M/S 1

F4 PP2 SECTION I (50 Marks)

1. Ballast, sand and cement are mixed in the ratio 6:4:3. The cost of 6 tonnes of sand is sh. 600, 7 tonnes of ballast cost sh. 9100, and 3 tonnes of cement cost sh. 24,000. Calculate the cost of 10 tonnes of the mixture to the nearest shilling. [3 marks]

$6+4+3=13$ tonnes

Cost of 6 tonnes of ballast $= \frac{6}{7} \times 9100$
$= Sh\ 7800$

" 4 tonnes of sand $= \frac{4}{6} \times 600$
$= Sh\ 4000$

" 3 tonnes of cement $= \frac{3}{3} \times 24000$
$= Sh\ 24000$

13 tonnes of Mixture cost $7800+4000+24000$ ✓
$= Sh\ 35,800$

$\therefore$ Cost of 10 tonnes $= \frac{10}{13} \times 35800$ ✓
$= Sh\ 27,538.4615$
$= Sh\ 27,538$ ✓

M1 - all costs

M1 - cost of 10 tonnes

A1

3

2. The 2nd term of an A.P is three times the 7th term. If the 9th term is 1, find the 1st term and the common difference. [3 Marks]

Let 2nd term be $a+d$ & 7th term be $a+6d$

$a+d=3(a+6d)$ ✓

$\therefore 2a+17d=0$ (i)

Let 9th term be $a+8d$ ✓

$\therefore a+8d=1$ (ii)

$2a+17d=0$
$2 \times (a+8d=1)$

$-\ \ 2a+17d=0$
$\ \ \ \ 2a+16d=2$
$d=-2$ ✓

$a+-16=1$
$a=17$

M1 - Equation

M1 - solving

A1 - values of

3

Given that $x = 2-\sqrt{3}$, simplify $x^2 - x\sqrt{3}$

(3 marks)

$x^2 \Rightarrow (2-\sqrt{3})^2$

$= 7-4\sqrt{3}$

$x\sqrt{3} \Rightarrow (2-\sqrt{3})\sqrt{3}$

$= 2\sqrt{3} \mp 3$

$\Rightarrow 7-4\sqrt{3}-(2\sqrt{3}\pm 3)$

$7-4\sqrt{3}-2\sqrt{3}+3$

$10-6\sqrt{3}$

M_1 - substitutions

M_1 - simplifying

A_1

3

Write the expansion of $\left(2-\frac{1}{4}x\right)^5$. Hence, use the expansion to find the value of $(1.96)^5$ correct to 3 decimal places

(4 Marks)

$2^5 + 2^4\left(-\frac{1}{4}x\right) + 2^3\left(-\frac{1}{4}x\right)^2 + 2^2\left(-\frac{1}{4}x\right)^3 + 2\left(-\frac{1}{4}x\right)^4 + \left(-\frac{1}{4}x\right)^5$

$32 + 16\left(-\frac{1}{4}x\right) + 8\left(\frac{x^2}{16}\right) + 4\left(-\frac{x^3}{64}\right) + 2\left(\frac{x^4}{256}\right) - \frac{x^5}{1024}$

$32 + 5\times(-4x) + 10\times\left(\frac{x^2}{2}\right) + 10\times\left(\frac{-x^3}{16}\right) + 5\times\left(\frac{x^4}{128}\right) - \frac{x^5}{1024}$

$32 - 20x + 5x^2 - \frac{5}{8}x^3 + \frac{5}{128}x^4 - \frac{x^5}{1024}$

M1

A1

Let $1.96 = (2-0.04)$

$\therefore -\frac{1}{4}x = -0.04$

$x = 0.16$

$\Rightarrow 32 - 20(0.16) + 5(0.16^2) - \frac{5}{8}(0.16^3) + \text{- - - - -}$

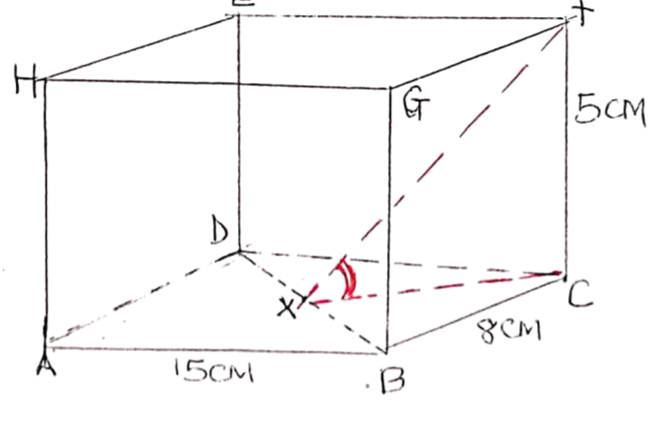
$32 - 3.2 + 0.128 - 0.00256 \qquad = 28.925$

