used for love charms by the Pythagoreans.

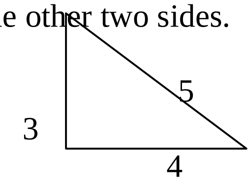
On discovery that gave the Pythagoreans a shock that rocked their beliefs that all could be explained in terms of pure numbers was their discovery of irrational numbers. Pythagoreans discovered that it is impossible to express $\sqrt{2}$ as the ratio of two integers. They even proved this in this way.

Proof that $\sqrt{2}$ cannot be written in the form p/q. Let it be possible that $\sqrt{2}$ can be expressed as a ration of two integers p/q i.e. let $\sqrt{2}$ =p/q where p and q have no common factor except 1.
Squaring both sides we have
$2 = p^2/q^2$ and so $2q^2 = p^2$
Then $2q^2$ is an even number
.: p is also an even number.
Let $p = 2r$ where r is an integer,
then $p^2 = 4r^2 = 2q^2$

or $2r^2 = q^2$
:. By similar argument *q* is also an even number.
:. *p* and *q* are even and so must have a common factor, contrary to the assumption.
:. →2 cannot be in the form *p/q*

The discovery of these incommensurable magnitudes was kept secret by the Pythagoreans. Legend had it that Happasus of Croton who revealed its discovery was drowned by the Pythagoreans. With this discovery of incommensurable magnitudes, the Greek mathematics concentrated from then, on almost entirely on geometry to the neglect of algebra and arithmetic. It is still the belief of the Pythagoreans that the essence of absolute truth is in geometry. Even algebra was transforEDU into geometric facts of simple equations, like $mx = ab$ where *a, b and m* are constants were solved by geometric constructions.

One everyday concept that was discovered by the Pythagoreans was the Pythagoreans theorem:
that in any right angled triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
Example: From the figure.



$5^2 = 3^2 + 4^2$

In geometry, there are infinite sets of Pythagorean triples. Examples are (6, 8, 10); (9,12,15) and (5, 12, 13).

3.2.3 Euclid of Alexandra (300BC)

Very little record was kept about Euclids' life. Even his birth place was not recorded. His fame came from his books on Geometry, published when he was a teacher in schools in Alexandra.

One of his most successful books, in fact, a very famous book is the book called "Euclid's. Elements: He is an author of other publications such as "The Data". The Division of Figures, the Phaenomena and the optics. But of these the Elements gave us the Euclidian geometry in our schools today.

The Elements is divided into 13 books or chapters; the first six are on elementary plane geometry; the next three on the theory of numbers. Book 10 is on the incommensurables and the last three are on solid geometry.

Euclid's theories on elementary geometry are based on five postulate and five common notions or axioms. Examples of postulates are: Let the following be postulated:

1. That a straight line is completely determined by two points.

2. That a straight line can be produced infinitely in both directions.

3. That a circle is determined completely by any three points not in a line.

4. That all right angles are equal.

5. That if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles the two straight lines of produced indefinitely meet on that side on which the angles are less than two right angles.

The common notion (or axioms) are:

1. Things equal to the same thing are equal.

2. If equals are added to equals the wholes are equal.

3. if equals be subtracted from equals the remainders are equal.

4. Things which coincide with one another are equal to one another.

5. The whole is greater than the part.

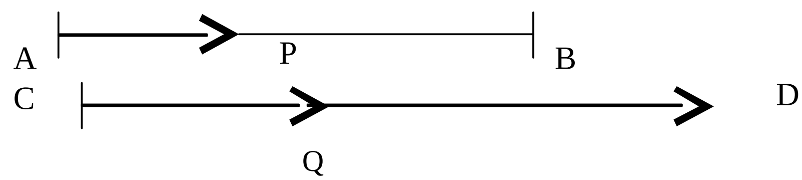
Euclid elements reigned without any challenges for many centuries. For 2000 years mathematicians tried in vain to prove the fifth postulate of Euclid. Attempts to prove the fifth postulate, which is commonly called the parallel postulate, led to the discovery by Lobachevsky (1793-1856) – a Russian mathematician, of the non-Euclidean geometry.

Most of the propositions in Book 1 of the Elements are well known to any student who has had a high school course in geometry. In it are congruent theorems, simple constructions using straight edge and compasses inequalities concerning angles and sides of a triangle, on properties of parallel lines leading to the fact that sum of angles of a triangle is two right angles, and parallelograms. The Book 1 closes in propositions 47 and 48 with the proof of the Pythagorean Theorem and its converse.

3.2.4 Napier John (1614-1420BC) and the Invention of Logarithm

John Napier was the first to publish his book (Mirifici Logrithmorum Canonis Descriplio i.e. A Description of the Marvelous Rule of Logarithms in 1614A.D. Johnst Buegi of Switzerland worked also on logarithms about the same time as John Napier but Napier's book came out first. Napier was not a professional mathematician as such, but his interest in writing certain aspects of mathematics – those relating to computation and trigonometry was high. Napier's "role" or "bones" were sticks on which items of the multiplication tables were carved in forms ready for multiplication. His other works were the "Napier's Analogues" and "Napier's rule of circular parts" which are devices to aid memory on spherical trigonometry.

Napier has no concept of a base for his system of logarithms is different from our logarithm of today. Napier's logarithms is explained in geometric terms as follows:



Let line segment AB of length 10, 000,000 and a ray CD be given as shown in diagram. Let a P point start from AB with variable speed decreasing a proportion to its distance from B. At the same time let Q start from C and move along CD with uniform speed equal to the rate with which P began from A.

If when P is at P then Q is at Q, then the variable distance CQ is the logarithm of the distance PB.

From this log 10,000,000 = 0 since at the start P is at A when Q is at C.

From this definition, CQ increases as PB decreases i.e. for Napier, his logarithm increases as his number decrease. But in our own base 10 logarithms, the logarithm increases as the number increases. This is so because Napier worked using a base 1/e which is less than 1.

In 1615, professor Henry Briggs – a professor of geometry at Oxford visited Napier to discuss Napier's Logarithms. In 1617, the year Napier died Briggs published his book on base 10 logarithm. This include a table of logarithms for numbers from 1 to 1000. This was extended to include common logarithms for numbers form 1 to 2000 and from

90000 to 100000 in a separate publication of 1624, from this later publication the words “mantissa” and “characteristic” are derived.

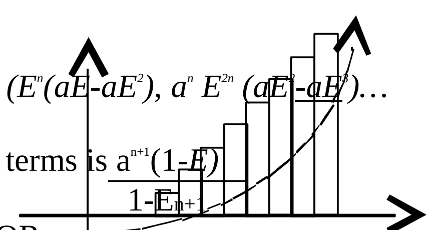
3.2.5 Fermat Pierre De and Differential Calculus

Fermat was a lawyer by profession, but by 1629 AD,he began to make discoveries of capital importance in mathematics. He discovered many theorems in analytic geometry one of which was in 1636 that “whenever in a final equation two unknown quantities are found, we have a locus, the extremity of one of these describing a line, straight or curved”. Most of his works were however published after his death.

Fermat had not only a method of finding the tangent to curves of the form $y=xm$ but he also in 1629, hit upon a theorem on the area under these curves – the theorem that Cavalieri published in 1635 and 1647.
Fermat’s method may be simplified as follows:
Let the curve be $y - x_n$
Let the area required be from
$x = 0$ to $x = a$

Fermat subdivided the area into infinite small rectangle by taking the points with abscissa $a, aE, aE^2, aE^3 \ldots$ where E is a quantity less than one.

