NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF EDUCATION

COURSE CODE: PED 144

COURSE TITLE: PRIMARY MATHEMATICS CURRICULUM AND METHODS

COURSE GUIDE

PED 144
PRIMARY MATHEMATICS CURRICULUM AND METHODS

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Published by
National Open University of Nigeria

Printed 2012

ISBN:

Printed by:

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COURSE OUTLINE

PED 144 Primary Mathematics Curriculum and Methods is a one-semester course. It is a two- credit unit course designed for 100 level undergraduate programme. The course consists of:

- the basic concept underlying the determination of objectives
- the selection and organisation of learning expert evaluation process
- the beginning and nature of merits as distinctive discipline
- assessment of mathematicsematics attainment formation and attitudes transferred from the primary schools
- individual differences in pupils' mathematics at teaching of numbers and fundamental properties of operation in arithmetic.

COURSE AIMS

The aim of this course is to introduce you to the basic fundamentals of primary mathematics curriculum. It is intended that mathematics education should contribute to the personal development of the students.

The overall aims of the course include to:

- introduce you to goals and objectives of mathematics education
- help you acquire the mathematics knowledge, skills and understanding necessary for personal fulfillment
- develop your problem-solving skills and creative talents, and introduce you to ideas of modelling
- develop your ability to handle abstractions and generalisations, and to recognise and present logical arguments
- further your powers of communication, both oral and written, and thus your ability to share ideas with other people
- foster your appreciation of the creative and aesthetic aspects of mathematics and your recognition and enjoyment of mathematics in the world around you
- enable you to develop a positive attitude towards mathematics as an interesting and valuable subject of study
- help to provide you with the mathematics knowledge, skills and understanding needed for continuing your education, and eventually for life and work
- promote your confidence and competence in using the mathematics knowledge and skills required for everyday life, work and leisure.

COURSE OBJECTIVES

There are overall objectives set out in order to achieve the aims set out for this course. In addition, each unit of this course has some performance objectives. These are included at the beginning of every unit. You may wish to refer to them as you study the unit in order to help you check your progress. You should also look at the unit. The wider objectives of this course, which if met, should have helped you to achieve the aims of the course as a whole are set out below. On successful completion of this course, you should be able to:

- explain the meaning and enumerate the goals and objectives of mathematics education
- discuss the aims and objectives of teaching mathematics
- discuss the features of the new 9-year basic mathematics curriculum
- discuss the components of effective mathematics instruction
- discuss mathematics instruction for students with learning difficulties
- explain Gagne's hierarchy of concept and meaning and mathematics learning
- explain Piaget theory of intellectual development and mathematics teaching and learning
- state and discourse teaching objectives using Bloom's taxonomy
- discuss and use innovations in teaching of mathematics
- identifying and discourse factors affecting learning
- discuss analysis of the teaching methods
- explain various classroom assessment technique
- state purposes and the tools of assessment.
- state and use basic number properties: associative, commutative, and distributive
- identify and use other number properties: identities, inverses, symmetry.

WORKING THROUGH THE COURSE

To complete this course, you are expected to read the study units, and other relevant books and materials provided by the National Open University of Nigeria.

Each unit contains self-assessment exercises and at certain points in the course, you are required to submit assignments for assessment purpose. At the end of the course, there is a final examination. This Course Guide lists all the components of the course, what you have to do, and how you should allocate your time to each unit in order that you may complete the course successfully and on time.

ASSIGNMENT FILE

There are fifteen (15) assignments in this course, covering all the units studied.This assignment file will be available at your study centre. You are expected to submit completed assignments in them. The marks you obtain for these assignments will count towards the final mark you obtain for this course. Further information on assignments will be found in the assignment file itself and also in the section on assessment.

THE COURSE MATERIALS

1). Course Guide
2). Study Units

The course consists of 15 units. These are made up of the concept of education, the goal of education, philosophy and its functions, relationship between philosophy and education, the philosophers, the curriculum, metaphysics, schools of thought, axiology, logic freedom and epistemology. This material has been developed to suit students in Nigeria.

ASSESSMENTS

There are three aspects of the assessments. The first is self-assessment exercise, the second is the tutor-marked assignments and the third is the final examination. You are advised to be sincere in attempting the exercise. You are expected to apply knowledge, information and skills that you have acquired during the course. The assignment must be submitted to your tutor for formal assessments in accordance with the deadline stated in your schedule of presentation.

TUTOR-MARKED ASSIGNMENT

There are fourteen tutor-marked assignments in this course, and you are advised to attempt all. Aside from your course material provided, you are advised to read and research widely using other references which will give you a broader viewpoint and may provide a deeper understanding of the subject.

Ensure that all completed assignments are submitted on schedule before the deadline. If for any reasons, you cannot complete your work on time, contact your tutor before the assignment is due to discus the possibility

of an extension. Except in exceptional circumstances, extension may not be granted after the due date etc.

FINAL EXAMINATION

The final examination for this course will be of three hours duration and have a value of 60% of the total course grade. All areas of the course will be assessed and the examination will consist of questions which reflect the type of self-testing, practice exercise and tutor-marked assignments you have previously encountered.

Utilise the time between the conclusion of the last study unit and sitting for the examination to revise the entire course. You may find it useful to review your self-assessment exercise, tutor-marked assignments and comments on them before the examination.

COURSE MARKING SCHEME

The work you submit will count for 40 % of your total course mark. At the end of the course however, you will be required to sit for a final examination, which will also count for 60% of your total marks.

HOW TO GET THE MOST FROM THIS COURSE

In distance learning, the study materials are specially developed and designed to replace the lecturer. Hence, you can work through these materials at your pace, and at a time and place that suits you best.
Visualise it as reading the lecture instead of listening to a lecturer.

Each of the study unit follows a common format. The first item is an introduction to the subject matter of the unit and how a particular unit is integrated with the other units and the course as a whole. Next is a set of learning objectives. These objectives let you know what you should be able to do by time you have completed the unit. Use these objectives to guide your study.

On finishing a unit, go back and check whether you have achieved the objectives. If made a habit, this will further enhance your chances of completing the course successfully.

The following is a practical strategy for working through the course:

- Read this Course Guide thoroughly
- Organise a study schedule, which you must adhere to religiously. The major reason students fail is that they get behind in their

course work. If you encounter difficulties with your schedule, please let your tutor know promptly

- Turn to each unit and read the introduction and the objectives for the unit.
- Work through the unit. The content of the unit itself has been arranged to provide a sequence for you to follow.
- Review the objectives of each study unit to confirm that you have achieved them. If you feel unsure about any of the objectives, review the study material or consult with your tutor
- When you are confident that you have achieved a unit's objectives, you can then start on the next unit. Proceed unit by unit through the course and try to pace your study so that you keep yourself on schedule.
- After submitting an assignment to your tutor for grading, do not wait for its return before starting on the net unit. Keep to your schedule. When the assignment is returned, pay particular attention to your tutor's comments.
- After completing the last unit, review the course and prepare yourself for final examination. Check that you have achieved the unit's objectives (listed at the beginning of each unit) and the course objectives listed in this Course Guide.

TUTORS AND TUTORIALS

There will be specific time made available for tutorial sessions in support of this course. You will be notified of the dates, time and location of these tutorials, together with the name and phone number of your tutor as soon as you are allocated a tutorial group. Your tutor will mark and comment on your assignments, keep a close watch on your progress and on any difficulties you might encounter. Do not hesitate to contact your tutor by telephone, e-mail or your discussion group (board) if:

- you do not understand any part of the study unit or the assigned readings
- you have difficulty with self tests or the exercises
- you have a question or problem with an assignment, comments on an assignment or with the grading of an assignment.

You should try your best to attend the tutorials. This is the only chance to have face-to-face contact with your tutor and to ask questions which are answered instantly. You can raise any problem encountered in the course of your study. To gain the maximum benefit from course tutorials, prepare a question list before attending them. You will learn a lot from participating in discussion actively.

SUMMARY

This course is designed to give to you some teaching skills that would help you improve your teaching techniques and thus produce students who will have confidence in learning mathematics. At the end of each unit are lists of books for references and further reading. While you may not procure or read all of them, they are essential complements to the course material. We, therefore, sincerely wish you the best and that you enjoy the course.

MAIN COURSE

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MODULE 1 BASICS ISSUES IN MATHEMATICS EDUCATION

UNIT 1 AIMS AND OBJECTIVES OF TEACHING MATHEMATICS

CONTENTS

1.0 INTRODUCTION

Every year, at the beginning of the semester, I ask my group of student-teachers “why do we teach mathematics? What do we want to achieve in our mathematics lessons?” These were the questions which I asked myself in joining the profession as a mathematics teacher. In fact, they are good questions which all teachers should ask themselves from time to time in their daily practice. Different teachers may have different answers to these questions. Some possible answers to the first question are “mathematics is important and useful in our daily life”; “mathematics is the basis for other subjects such as science and engineering”; “mathematics helps us develop logical thinking” and “mathematics helps us find the right way to solve problems”. Some even say “I like mathematics, so I would like to help my students appreciate the subject”. Each of these answers suggests a reason for the importance of mathematics or school mathematics in the teacher’s mind. Nevertheless, each answer is only a partial answer to the question. To look for a comprehensive answer, we inevitably need to address the question why mathematics is essential in our world.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain mathematics as an important part of understanding our world
- Prove that the subject and its applications in science, commerce and technology are important if students are to understand and appreciate the relationships and patterns of both number and space in their daily life
- express what mathematics is clearly and concisely
- explain that mathematics also help students to develop their capacity of reasoning so that they could think more logically and independently in making rational decisions.

3.0 MAIN CONTENT

3.1 Importance of Mathematics

In the past, teaching objectives in Mathematics were limited to having students memorise facts and obtain skills in manipulating and calculating numbers. Memorising of rules and mechanical manipulation of numbers were considered sufficient. Today, we emphasise skill in compilation as well as skill in mastery of ideas and understanding of operations. The application of knowledge and facts to new situations is the best criterion of effective learning. Applications need clear understanding, close study and concentrated attention. Hence the teacher of mathematics has to develop all these habits and attitudes in the pupils. There should be no insistence upon memorising facts. So the chief value of mathematics study is that it trains you in the use of reasoning power. Hence in teaching, the teacher should emphasise thinking and reasoning, rather than memory work and rote learning. To the students, the solving of a difficult problem is a discovery and constitutes training in such work. The chief aim of teaching mathematics is to develop these faculties that lead to discovery and inventions. The famous pedagogue, Schultze, remarks that "mathematical study trains the students in systematic and orderly habits and the pleasure connected with the successful conquering of a difficulty stimulate will power". It also cultivates the power of attention, for in mathematics, the slightest slack in attention is ruinous. Mathematics makes constant demands upon imagination (Prakash, 2011).

To enable students to cope confidently with the mathematics needed in their future studies, workplaces or daily life in a technological and information-rich society, the curriculum should aim at developing in the students:

- the ability to conceptualise, inquire, reason and communicate mathematically, and to use mathematics to formulate and solve problems in daily life as well as in mathematical contexts
- the ability to manipulate numbers, symbols and other mathematical objects
- the number sense, symbol sense, spatial sense and a sense of measurement as well as the capability in appreciating structures and patterns
- a positive attitude towards mathematics and the capability of appreciating the aesthetic nature and cultural aspect of mathematics

The main goals of teaching mathematics at the primary level (ages 6 to 12 years) are to help students to acquire:

a) the basic skills in numeracy
b) the ability to use these skills to solve problems
c) the ability to estimate and make or calculate approximations and d) the ability to interpret graphs and arrangements of numerical data

More specifically, the curriculum should be outlined so that students should be able to:

a) master the skills in writing numbers, counting and stating place value
b) acquire the basic skills in the four basic operations of adding, subtracting, multiplying and dividing
c) acquire the ability to measure, weigh, state time and specify the face value of currency
d) identify and state the shapes of objects and able to know the properties of square, rectangles, triangles, cuboids, cylinders, spheres, cones and pyramids
e) solve problems involving numbers, measurement, weight, money, distance, space and time
f) estimate and calculate approximations
g) record and read groups of data in the form of simple tables and graphs.

4.0 CONCLUSION

The study of mathematics contributes to the development of the individual and furthering a nation's scientific emancipation.

5.0 SUMMARY

Mathematics develops in the pupils the ability to acquire basic skills in numeracy and use these skills to solve problems. It also helps them to estimate and make or calculate approximations and to interpret graphs and arrangements of numerical data.

6.0 TUTOR-MARKED ASSIGNMENT

State five reasons for the teaching of mathematics in primary schools

7.0 REFERENCES/FURTHER READING

Curriculum Development Committee, Hong Kong. (1999). *Syllabus for Secondary Schools Mathematics* (Secondary 1-5). Hong Kong.

Education Department, Hong Kong. (1993). *Guide to the Secondary 1 to 5 Curriculum*. Hong Kong.

Mok, I.A.C. (2002). *Reflections on the Aims and Objectives of Teaching Mathematics: A Word to Mathematics Teachers at the Beginning of the Semester.*

Prakash, J. (2011). *Aims and Objectives of Teaching Mathematics.* http://www.preservearticles.com/201105216939/aims-and- objectives-of-teaching-mathematics.html

UNIT 2 FEATURES OF THE NEW 9-YEAR BASIC MATHEMATICS CURRICULUM

CONTENTS

1.0 INTRODUCTION

The school curriculum comprises all learning and other experiences that each school plans for its pupils. The National Curriculum is an important element of the school curriculum and a teacher's awareness of this is vital for effective usage of the curriculum.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain mathematics as a problem solving activities in all areas of life
- explain mathematics as a way of communication
- explain mathematics as a way of reasoning in diverse areas of life activities
- explain mathematics as a way of connections of ideas
- describe the format of primary mathematics curriculum.

3.0 MAIN CONTENT

3.1 Introduction to the New National Mathematics Curriculum for Basic Education Programme

Adeniyi (2007), in the revised edition of *National Mathematics Curriculum for Basic Education Programme* beginning from Basic 1 to 9 stated the general objectives for the 9-Year Basic Mathematics Curriculum thus: 'this revised edition of the National Mathematics Curriculum is for the Basic Education Programme beginning from Basic 1 to 9. In this new curriculum, there is no Primary Mathematics Curriculum for Junior Secondary curriculum. The two levels of education (Primaries 1-6 and JS 1-3) have been infused into Basic 1-9. Pupils are expected to continue their education from Basic one to Basic nine without interruption'.

This revised curriculum became necessary because the Universal Basic Education (UBE) Bill 2004 mandated a nine- year compulsory education. Second, the NEEDS and MDG goals necessitated also the need to revise this curriculum where necessary.

The revised National Mathematics Curriculum for Basic Education in Nigeria is focused on giving children the opportunity to:

1) acquire mathematical literacy necessary to function in an information age
2) cultivate the understanding and application of mathematics skills and concepts necessary to thrive in the ever-changing technological world
3) develop the essential element of problem solving, communication, reasoning and connection within their study of mathematics
4) understand the major ideas of mathematics, bearing in mind that the world has changed and is still changing since the first National Mathematics Curriculum was developed in 1977. There is need to incorporate such changes in the areas of Information and Communications Technologies (ICT), Population and Family Life Education, Environmental Degradation, Drug Abuse and HIV/AIDS.

   These gave rise to the need to make the curriculum more responsive to the survival and developmental needs of the Nigerian child. It should also be noted that this revised curriculum placed emphasis on affective domain and quantitative reasoning unlike the previous curriculum. This is to boost pupils' achievement in cognitive and psychomotor capabilities. The thematic approach was also adopted in selecting the content and learning experiences in the curriculum. This is because it is useful in accommodating new contents/programme without necessarily disrupting the entire content or curriculum structure. There are now six themes in this revised curriculum: Number and Numeration, Basic Operations, Measurement, Algebraic Process, Geometry and Mensuration and Everyday Statistics.