M1

A1

4

The figure below shows a cuboid labelled ABCDEFGH. Point X is the mid-point of BD. AB = 15CM, BC = 8CM and CF = 5CM



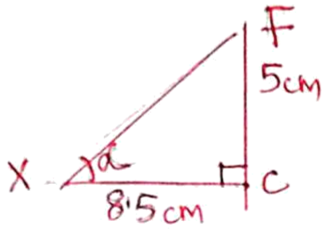
(a) Calculate the length XC [2 marks]

Diagonal $AC = \sqrt{15^2+8^2}$
$= \sqrt{289}$
$= 17$

$\therefore XC = \frac{1}{2} \times 17$
$= 8.5$ cm

M1 –
A1

(b) Calculate the angle $\angle FXC$ [2 marks]



$\tan\alpha = \frac{5}{8.5}$
$= 0.5882$
$\alpha = 30.4°$

M1 – ratio
A1
4

The area A of a sector of a circle of radius r varies jointly as r^2 and θ, the angle of the sector at the centre of the circle. If A = 30 cm², r = 8 cm, and θ = 24°, find A when θ = 48° and r = 4 cm. (3 marks)

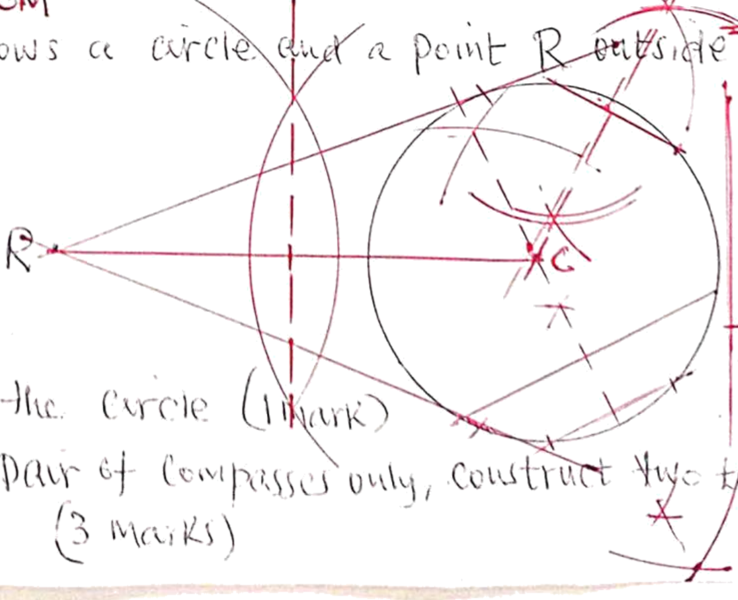
$A \propto r^2\theta$
$\therefore A = kr^2\theta$
$30 = k \times 64 \times 24$
$\therefore k = \frac{30}{1536}$
$= \frac{5}{256}$

$\Rightarrow A = \frac{5}{256} r^2\theta$

$\therefore A = \frac{5}{256} \times 16 \times 48$
$= 15 \text{ cm}^2$

B1 – Equation or constant got
M1 – substitution
A1
3

The figure below shows a circle and a point R outside the circle



B1 – any two chords bisected.

B1 – chords bisected
B1 – RC bisector
B1B1 – each tangent
4

a) Locate the centre of the circle (1 mark)
centre C

b) Using a ruler and a pair of compasses only, construct two tangents to the circle from R. (3 marks)

8. ... y in $(\log_{10} y)^2 = 3 - \log_{10} y^2$ (3 marks)

Let $\log_{10} y = u$

$u^2 = 3 - 2u$

$u^2 + 2u - 3 = 0$ ✓ — M_1 - Equation

$(u-1)(u+3) = 0$

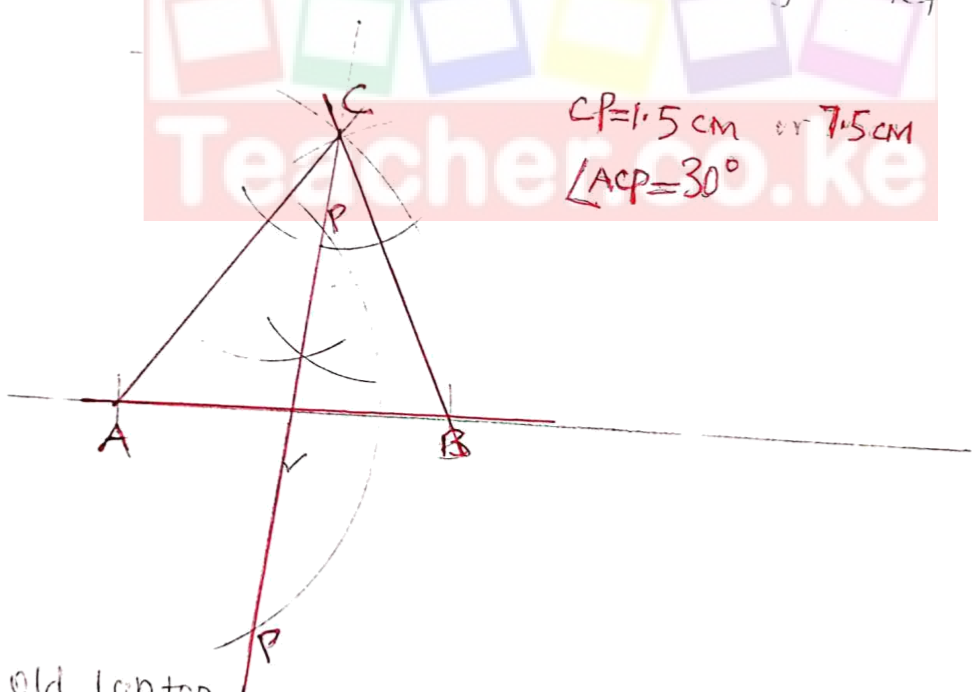
$\therefore u = 1$ or $u = -3$ ✓ — M_1 - Equations