The sum of all these rectangle is approximately equal to the area under the curve.



These areas are $a^n (a-aE), a^n (E^n(aE-aE^2), a^n E^{2n} (aE^2-aE^3) \ldots$

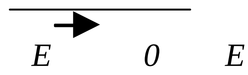
The sum of infinity of these terms is $\frac{a^{n+1}(1-E)}{1-E^{n+1}}$

OR $\frac{a^{n+1}}{1+E+E^2+E^3+\ldots+E^n}$

As E tends to one, the sum of these areas approach a limit $\frac{a^{n+1}}{n+1}$

which is the area under the curve $y = x_n$ *for* $x = 0$ *to* $x = a$. This work is the same as the process now known as differentiation for it is equivalent to finding.

Lim $F(X+E)-F(X)$



3.2.6 Blaise Pascal and the Discovery of Probability

Pascal Blaise was a gifted child in mathematics just like his father, but he abandoned mathematics for theology. But his father encouraged him in mathematics. At the age of sixteen in 1640 young Pascal published one of the most fruitful pages of history in “Essay Pour Les Comques”. This in essence states, that opposite sides of a hexagon inscribed in a conic intersect in three collinear points.

In 1654, his friend Chevalier de Mere, gave Pascal a problem to solve thus: “In eight throws of a die, a player is to attempt to throw a one, but after three unsuccessful trials, the game is interrupted, what is the probability of winning for each player?

Pascal wrote to Fermat on this problem and their resulting correspondence become the effective starting point for the modern theory of probability although neither Pascal nor Fermat wrote up their result. Their results were publication in 1657 by Huygens in a book “De ratiociniis in ludo aleae” (or “On Reasoning in Games of Dice”). Pascal was not the only person who worked on probability. We had works of Cardon a century before but these works were overlooked.

Pascal connected the study of probability with the arithmetic triangle which had been in existence for over 600 years before Pascal, revealing many properties of the triangle. Because of the numerous findings by Pascal about the arithmetic triangle, it is now known as Pascal’s triangle. Pascal originated and proved theorems about the triangle. In one of the proofs, Pascal in 1654, gave an eminently clear cut explanation of the method of mathematical induction.
Let us examine the Pascal’s triangle a little.

1
1 1
1 2 1
1 3 3 3 1 Row four
1 4 6 4 1
1 5 10 10 5 1

Take for example row four as shown. The sum of the numbers in row four is 1+3+3+1=8.

If a coin is tossed 3 times (one less than the row number) the chances are 1/8 that three heads will occur, 1/8 that exactly two heads will occur, 3/8 that exactly one that and 1/8 that no head will occur.

The theory of probability attracted many mathematicians in the early 18th century. One of them was Abraham De Moure who published more than 50 problems on probability as well as questions relating to life annuities. He discovered the theory of permutation and combinations from the principles of probability.

Probability theory grew into a very useful subject having applications in engineering, games of chance, science and business.

3.2.7 Popular Mathematical Quotations

This unit contains quotes on Mathematics in general.

3.3 Mathematics Is The Science Of Quantity.

Aristotle (BC 384-322)

3.3.1 This, therefore, is mathematics: she reminds you of the invisible form of the soul, she gives life to her own discoveries; she awakens the mind and purifies the intellect; she brings light to our intrinsic ideas; she abolishes oblivion and ignorance which are ours by birth.

Proclus (410-485)

3.2.2 Neglect of mathematics works injury to all knowledge, since he who is ignorance of it cannot know the other sciences or the things of this world.

Roger Bacon (1214-1294)

3.2.3 The advancement and perfection of mathematics are intimately connected with the prosperity of the state.

Napoleon I (1869-1821)

3.2.4 The moving power of mathematical invention is not reasoning, but imagination.

Augustus De Morgan (1806-1871)

3.2.5 In most sciences one generation tears down what another has built and what one has established another undoes. In mathematics alone each generation adds a new storey to the old structure.

Norman Hankel (1839-1873)

3.2.6 Mathematics started as a useful activity, and today it is more than it ever was. This is, however, an understatement. One should say: if it were not useful, mathematics would not exist.

Hans Fraudenthai in 1973

3.3 Popular Mathematics Anecdotes

1. Thales (649-550BC) was an ancient geometrician and astronomer. A story about him runs like this. One night, when walking out, he was looking so intently at the stars that he tumbled into a ditch. An old woman who was also walking beside him exclaiEDU:

    "How can you tell what is going on in the sky, when you can't see what is lying at your own feet?"

"A short account of the History of Mathematics" by W.W.R. Ball.

2. Felix Klein (1849-1925) presided over the mathematics department at Gottingen for a long time. A favorite joke bout him, in circulation during his late years, was the following:

    In Gottingen there are two kinds of mathematician – those who do what they want and not what Klein wants, and those who do what Klein wants and not what they want. Klein is not either King. Therefore, Klein is not a mathematician.

    Hilbert" by Constance Reid

3. Once when lecturing to class Lord Kelvin (1824-1902) used the word 'mathematics', and then interruption himself, asked his class: "Do you know what mathematics is?". Stepping to the blackboard he wrote upon it.

    Then putting his finger on what he had written, he turned to his class and said: "A mathematician is one to whom that is as obvious as that is as obvious as that two plus two makes four is to you. Liouville was a mathematician."

    "Life of lord Kelvin, "by s. f . Thomoson.

4. The death of Abraham De Moiver (1667-1754), famous for the theorem that bears his name, has a certain interest for psychologists. Shortly before his death he declared that it was necessary for him to sleep ten or twenty minutes longer

everyday. The day after he had reached the total of twenty-three hours, he slept exactly 24 hours and passed away in sleep.
` ‘’History of Mathematics” by W.W.R.Ball.

5. Albert Einstein (1879-1955) and Niels Bohr (1885-1962) were once arquing about quantum theory. The former would not accept the probabilistic approach. At best, he was prepared to accept the existence of quantum theory as a temporary expedient. “God does not throw dice” was his unshakable principle, to which Bohr retorted, “Nor is it our business to prescribe to God how He should run the world.”
“Physics and Seyond” by Werner Heisenberg

6. Srinivasa Remanujan (1887-1920), Who has been described by many as the most romantic mathematician of this century, was in love with each integer. When he was lying in a London Hospital one evening Professor G.H. Herdy (1377-1947) visited him. The latter told him that he came in a cab which had the number 1729 with the remark that the number seeEDU ratter dull to him. “No”,
Romanujan replied, “it is a very Interesting number.

It is the smallest number expressible as a sum of two cubes in to different ways.”

For example 1729 = $12^3 + 1 = 10^3 + 9^3$
= 144 (12)+1=1000+81(9)
= 1728 +1=1000+729
= 1729 = 1729

7. Augustus De Morgan (1806-1871) was explaining to an actuarian what was the chance that a certain proportion of some group of people would at the end of a given time be alive; and quoted the actuarial formula, involving II, which, in answer to a question, he explained stood for the ratio of the circumference of a circle to its diameter. His acquaintance, who had so far listened to the explanation with interest, interrupted him and exclaiEDU: “My dear friend, that must be a delusion; what can a circle have to do with the number of people alive at a given time?”
“Budget of Paradoxes” by Augustus De Morgan

4.0 CONCLUSION

In this unit, you have learned some briefs of some reknowned mathematicians and their areas of specializations. The unit also contained some popular quotations made by mathematicians of repute.

