MATHEMATICS FORM 4 PAPER 2 MARKING SCHEME

1. Use logarithm table to evaluate (4marks)

$\sqrt{\frac{75.4 \times 4.83^2}{0.00521}}$

No	Log
75.4	1.8774
4.83	0.6839
	2.5613
4.83	0.6839
	3.2452
0.00521	3.7168
	5.5284
	$\frac{5.5284}{5}$
1.276×10^1 ←	1.1057

$= 12.95$

B1 for all logs read correctly
M1 for addition and subtraction
M1 for dividing by 5
A1 for the answer

2. Make b the subject of the formula given that $a = \frac{bd}{\sqrt{Nb^2 - d}}$ (3 marks)

$a^2 = \frac{b^2d^2}{Nb^2 - d}$ ✓ M1

$Nb^2a^2 - a^2d = b^2d^2$ M1

$Nb^2a^2 - b^2d^2 = a^2d$

$b^2(Na^2 - d^2) = a^2d$ ✓

$b^2 = \frac{a^2d}{Na^2 - d^2}$ A1

$b = \pm\sqrt{\frac{a^2d}{a^2N - d}}$ ✓

3. Line PQ is the diameter of a circle such that the coordinates of P and Q are (2, 2) and (-2,-6) respectively. Find the equation of the circle in the form $ax^2 + ay^2 + bx + cy + d = 0$. (4marks)

$\left(\frac{-2+-2}{2}, \frac{-6+2}{2}\right)$ B1

Centre $(-2, 2)$ ✓

Radius $\sqrt{(-2+2)^2 + (2-6)^2}$ A1

$\sqrt{0+16} = \sqrt{16} = 4$ units ✓ M1

$(x+2)^2 + (y-2)^2 = 16$ ✓

$x^2 + 4x + 4 + y^2 - 4y + 4 = 16$

$x^2 + y^2 + 4x - 4y + 8 - 16 = 0$ A1

$x^2 + y^2 + 4x - 4y - 8 = 0$ ✓

4. Use completing the square method to solve the equation
$4 - 3x - 2x^2 = 0$ (3marks)

$-2x^2 - 3x + 4 = 0$

$x^2 + \frac{3}{2}x - 2 = 0$

$x^2 + \frac{3}{2}x - 2$

$x^2 + \frac{3}{2}x + \frac{9}{16} = 2 + \frac{9}{16} = \frac{41}{16}$

$\left(x + \frac{3}{4}\right)^2 = \frac{41}{16}$

$x + \frac{3}{4} = \pm\frac{\sqrt{41}}{4}$

$x = 0.8508$ or -2.3508

5. Given that $P=4-\sqrt{2}$ and $Q=2+\sqrt{2}$ and that $\frac{P}{Q}=a+b\sqrt{c}$, where a, b and c are constants, find the values of a, b and c. (3 marks)

$\frac{4+\sqrt{2}}{2+\sqrt{2}} \times \frac{2-\sqrt{2}}{2-\sqrt{2}} = \frac{4(2-\sqrt{2}) + \sqrt{2}(2-\sqrt{2})}{4-2}$ ✓ M_1

$\frac{8 - 4\sqrt{2} + 2\sqrt{2} - 2}{2}$ A_1

$\frac{6 - 2\sqrt{2}}{2} = 3 - \sqrt{2}$ ✓

B_1 for a, b and c

$a = 3$

$b = -1$

$c = 2$

6. The table below shows the temperature readings of four different solutions recorded by students to nearest $0.1^{\circ}C$ during a laboratory lesson. Calculate the percentage error in $\frac{P+Q}{S-R}$ to 3 d.p. (3marks)

Quantity	Temperature in $^{\circ}C$
P	22.5
Q	19.4
R	17.3
S	26.2

M_1 for Minimum and max

Max $\frac{22.55 + 19.45}{26.15 - 17.35} = \frac{105}{22} = 4.77$

Min $\frac{22.45 + 19.35}{26.25 - 17.25} = \frac{209}{45} = 4.64$ ✓

Actual $\frac{22.5 + 19.4}{26.2 - 17.3} = \frac{419}{89} = 4.71$

$\frac{4.77 - 4.64}{2} \times 100$ ✓ M_1

$\frac{11}{200} \div 4.71 \times 100$ 3

$= 1.168\%$ ✓ A_1

7. Use matrix method to solve the simultaneous equations

$2x + y = 10$

$2x + 2y = 14$ (3marks)