$\log_{10} y = 1$ or $\log_{10} y = -3$

$y = 10$ — $y = 10^{-3}$ or $\frac{1}{1000}$ ✓ — A_1 - Both

3

1. Triangle ABC is such that AB = 5 cm, BC = 4.5 cm and AC = 5.7 cm. Find a point P that is equidistant from AC and BC and which is 4 cm from A. Measure CP and angle ACP (3 marks)



CP = 1.5 cm or 7.5 cm

$\angle ACP = 30°$

B_1 – Δ ABC drawn

B_1 – Both loci located

B_1 – Both CP and $\angle$ACP

3

2. An old laptop has a cash price of sh 18,000. A trader offers a down payment of sh ~~4~~5000 and 15 monthly instalments of sh 1050 each. Calculate the rate of compound interest charged per month for the laptop. (3 marks)

Total instalments = ~~5000×~~ 15 × 1050

= Sh 15,750 — B_1

Borrowed amount = 18000 − 5000 = Sh 13,000

$$A = P\left(1+\frac{r}{100}\right)^n$$

$$15750 = 13000\left(1+\frac{r}{100}\right)^{15}$$

$$1.21 = \left(1+\frac{r}{100}\right)^{15}$$

$$\log 1.21 = 15\log\left(1+\frac{r}{100}\right)$$

$$\frac{\log 1.21}{15} = \log\left(1+\frac{r}{100}\right)$$

$$0.005519 = \log(1+0.01r)$$

$$1.013 = 1+0.01r$$

$$\therefore 0.01r = 0.013$$

$$r = 1.3\%$$

M1

A1

3

Find the inverse of the matrix $\begin{pmatrix}1 & 1\\ 3 & 1\end{pmatrix}$. Hence determine the point of intersection of the lines

$y + x = 7$

$3x + y = 15$ (3 marks)

$$\text{Det} = 1-3 = -2$$

$$\text{Inv.} = \frac{1}{-2}\begin{pmatrix}1 & -1\\ -3 & 1\end{pmatrix} = \begin{pmatrix}-\frac{1}{2} & \frac{1}{2}\\ \frac{3}{2} & -\frac{1}{2}\end{pmatrix}$$

(i) $x+y=7$

$3x+y=15$

$$\begin{pmatrix}1 & 1\\ 3 & 1\end{pmatrix}\begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}7\\ 15\end{pmatrix}$$

$$\therefore \begin{pmatrix}x\\ y\end{pmatrix} = \begin{pmatrix}-\frac{1}{2} & \frac{1}{2}\\ \frac{3}{2} & -\frac{1}{2}\end{pmatrix}\begin{pmatrix}7\\ 15\end{pmatrix} = \begin{pmatrix}\frac{15}{2}-\frac{7}{2}\\ \frac{21}{2}-\frac{15}{2}\end{pmatrix} = \begin{pmatrix}4\\ 3\end{pmatrix}$$

$x=4, y=3$

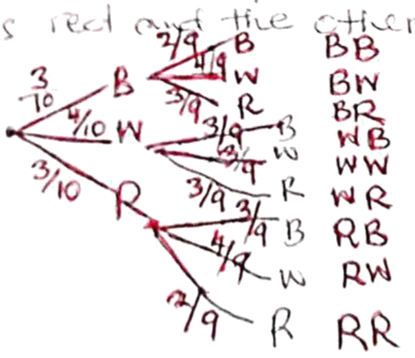
B1 - inverse

M1 - Equation

A1 - Both

3

A bag contains 3 blue marbles, 4 white marbles and 3 red ones. The marbles are drawn from the bag, one at a time without replacement. Find the probability that one of them is red and the other is white. (3 marks)



WR or RW

$$\frac{4}{10}\times\frac{3}{9}+\frac{3}{10}\times\frac{4}{9}$$

$$=\frac{24}{90}$$

$$=\frac{4}{15}$$

M1 - Tree or other sample space

B1

B1

3

(a) Draw the line of best fit for the values of E and C given in the table below: (3 marks)

