END TERM 1 EXAMINATIONS 2022

MATHEMATICS

FORM 3

INSTRUCTIONS TO THE CANDIDATES

a) Write your **name** and **Adm no.** in the spaces provided above.
b) This paper consists of **two sections; Section I and Section II.**
c) Answer **All** questions in **Section I** and only **Five** questions from **section II**
d) **All** answers and working **must** be written on the question paper in the spaces provided below each question.
e) Show all the steps in your calculations giving answers at each stage in the spaces provided below each question.
f) Marks may be given for correct working even if the answer is wrong.
g) Non-programmable silent electronic calculators and KNEC Mathematical tables may be used except where stated otherwise.
m) Candidates should answer questions in **English.**

For examiner's use only.

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	Total

Section II

17	18	19	20	21	22	23	24	Total

GRAND
TOTAL

SECTION I (50 MARKS)

*Answer **ALL** the questions in this section in the spaces provided*

1. Use logarithm tables to evaluate $\sqrt[3]{\dfrac{45.3\times0.00697}{0.534}}$ (4mks)

No	Std form	log
45.3	4.53×10^1	1.6561
0.00697	6.97×10^{-3}	$\bar{3}.8432^+$
		$\bar{1}.4993$
0.534	5.34×10^{-1}	$\bar{1}.7275^-$
		$\bar{1}.7718 \div 3$

$\dfrac{\bar{1}+0.92393}{\bar{3}+2.7718}$ M₁

$\dfrac{}{3}$ M₁

$\bar{1}.9239 \Rightarrow$ Antilog   M₁

8.392×10^{-1}

$= 0.8392$ A₁

(3mks)   4

2. Express $0.\dot{4}0\dot{7}$ as a fraction.

$1r = 0.407\,407\cdots$

$1000r = 407.\,407\,407\cdots$ M₁

$999r = 407$

$r = \dfrac{407}{999}$ M₁

$r = \dfrac{11}{27}$ A₁

M₁

M₁

A₁

3

(3mks)

3. Simplify $\dfrac{\sqrt5}{\sqrt7-\sqrt2}$ (3mks)

$\dfrac{\sqrt5}{\sqrt7-\sqrt2}\cdot\dfrac{(\sqrt7+\sqrt2)}{(\sqrt7+\sqrt2)}$ M₁

$\dfrac{\sqrt{35}+\sqrt{10}}{\sqrt{49}+\sqrt{14}-\sqrt{14}-\sqrt{4}}$

$\dfrac{\sqrt{35}+\sqrt{10}}{7-2}$ M₁

$\dfrac{\sqrt{35}+\sqrt{10}}{5}$ A₁

M₁

M₁

A₁

3

4. The lengths of wires were 30 m, 36 m and 84 m. Pieces of wire of equal length were cut from the three wires. Calculate the least number of pieces obtained.   (4mks)

G.C.D. required

2	30	36	84
3	15	18	42
	5	6	14

$2\times3 = 6$

Least number of pieces comparing the three

$\dfrac{30\,m}{6\,m} = 5$ pieces

$\dfrac{36\,m}{6\,m} = 6$ pieces

$\dfrac{84\,m}{6\,m} = 14$ pieces

∴ 5 pieces is the least.

B₁

B₁

B₁

B₁

4

5. The size of an interior angle of regular polygon is $3x°$. While its exterior angle is
   Find the number of sides of the polygon. (3mks)

$$3x° + (x - 20)° = 180°$$
$$4x° - 20° = 180°$$
$$4x° = 200°$$
$$x° = 50°$$

Size of exterior angle
$$= 50° - 20° = 30°$$

No. of sides $= \dfrac{360°}{30°}$

$= 12$ sides

M₁

M₁

A₁

3

6. Find the percentage error in calculating the volume of the cuboid whose dimensions are
   8.2cm by 6.2cm by 5.7cm (3mks)

$$V = 8.2 \times 6.2 \times 5.7$$

Individual absolute errors

$$\dfrac{0.1}{2} \qquad \dfrac{0.1}{2} \qquad \dfrac{0.1}{2}$$

$$0.05 \qquad 0.05 \qquad 0.05$$

R.E. in volume = sum of individual R.E.