This is a teaching curriculum. Thus, it provides maximal aid for the teacher by prescribing topics, objectives or expected learning outcomes stated in measurable terms, pupils and teachers activities and adequate evaluation guide. For this curriculum to be effective in achieving the purpose for which it is meant, the following recommendations are made:

1) it is strongly recommended that copies of these documents be made available to every primary school teacher and much emphasis placed on its use in order to achieve stated objectives
2) there is need to organise workshops for teachers, supervisors and inspectors on how to interpret and use the curriculum
3) The minimum qualification for teachers teaching in Basics 1-9 is National Certificate in Education (NCE). University graduates of first degree and above in various disciplines teaching in Basics 1-9 must have education qualification for the Basic Education Programme to succeed.

Finally, it is our hope that the revised National Mathematics Curriculum will achieve the goals and objectives of the Universal Basic Education Programme in Nigeria as contained in the National Policy on Education (2004) and the UBE bill of 2004.

3.2 Why Do We Teach Mathematics?

What do we want to achieve in our mathematics lessons? This is a good question which all teachers should ask themselves from time to time in their daily practice. Different teachers may have different answers to this question. Some possible answers to the first question are: “mathematics is important and useful in our daily life”; “mathematics is the basis for other subjects such as science and engineering”; “mathematics helps us develop logical thinking” and “mathematics helps us find the right way to solve problems”. Some may even say “I like mathematics, so I would like to help my students appreciate the subject.” Each of these answers suggests a reason for the importance of mathematics or school mathematics in the teacher’s mind. Nevertheless, each answer is only a partial answer to the question. To look for a comprehensive answer, we inevitably need to address the question why mathematics is essential in our world.

The approach taken in planning the mathematics curriculum in Nigeria is that the subject should be a friendly one and thus is planned or structured to meet the needs of students regardless of their abilities. This approach differs from the previous approach where mathematics is approached in a "specialised" manner. The curriculum is organised on six main themes: *number and numeration, basic operations, measurement, algebraic process, geometry and mensuration and everyday statistics*. These six blocks are chosen based on the belief that
in everyday living one is often faced with these elements in the order
listed. In addition, solving mathematical problems encountered in one's daily life becomes the overriding concern in the curriculum.

3.3 Mathematics as Problem Solving

Although the definition of problem solving may differ from that of NCTM's (1992), it, nevertheless, becomes the significant elements to be emphasised in the teaching and learning of mathematics. Teachers are expected to intentionally teach students on the heuristics of problem solving. Although teachers are free to choose the strategy suitable for their students, they are encouraged to follow those recommended by Polya (1974). Teachers are also encouraged to simulate mathematical problems based on their daily experiences. More specifically, teachers are expected to provide varied experiences through which students can work individually or in groups in tackling mathematical problems. The curriculum places heavy emphasis on the relationships between mathematics and real life problems. Problem solving in real contexts are considered essential in helping students appreciate mathematics. In short, problem solving becomes the focus in the curriculum.

3.4 Mathematics as Communication

The curriculum clearly states that one of the objectives in learning mathematics is to acquire the ability to communicate ideas through the use of mathematical symbols or ideas. An essential part of the curriculum is to help students attain the ability to comprehend mathematical statements encountered, for example, in the mass media.
For example, students are expected to be able to interpret the statistics used in various reports they encounter in the mass media. In mathematics lessons, students are encouraged to work in groups on certain projects or problems.

3.5 Mathematics as Reasoning

The main goal statement clearly states that the students need to develop the ability to think logically, systemically, creatively and critically..Although this is not clearly stated in the syllabus, teachers' guides and further elaboration of the syllabus specially encourage teachers to use approaches that can simulate mathematical thinking or reasoning. The use of statistics to critically examine information as part of the lesson, for example, can be said to be in correspondence with the aim of promoting the above thinking abilities.

3.6 Mathematical Connections

There is a strong emphasis in making connections within mathematics itself and across other subjects. In fact, the title of the curriculum suggests that making mathematical connections within itself or across other areas of study is strongly suggested. Making the connections between mathematics studied in class and material from everyday life or the environment are explicitly stated in the documents accompanying the syllabus. Through the introduction of certain facts concerning historical development in mathematics, the curriculum hopes that students should be able to see that mathematics has its origin in many cultures and is developed as responses to human needs that are both utilitarian and aesthetic.

Organisation of the 9-Year Basic Mathematics Curriculum Format

MARY ONE
ME: NUMBER AND NUMERATION

	Performance Objectives	Contents	Activities		Teaching and Learning materials	Evaluation Guide
			Teacher	Pupils		
le numbers	Pupils should be able to: 1. Sort and classify number of objects in a group or collection	i. Sort and classify objects leading to idea of 1-5	1. Brings objects such as: beans, bottle tops, buttons and nylon bags 2. Mixes the collections and asks pupils to sort them according to types	1. Bring various objects such as: beans, bottle tops, buttons and nylon bags to class 2. Sort them according to types	Courters: stones beans, bottle tops, buttons, leaves and nylon bags etc	Pupil to: 1. Sort given number of objects from a collection.
	2. Identify number of objects in a group or collection	ii. Identification of number of objects 1-5	1. Guides pupils to form groups: one for stones, two for bottle tops, three for beans, four for buttons and five for balls	Sort and classify the mixed collection by forming groups for objects e.g. pick a stone, pick two bottle tops etc	Counters: stones beans, bottle tops, buttons, leaves and nylon bags etc	2. Arrange given number of objects from a collection together.
	3. Count correctly up to 5	iii Reading of number 1-5	1. Asks pupils to show one bottle top, 2 bottle tops, up to 5 bottle tops 2. Reads number 1-5	Read the number 1-5		3. Read given numbers on the board
	4. Write correctly number 1-5	iv. writing of numbers 1-5	1. writes numbers 1-5 board 2. Leads pupils to write the numbers in order in their books	Write the number 1-5 in exercise book		4. Write numbers 1-5 on the board/exercise book
	5. Arrange numbers 1-5 in order of their magnitudes (quantities)	v. Ordering of number 1-5	Arranges numbers in order of their magnitude using counters and other objects	Use counters to arrange objects in magnitude or in ordering form		5. Order given numbers in order of their magnitudes form
	6. Appreciate the need for counting and ordering		Leads pupils to appreciate numbering in order of their magnitude	Appreciate the need for counting and ordering in everyday activities		6. State why counting and ordering are important

4.0 CONCLUSION

The new basic mathematics curricular emphasises that mathematics should be taught in connection with its usefulness as an everyday activities.

5.0 SUMMARY

Mathematics could be seen as a problem solving activities in all areas of life; as a way of communication; as a way of reasoning in diverse areas of life activities, and as a way of connections of ideas.

6.0 TUTOR-MARKED ASSIGNMENT

Describe the format of the new primary school curriculum in Nigeria and explain the linkages between each column.

7.0 REFERENCES/FURTHER READING

Bishop, A.J. (1991). 'Mathematical Values in the Teaching Process'. In: A.J. Bishop & S. Mellin-Olsen & J.V. Dormolen (Eds). *Mathematical Knowledge: Its Growth through Teaching*. Kluwer Academic Press.

National Council for Teachers of Mathematics (1992). *Curriculum and Evaluation Standards for School Mathematics***.** Reston, Val.

Zanzali, N.A.A. (2010). *Designing the Mathematics Curriculum in Malaysia: Making Mathematics More Meaningful.*

UNIT 3 COMPONENTS OF EFFECTIVE MATHEMATICS INSTRUCTION

CONTENTS

1.0 INTRODUCTION

Less is known about the components of effective mathematics instruction than the components of effective reading instruction, because research in mathematics is less extensive than in reading. However, conclusions can still be drawn from some very good studies that do exist, as well as from typical grade level expectations in mathematics. As is true for reading, there is no single "best" programme for teaching mathematics. Rather, certain key abilities involved in learning mathematics need to be addressed in instruction, with the importance of different abilities shifting somewhat across the elementary and secondary grades.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain how to develop pupils' ability in concept formations
- teaching through a concrete-to-representational-to-abstract sequence of instruction
- using concrete manipulative to teach mathematics
- state types of manipulative.

3.0 MAIN CONTENT

3.1 Effective Kindergarten through Grade Four Instruction

At these grade levels, general education instruction in mathematics should include development of the following mathematics-related abilities: concepts and reasoning (e.g., basic number concepts, meaning of operations such as addition, geometric concepts); automatic recall of number facts (e.g., memorisation of basic addition facts such as 3 + 4 so that children know answers instantly instead of having to count); computational algorithms (the written procedure or series of steps for solving more complex types of calculation, e.g., for two-digit addition with regrouping, calculation starts in the right-hand column and tens are "carried" from the ones to the tens column); functional mathematics (e.g., practical applications such as time and money); and verbal problem-solving (e.g., solving word problems).

Because progress in mathematics builds heavily upon previously learned skills, it is important for instruction to be clear, unambiguous, and systematic, with key prerequisite skills taught in advance. For instance, children should not be expected to develop automatic recall of addition facts if they do not understand the basic concept of addition or the meaning of the addition sign. It is also essential for children to have sufficient practice to acquire new skills. For example, although manipulative such as cubes or rods can be very helpful in developing basic concepts, many children will not spontaneously progress from accurately solving facts with manipulative to automatic recall of facts. Instead, most children benefit from practice activities focused specifically on helping them to memorise facts. Similarly, learning computational algorithms such as those used in long division or two-digit multiplication often requires considerable practice.

Scientific investigators interested in learning disabilities have identified several patterns that may be found in youngsters with mathematics disabilities. Some of these children have difficulties that revolve primarily around automatic recall of facts, coupled with good conceptual abilities in mathematics. This pattern characterises some children with reading disabilities. Another common pattern involves difficulties with computational algorithms; yet a third pattern involves visual-spatial difficulties, such as difficulty lining up columns or with learning spatial aspects of mathematics, such as geometry. Although effective general education instruction can help to prevent low mathematics achievement in many children, some youngsters with genuine mathematics disabilities will require more intensive, long-term instruction in order to be successful.

3.2 Teaching Primary Five and Beyond

In primary five and beyond, general education instruction in mathematics focuses a great deal on advanced concepts and reasoning (e.g., what a variable or a function is), learning of complex computational algorithms (e.g., those involved in adding and subtracting fractions and decimals), and more difficult kinds of verbal problem-solving (e.g., problems with multiple steps). By grade five, automatic recall of number facts is well-developed in most normally-achieving youngsters. However, youngsters with mathematics disabilities often continue to struggle with mathematics skills far below grade expectations, including not only automatic recall, but also many computational algorithms and mathematics concepts. A thorough evaluation that assesses a range of important mathematics skills is essential, because children can have different strengths and weaknesses even within the domain of mathematics, and knowing the pattern of strengths and weaknesses is central to instructional planning. For instance, a child who has good conceptual abilities but whose difficulties centre on automatic recall and computation will need a different kind of instructional programme than will one whose main difficulties are conceptual in nature.

As children advance into junior and secondary schools, tracking of students into different levels of mathematics (e.g., an accelerated track, a grade-level track, and a remedial track) could be easier. Also, science courses begin to draw more heavily on mathematics skills, and students with mathematics disabilities may begin to experience more difficulties in science. Providing intensive remediation of basic mathematics skills to students who need it remains essential in these classes, not only to help students acquire the skills needed for everyday life, but also because mathematics achievement serves as a gateway for higher education and for many occupations.

3.3 Teaching through a Concrete-to-Representational-to-Abstract Sequence of Instruction

The purpose of teaching through a concrete-to-representational-to-abstract sequence of instruction is to ensure that students truly have a thorough understanding of the mathematics concepts/skills they are learning. When students who have mathematics learning problems are allowed to first develop a concrete understanding of the mathematics concept/skill, then they are much more likely to perform that mathematics skill and truly understand mathematical concepts at the abstract level.

What is it?

- Each mathematics concept/skill is first modeled with concrete materials (e.g. chips, unifix cubes, base ten blocks, beans and bean sticks, pattern blocks).
- Students are provided many opportunities to practice and demonstrate mastery using concrete materials
- The mathematics concept/skill is next modeled at the representational (semi-concrete) level which involves drawing pictures that represent the concrete objects previously used (e.g. tallies, dots, circles, stamps that imprint pictures for counting)
- Students are provided many opportunities to practice and demonstrate mastery by drawing solutions
- The mathematics concept/skill is finally modelled at the abstract level (using only numbers and mathematical symbols)
- Students are provided many opportunities to practice and demonstrate mastery at the abstract level before moving to a new mathematics concept/skill.
- As a teacher moves through a concrete-to-representational-to-abstract sequence of instruction, the abstract numbers and/or symbols should be used in conjunction with the concrete materials and representational drawings (promotes association of abstract symbols with concrete & representational understanding)

What are the critical elements of this strategy?

- Use appropriate concrete objects to teach particular mathematics concept/skill (see Concrete Level of Understanding/ Understanding Manipulatives-Examples by mathematics concept area). Teach concrete understanding first.
- Use appropriate drawing techniques or appropriate picture representations of concrete objects (see Representational Level of Understanding/Examples of drawing solutions by mathematics concept area). Teach representational understanding second.
- Use appropriate strategies for assisting students to move to the abstract level of understanding for a particular mathematics concept/skill (see Abstract Level of Understanding/Potential barriers to abstract understanding for students who have learning problems and how to manage these barriers).
- When teaching at each level of understanding, use explicit teaching methods (see the instruction strategy Explicit Teacher Modeling).

How do I implement the strategy?

1. When initially teaching a mathematics concept/skill, describe and model it using concrete objects (concrete level of understanding).
2. Provide students many practice opportunities using concrete objects.
3. When students demonstrate mastery of skill by using concrete objects, describe and model how to perform the skill by drawing or with pictures that represent concrete objects (representational level of understanding).
4. Provide many practice opportunities where students draw their solutions or use pictures to solve problem
5. When students demonstrate mastery drawing solutions, describe and model how to perform the skill using only numbers and mathematics symbols (abstract level of understanding).
6. Provide many opportunities for students to practice performing the skill using only numbers and symbols.
7. After students have mastered performing the skill at the abstract level of understanding, ensure students maintain their skill level by providing periodic practice opportunities for the mathematics skills.

How does this Instructional Strategy Positively Impact Students who have Learning Problems?

- Helps passive learner to make meaningful connections
- Teaches conceptual understanding by connecting concrete understanding to abstract mathematics process
- By linking learning experiences from concrete-to-representational-to-abstract levels of understanding, the teacher provides a graduated framework for students to make meaningful connections.
- Blends conceptual and procedural understanding in structured way

3.4 Using Concrete Manipulative to Teach Mathematics

What is it?

The concrete level of understanding is the most basic level of mathematical understanding. It is also the most crucial level for developing conceptual understanding of mathematics concepts/skills. Concrete learning occurs when students have ample opportunities to manipulate concrete objects to solve problem. For students who have mathematics learning problems, explicit teacher modelling of the use of specific concrete objects to solve specific mathematics problems is needed.

Understanding manipulatives (concrete objects)

To use mathematics manipulatives effectively, it is important that you understand several basic characteristics of different types of mathematics manipulatives and how these specific characteristics impact students who have learning problems. As you read about the different types of manipulatives, refect on pictures of different manipulatives.

General types of mathematics manipulatives

Discrete - those materials that can be counted (e.g. cookies, children, counting blocks, toy cars, etc.).

Continuous - materials that are not used for counting but are used for measurement (e.g. ruler, measuring cup, weight scale, trundle wheel).

Suggestions for using discrete and continuous materials with students who have learning problems

Students who have learning problems needto have abundant experiences using discrete materials before they will benefit from the use of continuous materials. This is because discrete materials have defining characteristics that students can easily discriminate through sight andtouch. As students master an understanding of specific readiness concepts for specific measurement concepts/skills through the use of discrete materials (e.g. counting skills), then continuous materials can be used.

Types of manipulatives used to teach the Base-10 System/place-value (Smith, 1997):

Proportional - shows relationships by size (e.g. ten counting blocks grouped together is ten times the size of one counting block; a bean stick with ten beans glued to a popsicle stick is ten times bigger than one bean).