E	4	7	10	13	16	19
C	15.1	20.2	23.9	27.3	30.1	33.1

(marks)

(ii) Write the equation connecting E and C. (1 mark)

$C = ME + K$

$\therefore K = 10$

$M = \text{Gradient}$

using $(0, 10)$ and $(4, 15.1)$

$\text{Gradient} = \frac{15.1-10}{4-0}$

$= \frac{5.1}{4} = 1.275$

$\therefore C = \frac{51}{40}E + 10$

or

$C = 1.275E + 10$

Given that $\cos\theta = 0.6$ and θ is the acute angle, find

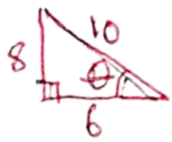
(a) θ (1 mark)

$\theta = 53.21^\circ$ ✓

B1

(b) Tan θ (2 marks)

$\cos\theta = \frac{6}{10}$



$\sqrt{10^2 - 6^2}$

$= 8$

$\therefore \tan\theta = \frac{8}{6}$ ✓

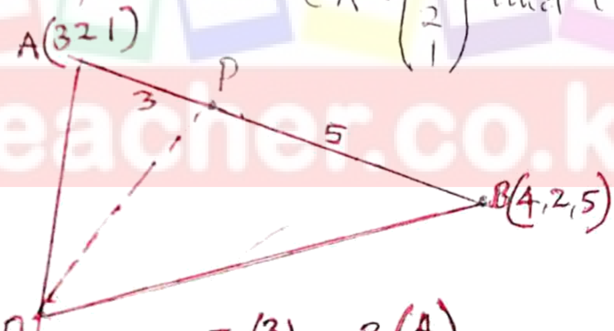
$= 1.333$ ✓

M1 - Ratio

A1

3

5. Find the position vector of a point p that divides AB in the ratio 3 : 5, where $OA = \begin{pmatrix}3\\2\\1\end{pmatrix}$ and $OB = \begin{pmatrix}4\\2\\5\end{pmatrix}$ (2 marks)



Applying ratio theorem: $\frac{5}{8}\begin{pmatrix}3\\2\\1\end{pmatrix} + \frac{3}{8}\begin{pmatrix}4\\2\\5\end{pmatrix}$ ✓

$\begin{pmatrix}\frac{15}{8}+\frac{12}{8}\\ \frac{10}{8}+\frac{6}{8}\\ \frac{5}{8}+\frac{15}{8}\end{pmatrix} = \begin{pmatrix}\frac{27}{8}\\ \frac{16}{8}\\ \frac{20}{8}\end{pmatrix} = \begin{pmatrix}3.4\\2\\2.5\end{pmatrix}$ ✓

Alt, $AB = B - A$

$= \begin{pmatrix}4\\2\\5\end{pmatrix} - \begin{pmatrix}3\\2\\1\end{pmatrix} = \begin{pmatrix}1\\0\\4\end{pmatrix}$

$OP = OA + AP$

$= \begin{pmatrix}3\\2\\1\end{pmatrix} + \frac{3}{8}\begin{pmatrix}1\\0\\4\end{pmatrix} = \begin{pmatrix}27/8\\2\\20/8\end{pmatrix}$

$= \begin{pmatrix}3.4\\2\\2.5\end{pmatrix}$

M1

A1

2

6. The positions of points A and B are A(25°N, 45°E) and B(25°N, 63°E) respectively. Calculate the distance between the points along the latitude. (Take the radius of the earth to be 6370 km) (2 Marks)

Let $\alpha = 63 - 45$

$= 18^\circ$

$AB = 60 \times 18 \cos 25$ ✓

$= 978.8$ nm ✓

Alternative in KM

$AB = \frac{18}{360} \times 2\pi \times 6370 \cos 25$ ✓

$= \frac{652925.73 \text{ KM}}{360}$ ✓

$= ~~18.13~~ \text{ KM}$ 1813.68 KM

M1 - Substitution

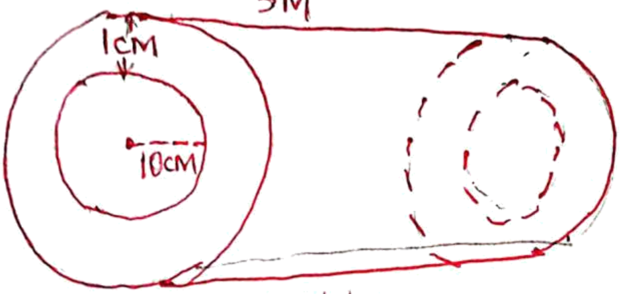
A1

2

SECTION 2 (50 Marks)

A metal water pipe has an internal radius of 10cm. The pipe is made of a metal of uniform thickness of 1cm.