$$\dfrac{0.05}{8.2} + \dfrac{0.05}{6.2} + \dfrac{0.05}{5.7}$$

$$= 0.0229$$

$\Rightarrow$ %age error $= 0.0229 \times 100 = 2.29\%$

M₁

M₁

A₁

3

7. A man wishes to save sh. 200 000 in 4 years time. Find the sum of the money he has to
   deposit now at 12% p.a. interest, compounded semi-annually to realize his goal. (3mks)

Rate $= \dfrac{1}{2} \times 12$ $n = 8$
$$= 6\%$$

compound Interest
$$A = P\left(1 + \dfrac{r}{100}\right)^n$$

$$200\,000 = P\left(1 + \dfrac{6}{100}\right)^8$$

$$200\,000 = 1.06^8 \, P$$

$$P = \dfrac{200\,000}{1.06^8}$$

$$= \text{sh. } 125\,482$$

M₁

M₁

A₁

3

8. Use completing the square method to solve $2x^2 + 4x + 1 = 0$ (3mks)

$$\dfrac{2x^2}{2} + \dfrac{4x}{2} + \dfrac{1}{2} = \dfrac{0}{2}$$

$$x^2 + 2x + \dfrac{1}{2} = 0$$

$$x^2 + 2x = -\dfrac{1}{2}$$

$$x^2 + 2x + \left(\dfrac{2}{2}\right)^2 = -\dfrac{1}{2} + \left(\dfrac{2}{2}\right)^2$$

$$(x + 1)^2 = \dfrac{1}{2}$$

$$x + 1 = \pm\sqrt{0.5}$$

$$x = -0.2929 \quad \text{or} \quad x = -1.707$$

M₁

M₁

A₁ for
   2 ans

3

9. Solve for x in: $(\log_2 \theta)^2 + \log_2 8 - \log_2 \theta^4 = 0$ (4mks)

$$(\log_2 \theta)^2 + 3\log_2 2 - 4\log_2 \theta = 0$$
let $\log_2 \theta = x$

$$x^2 + 3 - 4x = 0$$

$$x^2 - 4x + 3 = 0$$

$$x^2 - 3x - x + 3 = 0$$

$$x(x - 3) - 1(x - 3) = 0$$

$$(x - 1)(x - 3) = 0$$

$$x = 1 \quad \text{or} \quad x = 3$$

but
$$\log_2 \theta = x \qquad \text{also } \log_2 \theta = 3$$

$$\log_2 \theta = 1 \qquad \qquad \theta = 2^3$$

$$\therefore \theta = 2^1 = 2 \qquad \theta = 8$$

M₁

M₁

M₁

A₁ for 2
   ans

4

10. Determine the equation of a line that passes through (-2,5) and is parallel to the line whose equation is $5y + 2x = 10$ (4mks)