5.0 SUMMARY

In this unit you have been introduced to the following historical facts.

1. The important historical contributions of Thales of Miletus to mathematics are many. These include origin of deductive mathematics with which he proved geometric theorems.

2. Pythagoras- "the father for Greek geometry" was the founder of Pythagorean secret society. Some of his contributions to the history of mathematics are (a) the Pythagoras theorem. (b) many theories of number (c) the irrational numbers or what they referred to as the incommensurable numbers such as $\sqrt{2}$, or $\sqrt{5}$.

3. Euclid: The greatest contribution of Euclid to the History of mathematics was the Euclidian Elements.

4. John Napier is the founding father of logarithm but Professor Henry Briggs gave us base 10 logarithm.

5. Pierre Fermat did an original work on differential calculus and from his work the modern use of rate of change grew

6. The theory of probability was originated in an attempt by Pascal and his friend Fermat to solve a problem of throwing a die.

6.0 TUTOR MARKED ASSIGNMENT

1. State one number fact discovered by Pythagoras

2. Write one set of Pythagoras triples different from the examples given in this unit.

3. (a) Who classified number into amicable numbers?
   (b) Prove that 496 is a perfect number.

4. Who gave us logarithm to base 10?

5. Thales theorem states that … complete this theorem.

6. Who originated these two theorems?

   (i) A circle is bisected by a diameter.
   (ii) The base angles of an isosceles triangle are equal.

7. Postulational and axiomatic geometry are made popular by who?

8. Using the Pascal's triangle row5, make up probability chance problems similar to the ones using row four in this unit.

9. The greatest contribution to the history of mathematics by Euclid was the " Elements". Explain five facts about this word "The Elements".

10. What is one difference between a postulate and an axiom? Give one example each.

SELF ASSESSMENT EXERCISES

Self Assessment Exercise 1

Take any history of mathematics book in any Library and write out proposition 1 of Euclid elements Compare this with the distributive property of today.

7.0 REFERENCES/FURTHER READINGS

Boyer Carl B: A History of Mathematics New York John Wiley & Sons, 1968.

Odili G.A (1986) Teaching Mathematics in the Secondary School, Anachuna Educational Books.

UNIT 5 MATHEMATICS EDUCATION STRATEGIES

CONTENTS

1.0 INTRODUCTION

This chapter is made up of sections in which issues on mathematics teaching and learning are discussed in line with the objective stated below, Teaching is viewed as a problems –solving task which involves knowledge, comprehension. Application, analysis, synthesis, and evaluation. As such, the mathematics teacher should see him/her-self as a problem-solver who needs certain tools. The types of tools needed by the Mathematics teacher are discussed in this chapter. To help the student-teacher this chapter contains materials on (1) instructional methods,(ii) adolescent modes of thinking,(iii) sources and writing of instructional objectives,(iv) preparation of lesson notes and (v) peer teaching versus the real classroom situations. Since the student-teacher will definitely teach mathematics at both the junior and senior secondary schools in Nigeria, and he/she is a novice, the students teacher is asked to reflect back on his/her days as a pupil struggling to learn mathematics. You would be able to recall that your teacher probably used various means to teach mathematics textbooks, diagrams, instruction, audio-visual aids with a series of assignments and tests. As such you should be able to view mathematics teaching as a complex task and that there is no readymade formula for solving the complex problem. Your interaction with the real life situations will help you to device the formula you will need to deal with the situation. The material presented in this chapter will give you a good insight into the world of mathematics teaching and the satisfaction derivable from the activity.

Before proceeding further, ask yourself the question, "what kinds of things do teacher do in effort to help students learn the content of subject matter of mathematics? If you reflect upon your own experience as a pupil, certain things are sure to come to mind. You should be able to recall lecture, discussion, home work, question and answer, audio-visual aids, demonstration and laboratory work However, what do we mean by all these terms? if we are to communicate the ideas effectively we need to give working definitions of the terms as used in mathematics teaching and learning.

2.0 OBJECTIVES

Students are introduce to the teaching of mathematics at both the JSS and SS levels in this chapter. Thus, the following objectives are to be achieves after carefully studying the chapter. You are expected to be able to: at the end of this unit 1, you should be able to do the following:

1. Identify various teaching methods that are available for teaching mathematics at the JSS or SS level
2. Select suitable and appropriate instructional strategies for teaching specific contents of mathematics at the JSS or SS level
3. explain adolescents' way of thinking and how they may influence mathematics teaching and learning
4. Select and write suitable instruction objectives
5. Select materials for lessons, prepare lessons notes and execute the notes in mathematics classroom.

3.0 MAIN BODY

3.1 Study Approach

In order to benefic from this unit, you are expected to doe the following points

- Get hold of the mathematics curriculum for the primary school.
- Identified the content used in this unit as an example and take note of the class. Classes will not be indicated in these lessons.
- For each method discussed jot down the main ideas, its good point, its faults and precautions to be taken in using the methods.
- Give one more example of the use of the method as identified in the secondary school-mathematics curriculum.

3.2 DISCUSSION OF VARIOUS TEACHING METHODS

Research findings have shown that there is no one best method of teaching". Johnson and Rising (1971p.143) state that the method selected depends on "the topic, the class, the objectives and the procedures known to the teacher". There are, however some guidelines that may be considered by every teacher in selecting the method or strategy to be used;

- The teacher must be familiar with the method.
- The teacher should believe in the importance and the efficacy of the method.
- The method must be easily understood by the children
- The method must not leave the children passive in the class. The passive student cannot perform or utilize concepts learnt In the new 6-3-3-4 system of education.
- Do as I do "technique to arrive at the answer may not be suitable for the dynamic nature of the 6-3-3-4 mathematics curriculum. The "Do as I do technique breeds mathematics students who can arrive at the correct answer without being able to explain personally term, the meaning of the procedure to the answer. Such students cannot ask or answer "why" question in class. They therefore cannot transfer their knowledge to creative use.

For examples, a student who mechanically learnt to cancel like terms in fractions may come up with a right answer to problems such as, $\frac{16}{64} = \frac{1}{4}$ with the demonstrated wrong cancellation. The only way to discover such a mistake is to engage the students in a dialogue.

Some authors have summarized the principles behind effective teaching methods under three headings:

A **The Dynamic principles:** A learning concept grows in a dynamic way in the child's mind. For a learner he needs to handle, observe, experiment with materials, compare past experiences with the newly discovered ones and takes new decision-amending the old precepts or adding a new one. Therefore the child needs concrete materials objects, semi abstract diagrams of the objects, as well as some form of directed experimentation to arrive at required results.

B **The Constructive principle:** This is related to the dynamic principle in that it also requires practical situations before the use of reflective thinking by the child to analysis the patterns or common properties for generalizations Playing with materials first is what this principles implies by the words" constrictive"

C **Perceptual Variability principle** or multiple embodiment principle: this principle requires variability in methods, recognition of individual difference between students, The relational meaning of mathematical symbols and expressions must be emphasized. For example, the formular **D=2(a+b)** should be recognized as a possible formular for the perimeter of a rectangle. The use of **P=2(L+W)** should be compared with the former formular during the lessons on perimeter.

