THE KENYA NATIONAL EXAMINATIONS COUNCIL
Kenya Certificate of Secondary Education

121/1 **Paper 1**

MATHEMATICS

ALT A

Mar. 2022 – 2½ hours

Name .... M/Scheme .... **Index Number** ....

Candidate's Signature .... **Date** ....

Instructions to Candidates

(a) Write your name and index number in the spaces provided above.
(b) Sign and write the date of examination in the spaces provided above.
(c) This paper consists of **two** sections: **Section I** and **Section II**.
(d) Answer **all** the questions in **Section I** and only **five** question from **Section II**.
(e) **Show all the steps in your calculation, giving your answers at each stage in the spaces provided below each question**
(f) Marks may be given for correct working even if the answer is wrong.
(g) **Non-programmable** silent electronic calculators and KNEC mathematical tables may be used, except where stated otherwise.
(h) **This paper consists of 16 printed pages.**
(i) **Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.**
(j) **Candidates should answer the questions in English.**

For Examiner's Use Only

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

Grand Total

914016

Turn over

SECTION I (50 marks)

Answer all the questions in this section in the spaces provided.

1. Evaluate $\dfrac{1\frac{4}{5} \div \frac{2}{3} \text{ of } 2\frac{1}{4} - \frac{3}{10}}{\frac{5}{6} + \frac{22}{39} \times 1\frac{2}{11}}$ (4 marks)

N: $\frac{9}{5} \div \left(\frac{2}{3} \times \frac{9}{4}\right) - \frac{3}{10}$

$\left(\frac{9}{5} \div \frac{3}{2}\right) - \frac{3}{10}$

$\left(\frac{9}{5} \times \frac{2}{3}\right) - \frac{3}{10}$

$= \frac{6}{5} - \frac{3}{10}$

$\frac{12-3}{10} = \frac{9}{10}$

D: $\frac{5}{6} + \left(\frac{22}{39} \times \frac{13}{11}\right)$

$\frac{5}{6} + \frac{2}{3}$

$\frac{5+4}{6}$

$\frac{9}{6}$

$= \frac{3}{2}$

$\frac{N}{D} = \frac{9}{10} \div \frac{3}{2}$

$= \frac{9}{10} \times \frac{2}{3}$

$= \frac{3}{5}$

2. Two bells ring at intervals of 35 and 42 minutes respectively. The bells ring together at 8.48 a.m. Determine the time when the bells will ring together again. (3 marks)

LCM of 35 and 42:

$35 = 2^0 \times 3^0 \times 5^1 \times 7^1$

$42 = 2^1 \times 3^1 \times 5^0 \times 7^1$

$LCM = 2^1 \times 3^1 \times 5^1 \times 7^1$

$= 210$ Minutes

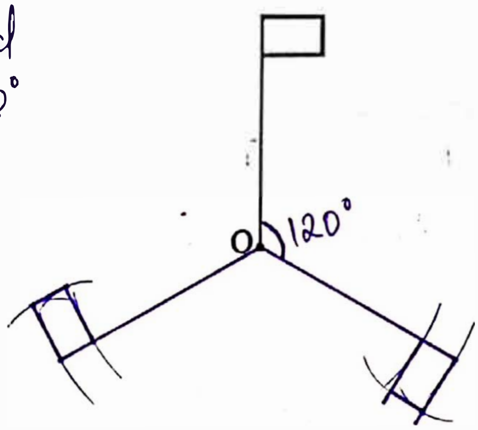
$= 3$ hours 30 minutes

Time $=$ 8.48
$\quad$ 3.30 +
12.18 p.m.

3. Complete the figure below to show a rotational symmetry of order 3 about O. (3 marks)

Angle of rotational Symmetry $= \frac{360^\circ}{3}$

$= 120^\circ$



4. Solve $\frac{5}{3} - 2x < 1 - \frac{2}{3}x \leq 2 - x$. Hence list the integral values that satisfy the inequalities. (3 marks)

$\frac{5}{3} - 2x < 1 - \frac{2}{3}x \; ; \; 1 - \frac{2}{3}x \leq 2 - x$

$\frac{5}{3} - 1 < 2x - \frac{2}{3}x \; ; \; x - \frac{2}{3}x \leq 2 - 1$

$\frac{2}{3} < \frac{4}{3}x \; ; \; \frac{1}{3}x \leq 1$

$\frac{1}{2} < x \; ; \; x \leq 3$

$\Rightarrow \frac{1}{2} < x \leq 3$

Integral values are 1, 2, 3

5. The size of two interior angles of an irregular polygon each measures 90°. All the other remaining interior angles each measure 150°.