for parallel lines

$M_1 = M_2$

$5y = -2x + 10$

$y = -\frac{2}{5}x + 2$

$M_1 = -\frac{2}{5}$ ✓

$M_2 = -\frac{2}{5}$ ✓

$P(-2, 5)$

$\frac{y-5}{x+2} = \frac{-2}{5}$ ✓

$5(y-5) = -2(x+2)$

$5y - 25 = -2x - 4$

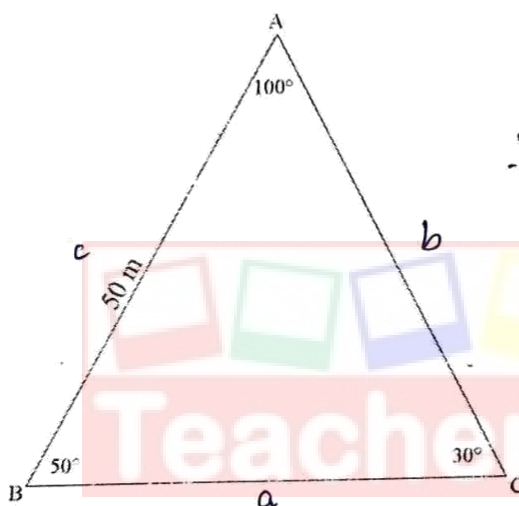$5y = -2x + 21$

$y = -\frac{2}{5}x + \frac{21}{5}$

M1

M1

M1

A1

—
4

11. In the figure below, if a circle is drawn passing through A, B and C what would be the radius of the circle (4mks)

$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} = 2r$

$\therefore \frac{c}{\sin C} = 2r$ ✓

$\frac{50}{\sin 30°} = 2r$ ✓

$100 = 2r$ ✓

$r = 50m$ ✓

M1

M1

M1

A1

—
4

12. Given that $\sin(x + 20°) = \cos(2x + 25)$ find the value of x and hence find the value of $\tan x$. (2mks)

complimentary angles

$(x+20)° + (2x+25)° = 90°$

$3x + 45° = 90°$

$3x = 45°$

$x = 15°$ ✓

$\tan x = \tan 15°$

$= 0.2679$ ✓

B1

B1

—
2

13. An arc of a circle of radius 4 cm is 5 cm long. Calculate the angle subtended by the arc at the center in radians (2mks)

$1^c = 4 cm$ ✓

$? s 5 cm$

$\frac{5 \times 1}{4}$
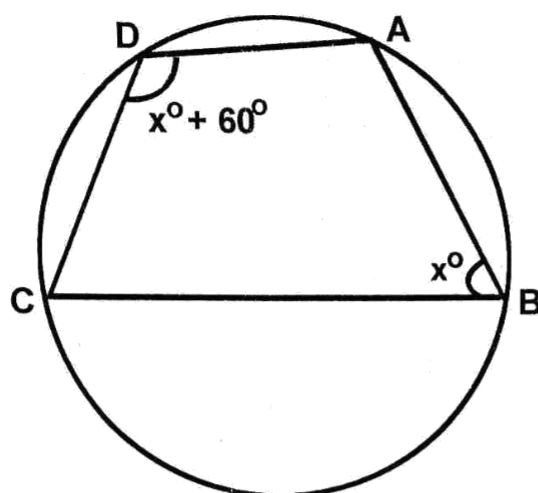
$= 1.25^c$ ✓

M1

A1

—
2

14. The figure below shows a quadrilateral ABCD which is cyclic. Solve for x.   (2mks)

$$x + 60^\circ + x^\circ = 180^\circ$$
$$2x^\circ + 60 = 180^\circ \qquad M1$$
$$2x^\circ = 120^\circ$$
$$x^\circ = 60^\circ \checkmark \qquad A1$$

2

15. Simplify $\dfrac{\log 25 + \log 625}{\log 5}$ (3mks)

$$\frac{\log 5^2 + \log 5^4}{\log 5}$$

$$\frac{2\log 5 + 4\log 5}{\log 5}$$

$$\frac{(2+4)\log 5}{\log 5} \checkmark$$

$$= 6 \checkmark$$

M1

M1

A1

3

16. Given that **OP** = 3i − 2j and **OQ** = 8i − 5j. Find |**PQ**| to 3 significant figures.   (3mks)

$$\vec{OP} = \begin{pmatrix} 3 \\ -2 \end{pmatrix} \qquad \vec{OQ} = \begin{pmatrix} 8 \\ -5 \end{pmatrix}$$

$$\vec{PQ} = \vec{OQ} - \vec{OP}$$

$$= \begin{pmatrix} 8 \\ -5 \end{pmatrix} - \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ -3 \end{pmatrix} \checkmark$$

$$|\vec{PQ}| = \sqrt{5^2 + (-3)^2} \checkmark$$

$$= \sqrt{34}$$

$$= 5.83 \quad (3 \text{ s.f.})$$

B1

M1

A1

3

SECTION II: (50 MARKS)

Answer only FIVE questions from this section in the spaces provided

17. Mr. Kobe is a civil servant who earns a monthly salary of Ksh.21200. He has a house allowance of Ksh.12000 per month, other taxable allowances are commuter Ksh.1100, medical allowance Ksh.2000. He is entitled to a personal relief of Ksh.1240 per month. Using the income rates below, solve the questions that follow.