Let us now consider specific example of methods.

3.2.1 Lecture Method

The teacher talks, perhaps with some use of the chalkboard, while the students listen quietly and sometimes take notes. Frequently at the end of such a class period the teacher asked," Any question? Your student may not ask any questions because they are seldom forthcoming.

Uses of Lecture

This method provides opportunity for the teacher to give verbal input that is not available to students in other forms (such as textbooks) or at appropriate level (Such as Vocabulary or conceptual sophistication). It is assuEDU that the words of the lecturer convey the same meaning to all the students. It is also assuEDU that the students are listening with the intention to comprehend and remember, since it is difficult for most secondary-school pupils to remain passive listeners for a long period of time the lecture (teacher) should either keep it short or be a dynamic lecture or both. The feedback giving and getting potential of this method is minimal, Because of that short attention span of pupils. Teachers are advised to use lecture method sparingly and for very brief time span. Other problem that are inherent in the lecture method are (a) the inability of a novice teacher to give a dynamic "Lecture" (b) the inability of pupils either at the JSS or SSS rapt attention, (c) the difficulty of identifying those who are confused and the point where they are confused, and (d) the inability of the pupils to completely comprehend the words of the lecturer. You may reflect upon your own experience when you were a student of mathematics at the secondary school levels.

3.2.2 Question and Answer Method

This is one the most common methods employed at the secondary school level. Pattern typically begins with the teacher asking a question, and then recognizing one student who answer. Next the teacher reacts verbally in some way to the student's response and asks a question of another students, who then responds.

As in the case with the lecture, the teacher may write something on the chalkboard and sometimes the students take notes. Thus, the working definition for Q/A: The teachers asks a questions: one student answers. The teacher reacts and asks another question which is responded to by a second student, and so forth.

Uses of Questions and Answer Method

Many teacher believes Q/A is a method that enables the teacher to find out who knows what, only one student responds at a time, a sample of only one out of fifty to seventy students. The questions and answer method is extremely valuable as a way to guide development thinking, to stimulate creative problem solving. To initiate discussions, and to stimulate quick recall of requisites needed for the days lesson. The Q/A method can be used effectively in combination with every other method. The kind of question posed the preamble to the question posed and the variety of ways used to encourage and accept responses are all skills that makes the differences between thoughtful interaction and dull sequences.

There are three major components of Q/A that needs special attention How to apply questions to obtain good responses.

A The Question:

1. Write down the major question in a developmental sequence and analyse the possible responses ahead of time.
2. Precede a Q/A sequence by a brief lecture, or demonstration designed to set the stage for the sequence.
3. Do not ask frequent yes-no questions or fill in the blank question such as "Does anyone know the answer to number five?
4. Increase the number of question requiring a phases or a sentence in response.
5. Do not try to elicit development thinking by the all-encompassing "what about" question, such as "what about the circle?"
6. Use a variety of opening question- phrases, such as "How?" "What" "why?"

B Getting responses from students

1. Pose the question before you call on someone.
2. Do not call on students in only one area of the room for all answers.
3. Ask weak and slower students low- level question
4. Save high-level questions for brighter students.
4 Do not direct a series of quick questions to students row by row (or in any clear pattern)
5 Wait at least five seconds prior to accepting responses to high-level question inform the students you are going to do this.
6 Tell the students that there is no penalty for incorrect or partially correct answer. Tell them it is not a quiz, but a learning experience.

C Handling Student Responses.

1. Ask another student to agree or disagree and give his/her reasons for doing so.
2. Take "straw vote" and follow up with a request for justification.
3. Frown a bit and ask, "Are you sure?"
4. Ask another student to add to the answer of the first student.
5. Ask the student to explain how he/she arrived at the solution
6. Ask if there is another way to solve the problem.
7. Do not accept mixed chorus responses
8. If a student cannot answer a difficult question, ask a contingency backup question on a lower level.
9. Refuse to accept responses that are not audible to all students
10. Give praise for partially correct responses to complicated questions.

The physical location of the teacher greatly influence the nature of verbal interaction in the class.

3.2.3 Discussion Method

Discussion refers to student to student- talk with occasional verbal intervention by the teacher.

Note that discussion is among students not between students and the teacher.

Uses:

Most activities labeled "discussions" are really lecture or question/answer in a discussion, as defined. The teacher usually initiates the interaction, but is only occasionally, the first essential ingredient for a discussion, a topic a question a problem, or a situation in which the students can share ideas and compare or contrast suitable topics, or, situations that have been used effectively by mathematics teacher in initialing discussion ?

The following illustrative example can help you to locate more topic in mathematics.

Topic, procedure purpose, original proofs. Small groups seek varied methods of proof, contrast efficiency, difficulty level of each, advantages of one over the other. Each group to be arranged by the teacher to reflect the range of group to discuss all problems on which any member needs helps. Consider reasons Range from very good to poor, behind errors, pinpoint strategies for correct solutions

This second major ingredient for a successful discussion is appropriate student prerequisites. When does a discussion end? A when the attention of sub-group members starts wandering, the discussion mode had already continued too long you may call a halt when all groups have made in roads on the topic and when starting of ideas seems desirable.

After all, students are responsible to the signal to end the discussion each spoken-person can report on that group's findings, then students from other groups can be asked to indicate agreement or disagreement. Finally, the teacher can help all to summarize class results.

3.2.4 Demonstration Method

Typically but not always the teacher shows something, such as a specimen or a model, while students watch, in some instances, one student may do the showing while others watch.

Uses:

Demonstration is used in combination with lecture, question/answer laboratory, the following examples illustrate just a few of the ways in which demonstrations can be used to enhance mathematics instruction.

Topic	Demonstration	Purpose
1. Symmetry	Teacher cuts orange of fruit in half	To show model radial symmetry
2. Units of measure	Teacher holds up tin of washing soap	To get attention on of milk, box practical problems
3. Geometry (locus)	Teacher stretches and shrinks shapes on cardboard	To illustrate path of moving points

To focus students' attention on a demonstration, you must first be sure that the object can be seen the specimen must be large enough to be visible from the rear, or the teacher must move about the room with a smaller object. White cardboard sheets or the over-head screen can provide an effective backdrop.

Although demonstration is often used simultaneously with other methods, a silent demonstration can be very effective. Such a demonstration might be used either to set the stage for or to further develop a topic.

3.2.5 Laboratory Method

This method does not refer to a place or a special class period in the weekly timetable, but to an activity. This activity may occur in a regular classroom, a specially equipped room, at home or out-doors. The key idea is that students manipulate concrete objects, specimens, or equipment under the direction of the teacher.

Uses of Laboratory and Discovery Methods

Since the students are manipulating equipment or material in this mode, they are doing so to collect data. Therefore, you might employ this method to help students reach a generalization, test a theory observe the application of a rule or learn and practice a psycho-motor skill.

Examples:

Materials	Activity	Purpose
1.Strips of Cardboard	Students to fold into segments	To teach Fractions e.g, 1/2 1/3,1/4,1/3,2/5
2. String mater stick tins of different shapes and sizes	Students measure circumference and diameter, record data and calculate	obtain the generalization that is a constant.