$\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$ ✓ M_1

$\frac{1}{2}\begin{pmatrix} 2 & -1 \\ -2 & 2 \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} \\ -1 & 1 \end{pmatrix}$ M_1

$\begin{pmatrix} 1 & -\frac{1}{2} \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 2 & 1 \\ 2 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 & -\frac{1}{2} \\ -1 & 1 \end{pmatrix}\begin{pmatrix} 10 \\ 14 \end{pmatrix}$ ✓

$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 - 7 \\ -10 + 14 \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ A_1

$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $x = 3$ ✓

$y = 4$

8. (a) Expand $(1+2x)^5$ to the fourth term. (1 mark)

$1 + 5(2x) + 10(2x)^2 + 10(2x)^3 + \ldots$

$1 + 10x + 40x^2 + 80x^3$ ✓ B_1

(b) Hence evaluate $(1.02)^5$ correct to 3 decimal places. (3 marks)

$(1.02)^5 = (1 + 0.02)^5$ ✓ B_1

$2x = 0.02$

$x = 0.01$ ✓

$1 + 10(0.01) + 40(0.01)^2 + 80(0.01)^3$ ✓ M_1

$1 + 0.1 + 0.004 + 0.000008$

1.104008 ✓ A_1

9. It is known that the value of land appreciate at 7% p.a in a town. John bought a plot in the town at Ksh 500,000. Given that he plans to sell the plot after 6 years, find out how much profit he expects to get. (Give your answer correct to the nearest thousand). (3marks)

$A = P\left(1 + \frac{r}{100}\right)^n$ M_1

$500,000(1 + 0.07)^6$ ✓

$500,000(1.07)^6$

$750,365.1759$ M_1

$750,365.1759 - 500,000$ ✓ 4

$250,365.1759$

$\approx 250,000$ ✓ A_1

10. The mass of a wire varies jointly with its length and with the square of its diameter. A section of the wire 500m long, with diameter 3mm has a mass of 31.5kg. what is the mass of 1000m of wire of diameter 2mm? (3marks)

$m \propto ld^2$

Where mass → m, length → l, diameter → d

$m = kld^2$ where k is a constant. M_1

$31.5 = 500 \times 3 \times 3k$ ✓

$k = \frac{31.5}{500 \times 9} = \frac{7}{1000}$ M_1

$m = \frac{7}{1000} \times 1000 \times 2 \times 2$ ✓

$= 28\,kg$ ✓ A_1

11. Mr. Gatua has a salary of sh.80000 per annum. He lives rent free in company house and is entitled to a monthly personal relief of sh.1056. Based on the tax rates given below, calculate his PAYE. (3 marks)

Taxable income (KE p.a.)	Rate	
1 - 1500	10%	$1500 \times 0.1 = 150$
1501 - 3000	15%	$1500 \times 0.15 = 225$
3000 - 4500	25%	$1500 \times 0.25 = 375$
Above 4500	35%	$100 \times 0.35 = 35$

✓ M_1

$\frac{80,000 \times 11.5}{20} = 4600$ kf

$\frac{(150 + 225 + 375 + 35) \times 20}{12} = 1308.33$ ✓ M_1

$1308.33 - 1056$

$= 252.33$

≈ 252 ✓ A_1

12. The third term and sixth term of a geometric series are $3\frac{1}{3}$ and $11\frac{1}{4}$ respectively. Calculate the common ratio and hence find its first term. (3marks)