Income in Ksh. per month	Rates in Ksh per sh 20
1 – 8,400	2
8401 – 18,000	3
18001 – 30,000	4
30001 – 36,000	5
36001 – 48,000	6
Above 48,000	7

Determine;

a) i) His monthly taxable income. (2mks)

$$= 21\,200 + 12\,000 + 1100 + 2000$$

$$= sh.\ 36,300$$

M1
A1
2

ii) Net tax (PAYEE) (5mks)

$$\frac{8400}{20} \times 20 = 840$$

$$\frac{9600}{20} \times 3 = 1440$$

$$\frac{12000}{20} \times 4 = 2400$$

$$\frac{6000}{20} \times 5 = 1500$$

$$\frac{300}{20} \times 6 = 90$$

$$= 6,270$$

Net tax = Gross – Relief

$$= 6270$$
$$-1240$$
$$sh.\ 5,030$$

B1
B1
B1
M1
A1
5

b) In addition to the PAYEE, the following deductions were made. Ksh.250 for NHIF, Ksh.120 service charges, he repays a loan at sh.4500 and contributes towards savings at sh.1800 every month. Calculate his net salary per month. (3mks)

$$\text{Net salary} = 36300 - (5,030 + 250 + 120 + 4500 + 1800)$$

$$= 36,300 - 11\,700$$

$$= sh.\ 24\,600$$

M1
M1
A1
3

18. Mr. Korir borrowed Khs 3,600,000 from the bank to buy a residential house. He was required to repay the loan with a simple interest for a period of four years. The repayment amounted to kshs 111 000 per month. Calculate;

a) The interest paid to the bank. (1mk)

$$S.I = (111\,000 \times 4) - 3,600,000$$

$$I = (111\,000 \times 12 \times 4) - 3,600,000 = 1,728,000$$

b) The rate per annum of the simple interest. (2mks)

$$1728\,000 = 3600\,000 \times \frac{R}{100} \times 4$$

$$R = \frac{1,728,000 \times 100}{3,600,000 \times 4}$$

$$= 12\%$$

c) The value of the house appreciated at the rate of 15% per annum. Calculate the value of the house after 4 years to the nearest hundreds. (3mks)

$$A = 3,600,000 \left(1 + \frac{15}{100}\right)^4$$

$$= 3\,600\,000 (1.15)^4$$

$$= sh.\ 6,296,422.5$$

$$= sh.\ 6,296,400$$

d) After n years, the value of the house was Kshs 8,327,019. Find the value of n. (4mks)

$$8,327,019 = 3,600,000 \left(1 + \frac{15}{100}\right)^n$$

$$2.313 = (1.15)^n$$

$$\log 2.313 = \log 1.15^n$$

$$\log 2.313 = n \log 1.15$$

$$n = \frac{\log 2.313}{\log 1.15}$$

$$= 5.9998$$

$$= 6\ years.$$

B1
1

M1

A1
2

M1

M1

A1
3

M1

M1

M1

A1
4

19. a) Complete the table below for the equation $y = x^2 - 6x + 5$ (2mks)

x	0	1	2	3	4	5	6
x^2	0	1	4	9	16	25	36
-6x	0	-6	-12	-18	-24	-30	-36
5	5	5	5	5	5	5	5
y	5	0	-3	-4	-3	0	5

B1 B1

—
2

b) Draw the graph of $y = x^2 - 6x + 5$ using the values in the table. (3mks)

check graph for accuracy

B1 B1 B1

—
3

(c) Use the graph to solve the equations

(i) $x^2 - 6x + 5 = 0$ (1mk)

$$y = x^2 - 6x + 5$$
$$- \quad 0 = x^2 - 6x + 5$$
$$\overline{\quad y = 0 \quad} \Rightarrow x = 1 \quad \text{or} \quad x = 5$$

B1

—
1

(ii) $x^2 - 6x + 7 = 0$ (2mks)

$$y = x^2 - 6x + 5$$
$$- \quad 0 = x^2 - 6x + 7$$
$$\overline{\quad y = -2 \quad}$$

check for line graph

$$\Rightarrow x = 1.55 \quad \text{or} \quad x = 4.4$$

B1

B1

—
2

(iii) $x^2 - 6.5x + 5 = 0$ (2mks)

$$y = x^2 - 6x + 5$$
$$- \quad 0 = x^2 - 6.5x + 5$$
$$\overline{\quad y = 0.5x \quad}$$