This method required careful preparation of material, instruction in safety precautions, organization and possible reorganization of the usual physical facilities in the classroom. Direction to students must be planned ahead some teachers write directions on an overhead projection sheet so that all students can refer to them repeatedly. The teacher can move systematically to each individual or group, listening asking question, and observing the way in which the student are proceeding. The teacher should correct imEDUiate errors committed by his/her students. This method is gaining popularity now mathematics mow because _ of the emphasis on productive application of mathematics. Many schools are establishing mathematics laboratories. This is in order because mathematics is a subject which has to be learnt by doing rather than by listening or seeing.

Essentially laboratory method' is very practical. The advantages are many

Every students participates, interest is aroused, and investigative capacities are developed .

Psychomotor skills are then acquired such as the use of fingers in drawing sawing, cutting or sewing.

Some disadvantages are:

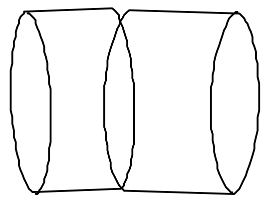
i. it is noisy
ii. it is time consuming
iii. it is expensive -the equipments may be costly.
vi. accidents may occur in laboratory lessons.

The teacher should be prepared to tackle these problems.

Model Lesson on Laboratory Method

Topic: To find the value of pie

Method: Take the pupils to the mathematics laboratory Give to each child a cylindrical can or an object with circular surface of different sizes.



Lets each pupils use any appropriate method to find the circumference c, of the cylinder and the diameter d of the circular end.

Let the pupils record their findings on the discovery chart shown below.

Student	C	d	$\frac{C}{d}$
1			
2			
3			

The students can use a thread or measuring calipers or other means, such as rolling the cylinder over a distance , to do the measurements. After all the pupils have entered their Readings get each one of them to calculate his ratio $\frac{u}{d}$ and record their results in the chart.

Through question it will be found that the value of the ration $\frac{c}{d}$ will be very to 3, 1, 4. This is the value of pie

3.2.6 Discovery Method

Guided discovery is an aid to problem solving, the teacher explains exactly what the students must do, allows them a free hand to carry out the activities, but gives suitable guides to prevent students from going astray. In this method, success is often assured provided the teacher gives sufficient hints to the discovery.

Another variation is open discovery: in this the teacher allows the students free hand to play with the materials and come up with whatever discovery that comes their way.

In this, the discoveries made by the students may be new to the teacher. The use of this in our system of mass education is minimal.

Guided discovery should be preferred

Most discovery lessons have discovery charts. We shall give example of this in the model lesson later.

Advantages of a Guided Discovery Lesson:

1. Students who use their energy to discover knowledge increase their ability organize resource in attacking problems, become more courageous at problem solving and receive self satisfaction at the success of discovery. (Burner 1960,p6 12)
2. Most students are likely to retain knowledge learned by discovery more than by other methods.
3. They are better able to transfer knowledge
4. Discovery can enhance students participation and interaction in class. Students puts in their best. No students is passive in a gilded discovery lesson.

Some Disadvantage of Guided Discovery

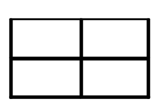
i. There is tendency for a discovery lesson to be noisy, the teacher should note this
ii. Discovery lesson is time consuming and the teacher must be ready to give hints at appropriate points to direct students to discovery and minimize time waste.
iii. The teacher should be aware that not all the students will come up with the discovery results. As soon as the majority of the students do so, he should use their results to bring other students to the results

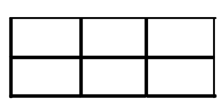
Model Lesson on Guided Discovery Method

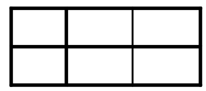
Topic: formular for finding area of rectangles. It is assuEDU that the other steps in a lesson plan shall be left to the students. We shall go straight to the content development step:

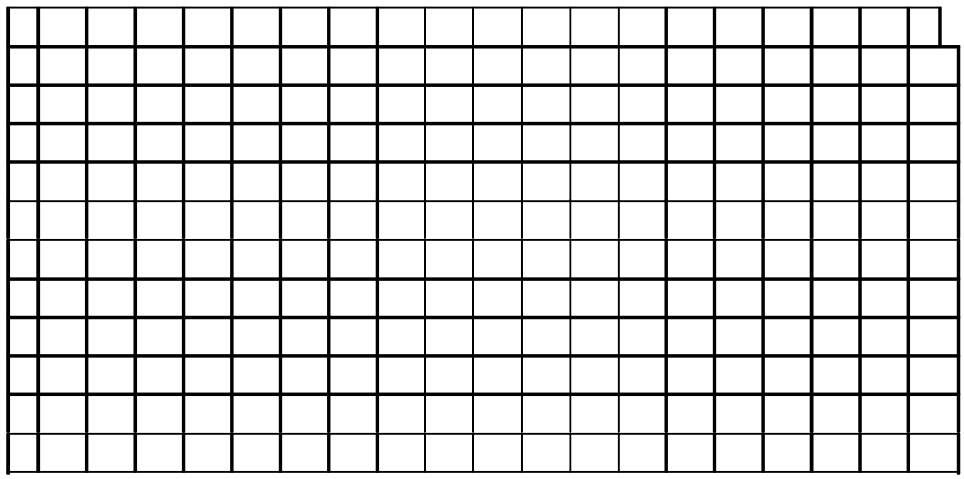
Contents

Teacher presents to the pupils a series of rectangles as shown (all dimensions are in centimeters)









Teacher find the areas of rectangles shown by counting the square and filling the discovery chart shown:-

Discovery Chart

(1) Rectangle	(2) Length cm	(3) Breadth (b) cm	(4) Area by Counting	(5) Area by a short method
A				
B				
C				
D				
E				
F				

Do not fill column 5 after you have found all the areas by counting.

Teacher does 1,2 along with the pupils to ensure they understand what is required of them He then allows them to go to the rest when all have finished a,b f the teacher asks the students to find if there is a short method to arrive at the area without counting, then let the students use the short method to fill column 5.

When the short method is found, ask the student to apply the short method to find

i. area of a rectangle 23 x 73cm.
ii. Area of a rectangle whose length is a cm and breadth is b cm.

By this it is expected that the area formula shall be discovery area = Length x Breadth

3.2.7 Supervised Practice Method

This method involves having students try to perform some task at their seats or at the chalkboard while the teacher observes their progress and gives help as needed.

Uses of Supervised Practice

The students must be involved in the practice of a learned rule, practice must follow relevant instruction. Here are a few suggestions aiEDU at helping you implement supervised practice:

1. Use several short periods rather than one long, marathon session at the end of class.
2. Sequence and cluster your practice examples so that all or almost all students can begin the work and so that no one is finished before you have some time to make a tour of most of the room.
3. Systematically move into the room to check the work of individual students. Use verbal and non-verbal signals to let students know you see their hands and will be with them in order. In the meantime, they could try another problem or be directed to share their question with another student.
4. Be aware of the entire class even while helping one student. Turn to face most of the class.

When should a teacher end a supervised practice session? Does everyone need supervised practiced session? You are advised to attempt this question.