Non-linked proportional - single units are independent of each other, but can be "bundled together (e.g. Popsicle sticks can be "bundled together in groups of 'tens' with rubber bands; individual unifix cubes can be attached in rows of ten unifix cubes each).

Linked proportional - comes in single units as well as "already bundled" tens units, hundreds units, and thousands units (e.g. base ten cubes/blocks; beans and bean sticks).

Examples of manipulative (concrete objects)

Suggested manipulatives are listed according to mathematics concept/skill area. Descriptionsof manipulative are provided as appropriate. A brief description of how each set of manipulative may be used to teach the mathematics concept/skill is provided at the bottom of the list for each mathematics concept area. This is not meant to be an exhaustive list, but this list does include a variety of common manipulative. The list includes examples of "teacher-made" manipulative as well as "commercially-made" ones. These are discussed under the following headings:

Counting/Basic Addition & Subtraction
Place Value
Multiplication/Division Positive
and Negative Integers Fractions
Geometry
Beginning Algebra

Counting/Basic Addition & Subtraction Pictures

- Colored chips
- Beans
- Unifix cubes
- Golf tees
- Skittles or other candy pieces
- Packaging popcorn
- Popsicle sticks/tongue depressors

Description of use: students can use these concrete materials to count, to add, and to subtract. Students can count by pointing to objects and counting aloud. Students can add by counting objects, putting them in one group and then counting the total. Students can subtract by removing objects from a group and then counting how many are left.

Place Value Pictures

- Base 10 cubes/blocks
- Beans and bean sticks
- Popsicle sticks and rubber bands for bundling
- Unifix cubes (individual cubes can be combined to represent "tens")

- Place value mat (a piece of tag board or other surface that has columns representing the "ones," "tens," and "hundreds" place values)

Description of use: students are first taught to represent 1-9 objects in the "ones" column. They are then taught to represent "10" by trading in ten single counting objects for one object that contains the ten counting objects on it (e.g. ten separate beans are traded in for one "beanstick" - a popsicle stick with ten beans glued on one side. Students then begin representing different values 1-99. At this point, students repeat the same trading process for "hundreds."

Multiplication/Division Pictures

- Containers and counting objects (paper dessert plates and beans, paper or plastic cups and candy pieces, playing cards and chips, cutout tag board circles and golf tees, etc.). Containers represent the "groups" and counting objects represent the number of objects in each group. (e.g. 2 x 4 = 8: two containers with four counting objects on each container) Counting objects arranged in arrays (arranged in rows and columns). Color-code the "outside" vertical column and horizontal row helps emphasise the multipliers

Positive and Negative Integers Picture

- Counting objects, one set light colored and one set dark colored (e.g. light and dark colored beans; yellow and blue counting chips; circles cut out of tag board with one side colored, etc.).

Description of use: light colored objects represent positive integers and dark colored objects represent negative integers. When adding positive and negative integers, the student matches pairs of dark and light colored objects. The color and number of objects remaining represent the solution.

Fractions Pictures

- Fraction pieces (circles, half-circles, quarter-circles, etc.)
- Fraction strips (strips of tag board one foot in length and one inch wide, divided into wholes, ½'s, 1/3's, ¼'s, etc.
- Fraction blocks or stacks. Blocks/cubes that represent fractional parts by proportion (e.g. a "1/2" block is twice the height as a "1/4" block).

Description of use: the teacher models how to compare fractional parts using one type of manipulative. Students then compare fractional parts.

As students gain understanding of fractional parts and their relationships with a variety of manipulatives, the teacher models and then students begin to add, subtract, multiply, and divide using fraction pieces.

Geometry Pictures

- Geoboards (square platforms that have raised notches or rods that are formed in an array). Rubber bands or string can be used to form various shapes around the raised notches or rods.

Description of Use: concepts such as area and perimeter can be demonstrated by counting the number of notch or rod "units" inside the shape or around the perimeter of the shape.

Beginning Algebra Pictures

- Containers (representing the variable of "unknown") and counting objects (representing integers) -e.g. paper dessert plates and beans, small clear plastic beverage cups 7 counting chips, playing cards and candy pieces, etc.

Description of use: The algebraic expression, "4x = 8," can be represented with four plates ("4x"). Eight beans can be distributed evenly among the four plates. The number of beans on one plate represent the solution ("x" = 2).

Suggestions for using manipulative (Burns: 1996):

- talk with your students about how manipulatives help to learn mathematics
- set ground rules for using manipulative.
- develop a system for storing manipulative.
- allow time for your students to explore manipulative before beginning instruction.
- encourage students to learn names of the manipulative they use
- provide students time to describe the manipulative they use orally or in writing. Model this as appropriate
- introduce manipulative to parents

Representational

What is it?

Examples of drawing solutions by mathematics concept level.

What is it?

At the representational level of understanding, students learn to problem-solve by drawing pictures. The pictures students draw represent the concrete objects students manipulated when problem-solving at the concrete level. It is appropriate for students to begin drawing solutions to problems as soon as they demonstrate they have mastered a particular mathematics concept/skill at the concrete level. While not all students need to draw solutions to problems before moving from a concrete level of understanding to an abstract level of understanding, students who have learning problems in particular typically need practice solving problems through drawing. When they learn to draw solutions, students are provided an intermediate step where they begin transferring their concrete understanding toward an abstract level of understanding. When students learn to draw solutions, they gain the ability to solve problems independently. Through multiple independent problem-solving practice opportunities, students gain confidence as they experience success. Multiple practice opportunities also assist students to begin to "internalise" the particular problem-solving process. Additionally, students' concrete understanding of the concept/skill is reinforced because of the similarity of their drawings to the manipulatives they used previously at the concrete level.

Drawing is not a "crutch" for students that they will use forever. It simply provides students an effective way to practice problem solving independently until they develop fluency at the abstract level.

Examples of drawing solutions by mathematics concept level

The following drawing examples are categorised by the type of drawings ("Lines, Tallies, and Circles," or "Circles/Boxes"). In each category, there are a variety of examples demonstrating how to use these drawings to solve different types of computation problems.

What is it?

Potential barriers to abstract understanding for students who have learning problems and how to manage these barriers

What is it?

A student who problem-solves at the abstract level, does so without the use of concrete objects or without drawing pictures. Understanding mathematics concepts and performing mathematics skills at the abstract level requires students to do this with numbers and mathematics symbols only. Abstract understanding is often referred to as "doing mathematics in your head." Completing mathematics problems where mathematics problems are written and students solve these problems using paper and pencil is a common example of abstract level problem solving. Potential barriers to abstract understanding for students who have learning problems and how to manage these barriers are:

1. students who are not successful solving problems at the abstract level may not understand the concept behind the skill.

Suggestions:

- re-teach the concept/skill at the concrete level using appropriate concrete objects (see Concrete Level of Understanding).
- re-teach concept/skill at representational level and provide opportunities for student to practice concept/skill by drawing solutions (see Representational Level of Understanding).
- provide opportunities for students to use language to explain their solutions and how they got them (see instructional strategy Structured Language Experiences).

2. Have difficulty with basic facts/memory problems.

Suggestions:

- regularly provide students with a variety of practice activities focusing on basic facts. Facilitate independent practice by encouraging students to draw solutions when needed (see the student practice strategies Instructional Games, Self-correcting Materials, Structured Cooperative Learning Groups, and Structured Peer Tutoring).
- conduct regular one-minute timings and chart student performance. Set goals with students and frequently review chart with students to emphasise progress. Focus on particular fact families that are mostproblematic first, and then slowly incorporate a variety of facts as the students demonstrate competence (see Evaluation Strategy Continuous Monitoring and Charting of Student Performance).
- teach students regular patterns that occur throughout addition, subtraction, multiplication, and division facts (e.g. "doubles" in multiplication, 9's rule - add 10 and subtract one, etc.)
- provide student a calculator or table when they are solving multiple-step problems.

3. Repeat procedural mistakes

Suggestions:

- provide fewer no's of problems per page.
- provide fewer numbers of problems when assigning paper and pencil practice/homework.
- provide ample space for students writing, cueing, and drawing. Provide problems that are already written on learning sheets rather than requiring students to copy problems from board or textbook.
- provide structure: turn lined paper sideways to create straight columns. Allow students to use dry-erase boards/lap chalkboards that allow mistakes to be wiped away cleanly. Color cue symbols; for multi-step problems, draw color-cued lines that signal students where to write and what operation to use; provide boxes that represent where numerals should be placed; provide visual directional cues in a sample problem; provide a sample problem, completed step by step at top of learning sheet
- provide strategy cue cards that students can use to recall the correct procedure for solving problem
- provide a variety of practice activities that require modes of expression other than only writing.

4.0 CONCLUSION

Students learning and mastery greatly depend on the number of opportunities a student has to respond. The more opportunities for successful practice that you provide (i.e. practice that does not negatively impact students learning characteristics), the more likely it is that your students will develop mastery of that skill especially when manipulatives are employed in teaching.

5.0 SUMMARY

Teaching through a concrete-to-representational-to-abstract sequence of instruction involves the use of manipulatives both concrete and representational. Concrete objects should be used when teaching the following topics in primary schools**:** Counting/Basic Addition and Subtraction; Place Value; Multiplication/Division; Positive and Negative Integers; Fractions; Geometry and Beginning Algebra.

6.0 TUTOR-MARKED ASSIGNMENT

i. In each case give five concrete objects that can be used to teach:
   a. Counting/Basic Addition and Subtraction
   b. Place Value/ Multiplication/Division c. Positive and Negative Integers
   d. Fractions e. Geometry
   f. Beginning Algebra.

ii. Briefly describe how you will use geoboard to teach area of a rectangle.

iii. List different types of manipulatives with at least two examples each.

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UNIT 4 MATHEMATICS INSTRUCTION FOR STUDENTS WITH LEARNING DIFFICULTIES

CONTENTS

1.0 INTRODUCTION

It is important for teachers to understand the characteristics of students with learning difficulties and be able to adopt instruction to their peculiar needs.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state four approaches for teaching students with learning difficulties in mathematics
- distinguish between the learning instructions for teaching students with learning difficulties.

3.0 MAIN CONTENT

3.1 Effective Mathematics Instruction for Students with Learning Difficulties in Mathematics — Four Approaches that Improve Results

Students have a variety of disabilities—most notably, learning difficulties. But other disabilities as well may occur such as mild mental retardation, AD/HD, behavioral disorders, and cognitive disabilities. Meta-analyses have found strong evidence of instructional approaches that appear to help students with disabilities improve their mathematics achievement. According to these studies, four methods of instruction show the most promise. These are:

- systematic and explicit instruction
- self-instruction
- peer -tutoring
- visual representation

Of course, to make use of this information, an educator would need to know much more about each approach. So let us take a closer look at them.

3.2 Explicit and Systematic Instruction

Explicit instruction, often called direct instruction, refers to an instructional practice that carefully constructs interactions between students and their teacher. Teachers clearly state a teaching objective and follow a defined instructional sequence. They assess how much students already know on the subject and tailor subsequent instruction,
based upon that initial evaluation of student skills. Students move through the curriculum, both individually and in groups, repeatedly practicing skills at a pace determined by the teacher's understanding of student needs and progress (Swanson, 2001). Explicit instruction has been found to be especially successful when a child has problems with a specific or isolated skill (Kroesbergen & Van Luit, 2003).

The Center for Applied Special Technology (CAST) offers a helpful snapshot of an explicit instructional episode (Hall, 2002), shown in Figure 4.1 below. Consistent communication between teachers and students creates the foundation for the instructional process. Instructional episodes involve pacing a lesson appropriately, allowing
adequate *processing* and *feedback* time, encouraging *frequent student responses*, and *listening* and *monitoring* throughout a lesson.

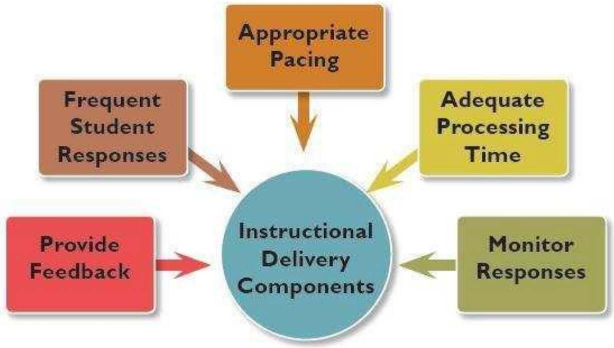

Figure 1. Standard instructional delivery components essential to all explicit instructional episodes (Hall, 2002).

Systematic instruction focuses on teaching students *how to learn* by giving them the tools and techniques that efficient learners use to understand and learn new material or skills. Systematic instruction, sometimes called "strategy instruction" refers to the strategies students learn that help them integrate new information with what is already known in a way that makes sense and be able to recall the information or skill later, even in a different situation or place. Typically, teachers model strategy use for students, including thinking aloud through the problem-solving process, so students can see when and how to use a particular strategy and what they can gain by doing so. Systematic instruction is particularly helpful in strengthening essential skills such as organisation and attention, and often includes:

- memory devices to help students remember the strategy (e.g., a first-letter mnemonic created by forming a word from the beginning letters of other words)
- strategy steps stated in everyday language and beginning with action verbs (e.g., read the problem carefully)
- strategy steps stated in the order in which they are to be used (e.g., students are cued to read the word problem carefully before trying to solve the problem)
- strategy steps that prompt students to use cognitive abilities (e.g., the critical steps needed in solving a problem) (Lenz, Ellis, & Scanlon, 1996, as cited in Maccini & Gagnon, n.d.).

All students can benefit from a systematic approach to instruction, not just those with disabilities. That is why many of the textbooks being published today include overt systematic approaches to instruction in their explanations and learning activities. That is also why NICHCY's first *Evidence for Education* was devoted to the power of strategy instruction. The research into systematic and explicit instruction is clear—the approaches taken together positively impact students' learning (Swanson, in press). The National Mathematics Advisory Panel Report (2008) found that explicit instruction was primarily effective for computation (i.e., basic mathematics operations), but not as effective for higher order problem solving. That being understood, meta-analyses and research reviews by Swanson (1999; 2001) and Swanson and Hoskyn (1998) assert that breaking down instruction into steps, working in small groups, questioning students directly, and promoting ongoing practice and feedback seem to have greater impact when combined with systematic "strategies." What does a combined systematic and explicit instructional approach look like in practice? Tammy Cihylik, a learning support teacher at Harry S. Truman Elementary School in Allentown, Pennsylvania, describes a first-grade lesson that uses money to explore mathematical concepts:

> [Students] use manipulatives, she explains, "looking at the penny, identifying the penny." Cihylik prompts the students with explicit questions: "what does the penny look like? How much is it worth?" Then she provides the answers herself, with statements like, "the penny is brown, and is worth one cent." Cihylik encourages students to repeat the descriptive phrases after her, and then leads them in applying that basic understanding in a systematic fashion. After counting out five pennies and demonstrating their worth of five cents, she instructs the students to count out six pennies and report their worth. She repeats this activity each day, and incorporates other coins and questions as students master the idea of value.

Within this example, the relationship between explicit and systematic instruction becomes clear. The teacher is leading the instructional process through continually checking in, demonstration, and scaffolding/extending ideas as students build understanding. She uses specific strategies involving prompts that remind students the value of the coins, simply stated action verbs, and metacognitive cues that ask students to monitor their money. Montague (2007) suggests that "the instructional method underlying cognitive strategy instruction is explicit instruction."

3.3 Self-Instruction

Self-instruction refers to a variety of self-regulation strategies that students can use to manage themselves as learners and direct their own behavior, including their attention (Graham, Harris, & Reid, 1992). Learning is essentially broken down into elements that contribute to success:

- setting goals
- keeping on task
- checking your work as you go
- remembering to use a specific strategy
- monitoring your own progress
- being alert to confusion or distraction and taking corrective action
- checking your answer to make sure it makes sense and that the mathematics calculations were correctly done.