(a) Find the volume, in cubic centimetres of metal used to make a portion of the pipe, 5m long (3 marks)



Extr Volume − Int Volume

$11^2\pi \times 500 - 10^2\pi \times 500$ — M1

$500\pi(121-100)$ — M1 - simplifying

32986.72cm^3 — A1

(b) Given that the metal used to make the pipe has a density of $8.5\ g/cm^3$, find the mass of the piece of pipe in kilograms. (3 marks)

$M = D \times V$

$= 8.5\ g/cm^3 \times 32986.72\text{cm}^3$ — M1

$= 280387.12\ gms \div 1000$ — M1 - conversions

$= 280.4$ kg — A1

(c) Water runs through the pipe at a rate of 30cm per second. Find the time taken to fill a water tank of capacity 50 000 Litres

Volume in 1 sec. $= \pi r^2 \times 30$

$= 10^2\pi \times 30$

$= 9424.78\text{cm}^3$ — M1

1sec $\longrightarrow 9424.78\text{cm}^3$

? $\longrightarrow 50000 \times 1000\ \text{cm}^3$ — M1

$= 5305.16$ sec

$= \frac{5305.16}{3600} = 1.474$ hrs — M1 - conversion

A1

10

18. The table below shows the distribution of masses of a sample of students in a certain school.

Mass (kg)	20-24	25-29	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69
No. of Pupils	1	5	9	11	20	20	19	8	4	3

Using 47 as a working mean, calculate

(a) The actual mean mass (5 marks)

class	x	f	d=x-A	fd
20-24	22	1	-25	-25
25-29	27	5	-20	-100
30-34	32	9	-15	-135
35-39	37	11	-10	-110
40-44	42	20	-5	-100
45-49	47	20	0	0
50-54	52	19	5	95
55-59	57	8	10	80
60-64	62	4	15	60
65-69	67	3	20	60
		$\Sigma f=100$		$\Sigma fd=-175$

$$\text{Mean } (\bar{x}) = a + \frac{\Sigma fd}{\Sigma f}$$

$$= 47 + \left(\frac{-175}{100}\right)$$

$$= 45.25\text{ kg}$$

B1 ✓ for x values
B1 ✓ for d
B1 ✓ for fd
B1 ✓ for Σfd
M1 - for substitution
A1

(b) The standard deviation of the masses (5 marks)

f	d	d^2	fd^2
1	-25	625	625
5	-20	400	2000
9	-15	225	2025
11	-10	100	1100
20	-5	25	500
20	0	0	0
19	5	25	475
8	10	100	800
4	15	225	900
3	20	400	1200
			$\Sigma fd^2=9625$

$$sd = \sqrt{\frac{\Sigma fd^2}{\Sigma f} - \left(\frac{\Sigma fd}{\Sigma f}\right)^2}$$

$$= \sqrt{\frac{9625}{100} - \left(\frac{-175}{100}\right)^2}$$

$$= \sqrt{69.25 - 3.063}$$

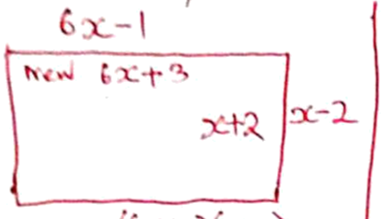
$$= 9.653\text{ kg}$$

B1 - for fd^2
B1 - for Σfd^2
M1 - for substitution
A1

10

The length and breadth of a rectangle are given as $(6x-1)$ and $(x-2)$ cm respectively. If the length and breadth are each increased by 4 metres, the new area is three times that of the original rectangle.

(a) Form an equation in x and solve it (4 marks)



New area $= (6x+3)(x+2)$

Old area $= (6x-1)(x-2)$ ✓

$\therefore (6x+3)(x+2) = 3(6x-1)(x-2)$ ✓ M1 - New lengths

$6x^2+12x+3x+6 = 3(6x^2-12x-x+2)$ M1 - Areas expressed

$6x^2+15x+6 = 3(6x^2-13x+2)$

$6x^2+15x+6 = 18x^2-39x+6$ M1 - Quadratic equation

$12x^2-54x=0$ ✓

$6x(2x-9)=0$

$x=0$ or $\therefore x = 4.5$ cm ✓ A1

(b) Find the dimensions of the original rectangle. (2 marks)

new 30 by 6.5

195

original $6\times4.5-1$
$=26$ ✓ M1 - Substitution

$4.5-2$
$=2.5$ ✓

Length 26 cm

Width 2.5 cm A1 - Both.

