NAME:………………………………………………………………CLASS:………ADM NO:…….

TEACHER .CO.KE
MATHEMATICS PAPER 2

OPENER EXAM
FORM4
TERM1 – 2022
TIME: 2hrs 30mins

INSTRUCTION.

Answer all questions in the spaces provided.

1. Factorize the expression $2x^2 + x - 15$. Hence solve the equation $2x^2 + x - 15 = 0$. (3mks)

$(6, -5)$
$2x^2 + 6x - 5x - 15$
$2x(x+3) - 5(x+3)$
$(2x-5)(x+3)$
$(2x-5)(x+3) = 0$
$2x - 5 = 0$
$\therefore x = 2\frac{1}{2}$ or

$x + 3 = 0$
$x = -3$

M1 – Factors
M1
A1
3

2. Two people each working for 8 hours a day can cultivate an acre of land in 4 days. How long would 6 people, each working 4 hours a day, take to cultivate 4 acres? (3mks)

People	2	6
Hours	8	4
Acres	1	4
Days	4	?

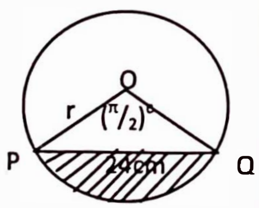
$2 : 6$
$8 : 4$
$4 : 1$

$\frac{\cancel{2}}{\cancel{6}_{3}} \times \frac{8}{\cancel{4}_{1}} \times \frac{\cancel{4}^{1}}{1} \times 4$

$10\frac{2}{3}$ days.

M1 Ratios
M1 – Accept other methods
A1
3

3. In the figure below, O is the centre of the circle $<POQ = (^{\pi}/_{2})^c$ and PQ=24cm.



a) i. Express <POQ in degrees. (1mk)

$2\pi^c \rightarrow 360^\circ$
$\left(\frac{\pi}{2}\right)^c \rightarrow ?$

$\frac{\pi}{2} \times \frac{1}{2\pi} \times 360$
$= 90^\circ$

B1

ii. Find the radius r of the circle. (1mk)

$r^2 + r^2 = 24^2$ | $r = \sqrt{288}$

$2r^2 = 576$ | $= 16.97$

B1

b) Find the area of the shaded segment. (2mks)

$A = \frac{\theta}{360}\pi r^2 - \frac{1}{2}ab\sin C$

$A = \frac{90}{360} \times \pi \times 16.97^2 - \frac{1}{2} \times 16.97^2 \sin 90$

$= 226.3 - 143.99$

$= 82.31\ cm^2$

M1 -substitutions

A1

2

4. The difference between the eight term and the fourth term of an AP is 24. The first term of this series exceeds the common difference by 4. Find the tenth term of the series. (3mks)

Let $8^{th} = a + 7d$

$\therefore 4^{th} = a + 3d$

$a + 7d - (a + 3d) = 24$

$\therefore 4d = 24$

$d = 6$

$a - d = 4$

$a - 6 = 4$

$\therefore a = 10$

$10^{th} = a + 9d$

$= 10 + 9 \times 6$

$= 64$

M1

M1

A1

3

5. Simplify $\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}}$, giving your answer in the form $a + b\sqrt{c}$, where a,b and c are real numbers. (3mks)

$\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} + \sqrt{2}} \times \frac{(\sqrt{3} - \sqrt{2})}{(\sqrt{3} - \sqrt{2})}$

Num.

$\sqrt{3}.\sqrt{3} - \sqrt{3}.\sqrt{2} - \sqrt{3}.\sqrt{2} + \sqrt{2}.\sqrt{2}$

$3 - 2\sqrt{6} + 2$

$5 - 2\sqrt{6}$

Den.

$\sqrt{3}.\sqrt{3} - \sqrt{3}.\sqrt{2} + \sqrt{3}.\sqrt{2} - \sqrt{2}.\sqrt{2}$

$3 - 2$

$= 1$

$\Rightarrow \frac{5 - 2\sqrt{6}}{1}$

$= 5 - 2\sqrt{6}$

m1

M1

A1

3

6. Find the first five terms of the expansion $(2-{}^{1}/_{x})^{8}$, hence, evaluate $(1.75)^{8}$ (3mks)

$2^8 + 2^7\left(-\frac{1}{x}\right) + 2^6\left(-\frac{1}{x}\right)^2 + 2^5\left(-\frac{1}{x}\right)^3 + 2^4\left(-\frac{1}{x}\right)^4 + ---$

$256 + 128\left(-\frac{1}{x}\right) + 64\left(\frac{1}{x^2}\right) + 32\left(-\frac{1}{x^3}\right) + 16\left(\frac{1}{x^4}\right) + ---$