$ar^2 = 10/3$

$ar^5 = 45/4$

$a = \frac{10}{3} / r^2$

$a = \frac{45}{4} / r^5$

$\frac{10}{3} \times \frac{1}{r^2} = \frac{45}{4} \times \frac{1}{r^5}$

$\frac{10}{3r^2} = \frac{45}{4r^5}$ ✓

$40r^5 = 135r^2$ M_1

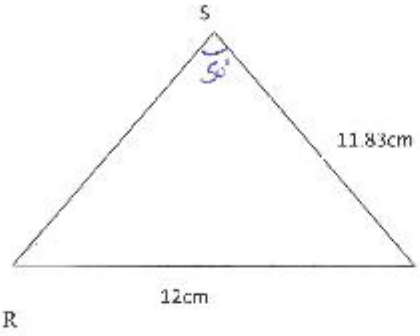
$r^3 = \frac{135}{40}$ M_1

$r = 3/2$

$a \times \frac{9}{4} = 10/3$ ✓

$a = \frac{40}{27}$. A_1 for both answers.

13. Use the figure below to answer the question that follows



Given that angle RSQ = 50°, SQ= 11.83 cm and QR=12cm.A circumcirle is drawn on the triangle.Find the radius of the circle (2marks)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2R$

$\frac{12}{\sin 50} = 2R$

$R = \frac{12}{2\sin 50}$ ✓ A1

$= 7.8324$ ✓ A1

14. A business man bought commodity A and commodity B at shs.60 and sh.72 respectively. In what ratio must he mix so that when he sells at shs 68. (3 marks)

$\frac{60A + 72B}{A+B} = 68$

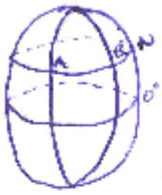
$60A + 72B = 68A + 68B$

$-8A = -4B$

$\frac{A}{B} = \frac{4}{8} = \frac{1}{2}$

$\therefore A:B = 1:2$

15. Points $A(x^\circ N, 30^\circ E)$ and $B(x^\circ N, 50^\circ E)$ are 1935 kilometres apart. Taking R= 6370 km and $\pi = \frac{22}{7}$, find the value of x. (3marks)



$\frac{20}{360} \times \frac{22}{7} \times 2 \times 6370 \cos x = 1935$ ✓ M1

$\cos x = \frac{1935 \times 7 \times 360}{20 \times 22 \times 2 \times 6370} = 0.8699$

M1

$x = \cos^{-1} 0.8699$ ✓

$= 29.55$ ✓ A1

16. Find the gradient function of the curve $y = \frac{1}{3}x^3 - 4x^2 + 9x + 4$ hence, find the gradient of the curve at point (1,-4) (3marks)

$\frac{dy}{dx} = x^2 - 8x + 9$ ✓ M1

$(1)^2 - 8(1) + 9$ ✓ M1

$1 + 9 - 8 = 2$ ✓ A1

17. Use a scale of 1:1 in both axes to draw the graphs of $y = x^2 - 6x + 7$ and $y = x - 2$ for the domain $0 \le x \le 6$. The point of intersection of the two functions satisfy a certain quadratic equation in x. Obtain the equation in x hence calculate it's solutions. Give answer correct to 2d.p. (10 marks)