(c) Express the increase in area as a percentage of the original area. (4 marks)

Original area $= 26\times2.5$
$= 65\ cm^2$

New area $= 65\times3$ ✓ both M1 - Both areas
$= 195\ cm^2$

Increase $= 195-65$
$= 130$ ✓ M1 - Increase

$\frac{130}{65}\times100$ ✓ M1 M1

200% ✓ A1 A1

10

21. The equation of a curve is given by $y = 1 + 3\sin x$.

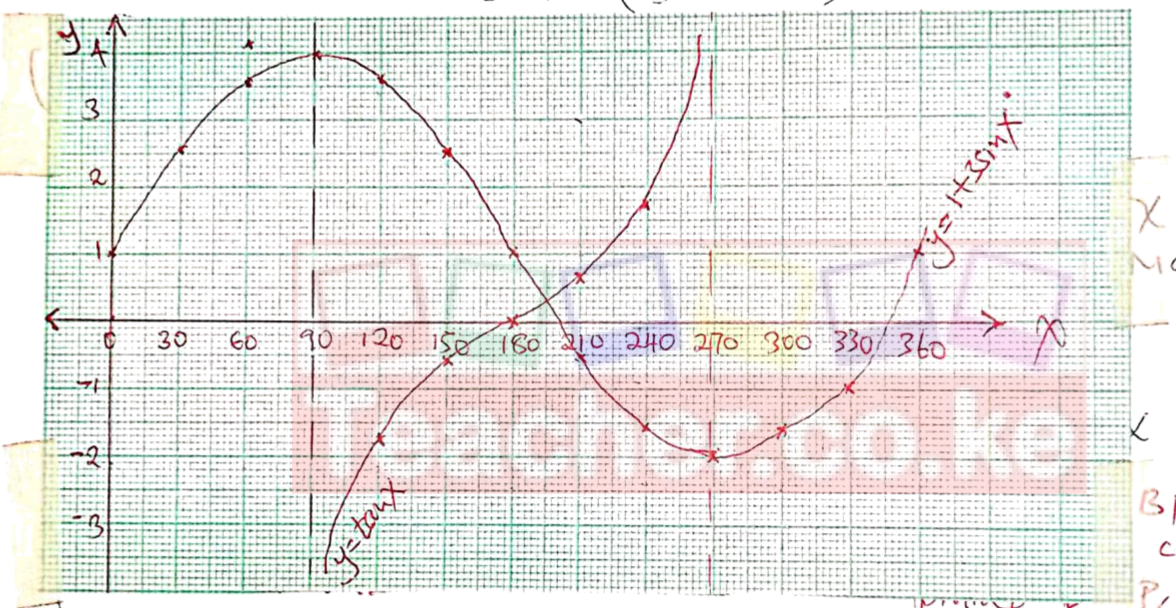
(a) Complete the table below for $y = 1 + 3\sin x$ correct to 1 decimal place (2 marks)

x°	0	30	60	90	120	150	180	210	240	270	300	330	360
$y = 1 + 3\sin x$	1	2.5	3.6	4	3.6	2.5	1	-0.5	-1.6	-2	-1.6	-0.5	1
$y = \tan x$				α	-1.7	-0.6	0	0.6	1.7	α			

(b)(i) On the grid provided, draw the graph of $y = 1 + 3\sin x$ for $0 \leq x \leq 360^\circ$. (3 marks)

B_2 - table all values or B_1 for at least four.

B_1 - table $y = \tan x$



P_1 C_1 S_1 wave of $1+3\sin x$

P_1 C_1 curve of $\tan x$

B_1 C_1 P_1

B_1 - Assymptotes shown

(d) Use the graphs to solve the equation $1 + 3\sin x = \tan x$ for $90^\circ \leq x \leq 270^\circ$ (3 marks)

Intersection point at $x = 195^\circ \pm 1^\circ$ B_1

10

Mrs. Kihara is a clerk on a basic monthly salary of Ksh 16,000. She also gets a house allowance of Ksh 12 000, a medical allowance of Ksh 3060 and a commuter allowance of Ksh 4635. She has a life insurance policy for which she pays Ksh 800 per month. She claims personal relief of Ksh 1056 and insurance relief of Ksh 120 per month. ~~Using the tax table below~~ Use the tax table below:

Income in £ per month	Rate (%)
1 – 484	10
485 – 940	15
941 – 1396	20
1397 – 1852	25
Over 1852	30

(a) Calculate Mrs. Kihara's taxable income in K£. (3 marks)

Ksh(16000+12000+3060+4635) ✓ M1

= 35 695

$\frac{35695}{20}$ = K£ 1784.75 ✓ M1 ✓ A1

(b) Using the table above, calculate her P.A.Y.E. (4 marks)

(space)

1st 484 ⇒ 484 × $\frac{10}{100}$ = £ 48.40

2nd 456 ⇒ 456 × $\frac{15}{100}$ = £ 68.40 ✓

3rd 456 ⇒ 456 × $\frac{20}{100}$ = £ 91.20

Last 388   388 × $\frac{25}{100}$ = £ 97.00

Total tax due   £ 305 ✓

305 × 20 = sh 6,100

PAYE = Total tax − reliefs

= 6100 − (1056 + 120) ✓

= sh 4924 ✓

(c) In addition to the PAYE, the following deductions are made on her pay every month:

NHIF Ksh. 600

Co-operative shares Ksh. 4 800

Calculate her net salary. (3 marks)

sh 35695 − (4924 + 600 + 4800) = sh 25,371 ✓✓

22\. A quadrilateral with vertices at K(1,1), L(4,1), M(2,3) and N(1,3) is transformed by a matrix $T = \begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix}$ to a quadrilateral K'L'M'N'.

a) Determine the coordinates of the image (3 marks)

$$\begin{pmatrix} 1 & 3 \\ 0 & 1 \end{pmatrix} \begin{array}{c} \begin{array}{cccc} K & L & M & N \end{array} \\ \begin{pmatrix} 1 & 4 & 2 & 1 \\ 1 & 1 & 3 & 3 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} K' & L' & M' & N' \end{array} \\ \begin{pmatrix} 1+3 & 4+3 & 2+9 & 1+9 \\ 0+1 & 0+1 & 0+3 & 0+3 \end{pmatrix} \end{array}$$

M1

M1

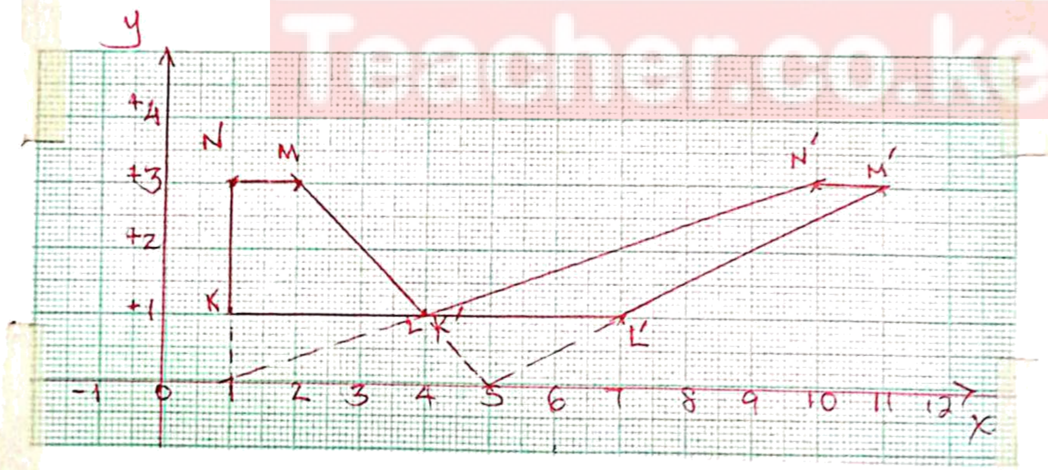
$$= \begin{array}{c} \begin{array}{cccc} K' & L' & M' & N' \end{array} \\ \begin{pmatrix} 4 & 7 & 11 & 10 \\ 1 & 1 & 3 & 3 \end{pmatrix} \end{array}$$

A1

(b) On the grid provided draw the object and the image (2 marks)

B1 – object

B1 – image



(c) Describe fully the transformation which maps KLMN onto K'L'M'N' (2 marks)

shear x-axis invariant and $M(2,3) \rightarrow M'(11,3)$

shear factor $\frac{9}{3} = 3$

B1 – invariant line must be stated.

B1 – $M \rightarrow M'$

(d) Find a matrix which maps K'L'M'N' onto KLMN (3 marks)

let Matrix be $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{array}{c} \begin{array}{cccc} K' & L' & M' & N' \end{array} \\ \begin{pmatrix} 4 & 7 & 11 & 10 \\ 1 & 1 & 3 & 3 \end{pmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} K & L & M & N \end{array} \\ \begin{pmatrix} 1 & 4 & 2 & 1 \\ 1 & 1 & 3 & 3 \end{pmatrix} \end{array}$$