Determine the number of sides of the polygon. (3 marks)

$(2n-4)90 = 2 \times 90 + (n-2)150$

$180n - 360 = 180 + 150n - 300$

$(180 - 150)n = 180 - 300 + 360$

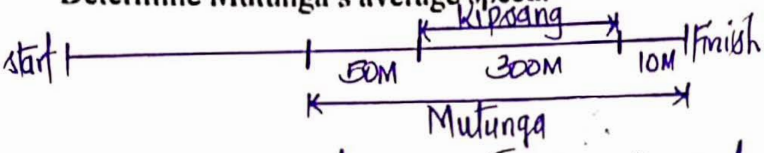
$30n = 240$

$n = 8$

8 sides.

6. In a race Kipsang maintained an average speed of 5 m/s. When he was 310 m to the finishing line, Mutunga was 50 m behind him. However, Mutunga finished the race 10 m ahead of Kipsang.

Determine Mutunga's average speed. (3 marks)



$\Rightarrow$ Mutunga covered 360M in the same length of time Kipsang took to cover 300M

$$\Rightarrow \frac{300}{5} = \frac{360}{x} \Rightarrow x = \frac{5 \times \cancel{360}^{6}}{\cancel{300}_{\cancel{60}_{1}}} \Rightarrow x = 6\,m/s \text{ (Mutunga's av. speed)}$$

7. Simplify $(4 + 2y)^2 - (2y - 4)^2$. (2 marks)

$$(16 + 16y + 4y^2) - (4y^2 - 16y + 16)$$

$$16 - 16 + 16y + 16y + 4y^2 - 4y^2$$

$$= 32y$$

8. A table is sold at Ksh 4 500 and a chair at Ksh 2 000. A salesman earns a commission of 8% on every table and 5% on every chair sold. On a certain week, he sold 3 more chairs than tables and his total earnings were Ksh 3 980.

Determine the number of chairs he sold that week. (3 marks)

Let the number of chairs sold be x

$\therefore$ No. of tables sold $= (x - 3)$

$$\text{Comm.} = \tfrac{8}{100} \times 4\,500(x-3) + \tfrac{5}{100} \times 2000(x) = 3\,980$$

$$360(x-3) + 100(x) = 3980$$

$$(360 + 100)x = 3980 + 1080 = 5060$$

$$\Rightarrow x = 11$$

542

2064

9. A translation T maps A(−6, 2) onto A′(3, 5).

(a) Determine the translation vector T. (1 mark)

Let $\underset{\sim}{T} = \begin{pmatrix} x \\ y \end{pmatrix}$, then

$\begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} -6 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 5 \end{pmatrix} \Rightarrow \begin{matrix} x = 9 \\ y = 3 \end{matrix}$

$\Rightarrow \underset{\sim}{T} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$

(b) A point P′(−4, 2) is the image of P under T. Determine the coordinates of P. (2 marks)

let P(a, b), then

$\begin{pmatrix} 9 \\ 3 \end{pmatrix} + \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -4 \\ 2 \end{pmatrix} \Rightarrow \begin{matrix} a = -13 \\ b = -1 \end{matrix}$

$\Rightarrow P(-13, -1)$

10. The cost of one litre of Petrol is Ksh 110. John's vehicle covers 12 km on one litre of petrol. He used Ksh 2 805 on petrol to travel from town A to town B. Jane's vehicle consumes 12.5 litres of Petrol for every 100 km travelled.