$y = x^2 - 6x + 7$

x	0	1	2	3	4	5	6
y	7	2	-1	-2	-1	2	7

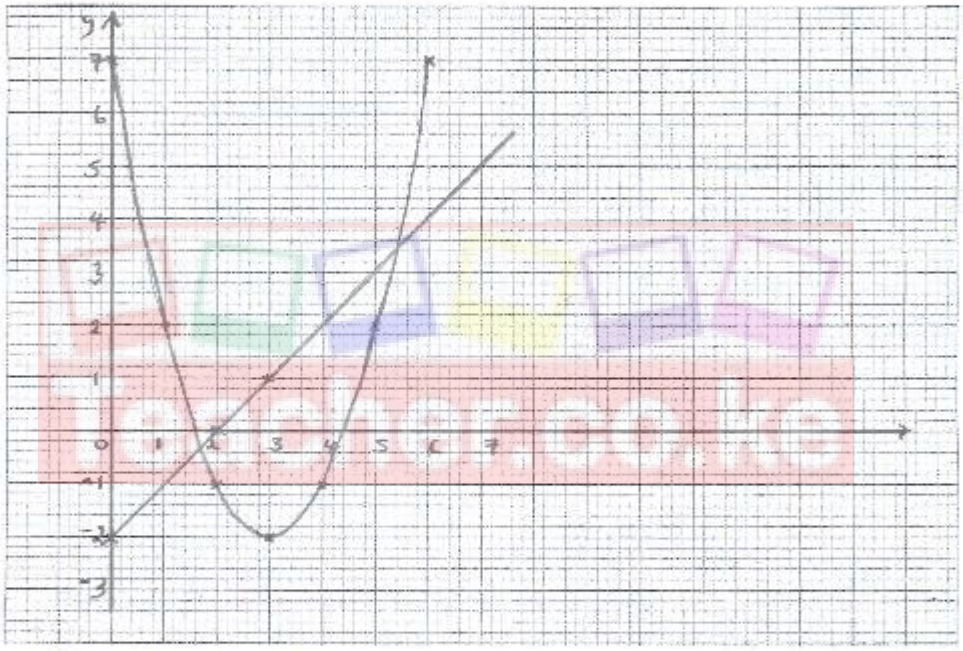
✓

$y = x - 2$

x	0	3	2
y	-2	1	0

✓

B, B, B,



S, P, C, L,

$x^2 - 6x + 7 = x - 2$ B,

$x^2 - 7x + 9 = 0$ ✓

$x = \frac{7 \pm \sqrt{49 - 4 \times 1 \times 9}}{2}$ ✓ M,

$x = \frac{7 \pm \sqrt{13}}{2}$

$x = 5.30$ or $x = 1.70$ } ✓ A,

8

18. Points A and B are centres of two equal circles of a radius 2 cm and 10 cm apart.
    i. Construct the two circles in the space given below. (1mark)
    ii. Construct the transverse common tangents to both circles. (4marks)
    iii. Calculate the length of the belt round the two circles. (Take $\pi = \frac{22}{7}$) (5marks)

B_1 for the two circles
B_1 " bisecting AB
B_1 " circle radius 4cm
B_1 " dotted lines
B_1 " transverse common tangent.

Length of belt

$\sqrt{5^2 - 2^2} = \sqrt{21} = 4.5826$ ✓

$\cos^{-1}\frac{2}{5} = 66.42°$

$180 - 66.42 = 113.58$ ✓

$113.58 \times 2 = 227.16$

$2\left(\frac{227.2}{360} \times \frac{22}{7} \times 4\right) + (4.5826 \times 4)$ ✓

$15.85 + 18.32$ ✓

34.18 ✓

B_1

B_1

M_1

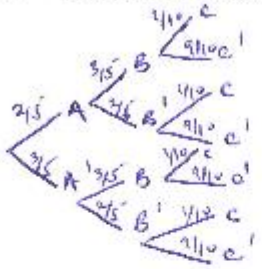
A_1

9

A_1

19. Albert, Bonny and Charles competed in a game of chess. Their probabilities of winning the game are $^2/_5$, $^3/_5$ and $^1/_{10}$ respectively.

(a) Draw a probability tree diagram to show all the possible outcomes. (2 marks)



B1 B1

(b) Calculate the probability that:

(i) No one loses the game. (2 marks)

$\frac{2}{5} \times \frac{3}{5} \times \frac{1}{10}$ ✓ M1

$\frac{3}{125}$ ✓ A1

(ii) Only one of them wins the game. (2 marks)

$\frac{2}{5} \times \frac{2}{5} \times \frac{9}{10}$ or $\frac{3}{5} \times \frac{3}{5} \times \frac{9}{10}$ or $\frac{3}{5} \times \frac{2}{5} \times \frac{1}{10}$ ✓ M1

$\frac{18}{125} + \frac{81}{250} + \frac{3}{125}$ ✓

A1

$= \frac{123}{250}$ ✓

(iii) At least one of them wins the game. (2 marks)

$1 - \left(\frac{3}{5} \times \frac{2}{5} \times \frac{9}{10}\right)$ ✓ M1

$1 - \frac{27}{125}$

$\frac{98}{125}$ ✓ A1

(iv) At most two of them lost the game. (2marks)