$\therefore$ $4a + b = 1$

$7a + b = 4$

$a = 1$

$b = -3$

Also $4c + d = 1$

$7c + d = 1$

$c = 0$

$d = 1$

Matrix $\begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$

M1

M1

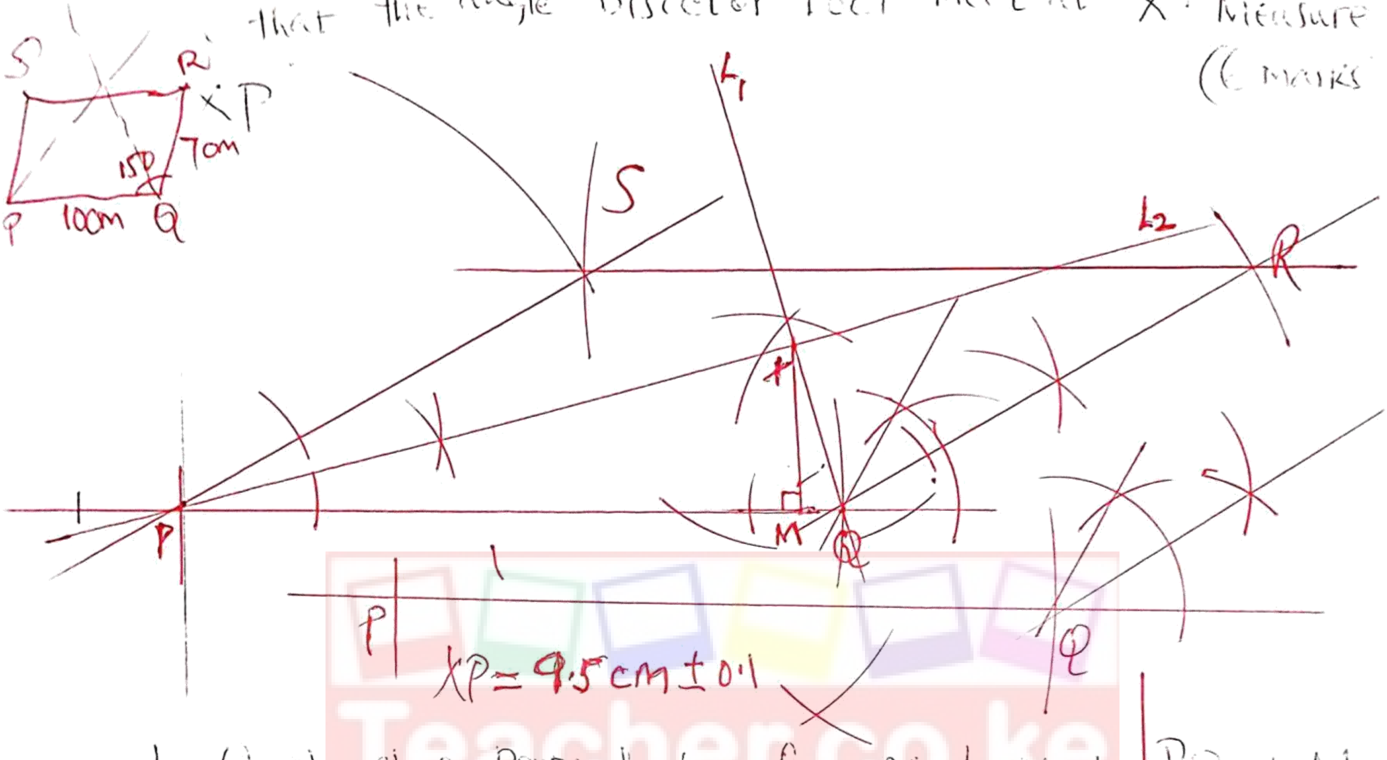
A1

Accept alt.

10

23. Using a ruler and a pair of compasses only:

(a) Construct a parallelogram PQRS, where PQ = 10cm, QR = 7cm and ∠PQR = 150. Bisect ∠PQR and ∠SPR that the angle bisector loci meet at X. Measure XP (6 marks)



XP = 9.5cm ± 0.1

(b) Construct a perpendicular from X to meet PQ at M. Measure XM. (2marks)

XM = 2.5cm ± 0.1

(c) Calculate the area of Δ PXQ (2marks)

$A = \frac{1}{2} \times 10 \times 2.5$

$= 12.5\ cm^2$

B1 – ∠PQR drawn
B1 – R located
B1 – S located
B1 – PQRS completed.
B1 – L1 & X located
B1 – L2 located
B1 – XP
B1 – XM drawn
B1 – XM measured.
M1 – Equation substituted
A1

10

A ... moves in a straight line in such a way that its distance s metres, after t seconds is given by the equation

$s = \frac{t^3}{3} - 3t^2 + 5t$. Find the times when

4. A body moves in a straight line such that at any time t seconds, its distance S metres from the starting point is given by $S = 8t - t^2$.

$V = \frac{ds}{dt} = 8 - 2t$

(a) How fast is the body moving at

(i) t = 1 second (2 marks)

$8 - 2\times1 = 6\,m/s$. B1

(ii) t = 3 seconds. (2 marks)

$8 - 3\times2$

$= 2\,m/s$. B1

(b) Calculate the maximum displacement from the starting point that the body achieves. (3 marks)

$\frac{ds}{dt} = 0$

$8 - 2t = 0$

$\therefore t = 4s$

but $S = 8t - t^2$

$S = 8\times4 - 4^2$

$= 32 - 16$

$= 16M$

M1 – $\frac{ds}{dt}$

M1 – substitution

A1

(c) Calculate the acceleration of the body during the motion. (1 mark)

$V = 8 - 2t$

$\therefore \frac{dv}{dt} = -2$

$= -2\,ms^{-2}$ ie. deceleration

B1

(d) After how long will the body be back at the starting point? (2 marks)

Displacement = 0

$8t - t^2 = 0$

$t(8 - t) = 0$

$\therefore t = 0$

or

$t = 8$

8 seconds.

M1 – Equation

A1 – Both.

10