When students discuss the nature of learning in this way, they develop both a detailed picture of themselves as learners (known as metacognitive awareness) and the self-regulation skills that good learners use to manage and take charge of the learning process. Some examples of self-instruction statements are shown on the next page.

To teach students to "talk to themselves" while learning new information, solving a mathematics problem, or completing a task, teachers should first model self-instruction aloud. They take a task and think aloud while working through it, crafting a monologue that overtly includes the mental behaviors associated with effective learning: goal-setting, self-monitoring, self-questioning, and self-checking. Montague (2004) suggests that both correct and incorrect problem-solving behaviors be modelled.

Modelling of correct behaviors helps students understand how good problem solvers use the processes and strategies appropriately. Modelling of incorrect behaviors allows students to learn how to use self-regulation strategies to monitor their performance and locate and correct errors. Self-regulation strategies are learned and practiced in the actual context of problem solving. When students learn the modelling routine, they then can exchange places with the teacher and become models for their peers.

The self-statements that students use to talk themselves through the problem-solving process are actually prompting students to use a range of strategies and to recognise that certain strategies need to be deployed at certain times (e.g., self-evaluation when you are done, to check your work). Because learning is a very personal experience it is important that teachers and students work together to generate self-statements that are not only appropriate to the mathematics tasks at hand but also to individual students. Instruction also needs to include frequent opportunities to practice their use, with feedback (Graham et al., 1992) until students have internalised the process.

3.4 Peer-Tutoring

Peer- tutoring is a term that is been used to describe a wide array of tutoring arrangements, but most of the research on its success refers to students working in pairs to help one another learn or practice an academic task. Peer -tutoring works best when students of different ability levels work together (Kunsch, Jitendra, & Sood, 2007). During a peer -tutoring assignment, it is common for the teacher to have students switch roles partway through, so the tutor becomes the tutee. Since explaining a concept to another person helps extend one's own learning, this practice gives both students the opportunity to better understand the material being studied.

Research has also shown that a variety of peer-tutoring programs are effective in teaching mathematics, including Classwide Peer -Tutoring (CWPT), Peer-Assisted Learning Strategies (PALS), and Reciprocal Peer -Tutoring (RPT) (Barley *et al.*, 2002). Successful peer-tutoring approaches may involve the use of different materials, reward systems, and reinforcement procedures, but at their core they share the following characteristics (Barley *et al.*, 2002):

- the teacher trains the students to act both as tutors and tutees, so they are prepared to tutor, and receive tutoring from their peers. Before engaging in a peer-tutoring program, students need to understand how the peer- tutoring process works and what is expected of them in each role.
- peer-tutoring programs benefit from using highly structured activities. Structured activities may include teacher-prepared materials and lessons (as in Classwide Peer- Tutoring) or structured teaching routines that students follow when it is their turn to be the teacher (as in Reciprocal Peer -Tutoring).
- materials used for the lesson (e.g., flashcards, worksheets, manipulatives, and assessment materials) should be provided to the students. Students engaging in peer tutoring require the same materials to teach each other as a teacher would use for the lesson.
- continual monitoring and feedback from the teacher help students engaged in peer tutoring stay focused on the lesson and improve their tutoring and learning skills.

Finally, there is mounting research evidence to suggest that, while low-achieving students may receive moderate benefits from peer tutoring, effects for students specifically identified with learning difficulties may be less noticeable unless care is taken to pair these students with a more proficient peer who can model and guide learning objectives (Kunsch, Jitendra, & Sood, 2007).

3.5 Visual Representations

Mathematics instruction is a complex process that attempts to make abstract concepts tangible, difficult ideas understandable and multifaceted problems solvable. Visual representations bring research-based options, tools, and alternatives to bear in meeting the instructional challenge of mathematics education (Gersten *et al.*, 2008).

Visual representations, broadly defined, can include manipulatives, pictures, number lines, and graphs of functions and relationships. "representation approaches to solving mathematical problems include pictorial (e.g., diagramming); concrete (e.g., manipulatives); verbal (linguistic training); and mapping instruction (schema-based)" (Xin & Jitendra, 1999, p. 211). Research has explored the ways in which visual representations can be used in solving story problems (Walker & Poteet, 1989); learning basic mathematics skills such as addition, subtraction, multiplication, and division (Manalo, Bunnell, & Stillman, 2000); and mastering fractions (Butler, Miller, Crehan, Babbitt, & Pierce, 2003) and algebra (Witzel, Mercer, & Miller, 2003).

Concrete-Representational-Abstract (CRA) techniques are probably the most common example of mathematics instruction incorporating visual representations. The CRA technique actually refers to a simple concept that has proven to be a very effective method of teaching mathematics to students with disabilities (Butler *et al.*, 2003; Morin & Miller, 1998). CRA is a three-part instructional strategy in which the teacher first uses *concrete* materials (such as colored chips, base-ten blocks, geometric figures, pattern blocks, or unifix cubes) to model the mathematical concept to be learned, then demonstrates the concept in *representational* terms (such as drawing pictures), and finally in *abstract* or *symbolic* terms (such as numbers, notation, or mathematical symbols).

During a fraction lesson using CRA techniques, for example, the teacher might first show the students plastic pie pieces, and explain that, when the circle is split into 4 pieces, each of those pieces is ¼ of the whole, and when a circle is split into 8 pieces, each piece is ⅛ of the whole. After seeing the teacher demonstrates fraction concepts using concrete manipulatives, students would then be given plastic circles split into equal pieces and asked what portion of the whole one section of that circle would be. By holding the objects in their hands and working with them concretely, students are actually building a *mental* image of the reality being explored physically.

After introducing the concept of fractions with concrete manipulatives, the teacher would model the concept in *representational* terms, either by drawing pictures or by giving students a worksheet of unfilled-in circles split into different fractions and asking students to shade in segments to show the fraction of the circle the teacher names.

In the final stage of the CRA technique, the teacher demonstrates how fractions are written using abstract terms such as numbers and symbols (e.g., ¼ or ½). The teacher would explain what the numerator and denominator are and allow students to practice writing different fractions on their own.

As the Access Center (2004) points out, CRA works well with individual students, in small groups, and with an entire class. It is also appropriate at both the elementary and secondary levels. The National

Council of Teachers of Mathematics (NCTM) recommends that, when using CRA, teachers should make sure that students understand what has been taught at each step before moving instruction to the next stage (Berkas & Pattison, 2007). In some cases, students may need to continue using manipulatives in the representational and abstract stages as a way of demonstrating understanding.

4.0 CONCLUSION

We have briefly examined four approaches to teaching mathematics to students with disabilities which research has shown to be effective. Each is worthy of study in its own right and the sources of additional information provided will help teachers, administrators, and families bring these research-based practices into the mathematics classroom.

When it is time to determine how you can best teach mathematics to your students, select an instructional intervention that supports the educational goals of those students based on age, needs, and abilities. Research findings can and do help identify effective and promising practices, but it is essential to consider how well-matched any research actually is to your local situation and whether or not a specific practice will be useful or appropriate for a particular classroom or child. Interventions are likely to be most effective when they are applied to similar content, in similar settings, and with the age groups intended for them. That is why it is important to look closely at the components of any research study to determine whether the overall findings provide appropriate guidance for your specific students, subjects, and grades—apples to apples, so to speak.

5.0 SUMMARY

Systematic and explicit instruction is a detailed instructional approach in which teachers guide students through a defined instructional sequence. Within systematic and explicit instruction, students learn to regularly apply strategies that effective learners use as a fundamental part of mastering concepts.

Self-instruction is a way by which students learn to manage their own learning with specific prompting or solution-oriented questions.

Peer tutoring, is an approach that involves pairing students together to learn or practice an academic task.

Visual representation uses manipulatives, pictures, number lines, and graphs of functions and relationships to teach mathematical concepts.

6.0 TUTOR-MARKED ASSIGNMENT

Distinguish between the followings types of instructions;: systematic and explicit instruction; self-instruction; peer tutoring and visual representation instructions.

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MODULE 2: Cognitive Development and Mathematics Learning

UNIT 1: Gagne's Hierarchy of Concept and Meaning and Mathematics Learning

UNIT 2: Piaget Theory of Intellectual Development and Mathematics

UNIT 3: Writing Objectives Using Bloom's Taxonomy

UNIT 4: Innovations in teaching of mathematics

UNIT 1: Gagne's Hierarchy of Concept and Meaning and Mathematics Learning

CONTENTS

1.0 Introduction

Cognitive development is a field of study in neuroscience and psychology focusing on a child's development in terms of information processing, conceptual resources, perceptual skill, language learning, and other aspects of brain development and cognitive psychology compared to an adult's point of view. A large portion of research has gone into understanding how a child imagines the world. A major controversy in cognitive development has been "nature versus nurture" or nativism versus empiricism. However, it is now recognized by most experts that this is a false dichotomy: there is overwhelming evidence from biological and behavioral sciences that from the earliest points in development, gene activity interacts with events and experiences in the environment. Another issue is how culture and social experience relate to developmental changes in thinking. Another question is phylogeny convergence or homology with non-human animals. Most aspects of learning and cognition are similar in humans and non-human animals. These issues propagate to nearly every aspect of cognitive development.

2.0 Objectives

At the end of the unit you should be able to:

- Identify Gagne's Hierarchy of Concept and Meaning
- Explain How the Present-day Mathematics Teaching Violates Gagne's Principle
- Discuss the implication of Gagne's Hierarchy to the teaching and learning of mathematics

3.0 Main content

3.1 Gagne's Hierarchy of Concept and Meaning

Robert Gagne in his book 'On the conditions of Learning', has given a taxonomy of learning types (Gagne, 1970Chap.4). that he has arranged hierarchically.

1. *Signal learning.* This is a type of associative learning that has been initially studied by Pavlov who has called it conditioned reflex. A subject that responds in a certain way (R) to a stimulus S1 is given two stimuli (S1 and S2) simultaneously. After sufficient number of repetitions he learns to give the response (R) to S2 even in the absence of S1. Much of the learning that we do without giving conscious thought is of this type. Much of the initial learning of early childhood is signal learning.

2. *Stimulus-response learning.* This is another type of associative learning that has been called trial and error learning by Thorndike. Skinner has used the term operant learning for it. It involves some goal or objective that the subject attempts to achieve. The process is essentially a successive approximation process. The initial efforts are almost random. The subject modifies his approach in every attempt. Each successful attempt is remembered while failed attempts are forgotten. The

success rate improves with more attempts. A good example is a child learning to walk. Initially he falls down often. But with more attempts he is able to master the skill.

3. *Chaining.* Chaining is the process of establishing a sequential connection of a set of stimulus-response pairs for the purpose of attaining a particular goal. For example, the opening of a lock involves a number of simpler steps connected in a sequence (locate the key-hole - insert the key - turn the key clockwise - watch for lever unlocking - take off the lock). Successful chaining requires prior learning of each component response. Algorithms are generally such chaining sequences.

4. *Verbal Association.* Human beings have the ability to encode and express knowledge through sound patterns.

Verbal association here refers to the most elementary kind of verbal behaviour - learning of verbal associations (object « name) and verbal sequences (chains of verbal associations).

5. *Multiple Discrimination Learning.* discrimination is the ability to distinguish between two or more stimulus objects or events. There are two different kinds of capabilities

involved. The first is where the learner is able to make different responses to different members of a collection of stimulus events and objects. The second type involves the capability of the learner to respond in a single way to a collection of stimuli belonging to a single set.

(This involves recognition of the defining rule for the set and responding accordingly.)

6. *Concept learning.* Concept learning involves discrimination and classification of objects**.** We will distinguish between two types of concept learning: concrete and abstract.

Concrete concepts are those that are formed through direct observation. For example, consider the edge of a table, the edge of a razor blade and the edge of a cliff. It is possible to formulate a rule that defines an edge. But the concept of edge is formed more easily through direct observation of several examples. A learner can respond to a set of stimulus objects in two ways – one by distinguishing among them and the other by putting them into a class and responding to any instance of that class in the same way. Both these types are examples of concept learning. The significance of concept learning is that it frees the learner from the control by specific stimuli.

7. *Principle (or rule) learning.* Some concepts are not concrete. They are based on rules that involve other concepts. So they have to be learnt through definition. Definitions are statements that express rules for classifying, i.e. rules that are applicable to any instance of a particular class. Definitions are used for objects as well as for relations. A salient feature of principle learning is that the learner cannot acquire the concept through memorizing its statements verbatim unless he knows the referential meanings of the component concepts. For example, $ax^2 + bx + c = 0$ is meaningless unless you understand what a, b, c, and x represented.

8. *Problem solving.* Problem solving, here, refers to something more than classroom mathematical drills. Also referred to as heuristics (Polya, 1957), the process of problem solving is one in which the learner discovers a combination of previously learnt rules that can be applied to achieve a solution for a novel situation. The following sequence of events is typically involved in problem solving.

(1) presentation of the problem, (2) definition of the problem, (3) formulation of hypothesis, (4) verification of hypothesis. The learning outcome of problem solving is a higher order rule that becomes a part of the student's repertory.

According to Gagne, cognition and concept formation is a multi-layered phenomenon, each layer consisting of a particular learning type. Signal learning, Stimulus-response learning, Chaining, Verbal Association and Multiple Discrimination Learning are all pre-requisites for the formation of concepts and the ability to solve problems. The process of concept formation involves all these eight processes. A very important point here is that if the learning has not been sufficiently accomplished at any level, then there is perceptible deterioration at all higher levels (Gagne and Wigand, 1970).

3.2 Rote Learning in the Context of Gagne's Hierarchy

Let us examine the different hierarchy levels of Gagne and see where the traditional methods of teaching fit. Signal learning, Stimulus-response learning, Chaining, Verbal Association and Multiple Discrimination Learning constitute the basic forms of learning. They are the basic building tools that enable the mind to acquire a working set-up for concept formation. It is this area where rote learning is most effective and insufficient learning at this level impairs the student's abilities for higher learning.

Signal learning refers to learning through unconditional association. When small children memorize alphabets and digit symbols, they are unconditionally associating the symbol sounds with their form. Since the child does not as yet possess any related pre-formed associations, this is the only learning alternative available at this stage. Rote learning is the most effective learning tool at this stage because it directly does what is required. Stimulus-response learning or Operand Conditioning is a process based on successive approximation. Once the basic nodal associations have been formed in the mind, a successive approximation process or shaping takes place on the basis of positive and negative reinforcements. In the traditional elementary education, this step is accomplished through a lot of oral exercises.

The next step, Chaining, is the process of combining a set of individual S-R's in sequence. In fact, the concept of *Sutra* developed in ancient India (Namita, 1996), is a formalisation of this step. *Sutra* has been identified with algorithm by Vernekar. The term 'algorithm' refers to a step by- step method for solving any problem (Rajaraman, 1980, p.3). According to Vernekar (1994), the basic idea of the algorithmic method is that the various steps in an operation

are arranged like beads in a thread (sutra). Thus sutra as well as algorithms refers to the same process as chaining or forming mental links.

The next step is verbal association. Most of the beginners' verbal associations are definitions and fact-snippets to be memorized. Here again, the rote methods are applicable. Although memorizing the vocabulary is a very boring job, once a student acquires good vocabulary through whatsoever means, its role in understanding verbal and written material cannot be denied.

3.3 How the Present-day Mathematics Teaching Violates Gagne's Principle

Present day curricula stress the role and necessity of concept formation in education (National Curriculum Framework 2000, 2005). This cognitive approach appears to be quite reasonable. A cognitive approach can be very useful in this context (Redish, 1994). At present, the heuristic constructivist approach is being implemented in the modern schools for teaching mathematics as well as other science subjects.

A majority of students who go to higher classes are found to be extremely poor in concepts (Agnihotri et al., 1994). Arons (1997) has pointed out several deep conceptual flaws in the thinking of average Physics students. Why do conceptual flaws occur? Assuming Gagne's model, the following learning types heavily rely on previously learned materials. (1) Chaining, (2) Multiple discrimination learning, (3) Concept learning, (4) Principle learning, and (5) Problem solving. And as we have seen, the kind of learning material that these learning types are based on is most effectively done through memorization.