3.2.8 Home Work Method or Assignment Method

This is any activity relevant to the achievement of objectives that students perform outside of the scheduled class sessions and without the supervision of the teacher. This definition encompasses such potentially meaningful activities as spending 20 minutes in the store after school to get data on prices of commodities. It is also leaves room for doing the even-odd numbered problems from a textbook when it makes sense.

When Giving Homework:

Choose only a few exercises or activities and make sure the ones you choose make sense to the students. Do not use homework as a punishment. Homework can be used to introduce the next day's work to the student. It can be used to recall the prerequisites needed for a new topic. What do you do with the home work?
When do you collect homework?
How do you collect it?

The teacher should not do the homework for the students. Do not ignore the homework. How do you deal with those who fail to do the homework? –Offenders may be asked to read relevant material in other texts instead of participating with their classmates. The teacher should refuse to allow, unprepared students to participate in that portion of the lesson where the homework is needed.

3.2.9 Use of Audiovisual Aids:

Actually all methods involve hearing, seeing, or both. However, this method typically means that students are viewing and hearing a motion picture, or television, seeing slides, a film strip, or a film loop, watching an overhead transparency, or listening to a recording of some type.

Uses of Audiovisuals:

Teacher's preparation is critical if this method is to help students learn. There are four major areas of preparation in the use of AVA.

(1) Use of the equipment
(2) Quality and suitability of the tape, slide, film, filmstrip, or transparency,
(3) Preparation of the students; and
(4) Follow-up and integration of the AVA into other methods.

Suggestion for using AVA

1. Use of equipment

(a) check out your school's rules and regulations and follow them.
(b) Dry-run equipment you will operate in the classroom in which it will be used. Check visibility and/or audibility from various areas of the room

.

2. Quality and suitability of the materials:

.

(a) Review all materials you hope to use and do not use material of poor quality or limited applicability.

(b) Be a ruler of, not a slave to, equipment and materials.

3. Preparation of the students:

(a) provide an introduction and an overview. Let the students know where and how AVAS contributes to the subject being studied.

(b) Explain to the students their responsibilities. e.g asking question thereafter.

4. Following-up and Integration of AVA Method:

(a) Plan alterrative ways of combining this method with others to get feedback, to give feedback:: and to keep attention on the task.

(b) Keep the particular AVA Materials alive after the lights go off. Conduct Q/.A or small-group discussions if suitable.

3.3 PROJECT METHOD

This may look like the laboratory method. The critical difference are that here the students are all doing different manipulative activities or varied library research, or different problem-solving task on an individual basis.

Uses of Individual Students' Project:

Projects can be students initiated or teacher initiated. They may be used to expose some topic in depth, to introduce relevant applications of the subjects being studied, to investigate historical background of the topic and so on.

3.3.1 Group Method

In most city schools particularly in the southern sections of Nigeria, the number of students in each class exceeds 50. For a teacher to prepare teaching aids for such a number is tedious and so the grouping method is recommended.

In this method, the class is arranged in groups and each group is given something specific to do. When there is equipment limitation, such as meter rule to measure objects in class, this method becomes very necessary. Another advantage in this method is that it makes for student-student interaction within the group. Group interests also breeds healthy competition. Each individual in a group helps to add to the general pool of thinking and so greater discovery is possible. Association with others is a necessary and useful educational objective which this method utilizes.

Some disadvantages may be noted:-

(i) There may be tendency for noise from groups
(ii) Supervision by the teacher is a little more difficult.

Generally, however the advantages of group method out weigh the disadvantages.

Topic: Estimation of length of the school lawn

Method: group 1: estimate the length of the school lawn
group 2: estimate the length of the chalk board
group 3: estimate the length of the goal post
group 4: estimate the length of the class room
group 5: estimate the length of the school building

In each group, each pupil will do his estimation privately but they come together to record their estimation in a chart thus:

Group 3 Goal post

Student	Estimate Length,	Actual Length	Error 1	Error 2
John				
James				
Bale				
Bob				
Ale				
Kola				
Sun				

Then they go together to get the actual measurement of the object. Each group finds the sum of errors$_2$.
The group where error is the smallest wins.

3.3.2 Feedback Systems

Many teachers assess their effectiveness in the classroom in terms of student reaction. Some teachers do this haphazardly, some do it systematically. The getting of feedback is important in the teaching and learning of Mathematics. What do we mean by feedback? The student's answer either verbally or non-verbally. The teacher gets feedback when he or she notes that a student has written the correct solution to a problem posed by the textbook, a student gets feedback while checking the results of a solution to a problem.

As such, the working definition of feedback is any information, sent to the teacher or student, verbal or non-verbal on the results of instruction. This means that both the teacher and the student need to get feedback if instruction (teaching and learning) is to be both efficient and effective. You as a Mathematics teacher, must plan to get feedback and to give feedback. Then when the teacher employs a sequence of two or more methods with the planned intention of getting and/or giving feedback, that type of sequence is referred to as a feedback strategy.

Getting Feedback

Go back to have another look at the various teaching methods treated earlier in this unit. You should note that each method does have a unique purpose in instruction. There are some methods with zero verbal feedback-getting potential. Such methods should be combined with those that have positive potential for getting feed-back. For example a short lecture can be followed by questions specifically designed to assess the listeners grasp of the lecture. Such questions can range from the "what did I say" type to "How" or "Why" types. The teacher might ask students to write the answers to appropriate questions then and there.

Further, the teacher may use straw votes to get feedback from many students. The teacher asks to see the hands of those who agree, who disagree, who are not sure.

Straw votes should frequently be followed by a series of questions to members of each faction probing the reason(s) behind their response, or a student demonstration in defence of a response. The following methods have high potential for getting and giving feedback:- Discussion, laboratory, individual project, supervised practice and home work.

The question and answer method has potential for getting feedback from a few students, while lecture and demonstration have nearly zero potential for feedback.

3.3.3 How to Use Feedback System

Sometimes the best imEDUiate use of feedback is to get more feedback. More feedback may imply a larger or broader sampling of students, or it may refer to the nature of the responses and the need to obtain more specific or more extensive responses. More extensive feedback getting is essential if instructional strategies are to be effectively implemented. Some of the more common uses of feedback are listed here:

1. After obtaining feedback on the number of students who possess the prerequisite learning for the next topic, the teacher may do the following:

   a) Reteach the prerequisite material if all or most students have given negative feedback to the teacher.
   b) Ask students to try a practice example. If there seems to be ambiguous feedback.
   c) Use appropriate recall Q/A and lecture to weave specific instance of prerequisites into the ongoing lesson if only a few students give negative feedback on the perquisites.

2. When beginning homework post mortem, the teacher displays or reads the answers and then gets feedback on the extent of the students problems with homework:

   a) if only a few students give the teacher negative feedback, the teacher may (i) use Q/A with members of the rest of the class, (ii) work with the small group of students while putting the larger group of students to work on other materials and (iii) tell the small group of students that they will be helped later in the class when supervised practice occurs.

How Teachers can Give Feedback to Students:

a. Gaining and controlling attention. The teacher gives students feedback on what to observe, its relevance to past work, and so on.
b. Informing the student of expected outcomes. The teacher tells students whether they will be expected to reproduce a deviation, apply a rule to typical problems, or write down observations made during film.
c. Recall of prerequisites. The teacher may tell students the relevant aspects of prerequisites needed for the day's work.
d. Presenting the new material
e. Guiding the new material
f. Providing feedback. The teacher corrects responses, modifies partially correct ones, and classifies in case of errors.
g. Appraising performance. As the students check out their learning in supervised practice, Q/A, or discussion, the teacher gives feedback as in (f)

Recall reinforcement system taught you in your psychology of learning. The feedback system is based on the reinforcement theory. If you want to become an effective Mathematics teacher, you must always use the feedback system during your lessons. Next we will consider the types of instructional objectives you can use to prepare your lesson notes.

