1. Matrices and Transformations

1. a) (i) On the grid provided, with the same scale on both axes, draw the square S whose vertices are (0, 0), (2, 0), (2,2) and (0, 2). (1 mk)

(ii) Find the coordinates and draw the image T of S under the transformation whose matrix A maps a point (x, y) onto (x', y')

where; $\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} 2x - y \\ x + 2y \end{pmatrix}$ (3 mks)

(iii) Draw the image U of S under the transformation whose matrix is

$B = \begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$ (2 mks)

(b) (i) Find the product AB and draw the image V of S under the transformation whose matrix is AB (3 mks)

(ii) Describe the single transformation that maps S onto V (1 mk)

2. On the grid provided, draw triangle PQR with P(2,3), Q(1,2) and R(4,1). On the same axes draw triangle $P^{11}Q^{11}R^{11}$ with vertices P^{11}(-2,3), Q^{11}(-1,2) and R^{11}(-4,1). (2mks)

(a) Describe fully a single transformation which will map triangle PQR onto triangle $P^{11}Q^{11}R^{11}$. (1mk)

(b) On the same plane, draw triangle $P^1Q^1R^1$ the image of triangle PQR under reflection in the line y = -x. (2mks)

(c) Describe fully a single transformation which maps triangle $P^1Q^1R^1$ onto triangle $P^{11}Q^{11}R^{11}$. (2mks)

(d) Draw triangle $P^{111}Q^{111}R^{111}$ such that it can be mapped onto triangle PQR by a position quarter about (0,0) (2mks)

(e) State all pairs of triangles that are oppositely congruent. (1mk)

3. a) Given the transformation matrices

T1 = $\begin{pmatrix} 2 & 1 \\ -1 & -2 \end{pmatrix}$ and T2 = $\begin{pmatrix} 3 & 1 \\ 1 & 3 \end{pmatrix}$

and that transformation T_1 followed by T_2 can be replaced by a single transformation T, write down the matrix for T. (3 marks)

a) Find the inverse of matrix T (2 marks)

b) The points A^{11}(7,-11), B^{11}(-7,-13), C^{11}(-8,16) and D^{11}(8,8) are the images of points A, B, C and D respectively under transformation T_1 followed by T2
Write down the co-ordinates of A, B, C, and D. (5 marks)

4. A(3, 7), B(5, 5), C(3, 1), D(1, 5)

a) On the grid provided in the next page, plot ABCD on a Cartesian plane (2mks)

b) A'B'C'D' is the image of ABCD under a translation $T\begin{pmatrix}-6\\-9\end{pmatrix}$. Plot A'B'C'D' and state its coordinates. (2mks)

c) Plot A"B"C"D", the image of A'B'C'D' after a rotation about (-1, 0) through a positive quarter turn. State its coordinates. (3mks)

d) A'"B'"C'"D'" is the image of A"B"C"D" after a reflection in the line y=x + 2. Plot A'"B'"C'"D'" and state its coordinates (3 mks)

5. A transformation represented by the matrix $\begin{pmatrix}-2 & 1\\1 & 2\end{pmatrix}$ maps P(0,0), Q(2,0), R(2,3) and S(0,3) onto P', Q', R', S'

a) On the grid provided draw the quadrilateral PQRS and P'Q'R'S' (4mks)

b) (i) Determine the area of PQRS (1mk)

(ii) Hence or otherwise find the area of P'Q'R'S' (2mks)

c) A transformation represented by the matrix $\begin{pmatrix}0 & -1\\-1 & 0\end{pmatrix}$ maps P'Q'R'S' onto P"Q"R"S". Determine the matrix of transformation that would map P"Q"R"S" onto PQRS (3mks)

6. A translator T maps P (8, -2) onto P^1 (-2, -3). Find the image of Q (6, -2) under the same translation. (3 mks)

7. The vertices of a triangle are A(2,5), B(4,3) and C(2,3). H represents a half turn rotation about the point (0,2).

a) Draw triangle ABC and A',B',C' under H (4 marks)

b) T represents a reflection in the lone x= 0 and K represent a translation $\begin{pmatrix}0\\-2\end{pmatrix}$. Find the coordinates of A'',B'',C'' of A',B',C' under TK. Hence draw A'',B'',C'' (4 marks)

c) Describe a single transformation that maps ABC onto A'',B'',C'' (2 marks)

8. Given triangle ABC with vertices A (-6, 5), B(-4, 1) and C(3, 2) and that A(-6, 5) is mapped onto A^1(-6, -4) by a shear with y-axis in variant. On the grid provided below;

(i) draw triangle ABC

(ii) draw triangle $A^1B^1C^1$, the image of triangle ABC, under the shear

(iii) determine the matrix representing the shear

(b) Triangle $A^1B^1C^1$ is mapped onto $A^{11}B^{11}C^{11}$ by a transformation defined by the matrix $\begin{pmatrix}-1 & 0\\3/2 & -1\end{pmatrix}$