In mathematics, one who has memorized the multiplication tables and rigorously practiced basic mathematical operations through oral methods is much more confident in higher mathematics because he has less stumbling blocks to overcome.

Modern school education has gradually done away with basic mathematical drills. So the pre-requisites for formation of higher concepts as pointed out by Gagne are not being fulfilled. Mathematical knowledge is cumulative in nature. So with a weak foundation the majority ofstudents are bound to display an overall weakness in their concepts. This, according to my view, is the main reason why many of today's students are weak in concepts.

4.0 Conclusion

Gagne's information processing model as shown above, that the rote and algorithmic methods could be used in traditional schools for effective building a strong base for formation of higher concepts. We should develop teaching methodology for mathematics and other subjects that incorporates rote learning in an effective way so that knowledge is better conveyed and represented in the mind of students. The rote learning of basic mathematical facts and word-meaning in primary schools will in particular be a very useful preparation for higher concepts. For better results a balance between heuristic approach and algorithmic approach will have to be established. We should also develop effective uses of sutra in mathematics teaching.

5.0 Summary

6.0 Tutor-Marked Assignment

1 List Gagne's Hierarchy of Concept and Meaning

2 What are the implications of Gagne's Hierarchy of concept and meaning to the teaching and learning of mathematics.

7.0 References/Further Readings

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UNIT 2: Piaget Theory of Intellectual Development and Mathematics

CONTENTS

1.0 Introduction

Jean Piaget (1896-1980) was a biologist who originally studied molluscs (publishing twenty scientific papers on them by the time he was 21) but moved into the study of the development of children's understanding, through observing them and talking and listening to them while they worked on exercises he set.

His view of how children's minds work and develop has been enormously influential, particularly in educational theory. His particular insight was the role of maturation (simply growing up) in children's increasing capacity to understand their world: they cannot undertake certain tasks until they are psychologically mature enough to do so. His research has spawned a great deal more, much of which has undermined the detail of his own, but like many other original investigators, his importance comes from his overall vision.He proposed that children's thinking does not develop entirely smoothly: instead, there are certain points at which it "takes off" and moves into completely new areas and capabilities. He saw these transitions as taking place at about 18 months, 7 years and 11 or 12 years. This has been taken to mean that before these ages children are not capable (no matter how bright) of understanding things in certain ways, and has been used as the basis for scheduling the school curriculum. Whether or not should be the case is a different matter

2.0 Objectives

- Identify Piaget's stages of intellectual development
- Distinguish between assimilation and accommodation
- Discuss the implication of Piaget's stages of intellectual development to the teaching and learning of mathematics

3.0 Main content

3.1 The Nature of Intelligence: Operative and Figurative Intelligence

Piaget believed that reality is a dynamic system of continuous change, and as such is defined in reference to the two conditions that define dynamic systems. Specifically, he argued that reality involves transformations and states. Transformations refer to all manners of changes that a thing or person can undergo. States refer to the conditions or the appearances in which things or persons can be found between transformations. For example, there might be changes in shape or form (for instance, liquids are reshaped as they are transferred from one vessel to another, humans change in their characteristics as they grow older), in size (e.g., a series of coins on a table might be placed close to each other or far apart) in placement or location in space and time (e.g., various objects or persons might be found at one place at one time and at a different place at another time). Thus, Piaget argued, that if human intelligence is to be adaptive, it must have functions to represent both the transformational and the static aspects of reality. He proposed that operative intelligence is responsible for the representation and manipulation of the dynamic or transformational aspects of reality and that figurative intelligence is responsible for the representation of the static aspects of reality.

Operative intelligence is the active aspect of intelligence. It involves all actions, overt or covert, undertaken in order to follow, recover, or anticipate the transformations of the objects or persons of interest. Figurative intelligence is the more or less static aspect of intelligence, involving all means of representation used to retain in mind the states (i.e., successive forms, shapes, or locations) that intervene between transformations. That is, it involves perception, imitation, mental imagery, drawing, and language. Therefore, the figurative aspects of intelligence derive their meaning from the operative aspects of intelligence, because states cannot exist independently of the transformations that interconnect them. Piaget believed that the figurative or the representational aspects of intelligence are subservient to its operative and dynamic aspects, and therefore, that understanding essentially derives from the operative aspect of intelligence.

At any time, operative intelligence frames how the world is understood and it changes if understanding is not successful. Piaget believed that this process of understanding and change involves two basic functions: Assimilation and accommodation.

3.2 Assimilation and Accommodation

Through studying the field of education Piaget focused on accommodation and assimilation. Assimilation, one of two processes coined by Jean Piaget, describes how humans perceive and adapt to new information. It is the process of taking one's environment and new information and fitting it into pre-existing cognitive schemas. Assimilation occurs when humans are faced with new or unfamiliar information and refer to previously learned information in order to make sense of it. Accommodation, unlike assimilation is the process of taking one's environment and new information, and altering one's pre-existing schemas in order to fit in the new information.

Through a series of stages, Piaget explains the ways in which characteristics are constructed that lead to specific types of thinking; this chart is called Cognitive Development. To Piaget, assimilation is integrating external elements into structures of lives or environments or those we could have through experience. It is through assimilation that accommodation is derived. Accommodation is imperative because it is how people will continue to interpret new concepts, schemas, frameworks, etc. Assimilation is different from accommodation because of how it relates to the inner organism due to the environment. Piaget believes that the human brain has been programmed through evolution to bring equilibrium, and to move upwards in a process to equilibriate what is not. The equilibrium is what Piaget believes ultimately influences structures because of the internal and external processes through assimilation and accommodation.

Piaget's understanding is that these two functions cannot exist without the other. To assimilate an object into an existing mental schema, one first needs to take into account or accommodate to the particularities of this object to a certain extent; for instance, to recognize (assimilate) an apple as an apple one needs first to focus (accommodate) on the contour of this object. To do this one needs to roughly recognize the size of the object. Development increases the balance or equilibration between these two functions. When in balance with each other, assimilation and accommodation generate mental schemas of the operative intelligence. When one function dominates over the other, they generate representations which belong to figurative intelligence.

Following from this conception Piaget theorized that intelligence is active and constructive. It is active in the literal sense of the term as it depends on the actions (overt or covert, assimilatory or accommodatory), which the thinker executes in order to build and rebuild his models of the world. It is also constructive because actions, particularly mental actions, are coordinated into more inclusive and cohesive systems, thus they are raised to more stable and effective levels of functioning.

3.3 Piagets' Stages of Intellectual Development

Sensorimotor stage

The **sensorimotor stage** is the first of the four stages in cognitive development which "extends from birth to the acquisition of language". "In this stage, infants construct an understanding of the world by coordinating experiences (such as seeing and hearing) with physical, motoric actions. Infants gain knowledge of the world from the physical actions they perform on it. An infant progresses from reflexive, instinctual action at birth to the beginning of symbolic thought toward the end of the stage. Piaget divided the sensorimotor stage into six sub-stages":0–2 years, Infants just have senses-vision, hearing, and motor skills, such as grasping, sucking, and stepping.---from Psychology Study Guide by Bernstein, Penner, Clarke-Stewart, Roy

Sub-Stage	**Age**	**Description**
1 *Simple Reflexes*	Birth- 6 weeks	"Coordination of sensation and action through reflexive behaviors". Three primary reflexes are described by Piaget: sucking of objects in the mouth, following moving or interesting objects with the eyes, and closing of the hand when an object

		makes contact with the palm (palmar grasp). Over the first six weeks of life, these reflexes begin to become voluntary actions; for example, the palmar reflex becomes intentional grasping.
2 *First habits and primary circular reactions phase*	6 weeks- 4 months	"Coordination of sensation and two types of schemes: habits (reflex) and primary circular reactions (reproduction of an event that initially occurred by chance). Main focus is still on the infant's body." As an example of this type of reaction, an infant might repeat the motion of passing their hand before their face. Also at this phase, passive reactions, caused by classical or operant conditioning, can begin.
3 *Secondary circular reactions phase*	4– 8 months	Development of habits. "Infants become more object-oriented, moving beyond self-preoccupation; repeat actions that bring interesting or pleasurable results." This stage is associated primarily with the development of coordination between vision and prehension. Three new abilities occur at this stage: intentional grasping for a desired object, secondary circular reactions, and differentiations between ends and means. At this stage, infants will intentionally grasp the air in the direction of a desired object, often to the amusement of friends and family. Secondary circular reactions, or the repetition of an action involving an external object begin; for example, moving a switch to turn on a light repeatedly. The differentiation between means and ends also occurs. This is perhaps one of the most important stages of a

		child's growth as it signifies the dawn of logic.
4 *Coordination of secondary circular reactions stages*	8–12 months	"Coordination of vision and touch--hand-eye coordination; coordination of schemes and intentionality." This stage is associated primarily with the development of logic and the coordination between means and ends. This is an extremely important stage of development, holding what Piaget calls the "first proper intelligence." Also, this stage marks the beginning of goal orientation, the deliberate planning of steps to meet an objective.
5 *Tertiary circular reactions, novelty, and curiosity*	12–18 months	"Infants become intrigued by the many properties of objects and by the many things they can make happen to objects; they experiment with new behavior." This stage is associated primarily with the discovery of new means to meet goals. Piaget describes the child at this juncture as the "young scientist," conducting pseudo-experiments to discover new methods of meeting challenges.
6 *Internalization of Schemes*	18–24 months	"Infants develop the ability to use primitive symbols and form enduring mental representations." This stage is associated primarily with the beginnings of insight, or true creativity. This marks the passage into the preoperational stage.

By the end of the sensorimotor period, objects are both separate from the self and permanent. Object permanence is the understanding that objects continue to exist even when they cannot be seen, heard, or touched. Acquiring the sense of object permanence is one of the infant's most important accomplishments, according to Piaget.

Preoperational stage

The preoperative stage is the second of four stages of cognitive development. Cognitive Development Approaches. By observing sequences of play, Jean Piaget was able to demonstrate that towards the end of the second year, a qualitatively new kind of psychological functioning occurs.

(Pre)Operatory Thought is any procedure for mentally acting on objects. The hallmark of the preoperational stage is sparse and logically inadequate mental operations. During this stage, the child learns to use and to represent objects by images, words, and drawings. The child is able to form stable concepts as well as mental reasoning and magical beliefs. The child however is still not able to perform operations; tasks that the child can do mentally rather than physically. Thinking is still egocentric. The child has difficulty taking the viewpoint of others. Two substages can be formed from preoperative thought.

- **The Symbolic Function Substage**

  Occurs between about the ages of 2 and 7.During 2-4 years old, kids cannot yet manipulate and transform information in logical ways, but they now can think in images and symbols. The child is able to formulate designs of objects that are not present. Other examples of mental abilities are language and pretend play. Although there is an advancement in progress, there are still limitations such as egocentrism and animism. Egocentrism occurs when a child is unable to distinguish between their own perspective and that of another person's. Children tend to pick their own view of what they see rather than the actual view shown to others.An example is an experiment performed by Piaget and Barbel Inhelder. Three views of a mountain are shown and the child is asked what a traveling doll would see at the various angles; the child picks their own view compared to the actual view of the doll. Animism is the belief that inanimate objects are capable of actions and have lifelike qualities. An example is a child believing that the sidewalk was mad and made them fall down.

- **The Intuitive Thought Substage**

Occurs between about the ages of 4 and 7. Children tend to become very curious and ask many questions; begin the use of primitive reasoning. There is an emergence in the interest of reasoning and wanting to know why things are the way they are. Piaget called it the intuitive substage because children realize they have a vast amount of knowledge but they are unaware of how they know it. 'Centration' and 'conservation' are both involved in preoperative thought. Centration is the act of focusing all attention on one characteristic compared to the others. Centration is noticed in conservation; the awareness that altering a substance's appearance does not change its basic properties. Children at this stage are unaware of conservation. Example, In Piaget's most famous task, a child is presented with two identical beakers containing the same amount of liquid. The child usually notes that the beakers have the same amount of liquid. When one of the beakers is poured into a taller and thinner container, children who are typically younger than 7 or 8 years old say that the two beakers now contain a different amount of liquid. The child simply focuses on the height and width of the container compared to the general concept.

Concrete operational stage

The **concrete operational stage** is the third of four stages of cognitive development in Piaget's theory. This stage, which follows the preoperational stage, occurs between the ages of 7 and 11 years and is characterized by the appropriate use of logic. Important processes during this stage are:

Seriation—the ability to sort objects in an order according to size, shape, or any other characteristic. For example, if given different-shaded objects they may make a color gradient.

Transitivity- The ability to recognize logical relationships among elements in a serial order, and perform 'transitive inferences' (for example, If A is taller than B, and B is taller than C, then A must be taller than C).

Classification—the ability to name and identify sets of objects according to appearance, size or other characteristic, including the idea that one set of objects can include another.

Decentering—where the child takes into account multiple aspects of a problem to solve it. For example, the child will no longer perceive an exceptionally wide but short cup to contain less than a normally-wide, taller cup.

Reversibility—the child understands that numbers or objects can be changed, then returned to their original state. For this reason, a child will be able to rapidly determine that if 4+4 equals t, t−4 will equal 4, the original quantity.

Conservation Conservation—understanding that quantity, length or number of items is unrelated to the arrangement or appearance of the object or items.

Elimination of Egocentrism—the ability to view things from another's perspective (even if they think incorrectly). For instance, show a child a comic in which Jane puts a doll under a box, leaves the room, and then Melissa moves the doll to a drawer, and Jane comes back. A child in the concrete operations stage will say that Jane will still think it's under the box even though the child knows it is in the drawer.

Children in this stage can, however, only solve problems that apply to actual (concrete) objects or events, and not abstract concepts or hypothetical tasks.

Formal operational stage

The formal operational period is the fourth and final of the periods of cognitive development in Piaget's theory. This stage, which follows the Concrete Operational stage, commences at around 11 years of age (puberty) and continues into adulthood. In this stage, individuals move beyond concrete experiences and begin to think abstractly, reason logically and draw conclusions from the information available, as well as apply all these processes to hypothetical situations. The abstract quality of the adolescent's thought at the formal operational level is evident in the adolescent's verbal problem solving ability. The logical quality of the adolescent's thought is when children are more likely to solve problems in a trial-and-error fashion. Adolescents begin to think more as a scientist thinks, devising plans to solve problems and systematically testing solutions. They use hypothetical-deductive reasoning, which means that they develop hypotheses or best guesses, and systematically deduce, or conclude, which is the best path to follow in solving the problem. During this stage the adolescent is able to understand such things as love, "shades of gray", logical proofs and values. During this stage the young person begins to entertain possibilities for the future and is fascinated with what they can be. Adolescents are changing cognitively also by the way that they think about social matters. Adolescent Egocentrism governs the way that adolescents think about social matters and is the heightened self-consciousness in them as they are which is reflected in their sense of personal uniqueness and invincibility. Adolescent egocentrism can be dissected into two types of social thinking, imaginary audience that involves attention getting behaviour, and personal fable which involves an adolescent's sense of personal uniqueness and invincibility.

The stages and causation

Piaget sees children's conception of causation as a march from "primitive" conceptions of cause to those of a more scientific, rigorous, and mechanical nature. These primitive concepts are characterized as magical, with a decidedly non-natural or non-mechanical tone. Piaget attributes this to his most basic assumption: that babies are phenomenists. That is, their knowledge "consists of assimilating things to schemas" from their own action such that they appear, from the child's point of view, "to have qualities which in fact stem from the organism." Consequently, these "subjective conceptions," so prevalent during Piaget's first stage of development, are dashed upon discovering deeper empirical truths. Piaget gives the example of a child believing the moon and stars follow him on a night walk; upon learning that such is the case for his friends, he must separate his self from the object, resulting in a theory that the moon is immobile, or moves independently of other agents. The second stage, from around three to eight years of age, is characterized by a mix of this type of magical, animistic, or "non-natural" conceptions of causation and mechanical or "naturalistic" causation. This conjunction of natural and non-natural causal explanations supposedly stems from experience itself, though Piaget does not make much of an attempt to describe the nature of the differences in conception; in his interviews with children, he asked specifically about natural phenomena: what makes clouds move? What makes the stars move? Why do rivers flow? The natures of all the answers given, Piaget says, are such that these objects must perform their actions to "fulfill their obligations towards men." He calls this "moral explanation.