Calculate the amount of money that Jane would use to travel from town A to B on the same road. (3 marks)

Distance from A to B $= \frac{2805}{110} \times 12 = 306$ KM

No. of litres used by Jane $= \frac{12.5}{100} \times 306 = 38\frac{1}{4}$ litres

Amount of money $= 38\frac{1}{4} \times 110 =$ Ksh 4 207.50

11. Solve for θ (2 marks)

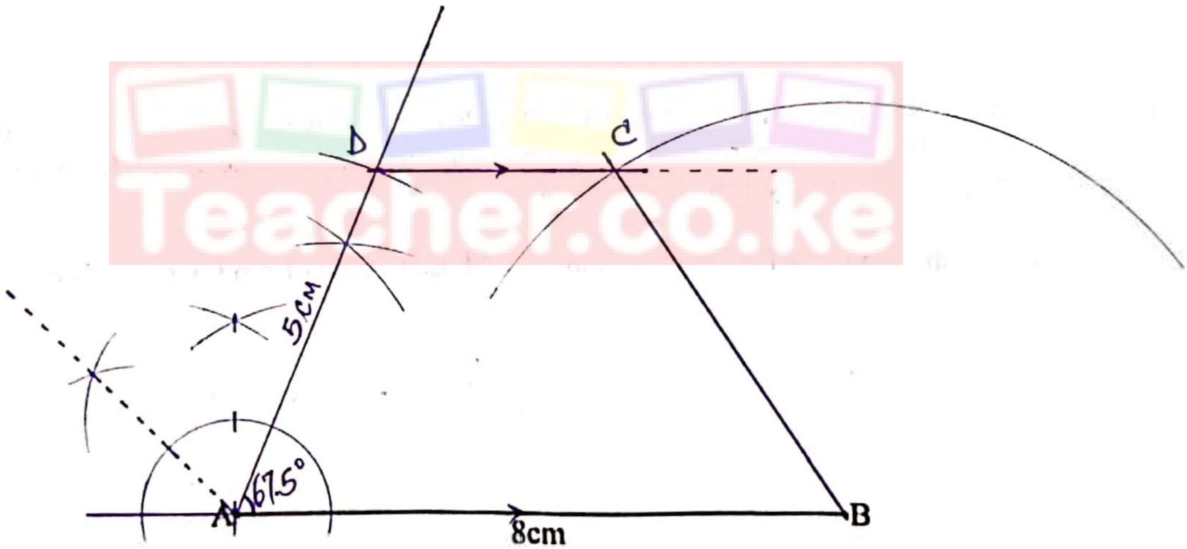
$\sin(2\theta - 15) = \cos 3\theta$.

$(2\theta - 15) + (3\theta) = 90°$

$5\theta = 105°$

$\theta = 21$

12. Line AB drawn below is a side of a trapezium ABCD.



(a) Using a ruler and pair of compasses only, complete trapezium ABCD in which AB is parallel to DC, $\angle BAD = 67.5°$, AD = 5 cm, BC = 5.5 cm and $\angle ABC$ is acute. (3 marks)

(b) Measure the length of DC. (1 mark)

3.2 CM

13. Ali left Mombasa for Nairobi on Tuesday at 2.30 a.m. He arrived in Mtito Andei after 3 hours 12 minutes. He stayed in Mtito Andei for 36 hours and then left for Nairobi. He took 5 hours 25 minutes to arrive in Nairobi.

Determine the day and time in the 12 hour system Ali arrived in Nairobi. (3 marks)

Time taken to travel $= 3h + 36h + 5h + (12+25)$ Minutes

$= 44h \ 37$ min.

Time of arrival $=$
$$\begin{array}{rr} 2 & 30 \\ 44 & 37 + \\ \hline 47 & 07\ h \end{array}$$
(this is 53 min. to Midnight)

$=$ Wednesday, 11.07 p.m.

14. The height of a cone is 12 cm. A frustrum whose volume is one eighth the volume of the cone is cut off. Determine the height of the frustrum. (3 marks)

Let vol. smaller cone be x, of larger cone;

$\therefore \frac{1}{8} + x = 1 \Rightarrow x = \frac{7}{8}$

V.S.F $= 1 : \frac{7}{8} = 8 : 7$

Let h be height of smaller cone;

$\sqrt[3]{\frac{8}{7}} = \frac{12}{h} \Rightarrow h = 11.477587$

$\Rightarrow$ Height of frustum $= 12 - h = 0.522413$ cm ≈ 0.5224 cm (4 sf)