Types of Instructional Objectives

What are instructional objectives?. An instructional objective is a statement that describes a desired student outcome of instruction in terms of observable performance, under given conditions. It is always phrased in terms of student performance, not in terms of teacher performance. Moreover, student performance must be described in terms of observable behaviour. You should view instructional objectives as guides to what the teacher will teach in the Mathematics class. Examine the following statements and point-out which of them represent clear statements of instructional objectives.

1. The teacher will illustrate the use of the calculator by a demonstration on a model calculator.
2. The students will understand how to use a calculator in multiplication work.
3. The students will observe a teacher's demonstration of the use of a calculator.
4. The students will write the product of any pair of 4-digit numbers. Given the use of a calculator.

3.3.4 Domains of Instructional Objectives

The name given to the three major areas of emphasis into which all instruction objectives may be classified is domain. The cognitive domain deals with recall or recognition of knowledge and the development of intellectual abilities and skills.

Changes in interest attitudes, values and the development of appreciation belong to the affective domain, the third domain, the psychomotor domain , refers to the manipulative or motor-skill area within each of these domains, there have been attempt to devise categorization systems called taxonomy. the purpose of such categorization are to make more explicit the varied levels of instructional objectives.

Taxonomy is a set of standard classified and these categories are related in a hierarchical fashion. Although all three domains are source of instructional objectives taxonomies in two of the domain, the cognitive and the affective are particular useful tools for the secondary school mathematics teacher.

The Cognitive Domain

The most widely used taxonomy of cognitive objectives is that of Benjamin Bloom and colleagues. Blooms taxonomy contains six major levels of classes. It is important to keep in mind that a taxonomy is a hierarchy. Therefore a level II objective implies that command of related level I behaviors is assuEDU. For example, if a student is expected to be able to use the formula for the area of a square in computing areas give the length of a side of a square. That student is expected to be able to recall the formula for the area of a square.

The Cognitive taxonomy

Level I: Knowledge the recall of material with little or no alternation required

Level ii: Comprehension the use of a specific rule, concept, method in a situation typical to those used in the class.

Level iii. Novel Application the selection and use of a learned rule, concept, method in a situation novel to the student.

Level iv: Analysis –Breaking down of material into its parts so that the relationships among its ideas are made explicit

Level v: Synthesis- Putting together parts so as to form a whole pattern or structure of ideas not clearly there before.

Level vi: Evaluation – Judgments about the values of material and method for given purposes.

Examples of how to state instructional objective based on the Bloom's taxonomy. The student will be able to

Level I State the formulas for the areas of a square, a rectangle given the name of the shape.

Level ii: Compute the areas of a square, a rectangle, a trapezoid or a general parallelogram, given appropriate lengths.

Level iii: Solving novel word problems which involve the use of the quadratic formula.

Level iv: List the facts, the relevant relationships and the irrelevant information contained in a novel word problem.

Level v: Write a novel word problem which includes the and use of at least three of the four area rules studied in the class.

Level vi: Write an assessment of the advantages and disadvantages of two different solution to a word problem, where accuracy and economy are desired criteria

The above contains example which a novice can use as a guide to writing objectives in the cognitive domain. However, there are certain steps which should be followed in writing objectives for a daily lesson.

Note:

1. Choose the topic or unit to be taught.
2. Study curriculum materials, syllabi and student textbooks with relevance to the lesson.
3. List the major content items (such as concepts, formulars, principles and the like).
4. Write instructional objectives directed toward each of the designated major content items
5. Classify the objectives by level and check these results against your assessment of the nature of the content and the intellectual development of the student.

To help beginners, the following performance verbs are given.

Add	describe	graph	list
Calculate	design	identify	multiply
Cite	diagram	induce	plan
Contrast	divide	interpret	plot
Deduce	explain	interpret	predict
Derive	extrapolate	justify	prove

	select	translate	write
solve	subtract	verify	

You are encourage to look for more performance verbs which are applicable in mathematics teaching and learning.

3.3.5 Affective Domain

The affective domain was constructed by David Krathwohi and his associate in 1964. three levels of the version are given below.

Level i: **Complying:-** Passive acceptance of role assigned by teacher no overt avoidance of activity

Level ii: **Responding:-** Voluntary participation in activity or selection of one activity out of several.

Level iii: **Valuing:-** Commitment to a value shown by consistent and stable response to objects, people, phenomena, and others.

As in the taxonomy of cognitive objectives, this taxonomy is also hierarchical. At the lowest level, the student may for example, merely sit quietly in a conduction of apparent listening during a lecture, rather than chatting with neighbors.

Examples of affective Objective

Level i : Participation in the lab-group role assigned by the teacher.

Level ii: Attempts an optional challenge problem on areas of regular polygons.

Level iii: Volunteers illustrative examples from non-assigned outside sources during a class discussion on the application of geometry to the real world.

Note:

The major difference between the level iii and level ii objectives lies in the evidence of student initiated interest over time.

3.3.6 The Psychomotor Domain

Note that the psycho, coveys the notion that motor (muscle) learning is intertwined with mind, soul and spirit, in mathematics the domain deals with manipulative skills to be learned by students.

You will be given a few of this domain to get you started decide whether or not each (1) contains a specific performance verbs (2) communicates clearly (3) places primary emphasis on coordinated muscle movements. And (4) makes obvious the material and equipment that would be made available to the learner.

a) Measure rectilinear shapes with a metric ruler to within-5mm

b) Describe how to operate a hand-held calculator to obtain sums, difference, products and quotients of integers

c) Adjust a georule to illustrate geometrically shaped letters of the alphabet

d) Manipulate a set of cut-out rectangular and square shaped to illustrate equivalent algebraic expressions.

You are now be ready to construct a few objectives in the psychomotor taxonomy on your own, think in terms of the manipulative skills associated with the use of devices such as compass, string, geoboard, graduated cylinders, stop watch, and pan balance. Then write at least four objectives relevant to a subject you expect to teach and submit them to a critique by a classmate.

You should have observed that we did not deal with a taxonomy within the psychomotor domain. There is a taxonomy for the domain, but it appears much more relevant to the primary school age group and physical education content than to secondary-school mathematics instruction. However, we can not ignore varying complexities of objectives aiEDU at motor skill development in mathematics. The three domains are gold mine for instructional objectives construction for daily lesson notes.

3.3.7 Daily Lesson Notes

How can daily lesson notes be designed so as to attend to the students? What right format is helpful to the beginner? A detailed written lesson note which includes key components of mathematics teaching and learning is an excellent base for a beginner, such lesson notes do not guaranty success, but their absence correlates highly with failure.

The lesson note format given below has stood the test of time and has proved workable in the hands of both novices and experienced teacher of mathematics. Analyse the format and note its characteristics i.e each category in the outline some items, such as date, topic and class period are simply identified for the teacher or supervisors. The remainder of the headings should make sense as a synthesis of method, feedback, objectives and characteristics of students and content. You should adjust the spacing to fit your needs.