3.4 Challenges to Piagetian stage theory

Piagetians' accounts of development have been challenged on several grounds. First, as Piaget himself noted, development does not always progress in the smooth manner his theory seems to predict. 'Decalage', or unpredicted gaps in the developmental progression, suggest that the stage model is at best a useful approximation. More broadly, Piaget's theory is 'domain general', predicting that cognitive maturation occurs concurrently across different domains of knowledge (such as mathematics, logic, understanding of physics, of language, etc.). During the 1980s and 1990s, cognitive developmentalists were influenced by "neo-nativist" and evolutionary psychology ideas. These ideas de-emphasized domain general theories and emphasized domain specificity or modularity of mind. Modularity implies that different cognitive faculties may be largely independent of one another and thus develop according to quite different time-tables. In this vein, some cognitive developmentalists argued that rather than being domain general learners, children come equipped with domain specific theories, sometimes referred to as 'core knowledge', which allows them to break into learning within that domain. For example, even young infants appear to be sensitive to some predictable regularities in the movement and interactions of objects (e.g. that one object cannot pass through another), or in human behavior (e.g. that a hand repeatedly reaching for an object has that object, not just a particular path of motion), as its be the building block out of which more elaborate knowledge is constructed. More recent work has strongly challenged some of the basic presumptions of the 'core knowledge' school, and revised ideas of domain generality—but from a newer dynamic systems approach, not from a revised Piagetian perspective. Dynamic systems approaches harken to modern neuroscientific research that was not available to Piaget when he was constructing his theory. One important finding is that domain-specific knowledge is constructed as children develop and integrate knowledge. This suggests more of a "smooth integration" of learning and development than either Piaget, or his neo-nativist critics, had envisioned. Additionally, some psychologists, such as Vygotsky and Jerome Bruner, thought differently from Piaget, suggesting that language was more

3.5 Post Piagetian and Neo-Piagetian stages

In the recent years, several scholars attempted to ameliorate the problems of Piaget's theory by developing new theories and models that can accommodate evidence that violates Piagetian predictions and postulates. These models are summarized below.

The neo-Piagetian theories of cognitive development, advanced by Case, Demetriou, Halford, Fischer, and Pascual-Leone, attempted to integrate Piaget's theory with cognitive and differential theories of cognitive organization and development. Their aim was to better account for the cognitive factors of development and for intra-individual and inter-individual differences in cognitive development. They suggested that development along Piaget's stages is due to increasing working memory capacity and processing efficiency. Moreover, Demetriou's theory ascribes an important role to hype-cognitive processes of self-recording, self-monitoring, and self-regulation and it recognizes the operation of several relatively autonomous domains of thought (Demetriou, 1998; Demetriou, Mouyi, Spanoudis, 2010).

- Postformal stages have been proposed. Kurt Fischer suggested two,Michael Commons presents evidence for four postformal stages: the systematic, metasystematic, paradigmatic and cross paradigmatic. (Commons & Richards, 2003; Oliver, 2004).
- A "sentential" stage has been proposed, said to occur before the early preoperational stage. Proposed by Fischer, Biggs and Biggs, Commons, and Richards.
- Searching for a micro-physiological basis for human mental capacity, Traill (1978, proposed that there may be "pre-sensorimotor" stages developed in the womb and/or transmitted genetically.

Postulated physical mechanisms underlying "schemes" and stages

Piaget himself (1967) considered the possibility of RNA *molecules* as likely embodiments of his still-abstract "schemes" (which he promoted as units of action) — though he did not come to any firm conclusion. At that time, due to work such as that of Holger Hydén, RNA concentrations had indeed been shown to correlate with learning, so the idea was quite plausible.

However, by the time of Piaget's death in 1980, this notion had lost favour. One main problem was over the protein which (it was assumed) such RNA would necessarily produce, and that did not fit in with observation. It then turned out, surprisingly, that only about 3% of RNA does code for protein (Mattick, 2001, 2003, 2004). Hence most of the remaining 97% (the “ncRNA”) could now theoretically be available to serve as Piagetian schemes (or other regulatory roles now under investigation). The issue has not yet been resolved experimentally, but its theoretical aspects have been reviewed; (Traill 2005 / 2008).

4.0 Conclusion

Piaget believed that all children try to strike a balance between assimilation and accommodation, which is achieved through a mechanism Piaget called equilibration. As children progress through the stages of cognitive development, it is important to maintain a balance between applying previous knowledge (assimilation) and changing behavior to account for new knowledge (accommodation). Equilibration helps explain how children are able to move from one stage of thought into the next.

5.0 Summary

Piaget's theory, however vital in understanding child psychology, did not go without scrutiny. A main figure whose ideas contradicted Piaget's ideas was the Russian psychologist Lev Vygotsky. Vygotsky stressed the importance of a child's cultural background as an effect to the stages of development.

Because different cultures stress different social interactions, this challenged Piaget's theory that the hierarchy of learning development had to develop in succession. Vygotsky introduced the term Zone of proximal development as an overall task a child would have to develop that would be too difficult to develop alone

6.0 Tutor-Marked Assignment

1 Identify Piaget's stages of intellectual development

2 Describe in details all the stages of Piaget stages of intellectual development.

3 What are the implications of Piaget's stages of intellectual development to the teaching and learning of mathematics.

7.0 References/Further Readings

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UNIT 3: Writing Objectives Using Bloom's Taxonomy

CONTENTS

1.0 Introduction

Behavioural objectives are means of conceiving instructional strategy in a form that requires a specification of what tasks the students are expected to be able to perform, under what conditions and how such tasks will be evaluated. The process of learning is an individual experience for each student. According to behaviourist school of psychology, learning takes place whenever an individual's behaviour is modified, that is when he thinks or acts differently; or when he has acquired new knowledge or a new skill and so forth. Thus the concept of behavioural objectives as a significant educational strategy is similar to the concept of operational definition of terms developed in science some years ago "to eliminate hypothetical concepts by defining a concept in terms of the steps or operations whereby the physical reality of the concept could be observe or measured" (Dressel, 1977).

2.0 Objectives

At the end of the unit you should be able to:

- state bloom's taxonomy of education objectives

- identify the attributes of each level of the taxonomy
- write behavioural objectives

3.0 Main content

3.1 Main Menu

Writing Objectives Using Bloom's Taxonomy

Various researchers have summarized how to use Bloom's Taxonomy. Following are four interpretations that you can use as guides in helping to write objectives using Bloom's Taxonomy

Bloom's Taxonomy divides the way people learn into three domains. One of these is the cognitive domain, which emphasizes intellectual outcomes. This domain is further divided into categories or levels. The key words used and the type of questions asked may aid in the establishment and encouragement of critical thinking, especially in the higher levels.

Level	**Level Attributes**	**Keywords**	**Questions**
1: Knowledge	Exhibits previously learned material by recalling facts, terms, basic concepts and answers.	who, what, why, when, omit, where, which, choose, find, how, define, label, show, spell, list, match, name, relate, tell, recall, select	What is ...? How is ...? Where is ...? When did ________ happen? How did _______ happen? How would you explain ...? Why did ...? How would you describe ...? When did ...? Can you recall ...? How would you show ...? Can you select ...? Who were the

			main ...? Can you list three ...? Which one ...? Who was ...?
2: Comprehension	Demonstrating understanding of facts and ideas by organizing, comparing, translating, interpreting, giving descriptions and stating main ideas.	compare, contrast, demonstrate, interpret, explain, extend, illustrate, infer, outline, relate, rephrase, translate, summarize, show, classify	How would you classify the type of ...? How would you compare ...? contrast ...? Will you state or interpret in your own words ...? How would you rephrase the meaning ...? What facts or ideas show ...? What is the main idea of ...? Which statements support ...? Can you explain what is happening . . . what is meant . . .? What can you say about ...? Which is the best answer ...? How would you summarize ...?
3: Application	Solving problems by applying acquired knowledge, facts, techniques and rules in a different way.	apply, build, choose, construct, develop, interview, make use of, organize, experiment with, plan, select, solve, utilize, model, identify	How would you use ...? What examples can you find to ...? How would you solve _______ using what you have learned ...? How would you organize _______ to show ...? How would you show your

			understanding of ...? What approach would you use to ...? How would you apply what you learned to develop ...? What other way would you plan to ...? What would result if ...? Can you make use of the facts to ...? What elements would you choose to change ...? What facts would you select to show ...? What questions would you ask in an interview with ...?
4: Analysis	Examining and breaking information into parts by identifying motives or causes; making inferences and finding evidence to support generalizations.	analyze, categorize, classify, compare, contrast, discover, dissect, divide, examine, inspect, simplify, survey, take part in, test for, distinguish, list, distinction, theme, relationships, function, motive, inference, assumption, conclusion	What are the parts or features of ...? How is _______ related to ...? Why do you think ...? What is the theme ...? What motive is there ...? Can you list the parts ...? What inference can you make ...? What conclusions can you draw ...? How would you classify ...? How would you categorize ...? Can you

			identify the difference parts ...? What evidence can you find ...? What is the relationship between ...? Can you make a distinction between ...? What is the function of ...? What ideas justify ...?
5: Synthesis	Compiling information together in a different way by combining elements in a new pattern or proposing alternative solutions.	build, choose, combine, compile, compose, construct, create, design, develop, estimate, formulate, imagine, invent, make up, originate, plan, predict, propose, solve, solution, suppose, discuss, modify, change, original, improve, adapt, minimize, maximize, delete, theorize, elaborate, test, improve, happen, change	What changes would you make to solve ...? How would you improve ...? What would happen if ...? Can you elaborate on the reason ...? Can you propose an alternative ...? Can you invent ...? How would you adapt ________ to create a different ...? How could you change (modify) the plot (plan) ...? What could be done to minimize (maximize) ...? What way would you design ...? What could be combined to improve (change) ...?

			Suppose you could _______ what would you do ...? How would you test ...? Can you formulate a theory for ...? Can you predict the outcome if ...? How would you estimate the results for ...? What facts can you compile ...? Can you construct a model that would change ...? Can you think of an original way for the ...?
6: Evaluation	Presenting and defending opinions by making judgments about information, validity of ideas or quality of work based on a set of criteria.	award, choose, conclude, criticize, decide, defend, determine, dispute, evaluate, judge, justify, measure, compare, mark, rate, recommend, rule on, select, agree, interpret, explain, appraise, prioritize, opinion, ,support, importance, criteria, prove, disprove, assess, influence, perceive, value, estimate, influence, deduct	Do you agree with the actions ...? with the outcomes ...? What is your opinion of ...? How would you prove ...? disprove ...? Can you assess the value or importance of ...? Would it be better if ...? Why did they (the character) choose ...? What would you recommend ...? How would you rate the ...? What would you cite to defend the actions ...? How would you evaluate

...? How could you determine ...? What choice would you have made ...? What would you select ...? How would you prioritize ...? What judgment would you make about ...? Based on what you know, how would you explain ...? What information would you use to support the view ...? How would you justify ...? What data was used to make the conclusion ...? Why was it better that ...? How would you prioritize the facts ...? How would you compare the ideas ...? people ...?

3 .2 Bloom’s Ranking of Thinking Skills

Knowledge	**Comprehension**	**Application**	**Analysis**	**Synthesis**	**Evaluation**
List, Name,	Summarize, Explain,	Solve,	Analyze,	Design,	Evaluate,
Identify,	Interpret, Describe,	Illustrate,	Organize,	Hypothesize,	Choose,
Show, Define,	Compare,	Calculate, Use,	Deduce,	Support,	Estimate,
Recognize,	Paraphrase,	Interpret,	Contrast,	Schematize,	Judge,

Recall, State, Visualize	Differentiate, Demonstrate, Classify	Relate, Manipulate, Apply, Modify	Compare, Distinguish, Discuss, Plan, Devise	Write, Report, Justify	Defend, Criticize

According to Benjamin Bloom, and his colleagues, there are six levels of cognition:

1. Knowledge: rote memorization, recognition, or recall of facts
2. Comprehension: understanding what the facts mean
3. Application: correct use of the facts, rules, or ideas
4. Analysis: breaking down information into component parts
5. Synthesis: combination of facts, ideas, or information to make a new whole
6. Evaluation: judging or forming an opinion about the information or situation

Ideally, each of these levels should be covered in each course and, thus, at least one objective should be written for each level. Depending on the nature of the course, a few of these levels may need to be given more emphasis than the others.

3.3 Examples of objectives written for each level of Bloom's Taxonomy and activities and assessment tools based on those objectives.

Common key verbs used in drafting objectives are also listed for each level.

Level	Level Attributes	Keywords	Example Objective	Example Activity	Example Assessment
1: Knowledge	Rote memorization,	list, recite, define, name,	"By the end of this course, the	Have students group up and	Use the following

	recognition, or recall of facts.	match, quote, recall, identify, label, recognize	student will be able to recite Newton’s three laws of motion.”	perform simple experiments to the class showing how one of the laws of motion works.	question on an exam or homework. “Recite Newton’s three laws of motion.”
2: Comprehension	Understanding what the facts mean.	describe, explain, paraphrase, restate, give original examples of, summarize, interpret, discuss	“By the end of this course, the student will be able to explain Newton’s three laws of motion in his/her own words.”	Group students into pairs and have each pair think of words that describe motion. After a few minutes, ask pairs to volunteer some of their descriptions and write these descriptions on the board.	Assign the students to write a simple essay that explains what Newton’s laws of motion mean in his/her own words.
3: Application	Correct use of the facts, rules, or ideas.	calculate, predict, apply, solve, illustrate, use, demonstrate,	“By the end of this course, the student will be able to calculate the kinetic	After presenting the kinetic energy equation in class, have the students pair off for just a	On a test, define a projectile and ask the students to “Calculate the kinetic energy of

		determine, model	energy of a projectile.”	few minutes and practice using it so that they feel comfortable with it before being assessed.	the projectile.”
4: Analysis	Breaking down information into component parts.	classify, outline, break down, categorize, analyze, diagram, illustrate	“By the end of this course, the student will be able to differentiate between potential and kinetic energy.”	Present the students with different situations involving energy and ask the students to categorize the energy as either kinetic or potential then have them explain in detail why they categorized it the way they did, thus breaking down what exactly makes up kinetic and potential	Give the students an assignment that asks them to outline the basic principles of kinetic and potential energy. Ask them to point out the differences between the two as well as how they are related.

				energy.	
5: Synthesis	Combining parts to make a new whole.	design, formulate, build, invent, create, compose, generate, derive, modify, develop	By the end of this section of the course, the student will be able to design an original homework problem dealing with the principle of conservation of energy.”	Tie each lecture or discussion to the previous lectures or discussions before it, thus helping the students assemble all the discreet classroom sessions into a unified topic or theory.	Give the students a project in which they must design an original homework problem dealing with the principle of conservation of energy.
6: Evaluation	Judging the value or worth of information or ideas.	choose, support, relate, determine, defend, judge, grade, compare, contrast, argue, justify, support, convince, select, evaluate	“By the end of the course, the student will be able to determine whether using conservation of energy or conservation of momentum would be more	Have different groups of students solve the same problem using different methods, then have each group present the pros and cons of the method they chose.	On a test, describe a dynamic system and ask the students which method they would use to solve the problem and why.

appropriate

forsolving a

dynamics

problem."