15. Solve the equation $8^{x+1} - 2^{3x-1} = 120$. (4 marks)

$2^{3(x+1)} - 2^{3x-1} = 120$

$2^{3x+3} - 2^{3x-1} = 120$

$2^{3x} \cdot 2^{3} - 2^{3x} \cdot 2^{-1} = 120$

Let 2^{3x} be k

$8k - \frac{1}{2}k = 120 \Rightarrow k = 16$

$\Rightarrow 2^{3x} = k = 16 = 2^4$

$\Rightarrow 3x = 4 \Rightarrow x = 1\frac{1}{3}$

16. A curve is given by $y = 2x^3 - 3x^2 - 12x + 12$.

(a) Find the gradient function of the curve. (1 mark)

$$\frac{dy}{dx} = 6x^2 - 6x - 12$$

(b) Determine the equation of the normal to the curve at the point $(1, -1)$, in the form $y = mx + c$, where m and c are constants. (3 marks)

grad. of curve at $x = 1$

$= 6(1)^2 - 6(1) - 12 = -12$

$\Rightarrow$ grad. of normal $= \frac{1}{12}$

$\therefore \frac{y+1}{x-1} = \frac{1}{12}$

$y = \frac{1}{12}(x-1) - 1$

$y = \frac{1}{12}x - \frac{13}{12}$

or

$y = \frac{1}{12}x - 1\frac{1}{12}$

SECTION II (50 marks)

Answer only five questions in this section in the spaces provided.

17. A factory packs fruit jam in cylindrical tins of radius 5 cm and height 15 cm. The tins are then packed into rectangular cartons each measuring 60 cm long, 30 cm wide and 30 cm high.

(a) Determine the maximum number of tins that can be packed in one carton. **(2 marks)**

$$= \frac{60 \times 30 \times 30}{\frac{22}{7} \times 5^2 \times 15}$$

$$= 45 \text{ tins}$$

(b) An empty carton and an empty tin weighs 560 g and 300 g respectively. The jam packed in one tin weighs 990 g. A pick-up which can carry a maximum of 600 kg is used to transport the jam.

Determine the maximum number of cartons the pick-up can carry. **(4 marks)**

Mass of a fully packed carton $= 560 + (300 \times 45) + (990 \times 45)$

$= 58610\text{g}$

$= 58.61\text{ kg}$

No. of cartons $= \frac{600}{58.61} = 10$ cartons

(c) The factory delivered a pick-up full of cartons of jam to a retailer. The factory sells one carton to a retailer for Ksh 2 880. The retailer sells each tin at Ksh 110.

Calculate the percentage profit made by the retailer. **(4 marks)**

B.P by retailer $= 10 \times 2\,880 =$ Ksh 28 800

S.P by retailer $= 10 \times 45 \times 110 =$ Ksh. 49 500

% profit Made $= \frac{49\,500 - 28\,800}{28\,800} \times 100$

$= 71.875\%$

18. (a) The length of each side of an equilateral triangle ABC is 10 cm. Calculate the area of the triangle, correct to 2 decimal places. **(2 marks)**

$A = \frac{1}{2} \times 10 \times 10 \sin 60^\circ$

$= 43.30 \text{ cm}^2$

OR:

$A = \sqrt{s(s-a)(s-b)(s-c)}$

$s = \frac{1}{2}(10+10+10) = 15$

$A = \sqrt{15 \times 5 \times 5 \times 5}$

$= 43.30 \text{ cm}^2$

(b) Triangle ABC in 18(a) forms the base of a solid triangular pyramid VABC. The perpendicular height of the pyramid is 15 cm.

Calculate the volume of the pyramid. **(2 marks)**

$\text{Vol} = \frac{1}{3} \text{BA} \times h$

$= \frac{1}{3} \times 43.30 \times 15$

$= 216.50 \text{ cm}^3$

(c) The pyramid VABC in 18(b) above is recast into a cone of base radius 3.5 cm.