$256 + 8\times128\left(-\frac{1}{x}\right) + 28\times64\left(\frac{1}{x^2}\right) + 56\times32\left(-\frac{1}{x^3}\right) + 70\times16\left(\frac{1}{x^4}\right) + -$

$256 - \frac{1024}{x} + \frac{1792}{x^2} - \frac{1792}{x^3} + \frac{1120}{x^4}$

M1 - expansion

M1 - substituting

$1.75 = (2-0.25)^8$

Let $-\frac{1}{2}x = -0.25$ $-\frac{1}{x} = -0.25$ $\therefore x = 4$

$256 - \frac{1024}{4} + \frac{1792}{4^2} - \frac{1792}{4^3} + \frac{1120}{4^4}$ $= 256 - 256 + 112 - 28 + 4.375$ $= 88.375$

A1

7. A quantity p is partly constant and partly varies as the square of Q. when Q=2,p=40 and when Q=3, P=65. Determine the value of p when Q=4. (3mks)

$P = K + CQ^2$

(i) $40 = K + 4C$

(ii) $65 = K + 9C$

(ii) − (i)

$25 = 5C$

$\therefore C = 5$

$K = 20$

$\Rightarrow P = 20 + 5Q^2$

When $Q = 4$

$P = 20 + 5\times4^2$

$= 100$

M1 - Equations

B1 - C and K values

B1

8. Given that $x = \begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}$ and $y = \begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix}$

Find:

a) XY (2mks)

$\begin{pmatrix} 1 & 0 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} 2 & -1 \\ 4 & -3 \end{pmatrix}$

M1

A1

b) $(XY)^{-1}$ (2mks)

Det. $= 2\times-3 - 4\times-1$

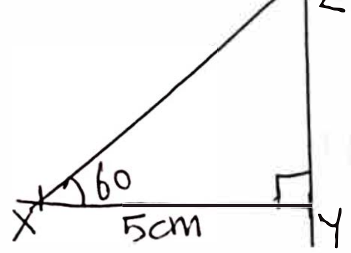
$= -2$

$\therefore (XY)^{-1} = \frac{1}{-2}\begin{pmatrix} -3 & 1 \\ -4 & 2 \end{pmatrix}$

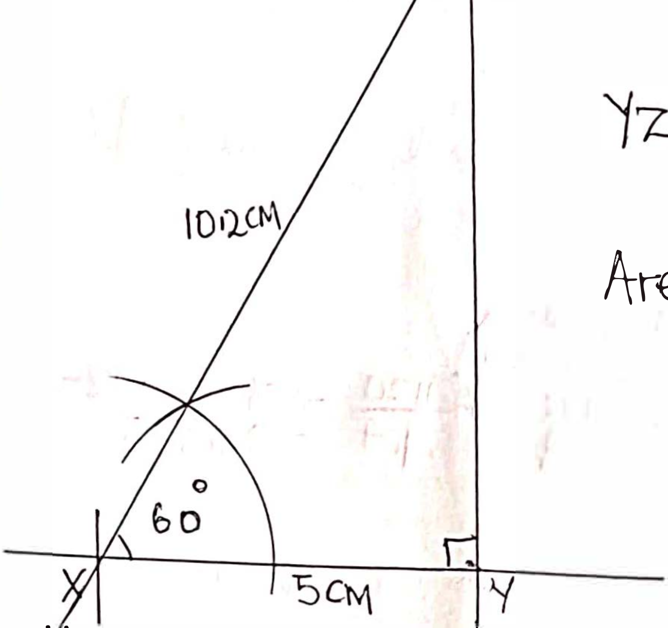
$= \begin{pmatrix} 3/2 & -1/2 \\ 2 & -1 \end{pmatrix}$

M1

A1



9. Construct the triangle XYZ in which XY=5cm, <YXZ = 60° and <XYZ= 90°. Measure YZ, hence use area= ½ absinθ to, calculate the area of triangle XYZ. (3mks)



$YZ = 10.2 \pm 1$ cm — B1 – ΔXYZ drawn

$Area = \frac{1}{2} \times 10.2 \times 5 \sin 60$ — M1 – correct subst.

