2. Matrices

1	$C = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ Let $\begin{pmatrix} 11 & 3 \\ 4 & 1 \end{pmatrix}\begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$ $(11a + 3c = 2)\times 1 \quad (11b + 3d = 4)\times 1$ $(4a + c = 3)\times 3 \quad (4b + d = 6)\times 3$ $11a + 3c = 2 \quad 11b + 3d = 4$ $12a + 3c = 9 \quad 12b + 3d = 18$ $a = 7 \quad b = 14$ $c = -25 \quad d = -50$ $\therefore C = \begin{pmatrix} 7 & 14 \\ -25 & -50 \end{pmatrix}$	M_1 M_1 A_1 3	Alternative $C = B^{-1}A$ ✓equations B^{-} $^{1}\begin{pmatrix} -1 & 3 \\ 4 & -11 \end{pmatrix}$ allow any two ✓solving of equations $\begin{pmatrix} -1 & 3 \\ 4 & -11 \end{pmatrix}\begin{pmatrix} 2 & 4 \\ 3 & 6 \end{pmatrix}$ or equivalent
2.	$\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 13 \\ 0 \end{pmatrix}$ *Det* $4 - {}^{-}9 = 13$ $\frac{1}{13}\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ -3 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{13}\begin{pmatrix} 2 & -3 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 13 \\ 0 \end{pmatrix}$ $\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ $x = 2,\ y = 3$	M1 M1 A1	
		03	

1.

$$\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & \frac{-3}{11} \end{pmatrix}\begin{pmatrix} 3 & 2 \\ 4 & -1 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} \frac{1}{11} & \frac{2}{11} \\ \frac{4}{11} & \frac{-3}{11} \end{pmatrix}\begin{pmatrix} 12 \\ 5 \end{pmatrix}$$

$$\begin{pmatrix} \quad \end{pmatrix} \quad \begin{pmatrix} \quad \end{pmatrix}$$

Premultiplication by the inverse.

Simplification.

C.A.O

$$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{matrix} a \\ b \end{matrix} = \begin{matrix} 2 \\ 3 \end{matrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$$

$a = 2$ ✓ and $b = 3$ ✓

2. $(x-3) - (2x) = 0$
$x-3-2x = 0$
$-2x + x - 3 = 0$
$-x - 3 = 0$
$x = 3$

3. $$\begin{pmatrix} 1 & 5 \\ 3 & 7 \end{pmatrix} \quad \begin{pmatrix} 7 & 3 \\ -4 & -2 \end{pmatrix} \quad = \quad \begin{pmatrix} -13 & -7 \\ -4 & -2 \end{pmatrix}$$

$$\text{Determinant} = +65 - 49 = 16$$

$$C^{-1} = \begin{pmatrix} 1 & -5 \\ 7 & -13 \end{pmatrix} \begin{pmatrix} 7 \\ \end{pmatrix}$$

4. $$\begin{pmatrix} 3 & 2 \\ 2 & 2 \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{matrix} 9 & -3 \\ 2 & 1 \end{matrix}$$

$3a + 2c = 9$
$\underline{2a + 2c = 2}$
$a = 7$
$c = -6$
$3b + 2d = -3$
$\underline{2b + 2d = 1}$
$b = -4$
$d = 4.5$
$\underset{\sim}{A} = \begin{pmatrix} 7 & -4 \\ -6 & 4.5 \end{pmatrix}$

5. $20x (-3 - {}^{-}8)$
100 area of 1^{st} image.
$100 \times (4 - {}^{-}3)$
700 area of 2^{nd} image

6. Det. $9 + 2 = 11$

$$A^{1} = \frac{1}{11} \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 3 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} \\ \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix} \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11} \begin{pmatrix} 22 \\ 22 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$P(2, 2)$

7 $PQ = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

$$\begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} 2 & 3 \\ 1 & 2 \end{pmatrix} \begin{pmatrix} 2 & -3 \\ -1 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 & 3 \\ 1 & q \end{pmatrix} \begin{pmatrix} 5 \\ -3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$$

$x = 1 \quad y = -2$

8. $\frac{1}{2} x - \frac{1}{4} y = 2$

$\frac{2}{5} + \frac{1}{6} = 6$

$2x \_ y = 8$

$12x + 5y = 180$

$10x - 5y = 40 \quad +$

$22x = 220$

$x = 10$

$\frac{1}{4} y = \frac{1}{2} (10)^{-2}$

$\frac{1}{4} y = 5\text{-}2 = 3$

$Y = 12$

9. $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\begin{pmatrix} -1 & -2 & -6 \\ 1 & 4 & 9 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} X^1 & Y^1 & Z^1 \\ -1 & -2 & -6 \\ 1 & 4 & 9 \end{pmatrix}$$