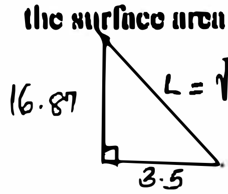
Calculate, correct to 2 decimal places:

(i) the height of the cone; **(2 marks)**

$\frac{1}{3} \times \frac{22}{7} \times 3.5^2 \times h = 216.50$

$h = 16.87 \text{ cm}$

(ii) the surface area of the cone. **(4 marks)**



$L = \sqrt{16.87^2 + 3.5^2} = 17.229 \text{ cm}$

$S.A = \pi r^2 + \pi r L \quad = \pi r(r + L)$

$= \frac{22}{7} \times 3.5(3.5 + 17.229)$

$= 228.019 \text{ cm}^2$

19. Elimu School bought 25 textbooks and 35 exercise books for Ksh 13 500 from bookshop A. From the same bookshop Soma School bought 21 textbooks and 38 exercise books and spent Ksh 1 300 less than Elimu School.

Take x to represent the price of a textbook and y to represent the price of an exercise book.

(a) Form two equations representing the above information. (2 marks)

$25x + 35y = 13\,500$
$21x + 38y = 12\,200$

(b) Use matrix method to determine the price of each item. (5 marks)

$$\begin{pmatrix} 25 & 35 \\ 21 & 38 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13\,500 \\ 12\,200 \end{pmatrix}$$

det. $= (25 \times 38) - (21 \times 35) = 215$

$$\text{Inv.} = \begin{pmatrix} \frac{38}{215} & \frac{-35}{215} \\ \frac{-21}{215} & \frac{25}{215} \end{pmatrix}\begin{pmatrix} 25 & 35 \\ 21 & 38 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{38}{215} & \frac{-35}{215} \\ \frac{-21}{215} & \frac{25}{215} \end{pmatrix}\begin{pmatrix} 13\,500 \\ 12\,200 \end{pmatrix} \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{38}{215} \times 13500 - \frac{35}{215} \times 12200 \\ \frac{-21}{215} \times 13500 + \frac{25}{215} \times 12200 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 400 \\ 100 \end{pmatrix} \Rightarrow x = \text{Ksh } 400 \text{ (Textbook)}, \quad y = \text{Ksh } 100 \text{ (Ex. book)}$$

(c) In bookshop B, the cost of a textbook was 5% less and that of an exercise book was 5% more than in bookshop A. Kasuku School bought the same number of textbooks and exercise books as Elimu School in bookshop B.

Calculate the difference in the amount spent by Kasuku School and Elimu School. (3 marks)

Bookshop B:

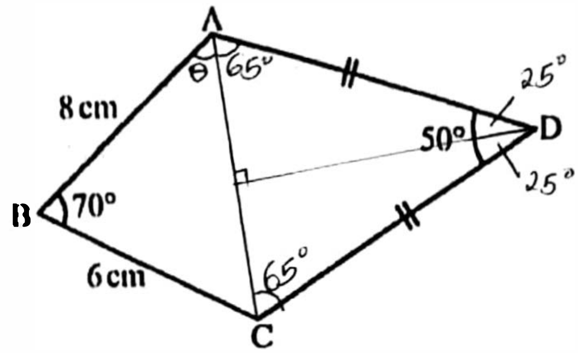
1 Textbook $= 0.95 \times 400 =$ Ksh 380

1 Ex. book $= 1.05 \times 100 =$ Ksh 105

Kasuku school spent $= (25 \times 380) + (35 \times 105)$
$=$ Ksh 13175

Difference in expend. $= |13175 - 13500|$
$=$ Ksh. 325

20. The figure below is a quadrilateral ABCD in which AB = 8 cm, BC = 6 cm, CD = AD, $\angle ABC = 70°$ and $\angle ADC = 50°$.



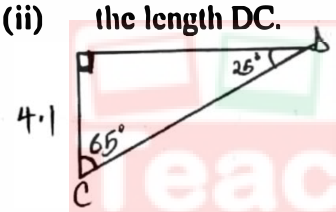
(a) Calculate, correct to one decimal place:

(i) the length AC. **(2 marks)**

$AC^2 = 8^2 + 6^2 - 2 \times 8 \times 6 \cos 70°$

$\Rightarrow AC = 8.2$ CM

(ii) the length DC. **(2 marks)**



$\sin 25° = \frac{4.1}{DC}$

$\Rightarrow DC = \frac{4.1}{\sin 25°}$

$= 9.7$ CM

Alt. 2

Let DA = DC = x cm.