$= 5.1 \times 5 \times 0.867$

$= 22.1\ cm^2$ — A1

3

10. The position vectors of points P,Q and R are $\begin{pmatrix}1\\-1\\3\end{pmatrix}$ $\begin{pmatrix}3\\3\\1\end{pmatrix}$ and $\begin{pmatrix}6\\9\\-2\end{pmatrix}$ respectively.

a) Find (i) PQ (1mk)

$PQ = Q - P$

$\begin{pmatrix}3\\3\\1\end{pmatrix} - \begin{pmatrix}1\\-1\\3\end{pmatrix} = \begin{pmatrix}2\\4\\-2\end{pmatrix}$ — B1

(ii) PR (1mk)

$PR = R - P$

$= \begin{pmatrix}6\\9\\-2\end{pmatrix} - \begin{pmatrix}1\\-1\\3\end{pmatrix}$

$= \begin{pmatrix}5\\10\\-5\end{pmatrix}$ — B1

b) Hence or otherwise, show that points P,Q and R are collinear. (2mks)

$PQ = \begin{pmatrix}2\\4\\-2\end{pmatrix} \Rightarrow 2\begin{pmatrix}1\\2\\-1\end{pmatrix}$

$PR = \begin{pmatrix}5\\10\\-5\end{pmatrix} \Rightarrow 5\begin{pmatrix}1\\2\\-1\end{pmatrix}$ — B1

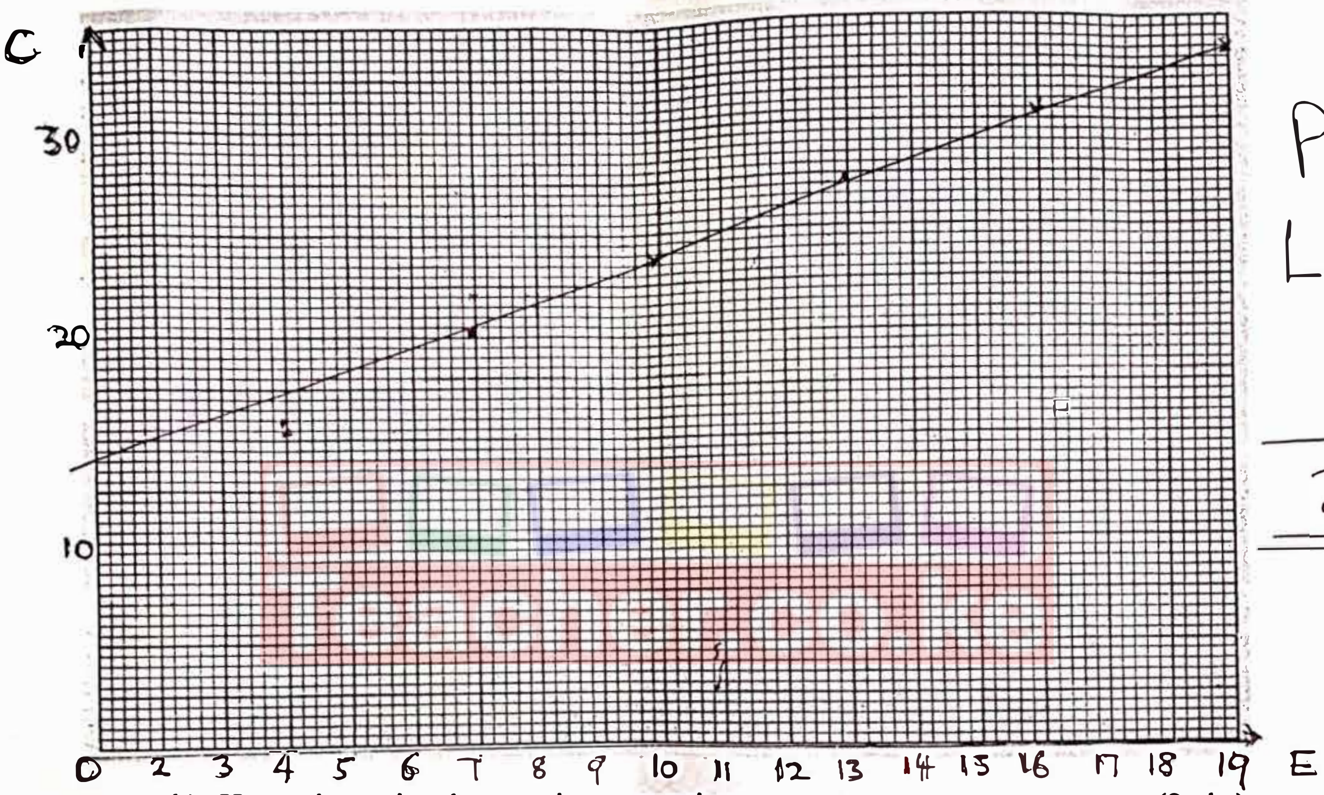
$\therefore \frac{1}{2}PQ = \frac{1}{5}PR$ — B1

