END TERM ONE EXAM- 2022
FORM FOUR

121/1
MATHEMATICS ALT A
PAPER 1
Kenya Certificate of Secondary Education (KCSE)

INSTRUCTIONS TO CANDIDATES

a) Write your name, Admission number and in the spaces provided at the top of this page.
b) Write name, admission number and class in the spaces provided above.
c) This paper contains TWO sections: section I and section II
d) Answer ALL the questions in Section I and only five questions from section II.
e) Show all the steps in your calculations, giving your answers at each stage in the spaces provided below each question.
f) Marks may be given for correct working even if the answer is wrong.
g) Non-programmable silent electronic scientific calculators and KNEC mathematical tables may be used except where stated otherwise
h) Candidates should check the question paper to ascertain that all pages are printed as indicated and that no questions are missing.

FOR EXAMINER'S USE ONLY:

Section I

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	TOTAL

Section II

17	18	19	20	21	22	23	24	TOTAL

GRAND TOTAL

SECTION 1 (50 marks)

Answer all the questions in this section in the space provided

1. Evaluated: (3mks)

$$\frac{2\frac{1}{2} \text{ of } 1\frac{3}{4} - 5\frac{1}{4}}{1\frac{2}{5} + 2(1\frac{1}{4} - 2\frac{3}{4})}$$

$$\frac{5}{2} \text{ of } \frac{7}{4} = \frac{35}{8}$$

$$\frac{35}{8} - \frac{21}{4}$$

$$\frac{35 - 42}{8} = \frac{-7}{8}$$

$$\frac{5}{4} - \frac{11}{4} = \frac{-6}{4}$$

$$\frac{-6}{4} \times 2 = \frac{-12}{4}$$

$$\frac{7}{5} + \frac{-12}{4} = -1\frac{3}{5}$$

$$\frac{-8}{5}$$

$$\frac{-7}{8} \times \frac{5}{-8} = \frac{35}{64}$$

2. Use logarithms to evaluate the following to 4 significant figures to: (4mks)

$$\left(\frac{95.75 \times 0.85}{4.524 + 1.234}\right)^{\frac{2}{3}}$$

No	std form	Log
95.75	9.575×10^1	1.9611 +
0.85	8.5×10^{-1}	$\bar{1}.9294$
5.758 -	5.758×10^0	$\bar{1}.9105$
		0.7603
		$\overline{1.1502} \times \frac{2}{3} = 0.7668$

$$5.845 \quad \leftarrow 5.845 \times 10^0 \leftarrow 0.7668$$

3. An electrician made a loss of 30% by selling a multi plug at sh.1400.what percentage profit would he has made if he sold the multi plug at sh. 2300 (3mks)
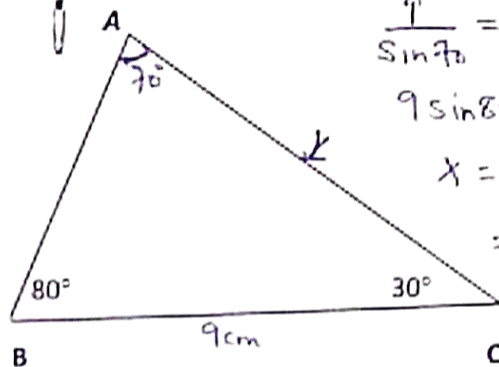
$$1400 \rightarrow \frac{70}{100}$$
$$100\%$$

$$\frac{1400 \times 100}{70\%} = 2000$$

$$100\% = 2000$$
$$= 2300$$

$$\frac{2300 \times 100}{2000} = 115\%$$

$$115 - 100 = 15\%$$

4. In the triangle ABC below, BC=9cm, angle ABC=80°.and angle ACB=30°.

$$\frac{9}{\sin 70} = \frac{x}{\sin 80°}$$

$$9\sin 80 = x\sin 70$$

$$x = \frac{9\sin 80}{\sin 70}$$

$$= 9.4321cm$$

$$A = \frac{1}{2} \times 9 \times 9.4331 \sin 30$$

$$= 21.22 cm^2$$

Calculate, correct to 4 significant figures, the area of the triangle.   (3mks)