check for line graph

x	0	2	4
y	0	1	2

$$x = 0.9 \quad \text{or} \quad x = 5.6$$

B1

B1

—
2

y-axis

5
4
3
2
1
0

A & S — B1
P — B1
C — B1
          3

$y = x^2 - 6x + 5$

$y = 0.5x$ B1

x-axis

1   2   3   4   5   6   7

-1
-2 $y = -2$ B1
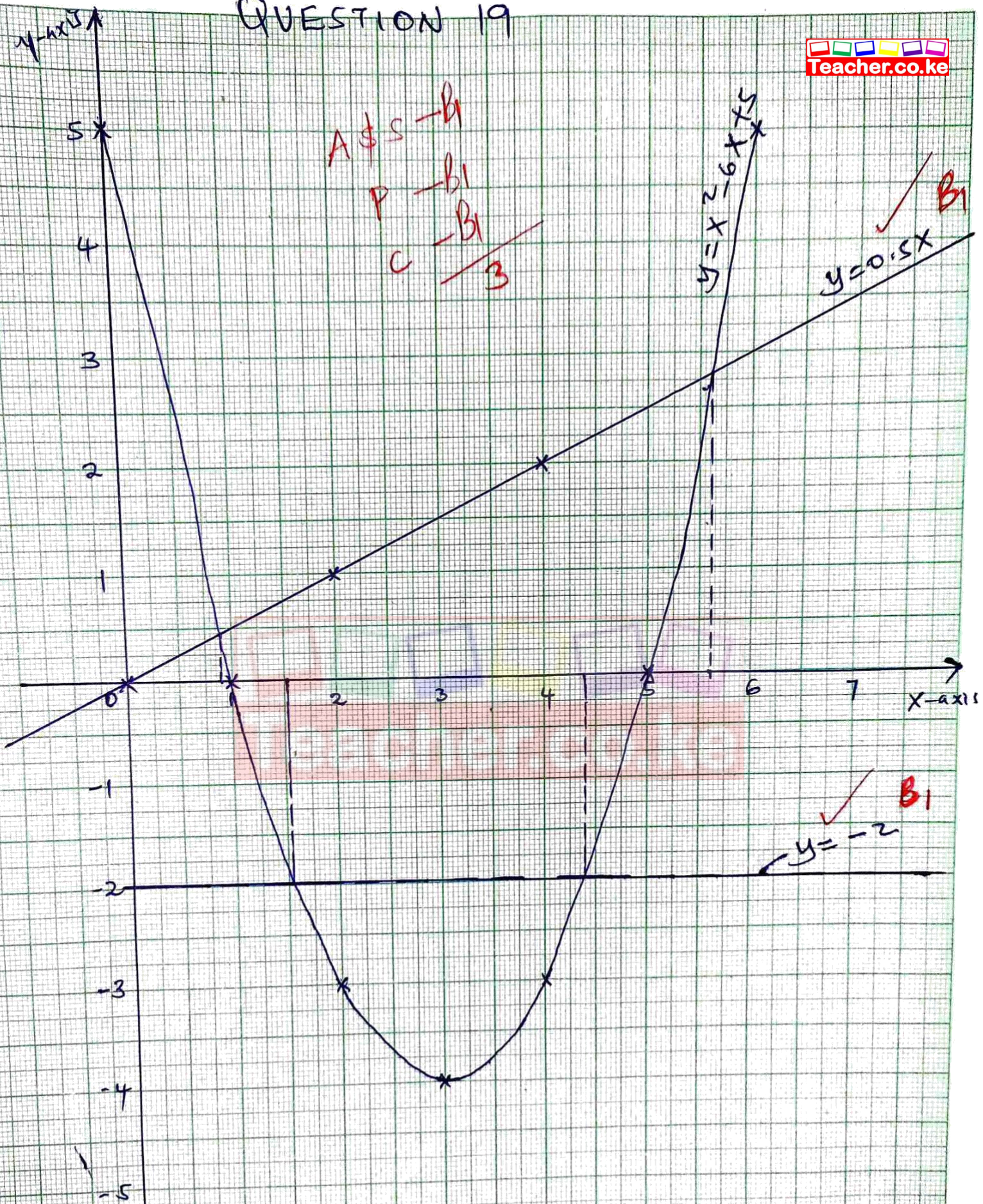-3
-4
-5

20. (a) A matatu travelling at 99km/h passes a checkpoint at 9.00 am. A police patrol travelling at 132km/h in the same direction passes through the police check point at 9.15 a.m. If the matatu and the police patrol car continue at their uniform speeds, calculate:

    i.   the relative speed of the two vehicles   (1mk)

$$132 - 99 = 33 \text{ Km/h}$$

B₁

    ii.   the distance between the two vehicles as at 9.15 a.m.   (2mks)

$$\frac{15}{60} \times 99 = 24.75 \text{ Km}$$

M₁

A₁

2

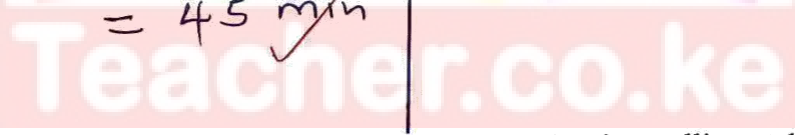   iii.   the time the police car will overtake the matatu.   (3mks)

$$\text{time} = \frac{\text{Distance}}{\text{speed}}$$

$$= \frac{24.75}{33}$$

$$= 45 \text{ min}$$

9.15 a.m.
+   45
———————
10.00 a.m.

M₁

B₁

A₁

3

(b) Two passenger trains A and B which are 240m apart and travelling at 164km/h and 88km/h respectively approach one another on a straight railway line. Train A is 150m long and train B is 100m long. Determine the time in seconds that elapses before the two trains completely pass each other.   (4mks)

Relative/Total distance

240 + 150 + 100

= 490 m

In km

= 0.49 km ✓

Relative speed

164 Km/h + 88 km/h

= ~~252~~ 252 kmh⁻¹

$$T = \frac{0.49}{252}$$

$$= 0.00194 \text{ hrs}$$

$$= 7 \text{ seconds.}$$

M₁

M₁

M₁

A₁

4

21. The marks scored in a form three Maths exam were recorded as follows

69	70	72	40	52	60	22	31	78	53	28	67
63	54	57	48	47	56	55	62	75	38	37	44
62	64	58	39	45	48	65	50	85	46	47	57
35	34	58	64	62	37	41	42	36	54	82	48
53	57	56	72	56	48	44	55	78	59	50	45

a) Starting with a class of 20 – 29, make a frequency distribution table for the above data

(2mk)

Class	Midpoint X	f	fX	Cf.
20 – 29	24.5	2	49	2
30 – 39	34.5	8	276	10
40 – 49	44.5	14	623	24
50 – 59	54.5	18	981	42
60 – 69	64.5	10	645	52
70 – 79	74.5	6	447	58
80 – 89	84.5	2	169	60

3190

b) What is the modal class of the test (1mk)

$$50 - 59 \checkmark$$

c) Calculate the mean of the data (4mk)

$$\bar{X} = \frac{\varepsilon f X}{\varepsilon f}$$

$$= \frac{3190}{60}$$

$$= 53.2$$

d) Calculate the median mark (3mk)

$$= 49.5 + \left(\frac{12}{18}\right) 10$$
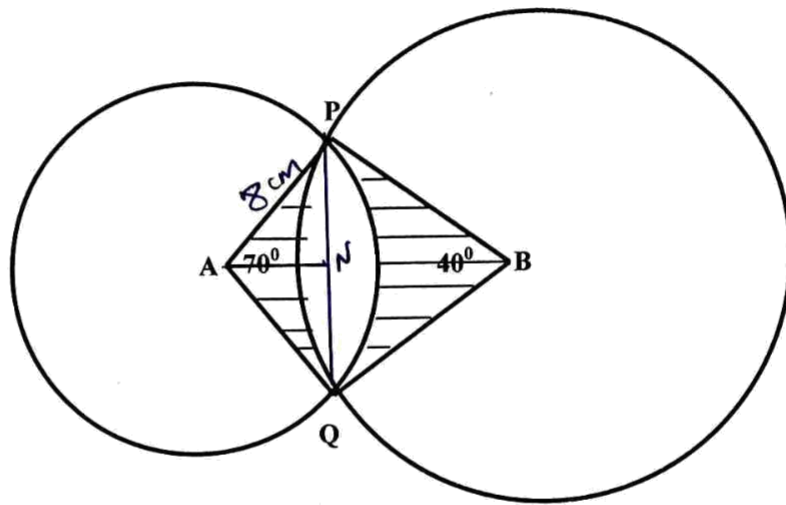
$$= 56.167$$
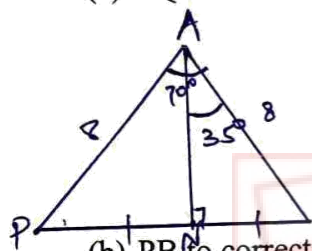
$$= 56.2 \checkmark$$