$$\begin{pmatrix} & & \\ & & \end{pmatrix}$$

$$= \begin{matrix} 1 & 4 & 9 \\ -1 & -2 & -6 \end{matrix}$$

Final image $X^{11}\ Y^{11}Z^{11}$

$X^{11}(1, -1)\ Y^{11}(4, -2), Z^{11}(9, -6)$

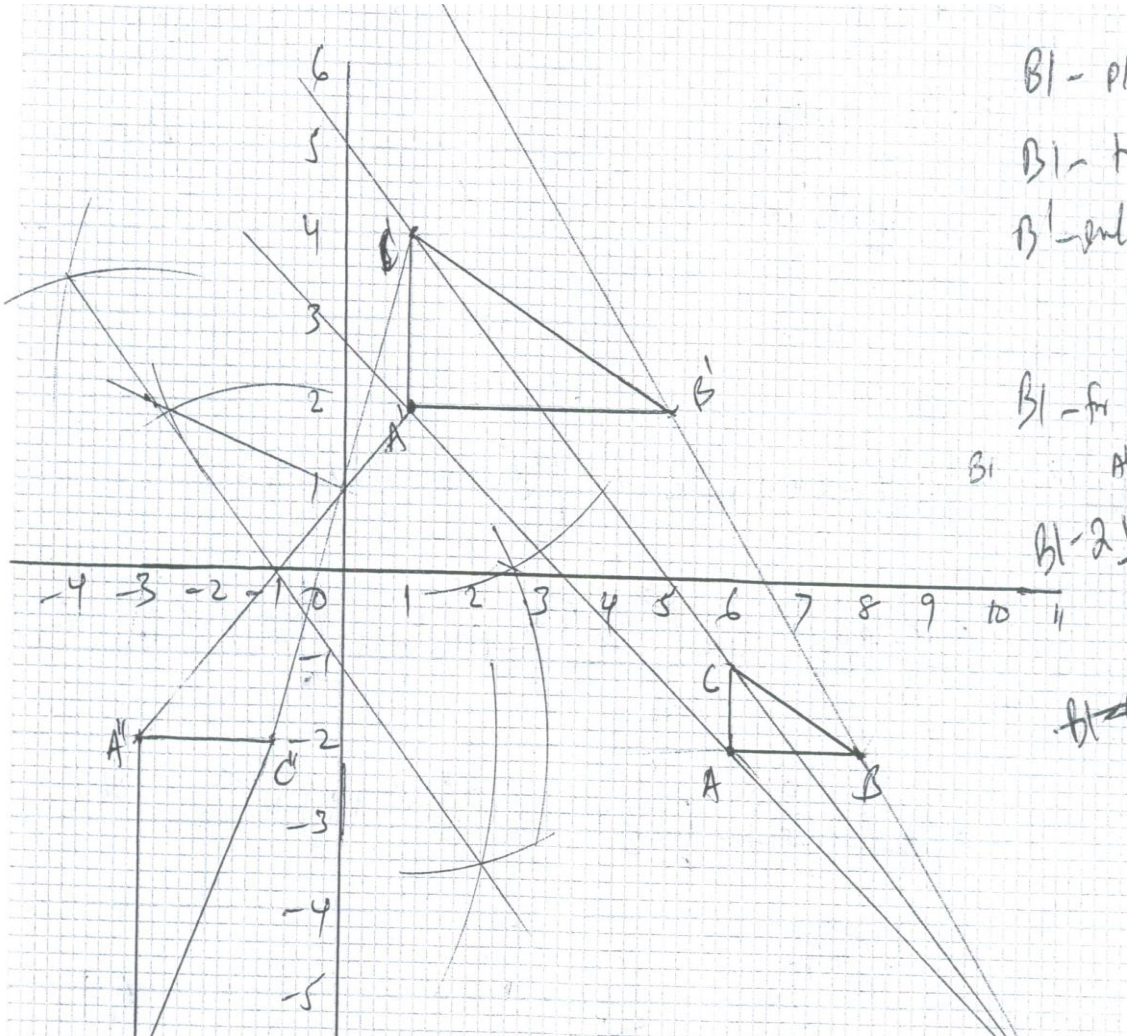
10.

$$a/: \quad \begin{matrix} P & Q \\ \begin{pmatrix} 2 & 2 \\ 0 & 4 \end{pmatrix} \end{matrix} \quad \begin{matrix} R \\ \begin{pmatrix} 5 & 6 & 4 \\ -1 & -1 & -\frac{1}{2} \end{pmatrix} \end{matrix} = \begin{matrix} A & B & C \\ \begin{pmatrix} 6 & 8 & 6 \\ -2 & 2 & -1 \end{pmatrix} \end{matrix}$$

(c) Centre (-3,2)

Angle + 90°

$$a) \quad \begin{pmatrix} 2 & 4 \\ 0 & 2 \end{pmatrix} \quad \begin{matrix} A & B & C \\ \begin{pmatrix} 5 & 6 & 4 \\ -1 & -1 & -\frac{1}{2} \end{pmatrix} \end{matrix} = \begin{matrix} A & B & C \\ \begin{pmatrix} 6 & 8 & 6 \\ 2 & 2 & -1 \end{pmatrix} \end{matrix}$$



11. $Det \quad 2 - -3 = 5$

$Area\ of\ A^IB^IC^I = 5 \times 15$

$= 75\ cm^2$

12. $A.S.F = \frac{110}{10} = 11$

$5X(X) - -6 = 11$

$5X^2 + 6 = 11$

$5x^2 = 5$

$X^2 = 1$

$X = \pm 1$

13. $Area\ of\ the\ image = Area\ of\ the\ object \times Det.$

$Det.\ (\Delta) = 15 - 18 = -3$

$54\ cm^2 = A \times -3$

$\frac{54}{3}\ cm^2 = A$

$Area\ of\ \Delta ABC = 18\ cm^2$

14. $Det. \quad 9 + 2 = 11$

$$A^1 = \frac{1}{11}\begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 3 & 2 \\ 3 & -1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 3 & -2 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 10 \\ 4 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{11}\begin{pmatrix} 22 \\ 22 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$P\ (2, 2)$