(i) Draw triangle $A^{11}B^{11}C^{11}$ on the same grid as ABC and $A^1B^1C^1$

(ii) Describe fully a single transformation that maps A^{11} $B^{11}C^{11}$

9. (a) Under a certain rotation A(2,0) is mapped onto A^1(-4, 2) and B(0,5) is mapped onto B^1(-9, o)

(i) On the grid provided plot the lines AB and A^1B^1 on the same axes

(ii) Hence determine by construction the co-ordinates of the centre and angle of rotation

(b) Under a quarter positive turn about the origin O, A^1 is mapped onto A^{11} and B^1 is mapped

onto B^{11}. Determine the co-ordinates of A^{11} and B^{11}

(c) Describe fully a single transformation which would map A to A^{11} and B to B^{11}

10. A transformation **T** is represented by the matrix $\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ and transformation **U** $\begin{pmatrix} \mathbf{0} & \mathbf{-1} \\ \mathbf{-1} & \mathbf{0} \end{pmatrix}$ by the matrix. Given that a rectangle has co-ordinates at A (1,2) B(6, 2), C(6, 4) and D (1, 4) and that under **T** the image of ABCD is $A_1B_1C_1D$ and under **U** the image of $A_1B_1C_1D$ is $A_2B_2C_2D_2$:

(a) Find the co-ordinates of $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$

(b) On the grid provided, plot ABCD, $A_1B_1C_1D_1$ and $A_2B_2C_2D_2$

(c) Describe the transformation represented by:-

(i) U

(ii) UT

(d) If $A_2B_2C_2D_2$ were to be transformed by a transformation represented by the matrix to map onto $A_3B_3C_3D_3$. What would be the area of $A_3B_3C_3D_3$

11. The vertices of a quadrilateral are A(2,2) B(8,2) , C (8,6) and D(6,4) under a rotation the images of vertices A and D are A(0,8) and D1(-2, 12).

(a) On the grid provided and using the same axes draw the quadrilateral ABCD and the points A^1 and D^1

(b) Determine the centre and angle of rotation

(c)Locate the points B^1 and C^1 under the rotation and complete the quadrilateral

12. A translation maps the point P(5, -3) onto P^1(2, -5)

(a) Determine the translation vector T (b)

A Point R^1 is the image of R(-2, -3) under the same translation in (a) above, find the magnitude of P^1R^1

13. Triangle ABC has vertices at A(0, -1), B(4, 3)and C(2,2).

(a) Find the coordinates of image triangle $A^1B^1C^1$ of triangle ABC under translation vector $\begin{pmatrix} 1 \\ -2 \end{pmatrix}$

(b) Given that triangle $A^{11}B^{11}C^{11}$ is the image of triangle $A^1B^1C^1$ under an enlargement scale factor 3, centre O(0,0) , find the coordinates of A^{11}, $B^{11}$and C^{11}

(c) If the area of triangle $A^1B^1C^1$ is 24 cm^2, calculate the area of triangle $A^{11}B^{11}C^{11}$

(d) Find the matrix that maps triangle $A^{11}B^{11}C^{11}$ onto triangle ABC

14. a) The triangle ABC where A (2,-1) B (1, 2) and C (4, 4) is reflected in the line X = 4 to give triangle $A_1B_1C_1$. Draw the two triangles on the graph provided and state the co-ordinates of $A_1B_1C_1$

b) Draw the triangle A_2 (5,6), B_2 (2,7) and C_2 (0,4). Given that triangle $A_2B_2C_2$ is the image of triangle $A_1B_1C_1$ under rotation, determine the centre and angle of this rotation

c) Show the image of triangle $A_2B_2C_2$, under an enlargement centre (0, 6) scale factor -1

15. (a) Find the co-ordinates for the image of point **P(6, -2)** under the transformation defined by :-

$x^1 = x - 3y$

$y^1 = 2x$

(b) (i) A quadrilateral ABCD has vertices A(4, -3), B(2, -3), C(4, -1) and D(5, -4). On the grid provided, draw the quadrilateral ABCD

(ii) $A^1B^1C^1D^1$ is the image of ABCD under a rotation through $+90^o$ about the origin. On the same axes, draw $A^1B^1C^1D^1$under the transformation

` (c) $A^2B^2C^2D^2$is the image of under $A^1B^1C^1D^1$ under another transformation by the matrix $\begin{pmatrix} \mathbf{1} & \mathbf{-2} \\ \mathbf{0} & \mathbf{1} \end{pmatrix}$

(i) Determine the co-ordinates of $A^2B^2C^2D^2$ and plot it on the same axes

(ii) Describe the transformation that maps $A^1B^1C^1D^1$onto $A^2B^2C^2D^2$

(d) Find a single matrix of transformation that would map $A^2B^2C^2D^2$onto ABCD

