CIRCULAR MOTION

MARKING SCHEME

1. A stone on a string is whirled in a vertical circle of radius 80 cm at a constant angular speed of 16 radians per second.

Calculate the speed of the stone along its circular path.

Speed = angular speed × radius (1)

= (16 radians per second) × (0.8 metre)

Speed = 12.8 m s^{-1} (1)

(2 marks)

Calculate its centripetal acceleration when the string is horizontal.

centripetal acceleration (1)

= $\omega^2 r$ = (16 rad s^{-1})^2 (0.80m)

Acceleration = 205 m s^{-2} (1)

(2 marks)

Calculate the resultant acceleration of the stone at the same point.

Why resultant acceleration = centripetal acceleration (1)

= $\omega^2 r$ = (16 rad s^{-1})^2 (0.80m) (1)

Resultant acceleration =205 m s^{-2} (1)

(3 marks)

Explain why the string is most likely to break when the stone is nearest the ground.

The tension in the string has its maximum value when the stone is nearest the ground (1)

because it equals centripetal force + weight (1)

(mass times centripetal acceleration)

(2 marks)
[Total 9 marks]

2. State the period of the Earth about the Sun.

1 year (1)

Use this value to calculate the angular speed of the Earth about the Sun in rad s^{-1}.

Angular speed = $\frac{2\pi}{T} = \frac{2\pi}{365 \times 24 \times 60 \times 60s}$

= 1.99 × 10^{-7}rad s^{-1} (1)

(2 marks)

The mass of the Earth is 5.98 × 10^{24} kg and its average disatance from the Sun is 1.50 × 10^{11} m. Calculate the centripetal force acting on the Earth.

Centripetal force = $m\omega^2 r$

= (5.98 × 10^{24} kg)(1.99 × 10^{-7}rad s^{-1})(1.50 × 10^{11} m) (1)

$= 3.55 \times 10^{22}$ N (1)

(2 marks)

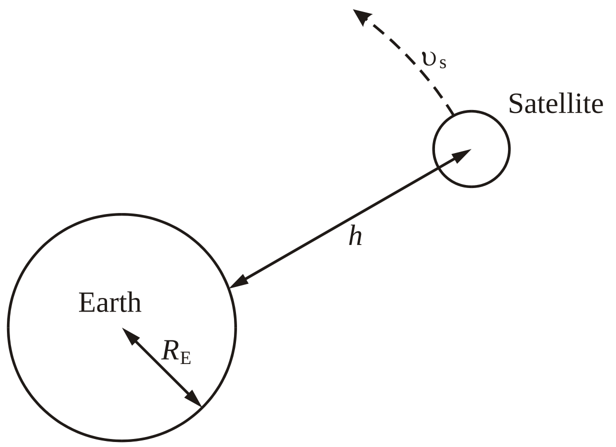
What provides this centripetal force?

The gravitational field of the sun.

(1 mark)

[Total 5 marks]

3. The diagram (not to scale) shows a satellite of mass m, in circular orbit at speed v_s around the Earth, *mass* M_E. The satellite is at a height h above the Earth's surface and the radius of the Earth is R_E.



Using the symbols above write down an expression for the centripetal force needed to maintain the satellite in this orbit.

$$F = \frac{m_s v_s^2}{R_E + h}$$ **(2)**

(2 marks)

Write down an expression for the gravitational field strength in the region of the satellite.

$$g = \frac{GM_E}{(R_E + h)^2}$$ **(2)**

State an appropriate unit for this quantity.

N kg $^{-1}$ (1)

(3 marks)

Use your two expressions to show that the greater the height of the satellite above the Earth, the smaller will be its orbital speed.

$$\frac{m_s v_s^2}{R_E + h} = \frac{GM_E m_s}{(R_E + h)^2}$$ **(1)**

$$v_s^2 = \frac{GM_E}{R_E + h}$$ **(1)**

Greater h Þ smaller v_s since G, M_E constant (1)

(3 marks)

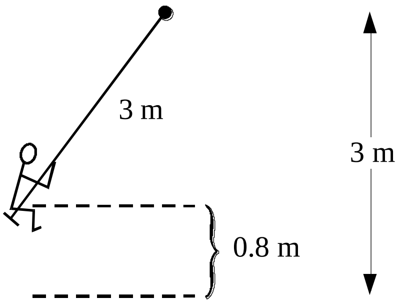
Explain why, if a satellite slows down in its orbit, it nevertheless gradually spirals in towards the Earth's surface.

As it slows $\frac{GM_E m_s}{(R_E + h)^2} > \frac{m_s v_s^2}{R_E + h}$ **(1)**

The "spare" gravitational force not needed to provide the centripetal acceleration pulls the satellite nearer to the Earth (1)

(2 marks)
[Total 10 marks]

4. A child of mass 21 kg sits on a swing of length 3.0 m and swings through a vertical height of 0.80 m.



Calculate the speed of the child at a moment when the child is moving through the lowest position.

Speed = $\sqrt{2gh}$

= $\sqrt{2 \times (9.81 ms^{-2})(0.8m)}$ **(1)**

Speed = **4.0 ms^{-1}** **(1)**

(2 marks)

Calculate the force exerted on the child by the seat of the swing at a moment when the child is moving through the lowest position.

mv^2/r **= 110 N**

mg = 206 N

∴ force = 316 N

(3 marks)

Explain why, as the amplitude of the motion increases, children may lose touch with the seat of the swing.

When the chain of the swing is horizontal, the weight of the child acts downwards (1)

centripetal force is zero (1)

(2 marks)
[Total 7 marks]

5. Angular speed

Use of $\omega = 2\pi/T$ 1

$\omega = 1.2 \times 10^{-3}$ [min 2 significant figures) [No ue as units given] 1

Free-body force diagram

Pull of Earth/Weight/mg/Gravitational Pull 1

Why satellite is accelerating

Resultant/Net/Unbalanced force on satellite must have an acceleration OR ΣF = ma. 1

Magnitude of acceleration

Use of $a = \omega^2 r$ OR $\upsilon^2 \div r$ 1

a = 9.36-9.42 OR 6.5 m s^{-2} 1

[Depends on which ω value used]

[6]