5.) Given that the exterior angle of a regular hexagon is 2x. Find the value of x . Hence find the size of each interior angle of the hexagon.   (3mks)

n = 6 sides

$$\frac{360}{n} = exterior\ angle$$

$$\frac{360}{6} = 2x$$

$$\frac{12x}{12} = \frac{360}{12}$$

$$x = 30°$$

exterior = 2x30 = 60°

180 - 60° = 120°

6. Two numbers t and s are such that $t^4 \times s^2 = 5625$. Find t and s (3mks)

```
5 | 5625
5 | 1125
5 | 225
5 | 45
3 | 9
3 | 3
  | 1
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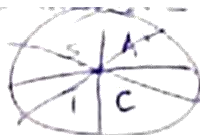
$$t^4 \times s^2 = 5^4 \times 3^2$$

$$t^4 = 5^4$$

$$t = 5$$

$$s^2 = 3^2$$

$$s = 3$$

7. Find the obtuse angle the line with equation $2y+5x+2=0$ makes with the x-axis. (3mks)

$$\frac{2y}{2} = \frac{-5x}{2} - \frac{2}{2}$$

$$y = -2.5x - 1$$

$$m = -2.5$$

Angle $= \tan^{-1}(2.5) = 68.20°$

$$180 - 68.20$$

$$= 111.80°$$

8. Simplify the expression (3mks)

$$\frac{12x^2 + ax - 6a^2}{9x^2 - 4a^2}$$

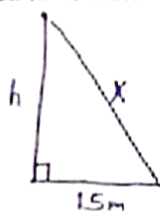$P = 12 \times -6 = -72$

$S = 1$

$12x^2 + 9ax - 8ax - 6a^2$

$3x(4x + 3a) - 2a(4x + 3a)$

$$\frac{(3x - 2a)(4x + 3a)}{(3x - 2a)(3x + 2a)}$$

$$= \frac{4x + 3a}{3x + 2a}$$

9. A plot is in the shape of a right angled triangle. The length of the shorter side is 15m and the area is 456.8m². Calculate the length of the longest side of the garden. (3mks)

$$\frac{1}{2} \times 15 \times h = 456.8$$

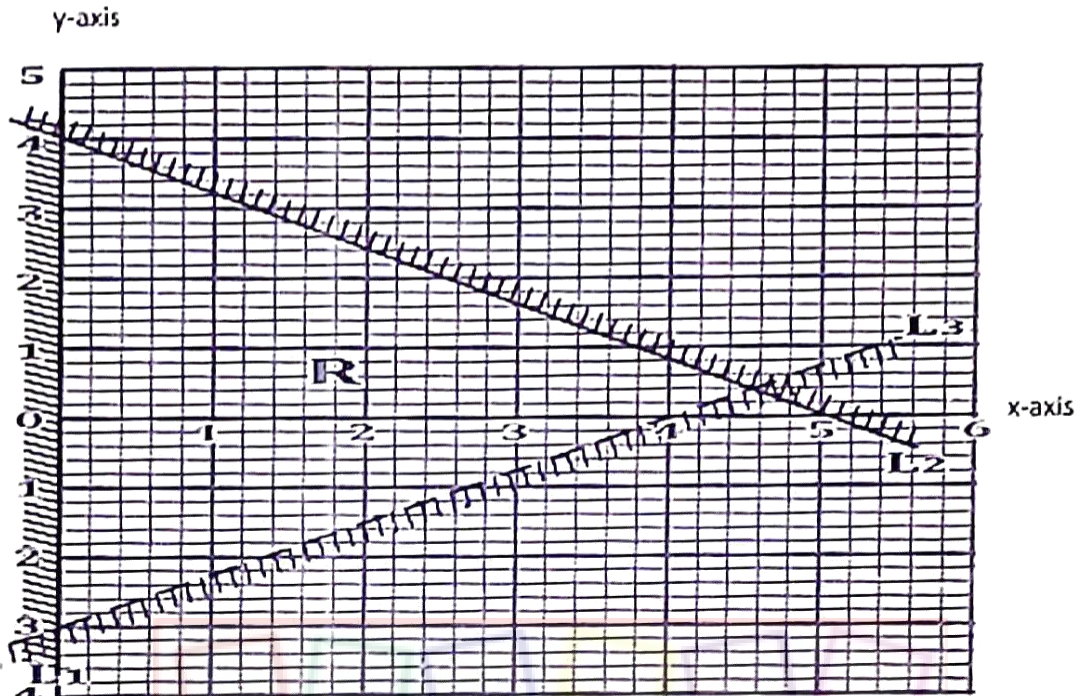$$7.5h = 456.8$$

$$h = 60.91m$$

$$X = \sqrt{60.91^2 + 15^2}$$

$$= 62.73m$$

10.) The diagram below shows a region R bounded by three lines L_1, L_2 and L_3 Form the three inequalities that satisfies the given region R   (3mks)

y-axis

L_1
$X \geq 0$

L_2
$\frac{x}{5} + \frac{y}{4} = 1$
$4x + 5y = 20$
$4x + 5y \leq 20$

L_3
$\frac{x}{4} + \frac{y}{3} = 1$
$-3x + 4y = -12$
$-3x + 4y > -12$

11. A bus travelled at an average speed of 63km/h left the station at 9.15 am. A car later left the same station at 10.00am and caught up with bus at 11.45 am. Find the average speed of the car.   (3mks)