4.0 Conclusion

Behavioural objectives have been defined as desired outcome of learning which is expressed in terms of observable and/or measurable behaviour or performance. In contrast to an educational aim which only stipulates changes that cannot be observed or measured, behavioural objectives spell out what the learner should be able to do as a consequence of the learning experiences associated with the objectives. It has been found that Bloom's taxonomy on cognitive domain lends itself to a number of adaptations suitable for formulating behavioural objectives in science instruction.

5.0 Summary

The six levels of cognition as proposed by Blooms are:

1. Knowledge: rote memorization, recognition, or recall of facts
2. Comprehension: understanding what the facts mean
3. Application: correct use of the facts, rules, or ideas
4. Analysis: breaking down information into component parts
5. Synthesis: combination of facts, ideas, or information to make a new whole
6. Evaluation: judging or forming an opinion about the information or situation

6.0 Tutor-Marked Assignment

1 List Blooms taxonomy of educational objectives

2 Pick a mathematics topic write behavioural objectives for each of the Blooms' levels of cognition.

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UNIT 4: INNOVATIONS IN TEACHING OF MATHEMATICS

CONTENTS

1.0 Introduction

2.0 Objectives

At the end of the unit you should be able to:

List and discuss at least four innovative methods of teaching mathematics

3.0 Main content

3.1 Need for Innovations in Teaching Mathematics

Though Mathematics being so important subject and occupying a central position since the Ancient period still it has not been the interest of many students. The gaps are found between aspiration and achievement. Mathematics is highly abstract. It is concerned with ideas rather than objects; with the manipulation of symbols rather than the manipulation of object. It is a closely-knit structure in which ideas are interrelated. Mathematical concepts are hierarchical and interconnected, much like a house of cards. Unless lower-level concepts are mastered, higher-level concepts cannot be understood. Students who discover some of the structures of mathematics, are often impressed by its beauty. They note the lack of contradiction, and they see how a new technique can be derived from one that has already been learned.

Teaching of mathematics is not only concerned with the computational knowhow of the subject but is also concerned with the selection of the mathematical content and communication leading to its understanding and application. So while teaching mathematics one should use the teaching methods, strategies and pedagogic resources that are much more fruitful in gaining adequate responses from the students then we have ever had in the past. The teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. The nature and quality of instructional material, the presentation of content, the pedagogic skills of the teacher, the learning environment, the motivation of the students are all important and must be kept in view in any effort to ensure quality in teaching-learning of mathematics.

Innovations and innovative practices in teaching mathematics, is discussed under teaching methods, strategies and pedagogic resources. The process of innovation is generally described as consisting of three essential steps, starting with the conception of an idea, which is then proposed and is finally adopted. Though many ideas have been conceived to bring about change in the teaching of mathematics, it is yet to be proposed and adopted. So, the innovations discussed may not be new in terms of the idea but is new in terms of practice.

Looking to the aims of teaching mathematics it can be seen that more focus is laid to the higher level of objectives underlying the mathematics subject, like critical thinking, analytical thinking, logical reasoning, decision-making, problem-solving. Such objectives are difficult to be achieved only through verbal and mechanical methods that are usually used in the class of mathematics. The verbal methods of instruction give all importance to speech and texts, to the book and to the teacher. From an historical point of view this method was majorly used until the end of the nineteenth century.

In one of these verbal methods teachers are simply satisfied with giving the mathematical rules to pupils and having them memorize it. They justify this method by saying pupils would not understand explanations. Their task is to transmit to their pupils the knowledge which has accumulated over the centuries, to stuff their memory while asking them to work exercises, e.g.

The rule of signs and formulas in algebra, students memorize this and remember it! Another verbal method involves explanation. Teachers who use this method assume that the mental structure of the child is same as the adult's. But a developmental stage according to Piaget is a period of years or months during which certain developments take place. Teachers think teaching must imply logic, and logic being linked to language, or at least to verbal thought, verbal teaching is supposed to be sufficient to constitute this logic. This method leads to series of explanations and students at the initial steps of logical explanations trying to understand and grasp but slowly the gap is created between the explanations transmitted by teacher and received

by students which lead to the poor understanding on part of students and they develop a fear of the subject: Math phobia. The Education Commission (1964-66) points out that "In the teaching of Mathematics emphasis should be more on the understanding of basic principles than on the mechanical teaching of mathematical computations". Commenting on the prevailing situation in schools, it is observed that in the average school today instruction still confirms to a mechanical routine, continues to be dominated by the old besetting evil of verbalism and therefore remains dull and uninspiring.

3.2 Innovations in Teaching Mathematics

Innovations in teaching of mathematics can be diversified in terms of Methods, Pedagogic Resources and Mastery Learning Strategy used in teaching-learning process.

1. Mastery Learning Strategy

Teaching Strategy is a generalized plan for a lesson and includes a specific structure to be followed.

B.S. Bloom has developed Mastery Learning Strategy. It is a new instructional strategy that is used for developing mastery learning and objectives of curriculum can be realized. It consists of different steps: division of content into units, formulation of objectives related to each unit, teaching and instruction are organized for realizing objectives of each unit, administering unit test to evaluate the mastery level and diagnose the learning difficulties, remedial instructions are given to remove the difficulties and attain mastery level by every student. This strategy plays an important role for learning of basics and fundamentals e.g. operations in different number systems – Natural numbers, Integers, Rational numbers, Real numbers.

2. Methods

Method is a style of the presentation of content in classroom. The following are the innovative methods that can be used to make teaching-learning process of Mathematics effective.

Inducto-Deductive Method

It is a combination of inductive and deductive method. Inductive method is to move from specific examples to generalization and deductive method is to move from generalization to specific examples. In classroom usually the instructions directly start with the abstract concepts and are being taught in a way that does not bring understanding on the part of majority of the students. Formulas, theorems, examples, results are derived, proved and used. But teacher needs to start with specific examples and concrete things and then move to generalizations and abstract things. Then teacher again needs to show how generalization can be derived and it holds true through specific examples. This method will help students for better understanding, students don't have to cram the things and will have long lasting effect.

Example: Pythagoras Theorem - In a right-angle □□ABC right angled at B, 2 *AB* + 2 *BC* = 2 *AC* (Considering right angle triangles of different measurement leading to generalization and then establishing it through the theoretical proof).

Analytico-Synthetic Method

It is a combination of Analytic and Synthetic method. Analytic is breaking down and moving from unknown to known and Synthetic is putting together known bits of information and moving from known to unknown. These methods are basically used in proving the results and solving sums. In textbooks mostly synthetic method is used, to prove something unknown we start with a certain known thing, but that leaves doubt in mind of students why we have started with that step and using this particular known thing. So teacher has to use combination in order to explain and relate each step logically.

Example: If *b a* = *d c* then prove that d(a-2ab) = b(c-2ad).

Synthetic Method Analytic Method

b a = d c □ *b a - 2a =d c - 2a* ***(Why??)**** □*d(a-2ab) = b(c-2ad)*

***the doubt raised in students mind is being solved with the help of analytic method

d(a-2ab) = b(c-2ad) □ *b a* −□2*ab* = *d c* −□2*ad* □ *b a - 2a = d c - 2a* □ *b a = d c*

Problem-Solving Method

This method aims at presenting the knowledge to be learnt in the form of a problem. It begins with a problematic situation and consists of continuous meaningful well-integrated activity. Choose a problem that uses the knowledge that students already have i.e. you as a teacher should be able to give them the problem and engage them without spending time in going over the things that you think they should know. After students have struggled with the problem to get solution, have them share their solutions. This method will help them in developing divergent thinking.

Example: Put a problem of finding the amount of water in a given container instead of deriving the formula of volume (cylinder filled with water).

Play-Way Method

This method consists of the activities that include a sort of fun or play and give joy to the students. Students don't realize that they are learning but in a way they are gaining knowledge through participating in different activities.

This method helps to develop interest in mathematics, motivates students to learn more and reduces the abstract nature of the subject to some extent.

Example: Mathematical games and puzzles.

Laboratory Method

Laboratory method is based on the principles of "learning by doing" and "learning by observation" and proceeding from concrete to abstract. Students do not just listen to the information given but do something practically also.

Principles have to be discovered, generalized and established by the students in this method. Students learn through hands on experience. This method leads the student to discover mathematical facts. After discovering something by his own efforts, the student starts taking pride in his achievement, it gives him happiness, mental satisfaction and encourages him

towards further achievement.

Example: Making and observing models, paper folding, paper cutting, construction work in geometry.

3. Pedagogic Resources

Pedagogic resources are the resources that a teacher may integrate in a method for the transaction of a particular content and draw upon to advance the students' learning.

Teaching Aids

Teaching aids are the materials used for effective teaching and enhancing the learning of students. It can be anything ready-made or made by the teacher or made by students. Different teaching aids should be used in teaching mathematics like Charts, Manipulatives, Programmed Learning Material (PLM), computers and television.

_ Charts – It can be used to display formulae, symbols, mathematical and geometrical figures. Charts can be used for making students familiar to the symbols and for memorization of basic formulae. Even it can be used to bring to the students two-dimension geometry and the graphical representation in a better way.

_ Manipulatives – They are objects or materials that involve mathematics concepts, appealing to several senses, which can be touched and moved around by the students (not demonstrations of materials by the teacher). Each student needs material to manipulate independently. With students actively involved in manipulating materials, interest in mathematics will be aroused. Canny (1984) has shown that mathematics instruction and students' mathematics understanding will be more effective if manipulative materials are used. Models can be used to make things concrete like three dimension figures in geometry.

_ Programmed Learning Material (PLM) – It is a self-learning material in which learner can proceed at his own pace. It has the characteristics of all sequential steps, learner's response, self-pacing, immediate feedback, reinforcement and self-evaluation. It is helpful in acquisition of concepts like fractions, number systems, etc. and can be used as a remedy for slow learners for a specific content.

_ Computers and Television – Computer can be used for multimedia presentation for the concepts that requires visualization and imagination. Computer can also be used for providing Computer Assisted Instruction (CAI), it is similar to PLM i.e. it is a computerized PLM. Television can be used to show some good mathematics education show.

Activities

Activities here include all such work where in students play an active role, has to interact with different resources and generate knowledge. It includes Quiz competition, Projects, Role play, Seminars, Discussion, Mathematics club, Assignment, Field trips, etc.

Name of the Activity Examples/Situations where Activity can be used

Quiz Competition

Logic, Properties of Numbers, Mathematical Rules and Results

Projects Contribution by Different Mathematicians

Role Play

Arithmetical concepts like Profit & Loss, Simple & Compound Interest

Seminars

Shortcuts through Vedic Mathematics,

Application of Mathematics in other Disciplines

Discussion

Properties of 'Zero', Difference between Rational and Irrational Numbers, Relating Different Concepts in Mathematics

Mathematics Clubs

Application of the concept studied, Preparing Models, Paper Folding (Origami)

Assignment Self-Study, Extension of Knowledge

Field Trips

Experiencing the Functional use of Mathematics in Bank,

Insurance Company

In any curriculum, content and presentation of content are the two most important and inseparable components. It is difficult to say anything definitely about which method and pedagogic resource is going to be most effective for presentation of a particular type of content. Selection of method and pedagogic resource depends on many factors like type of content, objectives to be achieved, level of the students, entry behaviour, availability of resources. Also acceptance of innovative methods and positive attitude of teachers towards it, is an important factor for the selection of method and pedagogic resource. The things included under innovations are existing in books, also there are researches which shows that some innovations are carried out in the classroom and has shown the positive effect on teaching learning process but their practical usage and implementation in classroom is not seen to the expected level.

3.3 Guidelines for a Teacher in Incorporating Innovations in Teaching Mathematics

For effective transaction of the curriculum and achievement of curricular objectives appropriate method and pedagogic resources should be used in providing learning experiences to the students.

_ A number of factors need to be considered while making use of a particular method and pedagogic resource: learners' capabilities, availability of resources, entry behavior, school environment, objectives to be achieved, the nature of content and the teacher's own preparation and mastery.

_ Decide on and plan in advance the innovative idea that the teacher would be incorporating to transact a particular concept so that loss of instructional time is prevented or minimized.

_ The immediate environment of the learner both natural and human should be used when and where possible for making learning concrete and meaningful.

_ Involve the students in the process of learning by taking them beyond the process of listening to that of thinking, reasoning and doing.

_ In order to promote self-study skills use of library and resource center needs to be encouraged.

_ Receiving regular feedback for teaching and learning should be an inbuilt component of teaching-learning process. Continuous and comprehensive evaluation has to be ensured as it plays an important role for the required modification in teaching-learning process.

_ Mathematics-teachers' organizations at different levels should be formed where sharing of ideas and experiences, developing resources in a collaborative manner and the mechanisms that enable teachers to carry out innovations is being discussed. Mathematics-teachers' organizations can be instrumental in establishing a climate of confidence in carrying out innovations and a positive attitude to new approaches in teaching mathematics.

_ Properly instruct and guide the students for carrying out different activities and precautionary measures should be taken so that students are not misguided.

Study mathematical journals and modern books of professional interest.

Any facilities of in-service training should be availed of for improving teaching of mathematics.

The teacher can always ask himself two questions: 1. 'Is there some new way in which I can present this material in order to make it more meaningful and more interesting?' 2. 'What activities, demonstrations, teaching aids, etc. would enrich the classroom presentation and direct attention of students to the important elements?' Once the teacher discovers innovative ways to arouse interest and enthusiasm in the class, he will be able to use these ideas again the following year, since those will be new and fascinating to a different class. But teacher should keep in mind that as time passes, the world undergoes a change, the environment surrounding students changes and their needs also changes, so one has to continuously go on modifying and discovering new ways of teaching which proves him a better teacher.

4.0 Conclusion

It is important that teachers to have a repertoire knowledge of several innovative ways of presenting mathematics to their students**.**

5.0 Summary

6.0 Tutor-Marked Assignment

(a) Why is innovative teaching necessary in mathematics classroom.

(b) Discuss as much as possible four innovative ways of teaching mathematics.

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MODULE 3 INSTRUCTIONAL METHODS IN MATHEMATICS

UNIT 1 FACTORS AFFECTING LEARNING

CONTENTS

1.0 INTRODUCTION

The search for an effective teaching method is a perennial concern and goal for a responsible educator. Teaching is not an end in itself, but rather a means to an end. Therefore, the effectiveness of a teaching method has to be evaluated by the degree of its attainment of specified goals. However, this means-end relationship is not a direct, linear one, but is intervened by a third variable, i.e. the learner and a set of elements associated with the learner's learning.

The choice of the most preferred instructional method is to be made on the basis of certain criteria, namely: (i) the objectives of the lesson, (ii) the nature of the topic, and (iii) factors affecting learning.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- identify several factors affecting the teaching and learning of mathematics
- state the components of a lesson plan.

3.0 MAIN CONTENT

3.1 Factors Affecting Learning

Teaching and learning are two sides of a coin. The understanding of the complex process of learning and some significant factors affecting learning will, therefore, enable a teacher to select a teaching method most suitable for productive learning on the part of his pupils.

(A) Motivation

The desire to learn is the primary prerequisite to the pursuit of knowledge in any field. Pupils with that desire will learn. However, pupils differ in motivation to fulfil the learning requirements. Pupils' motivation can be sustained when the learners consider learning tasks as having intrinsic worth or practical usefulness. Motivation is also improved when learners are encouraged and supported by compliments, evaluative comments, constructive suggestions and other forms of verbal feedback. Most students want to be liked by the teacher. The teacher's friendly approval and sympathetic understanding can go a long way to reinforce the pupil's effort to learn in the face of great difficulty. To sum up, motivation is important for learning. A teaching method should therefore possess, among other things, elements conducive to generating strong desire to learn in the learners.

(B) Intellectual Ability

Pupils vary widely in intellectual ability and aptitude for success in mathematics education. It is not inaccurate to expect pupils in primary schools in general with intelligence above average on account of a series of differential exposure prior to their admission to school. To have a knowledge of the distribution and range of intellectual abilities in the class will enable the teacher to pitch the initial level at which the subject should be taught, to understand the pace of learning of his pupils, and to individualise instruction for pupils of varying ability, when necessary.