$8.2^2 = x^2 + x^2 - 2x^2 \cos 50°$

$x = 9.7$ CM

(iii) the size of angle BAD. **(3 marks)**

$\frac{8.2}{\sin 70°} = \frac{6}{\sin \theta}$

$\sin \theta = \frac{6 \sin 70°}{8.2} \Rightarrow \theta = 43.4°$

$\angle BAD = 43.4° + 65°$

$= 108.4°$

(b) Calculate the area of the quadrilateral ABCD, correct to one decimal place. **(3 marks)**

$A = \frac{1}{2} \times 8 \times 6 \sin 70° + \frac{1}{2} \times 9.7 \times 9.7 \sin 50°$

$= 22.55 + 36.04$

$= 58.6\ CM^2$

21. (a) Solve for x

$(x-4)^2 = (x-8)(2x+7)$. (4 marks)

$x^2 - 8x + 16 = 2x^2 + 7x - 16x - 56$

$\Rightarrow 2x^2 - x^2 + 7x - 16x + 8x - 56 - 16 = 0$

$\Rightarrow x^2 - x - 72 = 0$

$(x+8)(x-9) = 0$

$x + 8 = 0$ or $x - 9 = 0$

$\Rightarrow x_1 = -8,\ x_2 = 9$

(b) John cycled 6 km from his home to school at an average speed of $(2x-3)$ km/h.

Peter walked 2.4 km from his home to the school at an average speed of x km/h. Peter took 16 minutes less than John.

Determine the time, in minutes, that John took to reach the school. (6 marks)

Time taken by John $= \left(\frac{6}{2x-3}\right)$ h

Time taken by Peter $= \left(\frac{2.4}{x}\right)$ h

$\therefore \left(\frac{6}{2x-3}\right) - \left(\frac{2.4}{x}\right) = \frac{16}{60}$

$\frac{6x - 2.4(2x-3)}{x(2x-3)} = \frac{16}{60}$

$(2x^2 - 3x)$

$6x - 4.8x + 7.2 = \frac{4}{15}(2x^2 - 3x)$

$15(1.2x + 7.2) = 4(2x^2 - 3x)$

$\Rightarrow 8x^2 - 30x - 108 = 0$

or $4x^2 - 15x - 54 = 0$

$\Rightarrow x = \frac{15 \pm \sqrt{15^2 - 4(4)(-54)}}{2 \times 4}$

$= \frac{15 \pm 33}{8} = -2.25$ or 6

$\Rightarrow x = 6$

John took $\frac{6}{2 \times 6 - 3} \times 60 = 40$ minutes

22. The position vectors of A and B are $\begin{pmatrix}-4\\6\end{pmatrix}$ and $\begin{pmatrix}-8\\2\end{pmatrix}$ respectively.

Point M is the midpoint of AB and point N is the midpoint of OA.

(a) Find:

(i) the vector AB. $= \underset{\sim}{OB} - \underset{\sim}{OA}$ (2 marks)

$= \begin{pmatrix}-8\\2\end{pmatrix} - \begin{pmatrix}-4\\6\end{pmatrix} \quad = \begin{pmatrix}-4\\-4\end{pmatrix}$

(ii) the coordinates of points M and N. (2 marks)

$M\left(\frac{-4-8}{2}, \frac{6+2}{2}\right) \rightarrow M(-6, 4)$

$N\left(\frac{0-4}{2}, \frac{0+6}{2}\right) \rightarrow N(-2, 3)$

or $\frac{1}{2}\underset{\sim}{OA} + \frac{1}{2}\underset{\sim}{OB}$ (Midpt theorem)

$\underset{\sim}{OM} = \frac{1}{2}\begin{pmatrix}-4\\6\end{pmatrix} + \frac{1}{2}\begin{pmatrix}-8\\2\end{pmatrix}$

$= \begin{pmatrix}-2\\3\end{pmatrix} + \begin{pmatrix}-4\\1\end{pmatrix} = \begin{pmatrix}-6\\4\end{pmatrix} \Rightarrow M(-6, 4)$

(iii) the magnitude of NM. (3 marks)

$\underset{\sim}{NM} = \underset{\sim}{OM} - \underset{\sim}{ON} = \begin{pmatrix}-6\\4\end{pmatrix} - \begin{pmatrix}-2\\3\end{pmatrix} = \begin{pmatrix}-4\\1\end{pmatrix}$

$|\underset{\sim}{NM}| = \sqrt{(-4)^2 + 1^2} = \sqrt{17}$ units

or 4.123 Units

(b) The coordinates of a point C is (2, *a*). Vector CA is parallel to vector OB.