$\begin{array}{r} 11.45 \\ - \quad 9.15 \\ \hline 2h \ 30mins \end{array}$

$2\frac{1}{2} \times 63 = 157.5km$

$\begin{array}{r} 11.45 \\ - 10.00 \\ \hline 1h \ 45mins \end{array}$

$157.5 \div 1\frac{3}{4}$

$= 157.5 \times \frac{4}{7}$

$= 90km/h$

12. A tourist came in Kenya and exchanged 1250 US dollars into Kenyan shillings at the rate shown below.

	Buying (Kshs)	selling (Kshs)
1 US dollar	105.5	110.8

He spent Ksh. 85400 after which he converted the remaining balance to US dollars. How much US dollars did he get back to the nearest dollar.　(3mks)

$$1 \text{ us dollar} = \text{sh } 105.5$$
$$1250 \text{ dollars} = ?$$

$$\frac{1250 \times 105.5}{1} = 131875$$

$$131875 - 85400$$

$$= \frac{46475}{110.8} = 419.449$$

$$= 419$$

13.a) Complete the table below for $y = x^2 + 5$　(1mk)

X	0	1	2	3	4	5	6
Y=x^2+5	5	6	9	14	21	30	41

b) Use the trapezoidal rule with 7 ordinates to estimate the area bounded by the curve $y = x^2 + 5$, x –axis , y-axis and x=6　(2mks)

$$7 \text{ ordinates} = 6 \text{ strips}$$

$$A = \frac{1}{2} \times 1 \left[(5 + 41) + 2 (6 + 9 + 14 + 21 + 30) \right]$$

$$= \frac{1}{2} \left[46 + 80 \right]$$

$$= \frac{1}{2} \times 206$$

$$= 103 \text{ sq units}$$

14. Given that $a = \begin{pmatrix} 6 \\ 2 \end{pmatrix}$, $b = \begin{pmatrix} -2 \\ -4 \end{pmatrix}$ and $3a-2b+2c = \begin{pmatrix} 32 \\ 20 \end{pmatrix}$, find c (3mks)

$$2c = 3\begin{pmatrix} 6 \\ 2 \end{pmatrix} - 2\begin{pmatrix} -2 \\ -4 \end{pmatrix} = \begin{pmatrix} 32 \\ 20 \end{pmatrix}$$

$$\begin{pmatrix} 18 \\ 6 \end{pmatrix} - \begin{pmatrix} -4 \\ -8 \end{pmatrix} + 2c = \begin{pmatrix} 32 \\ 20 \end{pmatrix}$$
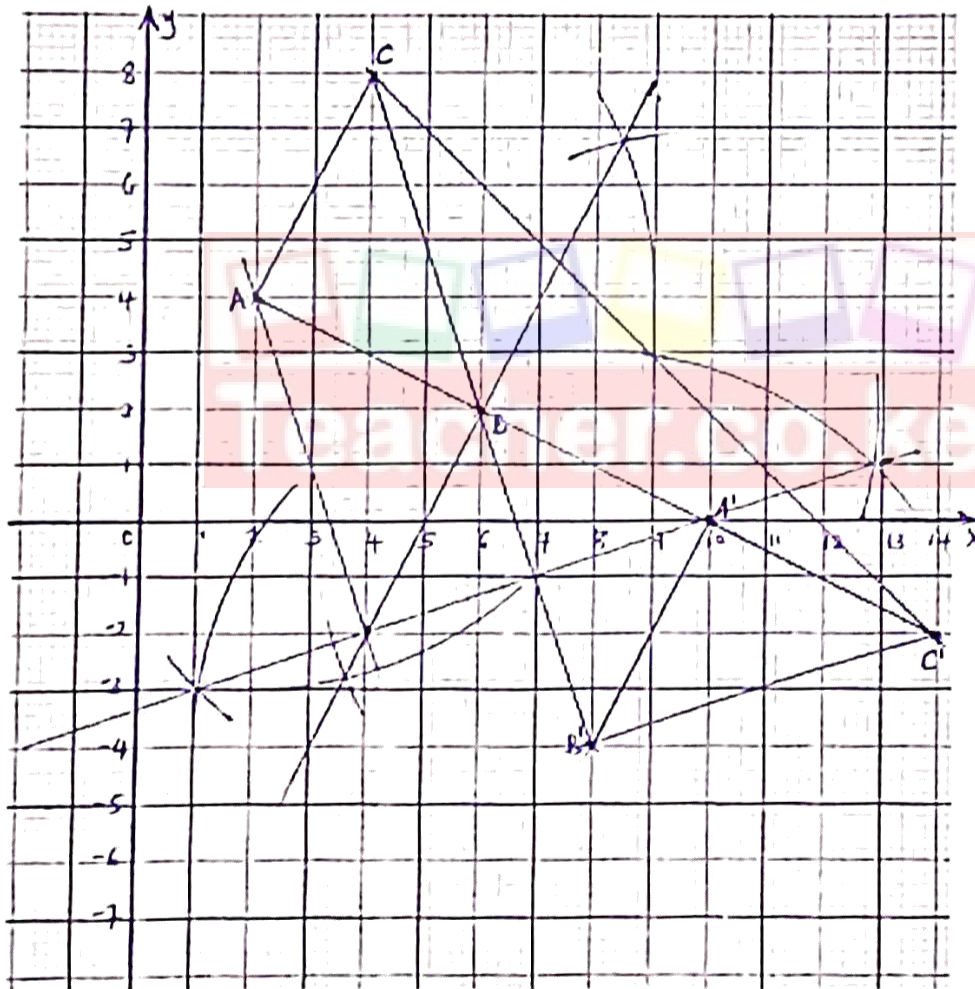
$$\begin{pmatrix} 22 \\ 14 \end{pmatrix} + 2c = \begin{pmatrix} 32 \\ 20 \end{pmatrix}$$

$$2c = \begin{pmatrix} 10 \\ 6 \end{pmatrix}$$