16. (a) Triangle **XYZ** has vertices **X**(2, -1) **Y**(4, -1) and **Z** (4,2). Triangle XYZ maps onto triangle $X^1Y^1Z^1$ under transformation $T_1 = \begin{pmatrix} 1 & -3 \\ 0 & 1 \end{pmatrix}$. Draw triangles XYZ and its image $X^1Y^1Z^1$ on the grid provided

(b) Another triangle $X^{11}Y^{11}Z^{11}$ is the image of $X^1Y^1Z^1$ after transformation $\mathbf{T_2} =$. Draw triangle $X^{11}Y^{11}Z^{11}$on the same set of axes

(c) Find the single transformation matrix **T** that maps triangle XYZ on to the final image $X^{11}Y^{11}Z^{11}$

(d) Given that the area of triangle XYZ is $15cm^2$, find the area of the triangle $X^{11}Y^{11}Z^{11}$

17. The quadrilateral A (2,1), B (4,1), C (4,4) and D (2,4) is mapped onto A'B' C'D' by a matrix M_1 such that A^1 (8,7), B^1 (14,7), C^1 (14,16) and D^1 (8,16) .

a) Draw both ABCD and A^1B^1 C^1D^1 on the same plane

b) Find the matrix of transformation that mapped ABCD onto A'B' C'D' and describe it fully

c) A^1B^1 C^1D^1 underwent another matrix transformation at N which is a translation that gave the image A^{11} B^{11} C^{11} D^{11}, Where A^{11} (7,9), B^{11} (13,9), C^{11} (13,18) and D^{11} (7,18). The transformation N is a translation . Find the translation

d) Draw A^{11} B^{11} C^{11} D^{11} on the same axes where ABCD and A^1B^1 C^1D^1 were drawn

18. a) On the grid provided. Plot the points A(2, -1) B (0, -3) C(2, -4) and D (4, -2) and join them to form a quadrilateral ABCD. What is the name of this quadrilateral?

b) The points A^1 (1, 2) B^1 (3, 0) C^1 (4, 2) and D^1 (2, 4) are the images of ABC and D under a certain transformation T_1. On the same grid draw quadrilateral $A^1B^1C^1D^1$ and describe transformation T_1 fully.

c) The points A^{11}(-2, -4) B^{11}(-6, 0) C^{11}(-8, -4) and D^{11}(-4, -8) are the images of $A^1B^1C^1D^1$ under transformation T_2. On the same grid draw quadrilateral $A^{11}B^{11}C^{11}D^{11}$ and describe the transformation T_2 fully.

d) On the same grid draw quadrilateral A^{111} B^{111} C^{111} D^{111}, the image of A^{11} B^{11} C^{11} D^{11} under a reflection in the x-axis. State the co-ordinates of A^{111} B^{111} C^{111} D^{111}.

19. The Points A^1B^1 and C^1 are the images of A(4, 1), B(0, -2) and C(-2, 4) respectively under a transformation represented by the matrix;

$$M = \begin{pmatrix} -1 & 1 \\ 2 & -3 \end{pmatrix}$$

(a) Write down the coordinates of A^1 B^1 and C^1

(b) A^{11} B^{11} and C^{11} are the images of A^1 B^1 and C^1 under another transformation whose Matrix is:

$N = \begin{pmatrix} 2 & -1 \\ 1 & 2 \end{pmatrix}$ Write down the coordinates of A^{11} B^{11} and C^{11}

(c) Transformation **M** followed by **N** can be represented by a single transformation **P**. Determine the matrix for **P**

(d) A matrix **P** is given by $\begin{pmatrix} 8 & 7 \\ 4 & 5 \end{pmatrix}$

Find P^{-1}

20. Triangle $A^1B^1C^1$ is the image of triangle ABC under a transformation represented by matrix $T = \begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$ If the area of triangle $A^1B^1C^1$ is $25.6cm^2$, find the area of the object

21. A point P(2, -4) is mapped into P^1(4, 0) under a translation.
Determine the image of point Q(-1, 2) under the same translation

22. The points A (2, 6), B (1, 1), C (2, 3) and D (4,0) are the vertices of quadrilateral ABCD.
(a) On graph paper plot the points A, B, C, and D and join them to form quadrilateral ABCD.

(b) The points A, B, C and D are the images of A^1, B^1, C^1 and D^1 respectively under an enlargement centre the origin and scale factor -2. On the same grid draw the image quadrilateral A^1 B^1 C^1 D^1.

(c) The points A^{11} B^{11} C^{11} and D^{11} are the images of ABCD respectively under reflection in the x – axis. On the same grid, locate the pints A^{11} B^{11} C^{11} and D^{11} and draw the second image quadrilateral A^{11} B^{11} C^{11} D^{11}.

(d) Quadrilateral A^{111} B^{111} C^{111} D^{111} is the image of ABCD under a certain transformation T. Describe transformation T fully.

23. T is a transformation represented by the matrix $\begin{pmatrix} 5\boldsymbol{x} & 2 \\ \boldsymbol{x} & -3 \end{pmatrix}$. Under T, a square of area $10cm^2$ is mapped onto a square $110cm^2$. Find the values of x