$PQ = \frac{2}{5}PR$ and P is common

11. In an experiment involving two variables E and C, the following results were obtained.

E	4	7	10	13	16	19
C	15.1	20.2	23.9	27.3	30.1	33.1

a) On the grid provided below plot the values of C against E hence use the plotted points to draw the line of best fit. (2mks)



P1 – all points

L1 – straight line

2

b) Hence determine the equation connecting E and C. (2mks)

$$\text{Gradient} = \frac{33.1 - 30.1}{19 - 16}$$

$$= \frac{3}{3} = 1$$

M1

C intercept = 14.1

$\Rightarrow C = 1 \times E + 14.1$

$C = E + 14.1$

B1

2

12. A and B are two points on the surface of the earth. Their position lie on the equator such that A (0°,30°W) and B(0°,60°E). calculate:

a) The distance in nautical miles between A and B. (2mks)

let $\alpha = 30+60$

$= 90°$ B1

$\therefore AB = 60 \times 90$

$= 5400$ nm B1

b) The time difference between A and B. (2mks)

$1° \rightarrow 4'$

$90° \rightarrow ?$

$90 \times 4 = 360'$ M1

$360 \div 60$

$= 6$ hrs A1

13. Find the value of y in the equation: (2mks)

$$\mathrm{Log}_7\left[\frac{1}{343}\right] = y$$

$\frac{1}{343} = 7^y$ M1 - index notation

$7^{-3} = 7^y$

$\therefore y = -3$ A1

2

14. Mwanyumba invested some money in a housing finance company that offered compound interest of 2% per annum. After 3 years, her money had accumulated to sh 106120.80. How much money had she invested? (3mks)

$A = sh.\ 106120.80$

$P = ?$

$r = 2\%$

$n = 3$

$A = P\left(1+\frac{r}{100}\right)^n$

$106120.80 = P\left(1+\frac{2}{100}\right)^3$ M1 Substitution

$\therefore P = \frac{106120.80}{1.061208}$ M1 Simplifying

$= sh.\ 100,000$ A1

3

15. Find the gradient of $y=x^2-3x$ at the point (2,-2) (2mks)

$\frac{dy}{dx} = 2x-3$ M1

at $x=2$,

Gradient $= 2\times2-3$

$= 1$ A1

16. An object starts from rest and its velocity is measured every second for 6 seconds as shown in the table below.

Time (t)	0	1	2	3	4	5	6
Velocity v (ms^{-1})	0	12	24	35	41	45	47
		y_0	y_1	y_2	y_3	y_4	y_n

Use the trapezium rule to calculate the area between t= 1 and t=6 seconds. (2mks)

$A = \frac{1}{2}h\{y_0 + y_n + 2(y_1 + y_2 + y_3 + y_4 + y_5)\}$

$= \frac{1}{2}\times1\{0+47 + 2(12+24+35+41+45)\}$ M1

$\frac{1}{2}(47+314)$

$= ~~157~~\ 180.5$ sq units A1

$h=1$

$\frac{1}{2}(1)\{12+47+2(24+35+41+45)\}$

$\frac{1}{2}\times\{59+290\}$

$= 174.5$ sq units

SECTION II (50 marks)

17. The table below shows the income tax rates for a certain year.

Monthly taxable income in Ksh.	Tax rate (%) in each shilling
1-11180	10
11181-21714	15
21715-32248	20
32249 – 42782	25
Over 42782	30

a) During the year, Obonyo's monthly income was as follows;

Basic salary Ksh 40,000

House allowance Ksh 11,090

Commuter allowance Ksh 7,000

Calculate;

i. Obonyo's total monthy taxable income. (2mks)

$40000 + 11090 + 7000$

$= \text{Ksh } 58,090$

M1

A1

2

ii. The total income tax charged on Obonyo's monthly income. (6mks)

1st slab $11180 \times \frac{10}{100} = 1118$

2nd " $10534 \times \frac{15}{100} = 1580.1$

3rd " $10534 \times \frac{20}{100} = \cancel{2633.5}\ 2106.8$

4th " $10534 \times \frac{25}{100} = 2633.5$

5th " $15308 \times \frac{30}{100} = 4592.4$

M4 – all correct

Total tax $= 1118 + 1580.1 + 2106.8 + 2633.5 + 4592.4$

$= 12,030.80$

M1

A1

6

b) Obonyo's net monthly tax was Ksh 10,750.80. Determine the monthly tax relief allowed. (2mks)