$1 - \frac{123}{250}$ ✓ M1

$\frac{127}{250}$ ✓ A1 10

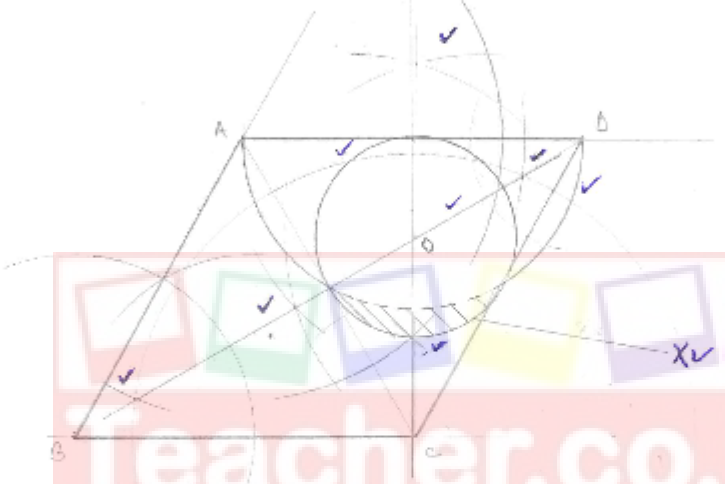
20. Construct rhombus ABCD such that AB=BC= 6cm and ∠ABC=60°.

(a) Measure BD. 11.3cm (1 mark)

(b) On the same diagram, construct the inscribed circle of triangle ACD. (3marks)

(c) Construct the locus of points equidistant from A and C. (3 marks)

(d) If x is a point on the circle in b above such that AX=XD and ∠AXD is acute, find the locus of X and make it on the diagram. (3 marks)



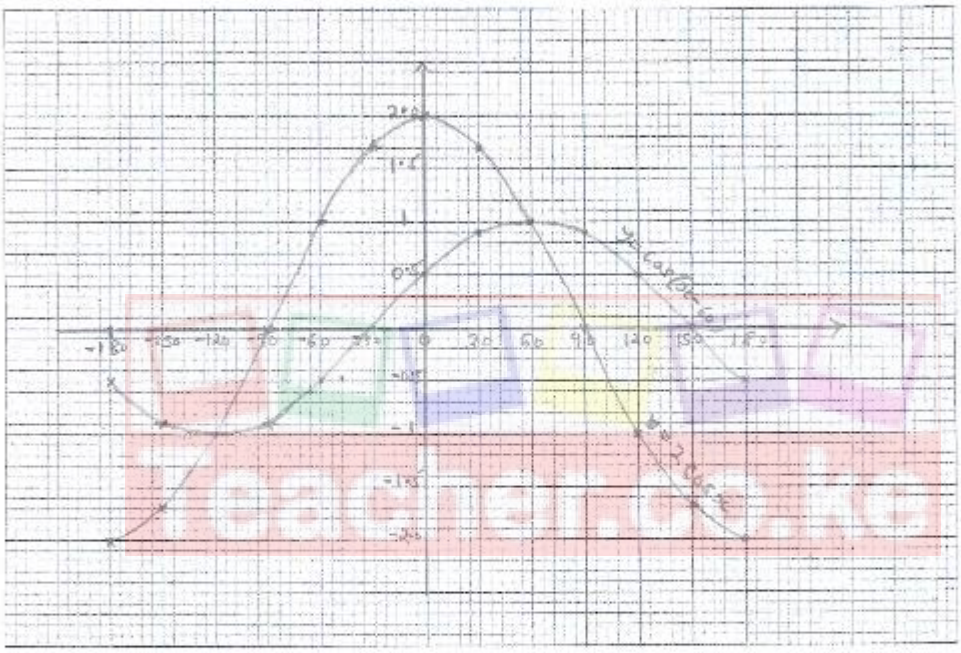
B_1 - for 60° constructed
B_1 for rhombus
B_1 B_1 for bisecting ∠ACD/∠CDA/∠DAC
B_1 for the circle centre O
B_1 for Bisecting AC
B_1 for Bisecting AD
B_1 for semi circle diameter AD
B_1 for the broken line identifying X
B_1 for locus of X → Shaded Region