Determine the value of *a*. (3 marks)

$\underset{\sim}{CA} = \underset{\sim}{OA} - \underset{\sim}{OC} = \begin{pmatrix}-4\\6\end{pmatrix} - \begin{pmatrix}2\\a\end{pmatrix} = \begin{pmatrix}-6\\6-a\end{pmatrix}$

$\underset{\sim}{OB} = \begin{pmatrix}-8\\2\end{pmatrix}$

If $\underset{\sim}{OB} = k\underset{\sim}{CA}$, then $k = \frac{8}{6} = \frac{4}{3}$

$\Rightarrow (6-a) \times \frac{4}{3} = 2$

$\Rightarrow a = 4.5$

23. The masses of 40 adults who attended a health clinic were recorded as follows.

64	50	58	73	51	42	58	46
58	60	45	48	69	48	50	43
52	64	58	46	59	54	41	61
73	49	74	53	44	73	53	67
62	47	66	52	60	61	54	70

(a) Complete the frequency distribution table below for the above information. Use classes of size 5 starting with the class 40 – 44. **(4 marks)**

Tally	Mass *(kg)*	Frequency *(f)*	Mid points *(x)*	*fx*	*cf*
////	40 – 44	4	42	168	4
𝍸 //	45 – 49	7	47	329	11
𝍸 ///	50 – 54	8	52	416	19
𝍸 /	55 – 59	6	57	342	25
𝍸 //	60 – 64	7	62	434	32
///	65 – 69	3	67	201	35
𝍸	70 – 74	5	72	360	40
		40		2250	

(b) State the modal class. **(1 mark)**

50 - 54 Kg

(c) Estimate:

(i) the mean mass. **(2 marks)**

$$\bar{x} = \frac{\Sigma fx}{\Sigma f} = \frac{2250}{40}$$

$$= 56.25 \text{ Kg}$$

(ii) the median mass. **(3 marks)**

L $\frac{40}{2}$ = 20^{th} adult had median mass

$$\text{Median} = 54.5 + \left(\frac{20-19}{6}\right) \times 5$$

$$= 55\tfrac{1}{3} \text{ kg} \quad \text{or} \quad 55.\dot{3} \text{ Kg}$$

24. The equation of a curve is given as $y = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x - \frac{1}{3}$.

(a) Find:

(i) the value of y when $x = -2$. (2 marks)

$y = \frac{1}{3}(-2)^3 - \frac{1}{2}(-2)^2 - 2(-2) - \frac{1}{3}$

$= -1$

(ii) the equation of the tangent to the curve at $x = -2$. (4 marks)

$\frac{dy}{dx} = x^2 - x - 2 \Big|_{x=-2}$ gradient $= 4$

$(-2, -1)$, $g = 4$

$\frac{y+1}{x+2} = \frac{4}{1} \quad \Rightarrow y = 4x + 7$

(b) Determine the coordinates of the turning points of the curve. (4 marks)

$\frac{dy}{dx} = x^2 - x - 2 = 0$

$\Rightarrow (x-2)(x+1) = 0$

$\Rightarrow x - 2 = 0 \Rightarrow x_1 = 2$

or $x + 1 = 0 \Rightarrow x_2 = -1$

when $x = 2$, $y = \frac{1}{3} \times 8 - \frac{1}{2} \times 4 - 2 \times 2 - \frac{1}{3} = -3\frac{2}{3}$

when $x = -1$, $y = \frac{1}{3}(-1)^3 - \frac{1}{2}(-1)^2 - 2(-1) - \frac{1}{3} = \frac{5}{6}$

turning points: $(-1, \frac{5}{6})$ and $(2, -3\frac{2}{3})$

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