Relief $= 12030.80 - 10,750.80$

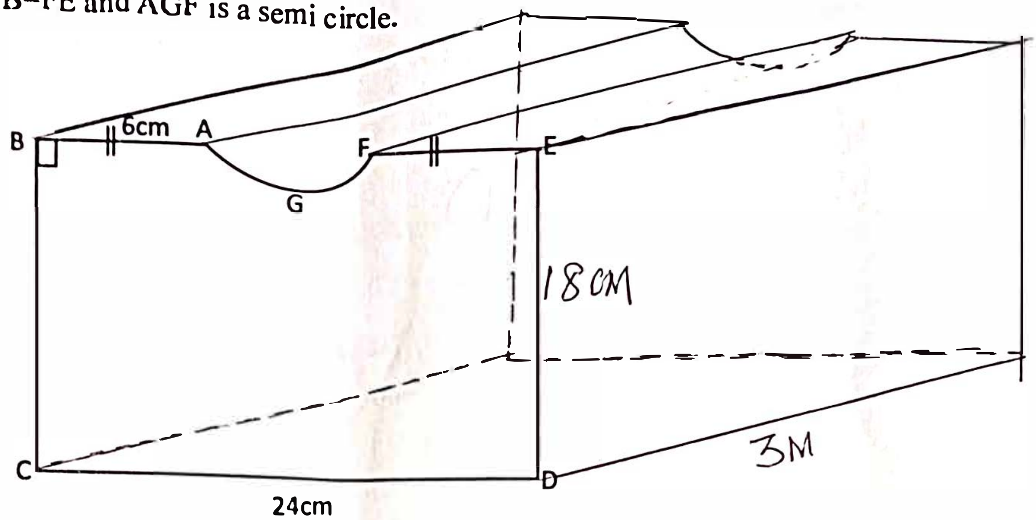
$= \text{Ksh. } 1,280$

M1

A1

2

18. The figure below represents a cross-section of a concrete prism whose length is 3m. Given that AB=FE and AGF is a semi circle.



a) Calculate the total surface area of the prism. (5mks)

Diameter AF $= 24 - (6 \times 2) = 12cm$.
$\therefore AGF = \frac{\pi D}{2} = 6\pi = 18.85cm$

Curved S.A. $= 18.85 \times 300 = 5654.87 cm^2$
Cross section area $= (24 \times 18 - \frac{1}{2}\pi \times 6^2) \times 2$ M1
$= (432 - 56.55) \times 2$
$= 750.9 \ cm^2$

Base area $= 24 \times 300 = 7200 cm^2$
Side faces $= (18 \times 300) \times 2$ M1
$= 10,800 \ cm^2$

Top faces $= 6 \times 300 \times 2$
$= 3600 \ cm^2$

Total $= 5654.87$
750.9
7200 +
10800
3600
$28005.77 cm^2$

M1
M1 - for addition
A1
5

b) Determine the volume of the concrete used to make the prism. (3mks)

V = Cross section Area × Length
$= 750.9 \times 300 \ cm^3$
$= 225,270 \ cm^3$

M1 M1
A1
3

c) Given that $1cm^3$ of concrete weighs 6.8 grams, find the mass of the prism in kg. (2mks)

$1 cm^3 \rightarrow 6.8 g$
$225270 cm^3 \rightarrow ?$

$\frac{225270 \times 6.8}{1000}$
$= 1531.836 Kg$.

M1
A1
2

19. Wamuyu purchased (2x-1) identical pens for ksh. 180. Nyambura purchased (3x+1) identical pencils for ksh. 200.

a) Write an expression for the;

i. Price of one pen. (1mk)

$$\frac{180}{(2x-1)}$$

M1

ii. Price of one pencil (1mk)

$$\frac{200}{(3x+1)}$$

M1

b) A pen costs Ksh 4 more than a pencil. Form an equation to represent the information above and hence solve for x. (4mks)

$$\frac{200}{3x+1} + 4 = \frac{180}{2x-1}$$

M1 Equation

$$\frac{200+12x+4}{3x+1} = \frac{180}{2x-1}$$

$$(204+12x)(2x-1) = 180(3x+1)$$

$$24x^2 - 144x - 384 = 0$$

M1 - Quadratic equation

$$x^2 - 6x - 16 = 0$$

$$(x-8)(x+2) = 0$$

M1 - Factors

$$\therefore x = 8 \text{ or } -2 \qquad x = 8$$

A1

4

c) Later the price of a pen went up by 25% while that of a pencil remained unchanged. A school bought 30 pencils and 16 pens. Determine the total amount of money spent by the school. (4mks)