21. (a) Complete the table below. (2marks)

X	-180°	-150°	-120°	-90°	-60°	-30°	0°	30°	60°	90°	120°	150°	180°
Y=2cosx	-2	-1.73	-1	0	1	1.73	2	1.73	1	0	-1	-1.73	-2
Y=cos(x-60)	-0.5	-0.9	-1	-0.9	-0.5	0	0.5	0.9	1	0.9	0.5	0	-0.5

B1 B1

(b) On the same axes plot the graphs of y= cos(x-60°) and y = 2 cosx (use a scale of 1unit for 30° on the x axis and 1 unit for 0.5 units on the y axis) (4mrks)



S1 P1 C1 C1

(c) Describe the transformation which maps y= cos(x-60°) to y = 2cosx. (2marks)

Translation by translation vector $\begin{pmatrix} -60 \\ 0 \end{pmatrix}$ followed by a stretch, stretch factor 2, x-axis invariant

B1 B1 given jointly

(d) State the period and amplitude of each of the waves above. (1mark)

	Amplitude	Period
Y=2cosx	2	360°
Y=cos(x-60)	1	360°

B1 for both

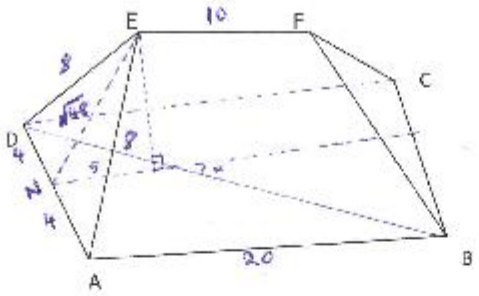
(e) Using the graph above determine the values of x for which

cos(x-60°) - 2cosx = 0 (1mark)

x = -120° and 60°

B1 for 2 both

22.



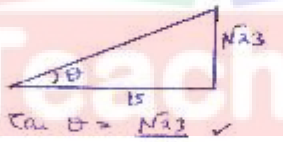
The roof of a building is as shown in the figure above with a rectangular base ABCD. AB = 20m and AD = 8m. The ridge EF = 10m and is centrally placed. The faces ADE and BFC are equilateral triangles. Calculate

(i) The height of E above the base ABCD (2 marks)

Height $\sqrt{48-25}$ ✓ M_1

$\sqrt{23} = 4.7958$ ✓ A_1

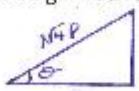
(ii) The angle between the planes ABCD and ADF (3 marks)



$\tan\theta = \frac{\sqrt{23}}{15}$ ✓ M_1 M_1

$\theta = \tan^{-1}\frac{\sqrt{23}}{15}$ ✓ $= 17.73^\circ$ ✓ A_1

(iii) The angle between the planes AED and ABCD (2 marks)

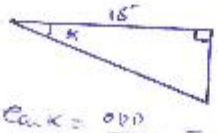


M_1

$\cos\theta = \frac{5}{\sqrt{48}}$ ✓

$\theta = \cos^{-1}\frac{5}{\sqrt{48}} = 43.81$ ✓ A_1

(iv) The acute angle between lines DB and EF (3 marks)



$\tan x = \frac{\text{opp}}{\text{adj}} = \frac{4}{15}$ ✓ M_1

M_1 13

$x = \tan^{-1}\frac{4}{15} = 14.93^\circ$ ✓ A_1

23. Kiprop has at least 50 ares of land on which he plans to plant potatoes and cabbages. Each are of potatoes requires 6 men and each are of cabbages requires 2 men. The farmer has 240men available and he must plant at least 10 ares of potatoes. The profit on potatoes is kshs.1200 per are. If he plants x ares of potatoes and y ares of cabbages;