$$C = \frac{1}{2}\begin{pmatrix} 10 \\ 6 \end{pmatrix}$$

$$= \begin{pmatrix} 5 \\ 3 \end{pmatrix}$$

15). A triangle T with vertices A(2,4), B(6,2) and C(4,8) is mapped onto triangle T' with vertices A'(10,0), B'(8,-4) and C'(14,-2) by a rotation .

a) on the grid provided draw triangle T and its image T' (2mks)

b) Determine the centre and the angle of rotation that maps T onto T'' (2mks)

Centre of Rotation = (4 -2)
Angle of Rotation = -90°

16. A small cone of height 8cm is cut off from a bigger cone leaving a frustum of height the 16cm. If the volume of the smaller cone is 160cm³, find the volume of the frustum. (3mks)

$$L.S.f = \frac{24}{8} = \frac{3}{1}$$

$$V.S.f = \left(\frac{3}{1}\right)^2 = \frac{27}{1}$$

$$\frac{27}{1} = \frac{X}{160}$$

$$X = 27 \times 160 = 4320cm^3$$

$$4320 - 160 = 4160cm^3$$

Section II (50MKS)

Answer any five questions in this section in the space provided

17. Three businessmen, Hassan, Mutua and Wanyonyi decided to start a business. The initial capital which was needed was Ksh. 4,000,000 of which they were able to raise 30% by making contributions in the ratio 3:3:2 respectively. The rest of the amount was obtained from a bank and was to be paid back within one year with an interest of 25% in the same ratio 3:3:2. The three men were to share the profit of the business in the ratio of their contribution. During the year, the business realized a profit of ksh.4, 800,000.

a) How much of the initial amount did Wanyonyi raise?   (3mks)

$$0.3 \times 4000000 = 1,200,000$$

$$\frac{2}{8} \times 1200000 = 300,000$$

b) How much did Hassan pay to the bank at the end of the year?   (4mks)

$$4000000 - 1200000$$

$$= 2,800,000 \times 1.25$$

$$= 3,500,000$$

$$\frac{3}{8} \times 3500000$$

$$= 1,312,500$$

C) After paying the bank at the end of the year, how much was Mutua left with?   (3mks)

$$Profit = \frac{3}{8} \times 4,800,000 = 1,800,000$$
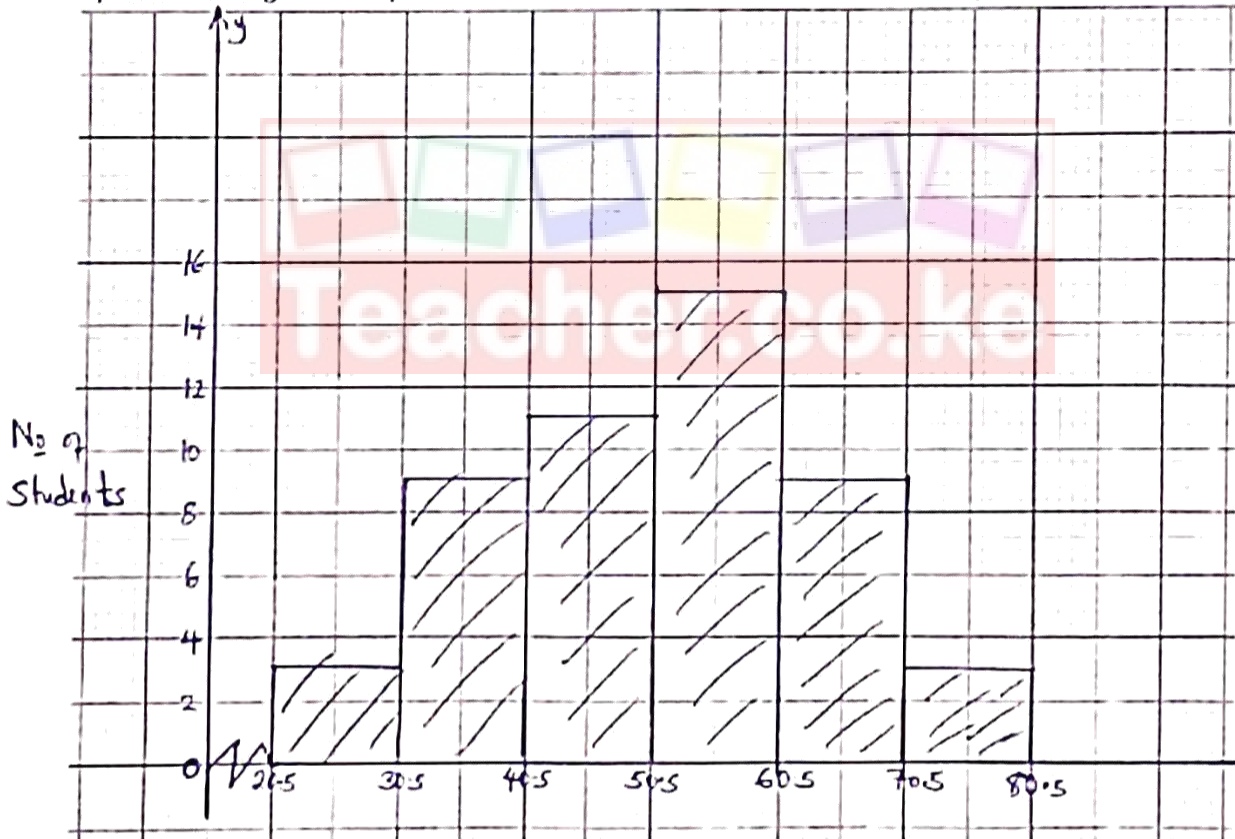