$\therefore$ Price of pen $= \frac{180}{2\times8-1}$

$= \text{Sh}.12$

M1 - for both

Price of Pencil $= \frac{200}{3\times8+1}$

$= \text{Sh}.8$

New Pen's Price $= \frac{125}{100}\times 12$

$= \text{Sh}.15$

M1 - for new Pen's Price

$\Rightarrow 30\times8 + 16\times15$

M1

$240 + 240$

$= \text{Sh}.480$

A1

4

20. The first term of an arithmetic sequence is 2. The first term of a geometric sequence is also 2. The common ratio of geometric sequence equals the common differences of the arithmetic sequence. By taking d as the common difference and r as the common ratio.

a) Write an expression connecting d and r. (1mk)

$d = r$

B1

b) The third term of the geometric sequence exceeds the square of the first term of the arithmetic sequence by 124. Find:

i. The common ratio (3mks)

$3^{rd}\text{ term} = ar^2$
$= 2r^2$
$2r^2 - 2^2 = 124$
$2r^2 = 128$
$r^2 = 64$

$\therefore r = \pm 8$

M1 - eqn.
M1
A1
3

ii. The common difference (1mk)

$d = r$
$\therefore d = \pm 8$

B1

c) Determine;

i. The first ten terms of the arithmetic sequence. (2mks)

When $d = 8$ 2, 10, 18, 26, 34, 42, 50, 58, 66, 74

When $d = -8$ 2, −6, −14, −22, −30, −38, −46, −54, −62, −70

B2

ii. Hence or otherwise, find the sum of the first 10 terms of the arithmetic sequence. (3mks)

If $d = 8$,

$S_{10} = \frac{10}{2}\{2 \times 2 + 9 \times 8\}$
$= 5(4 + 72)$
$= 5 \times 76$
$= 380$

M1
M1
A1
3

If $d = -8$,

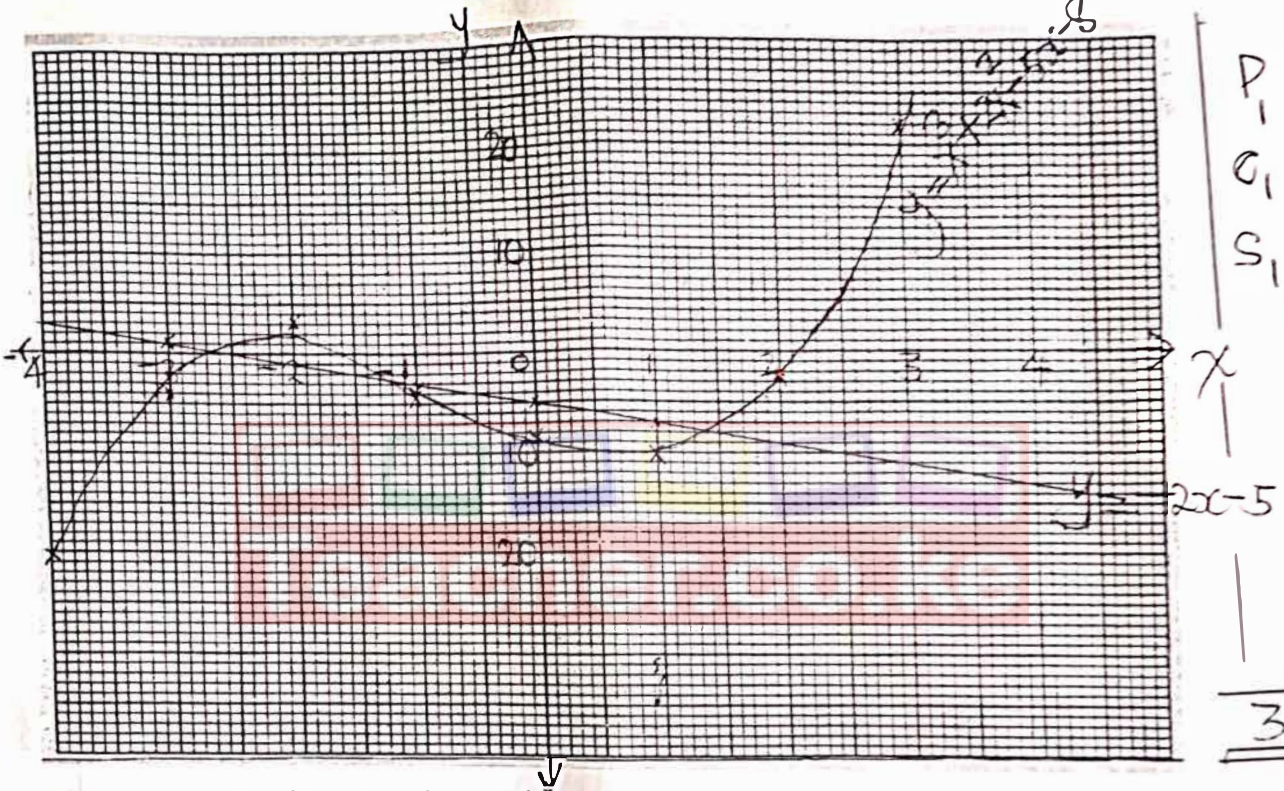
$S_{10} = \frac{10}{2}\{2 \times 2 + 9 \times -8\}$
$= 5\{4 - 72\}$
$= -340$