B1
B1
2
B1
B1
B1
M1
A1
4
B1
B1
A1
3

22. The diagram below shows two circles, centre A and B which intersect at points Angle PAQ = 70°, angle PBQ = 40° and PA = AQ = 8cm.

Calculate

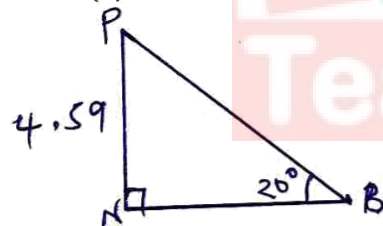(a) PQ to correct to 2 decimal places (2mks)

$$\sin 35° = \frac{NQ}{8}$$

$$NQ = 8 \sin 35°$$

$$= 4.5886 \text{ cm}$$

$$PQ = 2 \times 4.5886$$

$$= 9.1772$$

$$= 9.18 \text{ cm}$$

M₁
A₁
2

(b) PB to correct to 2 decimal places (2mks)

$$\sin 20° = \frac{4.59}{PB}$$

$$PB = \frac{4.59}{\sin 20°}$$

$$PB = 13.42 \text{ cm}$$

M₁
A₁
2

(c) Area of the minor segment of the circle whose centre is A (2mks)

$$= \frac{70}{360} \times \frac{22}{7} \times 8^2 - \frac{1}{2} \times 8^2 \times \sin 70°$$

$$= 39.1111 - 30.07$$

$$= 9.04 \text{ cm}^2$$

M₁
A₁
2

(d) Area of shaded region (4mks)

$$\text{Segment Area} = \frac{40}{360} \times 13.42 \times 13.42 \times \frac{22}{7} - \frac{1}{2} \times 13.42 \times 13.42 \times \sin 40°$$

$$62.89 - 57.88$$

$$= 5.01$$

$$\text{Area of shaded} = [(30.07 + 57.88) - (9.04 + 5.01)]$$

$$87.95 - 14.05 = 73.9 \text{ cm}^2$$

M₁
A₁

M₁
A₁
4

23. a) Complete the table below for $y = \cos x$ for $0° \leq x \leq 360°$ (2mks)

x°	0	30	60	90	120	150	180	210	240	270	300	330	360
y=cos x	1.0	0.87	0.5	0.0	-0.5	-0.87	-1.0	-0.87	-0.5	0	0.5	0.87	1.0

②

b) Draw the graph of $y = \cos x$. (4mks)

c) Use the graph in (b) above to to solve for the values of x in the following (2mks)

   i. $\cos x = 0.74$

$$X = 42 \pm 3°, \quad 319.5° \pm 3° \checkmark$$

B₁

   ii. $\cos x = -0.5$

$$X = 120°, \quad 240° \checkmark$$

B₁

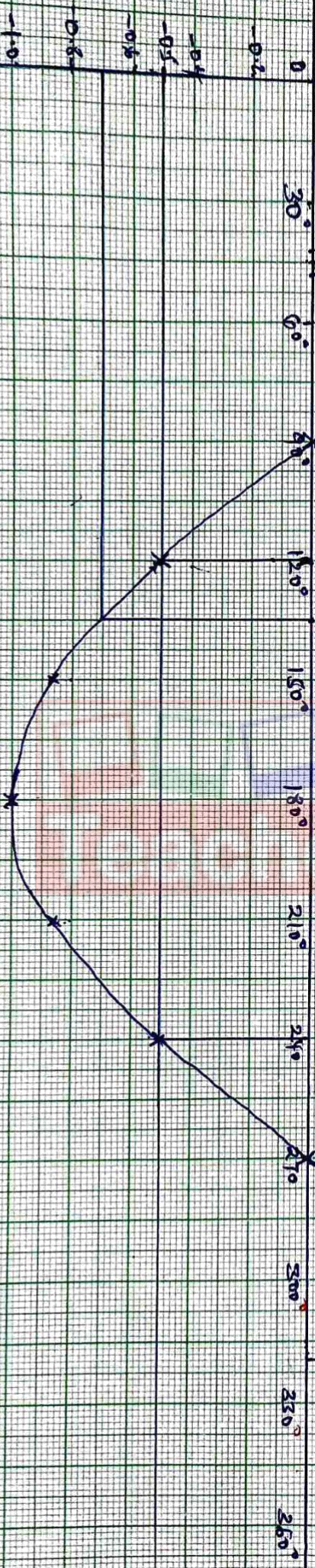d) Use the graph in (b) above to find the cosine values of the following angles; (2mks)