$$1,800000 - 1,312,500$$

$$= Sh\ 487,500$$

18. The marks scored by 50 students in a geography examination are as follows:

60	54	40	67	53	73	37	55	62	43
44	41	39	32	45	58	48	67	39	51
46	59	40	52	61	48	23	60	59	47
65	58	74	47	40	59	68	51	50	50
71	51	26	30	38	70	46	40	51	42

a) Prepare a frequency distribution table using a class interval of 10 starting from 21-30 (3mks)

Marks	f	Tally
21 - 30	3	III
31 - 40	9	HHH IIII
41 - 50	11	HHH HHH I
51 - 60	15	HHH HHH HHH
61 - 70	9	HHH IIII
71 - 80	3	III
	50	

b) Draw a histogram to represent the distribution (4mks)

c) Use your histogram to estimate the modal class (1mk)

$$51 - 60$$

d) Using the histogram estimate the median of the distribution of the data. (2mk

$$\frac{1}{2} \times 50 = 25^{th} \text{ position} \qquad X = \frac{2}{15} = 0.1333$$

$$25 - 23 = 2$$

$$X \times 15 = 2 \qquad \text{Median} = 50.5 + 0.1333$$

$$= 50.633$$

19. A petrol tanker has a cross-section in the shape shown below. It is used to transport petrol. Its internal length is 7m while its internal radius is 3.5 m. Obtuse angle POQ = 144°. On one of its trips, it was filled to capacity. Taking $\pi = \frac{22}{7}$
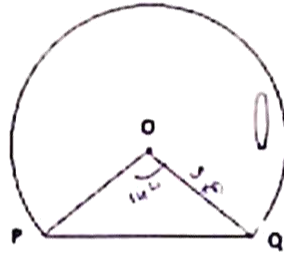
(a) Calculate the volume of petrol in the tanker in

(i) m³   (2 marks)

$$\left(\left(\frac{22}{7} \times 3.5 \times 3.5 \right) - \left(\frac{22}{7} \times \frac{144}{360} \times 3.5 \times 3.5 - \frac{1}{2} \times 3.5 \times 3.5 \times \sin 144 \right) \right) \times 7m$$

$$(26.7)\, 7$$

$$= 186.9 m^3$$

(ii) litres   (1 mark)

$$186.9 \times 1000 \qquad = 186900 \text{ litres}$$

(b) In the parking lot at night, a third of the petrol was stolen.

i) How many litres of petrol was the owner left with?   (2 marks)

$$\frac{2}{3} \times 186900$$

$$= 124600 \text{ litres}$$

ii) What was the mass of the remaining petrol given that one cubic metre of petrol has a mass of 700kg?   (3 marks)

$$1m^3 \longrightarrow 700kg$$