21. a) Given that $y=x^3+2x^2-5x-8$, complete the table below; (2mks)

X	-4	-3	-2	-1	0	1	2	3
X^3	-64	-27	-8	-1	0	1	8	27
$2x^2$	32	18	8	2	0	2	8	18
-5x	20	15	10	5	0	-5	-10	-15
-8	-8	-8	-8	-8	-8	-8	-8	-8
Y	-20	-2	2	-2	-8	-10	-2	22

B2 - All
otherwise B1

b) On the grid provided below, using a suitable scale draw the graph of $y=x^3+2x^2-5x-8$ for the values of x in the range $-4 \le x \le 3$. (3mks)



P1
C1
S1
Smooth curve

3

c) Use your graph to solve the equations. (2mks)

$x^3+2x^2-5x-8=0$

Line $y=0$

Solutions $x=-2.8$ or $x=-1.3$ or $x=2.1$

B1 - line $y=0$

B1 - solutions

2

d) By drawing a straight line, use the graph to solve the equation $-x^3-2x^2+3x+3=0$ (3mks)

$y=x^3+2x^2-5x-8$

$0=-x^3-2x^2+3x+3$ +

Line $y = -2x-5$

X	
Y	

$y=-2x-5$

Solutions $x=-2.7$ or $x=-0.8$ or $x=1.5$

B1 - line

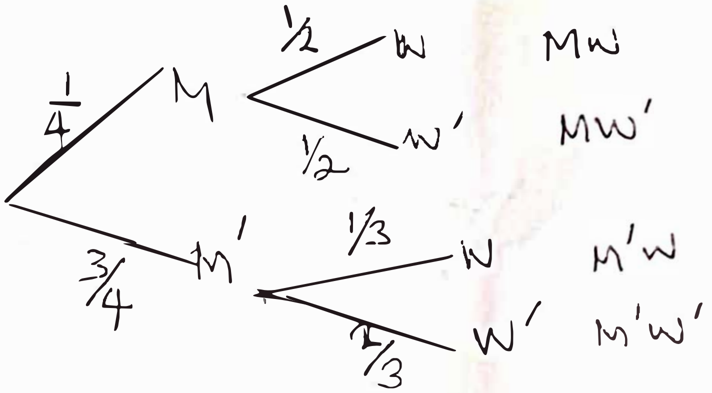
L1 - line drawn

B1 - solution

3

22. Alberto LUCCI is an Italian tourist who regularly visits Masai Mara while on tour in Kenya. The probability that Alberto goes to Maasai Mara is 1/4 , if he visits Maasai Mara, the probability that he sees a wild beast is ½. If he does not go to Maasai Mara, the probability that he will see a wild beast is $^1/_3$

a) Draw a tree diagram to show the above outcome. (3mks)



M1 Branch

M1 Branch

M1 – outcomes

3

b) Find the probability that Alberto will:

i. Visit Maasai Mara and see a wild beast. (2mks)

$$P(MW) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8}$$

M1

A1

2

ii. Not visit Maasai Mara and see a wild beast. (2mks)

$$P(M'W) = \frac{3}{4} \times \frac{1}{3}$$

$$= \frac{1}{4}$$

M1

A1

2

iii. See a wild beast. (3mks)