   i. 48°

$$\cos 48° = 0.66 \checkmark$$
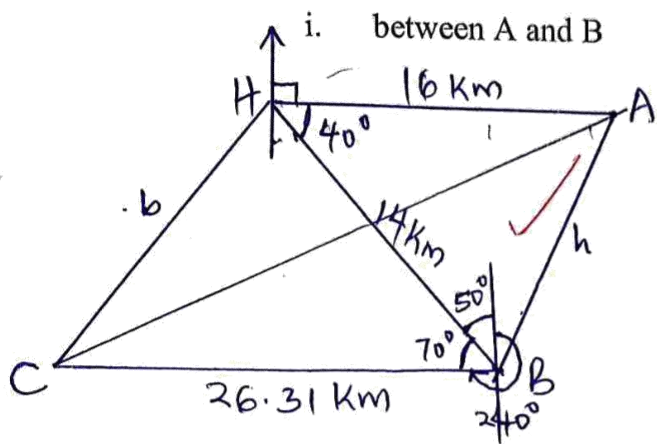
B₁

   ii. 135°

$$\cos 135° = -0.7 \checkmark$$

B₁

QUESTION 23 (b)

$y = \cos x$

X°

24. Three ships A, B, and C are approaching a harbor H. Ship A is 16 km from the harbor on a bearing of 090^0. Ship B is 14 km from the harbor on a bearing of 130^0, and ship C is 26.31 km to the west of B and on a bearing of 240^0 from the harbor. Without using scale drawing, calculate the distance:

i.   between A and B

$$AB = h \qquad \text{(3mks)}$$

$$h^2 = 14^2 + 16^2 - 2 \times 14 \times 16 \cos 40^0$$

$$= 452 - 343 \cdot 188$$

$$= 108 \cdot 812$$

$$h = \sqrt{108 \cdot 812}$$

$$= 10 \cdot 43 \ Km$$

$B,$ ske

M_1

A_1

$\boxed{3}$

ii.   of ship C from the harbor   (3mks)

Angle at $B = 360^0 - (240^0 + 50^0)$

$$= 70^0$$

$$CH = b$$

$$b^2 = 14^2 + 26 \cdot 31^2 - 2 \times 14 \times 26 \cdot 31 \cos 70^0$$

$$= 888 \cdot 2161 - 251 \cdot 96$$

$$= 636 \cdot 257$$

$b = 25 \cdot 22 \ Km$

B_1

M_1

A_1

$\boxed{3}$

iii.   between A and C   (4mks)

Angle at B

$$\frac{h}{\sin H} = \frac{16}{\sin B}$$

$$\frac{10 \cdot 43}{\sin 40^0} = \frac{16}{\sin B}$$

$$\sin B = \frac{16 \sin 40^0}{10 \cdot 43}$$

$$= 0 \cdot 9861$$

$$B = \sin^{-1} 0 \cdot 9861$$

$$B = 80 \cdot 4^0$$

$$\angle CBA = 70^0 + 80 \cdot 4^0$$

$$= 150 \cdot 4^0$$

$$AC^2 = 26 \cdot 31^2 + 10 \cdot 43^2 - 2 \times 26 \cdot 31 \times 10 \cdot 43 \cos$$

$$= 801 \cdot 001 - (-477 \cdot 20)$$

$$= 1278 \cdot 203$$

$$AC = \sqrt{1278 \cdot 203}$$

$$= 35 \cdot 75 \ KM$$

M_1

M_1

M_1

A_1

$\boxed{4}$