$$124.6m^3 $$

$$\frac{124.6 \times 700}{1} = 87220 \text{ Kg}$$

(iii) At the weigh bridge, any vehicle carrying excess of 50,000 Kg was charged Sh. 12.50 for every extra kilogram. How much fine did the owner of the tanker pay? (2 marks)
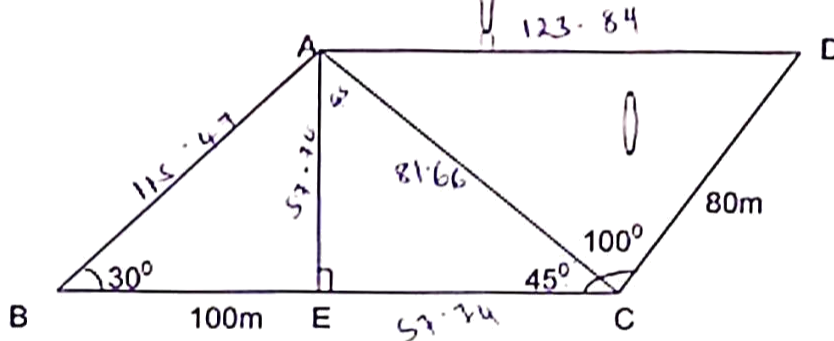
$$87220 - 50000 = 37220$$

$$1K = 12.50$$
$$37220$$

$$= 465250 \text{ shillings}$$

20. The figure below represents a quadrilateral piece of land ABCD divided into three triangular plots. The lengths BE and CD are 100m and 80m respectively. Angle ABE = 30^0, ACE = 45^0 and ACD = 100^0.

(a) Find to four significant figures:

(i) The length of AE.   (2mks)

$$\tan 30 = \frac{AE}{100}$$

$$AE = 57.74 \, m$$

(ii) The length of AD.   (2mks)

$$AC^2 = (57.74)^2 + (57.74)^2$$

$$AC = 81.66 \, m$$

Cosine rule

$$AD^2 = 81.66^2 + (80)^2 - (2 \times 81.66 \times 80 \cos 100)$$

$$= 123.84 \, m$$

(iii) The perimeter of the piece of land.   (3mks)

$$\cos 30 = \frac{100}{AB} = AB = 115.47$$

$$\text{Perimeter} = 115.47 + 123.84 + 80 + 57.74 + 100 = 477.08 \, m$$

(b) The plots are to be fenced with five strands of barbed wire leaving an entrance of 2.8m wide to each plot. The type of barbed wire to be used is sold in rolls of lengths 480m. Calculate the number of rolls of barbed wire that must be bought to complete the fencing of the plots.   (3mks)

$$2.8 \times 3 = 8.4$$

$$477.08 - 8.4 = 468.68$$

$$\frac{468.68}{480} \times 5 = \times 5 = 5$$

$$= 5 \text{ rolls}$$

21. A straight line L_1 has a gradient $-\frac{1}{2}$ and passes through point P (-1, 3). Another line L_2 passes through the points Q (1,-3) and R (4, 5).

Find:

(a) The equation of L_1 (2 marks)

$$-\frac{1}{2} = \frac{y-3}{x+1}$$
$$2y - 6 = -x - 1$$
$$2y = -x + 5$$
$$y = \frac{-x}{2} + \frac{5}{2}$$
$$y = \frac{-x}{2} + 2\frac{1}{2}$$

(b) The equation of L_2 in the form ax + by + c = 0   (3 marks)

$$m = \frac{5 - -3}{4 - 1} = \frac{8}{3}$$

$$8x - 3y - 17 = 0$$