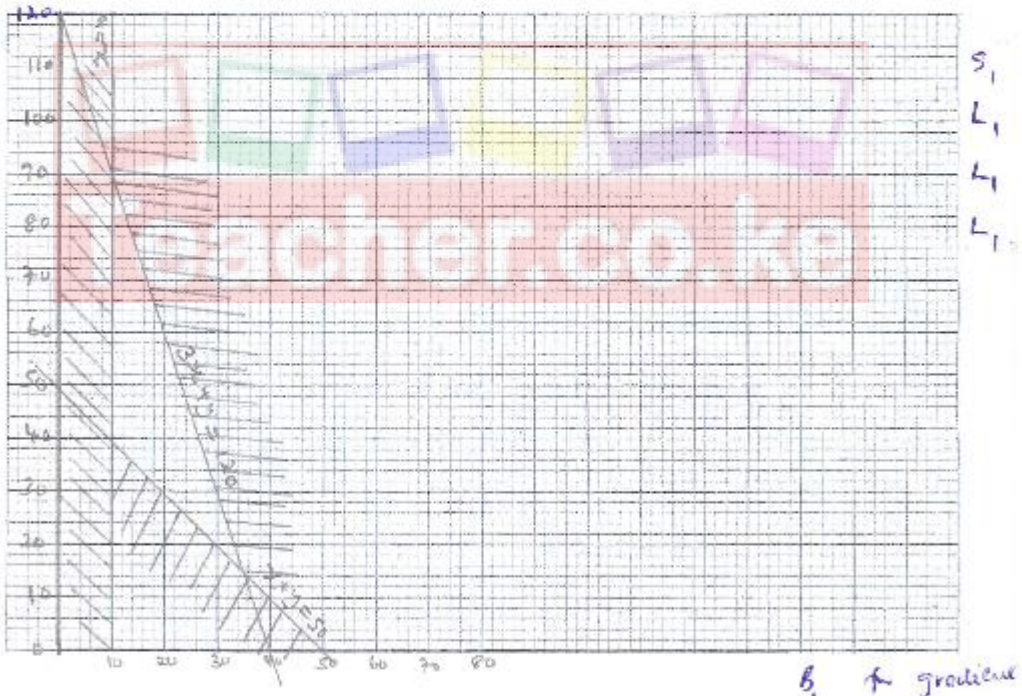
(a) Write down 3 in equalities in x and y to describe the information.(2 marks)

(b) Represent these in equalities graphically. (use a scale of 1:10 for both axes) (4 marks)

(c) Use your graph to determine the number of acres for each vegetable which will give maximum profit. (4 marks)

(a) $x + y \leq 50$
$x \geq 10$
$3x + y \leq 120$ ✓

B1 B1



S1
L1
L1
L1

B1 for graduated axes
B1 for 3 correct lines

14

(c) (10, 90) ✓B1

kshs 118,000 ✓B1

24. (a) Complete the table below for $y = x^2 - 3x + 5$ in the range $2 \le x \le 8$ (2marks)

x	2	3	4	5	6	7	8
y	3	5	9	15	23	33	45

✓ B_1

(b) Use the trapezium rule with six strips to estimate the area enclosed by the curve, x-axis and the lines x=2 and x=8. (2marks)

$h = \frac{8-2}{6} = 1$ ✓ B_1

$\frac{1}{2} \times 1 \{(3+45) + 2(5+9+15+23+33)\}$ ✓ M_1

$\frac{1}{2}\{48 + 170\}$

$\frac{1}{2}\{218\}$ A_1

$= 109$ ✓ sq units

(c) Find the exact area of the region given in (b). (4marks)

$\int_2^8 (x^2 - 3x + 5)\,dx = \left[\frac{x^3}{3} - \frac{3x^2}{2} + 5x + c\right]_2^8$ ✓ M_1

$\left[\frac{512}{3} - \frac{192}{2} + 40 + c\right] - \left[\frac{8}{3} - \frac{12}{2} + 10 + c\right]$ ✓ M_1 M_1

108 sq units ✓ A_1

(d) Calculate the percentage error in the area. (2marks)

$109 - 108 = 1$

$\frac{1}{108} \times 100$ ✓ M_1

$= 0.9259\%$ ✓ A_1