Under content item are listed the labels of the concepts, the rule statements, use of rules, or ideas and so on, in the sequence in which you expect to introduce these. Across from each content item in column 3 would be a description of the sequence of methods designed to achieved the stated objectives (s) with respect to that content item, if materials other than standard chalk and chalkboard are to be used, these are listed opposite the related strategy in column 3. In addition, two columns labeled feedback strategies are added in order to give particular emphasis to the need to plan ways to get feedback as well as ways give feedback. These columns should contain the strategies you have chosen in order to elicit feedback at critical times and to give certain kinds of feedback during the lesson. Finally column 5 should contain your time estimates for a strategy aiEDU at a particular objectives. After the lesson has been taught, these estimates should be checked against actual times to assure more realistic planning of time use in the future. Now turn to figure 1. and read over the overview given before the figure.

Topic……………………….
Date ………………………
Class period ………………

Instructional Objectives				Number of Students Other routines
1 content item	2 SPECIAL Material Equipment	3 instructional strategies I,e methods	4 Feedback Strategies Get Give	5 Time Estimate

Fig. 1. Daily lesson note format

To help the beginning mathematics teacher, examples of instructional strategies for key learning items in mathematics are provided below.

3.3.8 Instructional Strategies

1. Instructional strategy pattern for concepts

   show differentiated examples and non-examples in sequence get essential and non-essential characteristics, identified, elicit operational definition, model by analogy elicit generation of the concept to a variety of specific instance not previously used.

2. Instructional strategy pattern for vocabulary

   Show object (or exemplify idea) then pronounce name and spell board have students do likewise, repeat (practice) give name and elicit statement of idea or description of objectives have students use in content

3. Rule/ Principle Learning

Rules and principles, the heart of mathematics, depend on concept learning, the formula for finding the area of a triangle might be memorized without any prior learning but formula itself is only a statement of a rule just as the label "Triangle" is the name of a concept.

Guide students to review prerequisites, indicate nature of expected terminal performance; cue (via questions, lab-work, applications) dates (by student) provide a model of correct performance, have students demonstrate instances of the rules in a variety of situations, fix students, maintain skills by spaced and varied drill.

4. Instructional strategy pattern solving

Present problem, teacher may question students to elicit alternatives proaches and will emphasize the desirability of a variety of some strategies, arranges for individual work or small-group discussion, resemble class, ask students to weight the advantages and disadvantages of the proposal which resulted from the group discussion or indebt work. Study the example all-over and attempt constructing a daily lesson note on one of the key items.

4.0 CONCLUSION

In this unit, you have studied materials on the various teaching methods, their definition in operational terms and their uses. No single method is adequate or sufficient in the teaching of mathematics. None is superior to the other. A combination of methods in a sequence in order to achieve stated instructional objectives is called an instruction strategy. As a teacher, you have to combine several of the methods to help you achieves your stated objectives. Further, you should have noticed the methods belong to two families:

(1) Talking methods and (2) showing and doing methods. In either of the two intensity of teacher's or student's participation varies from one method to the other. Can you find out this? Note that in discussion, students are very active, while the teacher is a little passive. In lecture methods, Q/A Lies in-between active and passive participation by both teacher and the students.

Also in the unit, we discussed the importance of feedback strategies in the execution of lesson notes. The feedback systems hinges on the theory of reinforcement in learning psychology. The provision of positive and negative reinforces, and the scheduling of these, by the teachers, the feedback systems allows the teachers to assess the progress being made in the achievement of already stated instructional objectives of the lesson. In addition, we discussed the importance of instructional objectives and their sources. The three domains, cognitive, affective and the psychomotor which are hierarchical in pattern. The cognitive and affective taxonomies are very embedded in the teaching and learning at the secondary school level. The psychomotor domain has a few application too. Writing instructional objectives, is an art the beginning mathematics teacher should learn and cultivate that art. Objectives guide the teacher and they are stated in terms of students' performance. Usually instructional objectives are task specific and the conditions under which the task will be perforEDU are always given.

The uses of instructional objectives in daily lesson notes were discussed in the unit. Look at the format of lesson note given in figure 1. It may appear tedious but it is good for a resourceful and creative mathematics teacher. The major categories are five- (1) the content items, topics to be taught is broken in to bits and organized in a sequences till you get to the last bit, it is close to the step style of constructing lesson notes, (2) special material or equipment (note your chalk and chalkboard), (3) instructional strategy to be used to achieve the objective attached to each content item (bits of topic).(4) Feedback strategies, getting and giving of feedback and (5) time estimate, this to guide you when executing your lesson note in the class and to help you plan subsequent notes. Try this format during your peer teaching exercise. Ask a classmate to time you on each item of content you are teaching

5.0 SUMMARY

In this unit we have explained the meaning, the advantages, and the disadvantages of the following methods

(i) Discovery method
(ii) Group method
(iii) Laboratory method
(iv) Questioning or Dialogue method
(v) Lecture method

6.0 TUTOR-MARKED ASSIGNMENT

Attempt the following tasks in order to test knowledge and understanding of the subject matter of mathematics

1. Name and define nine common teaching methods in mathematics.
2. List at least two uses and two abuses of each of the teaching methods and provide reasons for your choices

3. List at least three variables in teacher's behaviors that could account for differences in students 'performance in mathematics
4. Design effective feedback strategies that can be incorporated into actual lesson you will present to students of mathematics
5. Define instructional objectives
6. Below are statements purported to be instructional objectives, React to each only in terms of whether or not it satisfies all three criteria for instructional objectives use an (x) to indicate your judgment.

NO
Uncertain
Yes

(a) identify the primes smaller than 100
(b) Knows the Pythagorean theorem
(c) Appreciate the symmetry forms in physical objects
(d) Derive the formula for the Sin (A+B). when A.B and A+B are each acute
(e) Factorise the difference of Two square is a monomial
(f) Graph any function of the Form Y=ax+b,a, not equal to zero
(g) List four ways to prove triangles Congruent
(h) Explain the "delta process" used In obtaining the first derivative.

SELF ASSESSMENT EXERCISES

Self Assessment Exercise 1

State two important uses of the Discovery chart in a discovery lesson

Self Assessment Exercise 2

What is the best way to handle a student who gives a wrong answer to a question?

7.0 REFERENCES / FURTHER READING

Bloom, B.S (Ed). (1956). Objectives, Handbook 1: David Mckay Taxonomy of Education Cognitive Domain, new york:

Bruner, J,S (1977). The Process OF Education. Cambridge, Mass: Harvard University Press.

Farrell, M.A & Farmer, W.A (1980). Systematic instruction in mathematics for the middle and high school Years Reading, Mass: Addison Wesley Publishing Company, Chapter: 1,2,5,7,

Gagne, R,M (1975). Essentials, of Learning for instruction. Hinsdale I llinois: Dryden Press.

Krathwohi, D, Bloom, B & Mass, B (1964). Taxonomy of Educational Jectvies, Handbooks ll Affective Domain New York: David Mckay

Krulik, S. & Welse, I,B. (1975): Teaching Secondary School Mathematics Philadelphla: W.B Sanders Company .

Manger,R, F, (1962). Preparing Instructional Objectives, Belmant. Caliform Fearon Publishers.