$$\frac{8}{3} = \frac{y-5}{x-4}$$
$$3y - 15 = 8x - 32$$
$$3y = 8x - 17$$

(c) The equation of a line passing through a point S (0, 5) and is perpendicular to L_2.   (3 marks)

$$m_2 = \frac{-1}{8/3} = \frac{-3}{8}$$

$$\frac{-3}{8} = \frac{y-5}{x}$$
$$-3x = 8y - 40$$
$$3x + 8y - 40 = 0$$

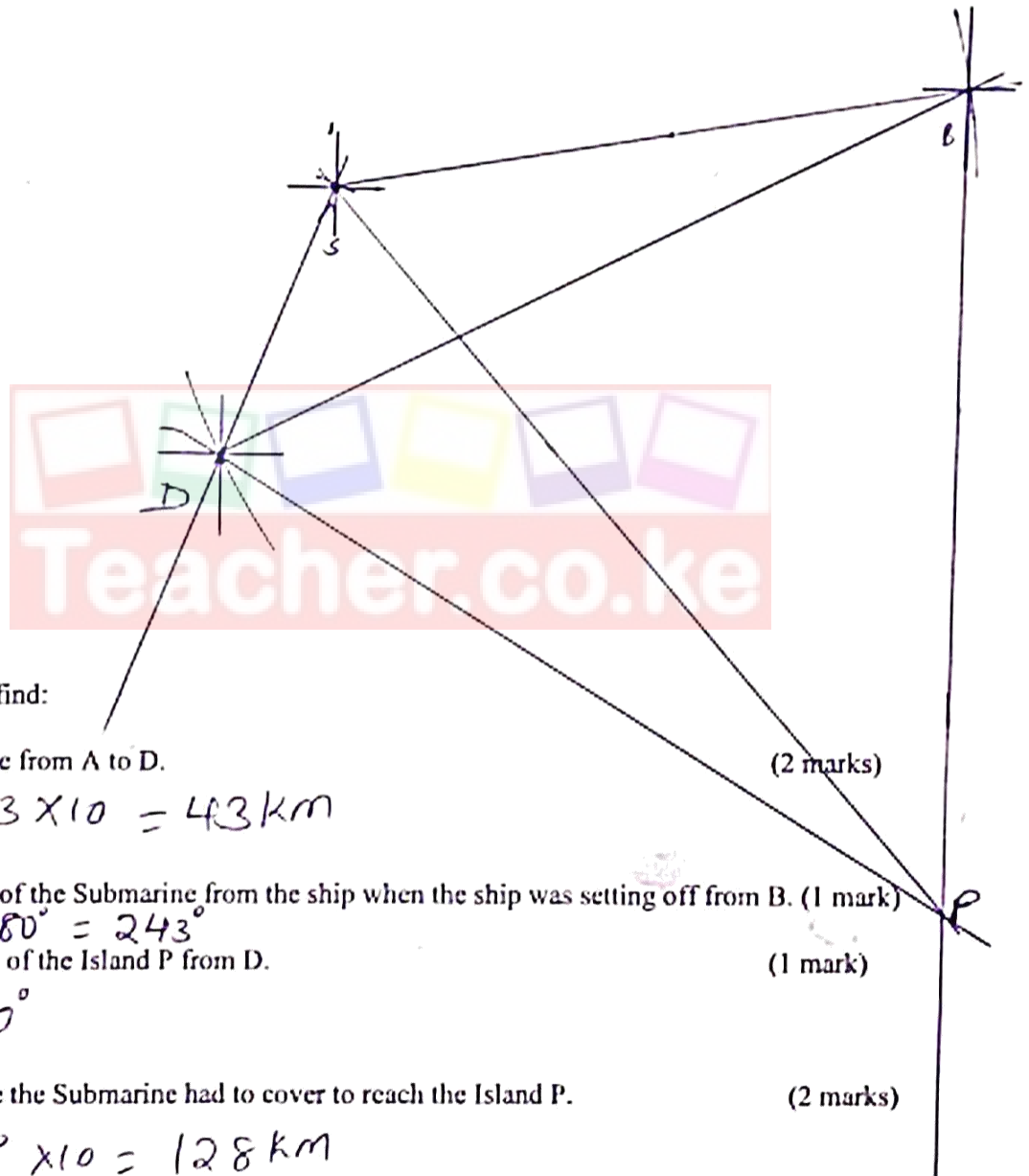(d) The equation of a line through R parallel to L_1.   (2 marks)

$$-\frac{1}{2} = \frac{y-5}{x-4}$$
$$2y - 10 = -x + 4$$
$$2y = -x + 14$$
$$2y + x - 14 = 0$$

22. a) A port B is on a bearing 080° from a port A and a distance of 95 km. A Submarine is stationed at a port D, which is on a bearing of 200° from A, and a distance of 124 km from B. A ship leaves B and moves directly Southwards to an Island P, which is on a bearing of 140° from A. The Submarine at D on realizing that the ship was heading to the Island P, decides to head straight for the Island to intercept the ship. Using a scale of 1 cm to represent 10 km, make a scale drawing showing the relative positions of A, B, D and P.   (4 marks)

Hence find:

b) The distance from A to D.   (2 marks)

$$4 \cdot 3 \times 10 = 43 km$$

c) The bearing of the Submarine from the ship when the ship was setting off from B. (1 mark)

$$63 + 180° = 243°$$

d) The bearing of the Island P from D.   (1 mark)

$$120°$$

e) The distance the Submarine had to cover to reach the Island P.   (2 marks)

$$12.8 \times 10 = 128 km$$

23. (a) Find the inverse of the matrix: (1mk)

$$A = \begin{pmatrix} 4 & 3 \\ 3 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-1}\begin{pmatrix} 2 & -3 \\ -3 & 4 \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}$$

b) Amina bought 20 bags of oranges and 15 bags of mangoes for a total of sh. 9,500. Nafula bought 30 bags of oranges and 20 bags of mangoes for a total of sh. 13,500. If he price of a bag of oranges is X and that of mangoes is y:

i) Form two equations to represent the information above. (2mks)