$$P(MW) \text{ or } P(M'W)$$

$$\frac{1}{4} \times \frac{1}{2} + \frac{3}{4} \times \frac{1}{3}$$

$$\frac{1}{8} + \frac{1}{4}$$

$$= \frac{3}{8}$$

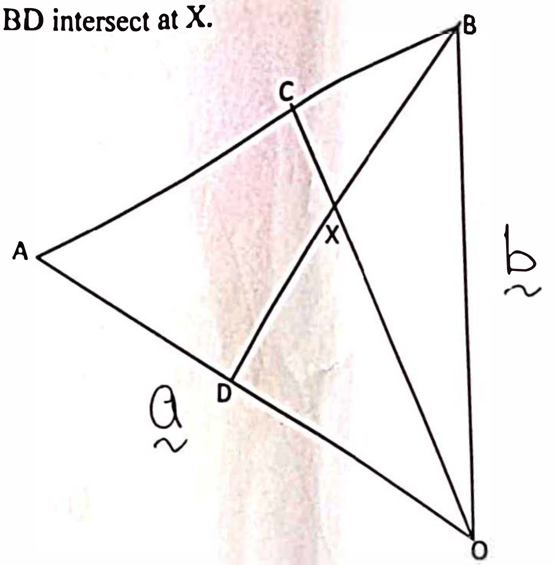
M1

M1

A1

3

23. In the figure below C is a point of AB such that AC:CB=3:1 and D is the mid-point of OA. OC and BD intersect at X.



Given that OA = $\underset{\sim}{a}$ and OB = $\underset{\sim}{b}$

a) Write the vectors below in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$.

i. AB (1mk)

$AB = AO + OB$
$= -\underset{\sim}{a} + \underset{\sim}{b}$

B1

ii. OC (2mks)

$OC = OA + AC$
$= \underset{\sim}{a} + \frac{3}{4}AB$
$= \underset{\sim}{a} + \frac{3}{4}(-\underset{\sim}{a} + \underset{\sim}{b})$
$= \frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b}$

B1
B1

iii. BD (1mk)

$BD = BO + OD$
$= -\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{a}$
$= \frac{1}{2}\underset{\sim}{a} - \underset{\sim}{b}$

B1

b) Given that BX=hBD, express OX in terms of a,b and h. (1mk)

$BX = h(\frac{1}{2}a - b)$
$= \frac{1}{2}h\underset{\sim}{a} - h\underset{\sim}{b}$

$OX = \underset{\sim}{b} + \frac{1}{2}h\underset{\sim}{a} - h\underset{\sim}{b}$
$= \frac{1}{2}h\underset{\sim}{a} + (1-h)\underset{\sim}{b}$

B1

c) If OX=kOC, find the values of h and k. (4mks)

$OX = k(\frac{1}{4}\underset{\sim}{a} + \frac{3}{4}\underset{\sim}{b})$
$= \frac{k}{4}\underset{\sim}{a} + \frac{3k}{4}\underset{\sim}{b}$

$\Rightarrow \frac{1}{2}h\underset{\sim}{a} + (1-h)\underset{\sim}{b} = \frac{k}{4}\underset{\sim}{a} + \frac{3}{4}k\underset{\sim}{b}$

(i) $\frac{1}{2}h = \frac{1}{4}k$ (ii) $1-h = \frac{3}{4}k$

$\therefore h = \frac{1}{2}k$
$1 - \frac{1}{2}k = \frac{3}{4}k$
$1 = \frac{5}{4}k$
$\therefore k = \frac{4}{5}$
$h = \frac{2}{5}$

M1 – Equation
M1 – Equations
M1 – Substitutions
A1 – Both

d) Hence write the ratio OX:XC (1mk)

4 : 1

B1

10

24. A particle moves in a straight line in such a way that its distance s metres from a fixed point O after t seconds is given by $S = \frac{1}{3}t^3 - \frac{3t^2}{2} + 2t$

a) Find the position of the particle when it is momentarily at rest. (5mks)

Velocity = 0

$\frac{ds}{dt} = t^2 - 3t + 2$

$\therefore t^2 - 3t + 2 = 0$

$(t-1)(t-2) = 0$

$\Rightarrow t = 1$ and $t = 2$

Its positions at these times

$\frac{1}{3}(1)^3 - \frac{3}{2}(1)^2 + 2(1) = 5/6\,M$

and

$\frac{1}{3}(2)^3 - \frac{3}{2}(2)^2 + 2(2) = \frac{4}{6}M$

M1 - ds/dt

M1 - Equation

A1 - t value

M1 - substitution

A1

5

b) Calculate the acceleration of the particle at this time when it is momentarily at rest. (2mks)

$V = t^2 - 3t + 2$

Acceleration $= \frac{dv}{dt}$

$\Rightarrow a = 2t - 3$

When $t = 1$ sec.

$a = 2 \times 1 - 3 = -1\,m/s^2$

When $t = 2s$

$a = 2 \times 2 - 3 = 1\,m/s^2$

M1 - $\frac{dv}{dt}$

A1

c) Find the maximum or minimum velocity of the particle. (3mks)

At maximum/minimum, $a = 0$

$\Rightarrow 2t - 3 = 0$

$2t = 3$

$\therefore t = 3/2$ sec.

1	$\frac{dv}{dt}$	2
−	0	+
\	—	/

$\therefore$ At $3/2$ sec., velocity is a minimum.

M1 - Equation

A1

B1

3