$$20x + 15y = 9500 \rightarrow 4x + 3y = 1900$$
$$30x + 20y = 13500 \rightarrow 3x + 2y = 1350$$

ii) Hence use the matrix A^{-1} above to find the price of one bag of each item. (4mks)

$$\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1900 \\ 1350 \end{pmatrix}$$

$$\begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 3 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2 & 3 \\ 3 & -4 \end{pmatrix}\begin{pmatrix} 1900 \\ 1350 \end{pmatrix}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -3800 + 4050 \\ 5700 - 5400 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 250 \\ 300 \end{pmatrix}$$

$$x = 250$$
$$y = 300$$

(c) The price of each bag of oranges was increased by 10% and that of mangoes reduced by 10%. The businesswomen (Amina and Nafula) bought as many oranges and as many mangoes as they bought earlier. Find the total cost of oranges and mangoes that each businesswomen bought after the percentage change. (3mks)

New price of oranges = $\frac{110}{100} \times 250 = 275$

New price of mangoes = $\frac{90}{100} \times 300 = 270$

Amina = $20 \times 275 + 15 \times 270 = 9550$

Nafula = $30 \times 275 + 20 \times 270 = 13650$

24. The displacement, s metres, of a moving particle from a point O, after t seconds is given by, $s = t^3 - 5t^2 + 3t + 10$

a) Find s when t=2   (2mks)

$S = (2)^3 - 5(2)^2 + 3(2) + 10$

$= 4m$

b) Determine:
i) the velocity of the particle when t=5 seconds;   (3mks)

$V = \frac{ds}{dt} = 3t^2 - 10t + 3$

$V = 3(5)^2 - 10(5) + 3$

$= 28m/s$

ii) the value of t when the particle is momentarily at rest   (3mks)

At rest, $V = 0$

$3t^2 - 10t + 3 = 0$

$t = \dfrac{10 \pm \sqrt{100 - 4(3)(3)}}{2 \times 3}$ $\dfrac{10 \pm \sqrt{64}}{6}$

$\dfrac{10 \pm 8}{6} = t = 3$ or $\frac{1}{3}$

$t = 3$ or $\frac{1}{3}$

c) find the time, when the velocity of the particle is maximum   (2mks)

$A = 6t - 10$

$6t - 10 = 0$

$6t = 10$

$t = \frac{5}{3}$ $t = 1\frac{2}{3}$