(C) Background, Experience and Attitude

Pupils' variations in social, economic and cultural background give rise to their different reactions to certain learning situations. Wholesome respect for and a sympathetic understanding of the learner's attitude will result in a speedier change of the erroneous attitude than disrespect and coercion. The instructor who notes and studies such differences among his pupils will be able to adjust course activities and

teaching techniques
in the interest of the class learning. Research shows that past learnin

experiences and personality differences also account for different responses to learning stimuli. Three types of personality are identified:

i) Those pupils who are insecure and who want more direction,
ii) Those pupils who are independent and who want more autonomy, and
iii) Those pupils who are satisfied and adjustible.

The "insecure" pupil is said to be less favourably disposed toward non- directive, non-guided teaching than either of the other two types. The "satisfied" pupil is amenable to both directive and permissive teaching methods. The "independent" pupil is confident, verbal and prefers to have permissive teaching and autonomy.

(D) Communication

Learning will be enhanced when what is said and demonstrated is clear and unambiguous. Effective teaching is determined in great measure by the art of communication, the transmission of thought from one mind to others.

The importance of clear communication for teaching is propounded by Gilbert Highet who remarked that, "let him (the teacher) be good at communication, and even if he is a mediocre scholar, he can be an excellent teacher." In another sense, communication also means the flow of interaction from the students to the instructor. Pupil participation in class will certainly serve to clarify points of ambiguity, disagreement, and misunderstanding. Differentkinds of teaching procedures afford various amount of opportunity for pupil participation. Teaching techniques vary in the extent to which a genuine two-way communication between the teacher and the class is permitted. Findings in research studies tend to show a high positive correlation between communication and effective teaching-learning

(E) Anxiety

The effect of anxiety upon learning has attracted the attention of many educators.

Spence and Taylor concluded in their research that a high level of anxiety will facilitate simple learning, but beyond an optimal point, will hamper complex learning. Since academic situations sometimes produce high level of anxiety, we might expect damaging effects. Anxiety is related to uncertainty. We therefore expect that anxious people work most effectively in a highly structured situation. Susceptibility to anxiety varies from individual to individual. For some students, certain course requirements and assignment demands may provoke anxiety of a disabling proportion, and their learning progress will consequently become seriously retarded. The teacher, sensitive to the anxiety level of his class, will alertly match learning situation to an appropriate choice of teaching methods.

3.2 Components of a thorough Lesson Plan

For every lesson, careful consideration should be given to the following:

- The mathematical content of the lesson- what skills or concepts are being developed or mastered as a result of the lesson? Often, teachers who plan effective lessons back-map the content from asking "exactly what do I expect my students to know or be able to do at the end of this lesson?"
- The mathematical tasks of the lesson - what specific questions, problems, tasks, investigations, or activities will students be working on during the lesson? Often, this includes the worksheets that are prepared for the lesson and the references or materials that are needed.
- Evidence that the lesson was successful- deliberate consideration of what performances will convince you (and any outside observer) that most, if not all, of your students have accomplished your objective.

4.0 CONCLUSION

The teacher wishing to do his best will have discovered that there is a larger aim – that of awakening the interest of the students, of bringing them to react and to delight in the use of the mind, to enjoy the process of gaining information in order to follow ideas to see where they lead. The true teacher will accept this as his responsibilities - to increase his capacity to lead the student through his subject, not merely to know, not merely to parrot, but to use his mind and to feel comfortable in doing so, so that he can more effectively magnetise young minds and give them that electric current of curiosity and questioning that is the precious possession of the truly educated man.

5.0 SUMMARY

There is no doubt that several factors militate against the teaching and learning of mathematics These lead to poor performance in mathematics and other physical sciences and some have indirect and direct influences. Factors with a direct influence related to teaching strategies, content knowledge, motivation, laboratory use, and non-completion of the syllabus content over a year. On the other hand, indirect influences may be attributed to (a) the role played by parents in their children's education, and (b) general language usage.

6.0 TUTOR-MARKED ASSIGNMENT

Identify and discuss five factors militating against the learning of mathematics.

7.0 REFERENCES/FURTHER READING

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UNIT 2 ANALYSIS OF THE TEACHING METHODS

CONTENTS

1.0 INTRODUCTION

The traditional passive view of learning involves situations where the material is delivered to students using a lecture-based format. In contrast, a more modern view of learning is constructivism, where students are expected to be active in the learning process by participating in discussion and/or collaborative activities (Fosnot, 1989). Results of recent studies concerning the effectiveness of teaching methods favour constructivist and active learning method and it is important that mathematics teachers are exposed to all types of teaching methods.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- list methods of teaching mathematics
- employ the various methods in teaching primary mathematics
- distinguish between the various methods
- state the advantages and disadvantages of each methods in the teaching of mathematics.

3.0 MAIN CONTENT

Having discussed the criteria for evaluating teaching methods, namely: (i) objectives of a mathematics education and (ii) factors affecting learning elsewhere in this module, it is important to discuss some most commonly used methods of teaching. Mathematics can present an overwhelming challenge for some students, while others may breeze through mathematics-related course work. Use a variety of teaching methods to present mathematics lessons that will help students of varied backgrounds and learning styles comprehend the information before them.

3.1 The Lecture Method

The most traditional, long-established method of teaching is lecturing. It is alleged to have first been employed by the Sophists who travelled throughout Greece speaking on various topics upon request. Nowadays, this method is universal at all levels of teaching-learning situations. In this method, the teacher talks more or less continuously to the class. The class listens, takes notes of the facts and ideas worth remembering, thinks over them later; but the class does not converse with the teacher. G. J. Umstattd described this type of lecturing as an "uninterrupted verbal presentation by an instructor." In the less formal lectures, the class is invited to ask a few questions but these are largely for the sake of clarification, not of discussion. The essence of this kind of teaching and its purpose are for a steady transmission of information from the teacher to the students.

The advantages of the lecture method are:

i) it gives students the information not elsewhere available. This is especially true when the lectures are based on the unpublished research projects and on the crystallised wisdom out of the life-long academic pursuits of the instructor
ii) it summarises, synthesises and organises for the students the content of numerous articles and books, which represents years of laborious work on the part of the instructor
iii) it points out relationships and salient points that even abler students might not sense or not fully comprehend until amplified by the instructor. Thus, the student's learning progress will be accelerated and their level of understanding will be elevated
iv) it widens the intellectual horizons of the student, making it possible for the learner to gradually move toward acquisition of self-discovery and self-understanding
v) it enables the instructor to correct error in literature and articles read by the student
vi) it affords opportunities for an instructor to explain a particularly equivocal ambiguous point of idea, or a complicated,

difficult, abstract process or operation, thus unnecessary obstacles to learning are removed

vii) it resolves conflicting points of view and clarifies misunderstanding of different schools of thought

viii) it enlivens the learning situation by adding the voice, gesture and the personality of the teacher.

Limitations of the lecturing procedure:

i) it wastes the student's time if the lectures are repetitive of what is found in the assigned reading or textbooks, or if the lectures contain obsolete materials. This is most likely when the instructor is overburdened with a multitude of administrative responsibilities and community commitments or is too preoccupied with his own research to bring his lecture-notes up- to-date

ii) it gives the students no opportunity to express their reactions and is therefore less "democratic" than other procedures in teaching. This lack of class participation dampens the learner's motivation to learn and impedes learning progress

iii) it promotes the authoritarian role of instruction and. minimises the importance of the student's spirit of curiosity and scientific inquisitiveness. It discourages critical thinking and initiative. The result might turn the learner into a passive, apathetic individual; being satisfied to do minimal work necessary for passing the course

iv) it tends to widen the gap between the instructor and the students by setting them apart and on different levels in the classroom

v) it bores the students, especially when the instructor has a hypnotic, monotonous voice which lulls the class into sleep. This may result in distaste for learning on the part of the students.

The shortcomings of the lecturing method are quite serious and it’s not especially good for teaching mathematics. It, as an instructional device, is far from being able to realise the widely accepted objectives of mathematics education. The one-way process of formal lecturing is the source of all its demerits. It ignores all important principles of learning, from starting "where the students are" to "involving the students actively in the learning process". The lack of opportunity for students to receive feedback weakens motivation and blocks communication not only between the teacher and the learners, but also among the fellow-students.

Due to such misgivings or criticisms, the lecturing method has been modified in order to better fulfil its educational mission. The modifications aim at diluting the authoritarian nature of the "one-way" lecturing by allowing for some sort of interaction between the instructor and the class Thus, lectures are often interspersed by brief discussion periods or followed by such innovations as buzz groups or brainstorming sessions. In effect, the efficacy of lecturing depends largely upon who delivers the lectures, what information has to be imparted and how the instructor presents them. The well-prepared, sensitive, student-centred instructor, who is familiar with the background and individualised needs of his students, for example, may provide through his presentation, unique and lasting contributions to learning. He may select for emphasis points he knows to be needed by his students. He may invite questions and comments to ensure that the students have grasped the main theme of his lecture and that their desire to learn is at least sustained.

3.2 The Discussion Method

The discussion method owes its origin again to the Greek philosopher-educators, particularly Plato, who rebelled against the authoritarian type of lecturing system of the Sophists. This teaching device is basic to the democratic process and it involves an entire class in an extended interchange of ideas between the teacher and the learners and concurrently among the fellow-learners. Although the members in the class approach the discussion topic with many and varying points of view, the group leader, i.e. the instructor, will tend to focus the discussion in the direction most conducive to effective and purposeful learning. The growing emphasis on critical thinking and problem-solving in academic instruction accounts in part for the current increased attention to the discussion method as a medium of instruction.

Let us examine the characteristics of this instructional procedure, in the same manner as we did for the lecturing method, in order to assess its effectiveness.

The advantages of the method are first summarised for analysis:

i) it promotes interest by giving the students a share in the responsibility for the course and in search for knowledge. This compels the students to be active learners, and is quite contrary to the lecturing method
ii) it motivates the learners by keeping the work within their intellectual bounds and by allying it with their aptitude
iii) it enables the instructor to constantly appraise the students' understanding of the issues under discussion, as he leads the class into the higher levels of the course
iv) it sharpens the students' ideas and concepts by forcing them

to express them in their own words. This facilitates intellectual comprehension and application of new knowledge to life-situations. Integration of learning with experience will be brought about

v) it permits the students to challenge statements with which they disagree or which they misunderstand, thereby facilitating the process of self-discovery and self-understanding and developing the sense of self-assertiveness

vi) it develops in the students the skills essential to effective group discussion and verbal communication.

The discussion method forces the students out of their classroom lethargy, so that every learner will react either in support or in opposition to the issue under discussion. Each student will learn to feel free to express his opinions, to argue with mutual respect and to defend his own stand in the light of logic and rationality. This method is able to achieve more than the lecturing method in terms of the objectives of a college general education. It develops facility in oral expression, critical and creative thinking, and intellectual and imaginative problem-solving ability.

The discussion method sustains and strengthens most of the elements essential for productive learning. The provision of feedback and class participation heightens the learner's motivation, facilitates the intellectual grasp of abstract concepts and the learning of problem-solving skills. The movement in the group discussion sensitises the instructor to the class reactions to the learning situation, thus making it possible for him to pitch a level of learning appropriate to the readiness of the majority.

There is, however, a negative side to the discussion method as an instructional device.

i) It requires knowledge and skill of group dynamics and group handling in order to produce effective, orderly discussion. This skill takes time and practice to develop, and not every individual can become a good discussion leader even given time and practice.

ii) It makes more demands on the instructor as a group leader than as a lecturer. In the discussion, the role of the instructor varies from one of authority to one of a member of the group. How the instructor interprets his role in any situation will depend on his image of himself from which his personality derives and from which his actions follow. The assumption of an inappropriate role by the instructor will seriously negate the value of the discussion method.

iii) It depends substantially for its success on the cultural, psychological and experiential backgrounds of the learners. The free, open-minded interchange of ideas and opinions may not be evoked from the learners due to disadvantaged ethnical origin, emotional inhibition and inadequate life exposure.

In an actual discussion situation, any of the aforesaid drawbacks is likely to occur in varying degrees and in varying combination; the efficacy of the discussion method as a teaching tool will accordingly be adversely affected. If, for example, a discussion topic is poorly chosen, that is, it is not related to the discussants' background, experience and interest and it itself not controversial enough to evoke arguments from various angles, the discussion period will likely be characterised by a low level of intellectual exchange among the discussants, and interspersed with embarrassing pauses and silences. In case the instructor is an inefficient discussion leader, unable to direct and control the group, the discussion will probably end up in a disorderly, chaotic battle of verbal comments, adding more confusion to the students than before the discussion. Should the instructor take an authoritarian role, on the other hand, the entire class will be submitted to his despotic dominance and prejudices, with the resultant effect that the discussion is conducted in the instructor's interest rather than for the learning needs of the students. Under all these circumstances, little or no learning may take place.

3.3 The Tutorial Method

This method of teaching was said to have been invented by Socrates. It sprang from the character of the Greek people who loved asking questions and arguing them out. Socrates thought that teaching might mean, not pouring new ideas into an empty brain, but drawing out from the mind those ideas that lay concealed. This was done by asking the student a series of questions. In the process of answering questions on the part of the student, he was made to realise that knowledge and truth were in the student's own power to find, if he cares to search long and hard enough. It is in the combination of these two assumptions, namely: the critical method and the positive purpose of self assertion, where the essence of the tutorial method lies.

In our contemporary university or college teaching, tutorial, according to G. J. Umstattd, is instruction for an individual or at most for a small group of students of three, with special attention to personal interests and abilities. This aim for tutorial is to provide challenges and stimulation in order that the student may develop his optimal potentials. It is a two-way process in which the instructor, normally known as the tutor and the student engage in a rigorous intellectual exercise. A great amount of reading, thinking and independent work is done by the student, of course, with the assistance of the tutor. A written report of considerable quality will be submitted to the tutor prior to the tutorial session. The tutor challenges, criticises or stimulates the students during the regular tutorial sessions which are held at regular intervals. The merits of such an instructional device are summarised below.

i) It provides a means through which the learner's individualised needs, interests, aptitude and experience will be taken into account in the instructional process. This feature of the tutorial method is highly congruent with the ethics of our democratic ideology and with the educational principle that individualisation of the learner is respected.

ii) It permits the learner to acquire a gradual, orderly and sequential progression from a broad and relatively simple level to one of much greater depth and complexity. This is consonant with the learning principle of sequential progression elaborated by Raph W. Tyler.

iii) It makes possible immediate feedback, thereby promoting the learner's motivation; and the tutor's comments stimulate the learner's capacity for critical and analytical thinking.

iv) It encourages active, analytic, independent learning in the student. This is especially a highly valued contribution of higher education to an individual in the contemporary society. The constantly and rapidly changing technology and social conditions soon make obsolescent certain concepts that are valid a short time ago and requires a dynamic, imaginative approach to problem-solving.

v) It trains oral presentation, articulation and intellectual self-defence. Under the relentless verbal bombardment of the tutor during the one-hour tutorial session, the student is compelled to defend his assertions in the best manner his natural endowment, knowledge and humour permit him.

vi) It magnetises the learner's imagination and teaches the learner to examine any issue from a much broader perspective. This type of intellectual training is extremely conducive to achievement of integration of learning in the total curriculum.

vii) It enables the students to come to grasp more quickly with abstract concepts and complicated problem-solving skills.

It is apparent that the tutorial method, as a teaching tool, is highly favourable in assuring the realisation of the widely held objectives of a college general education, namely: acquisition of self-discovery and self-understanding, critical and analytical thinking, creative self-expression, intelligent application of knowledge to life-situations, and oral and written facility. It also measures well against the criteria necessary for an effective learning process. The tutor, by virtue of the one-to-one relationship with his student, is able to make appropriate demands on the learner in accordance with the latter's individualised

background, experience and potentials. The increased personal attention and contact from the tutor enables the learner to feel the tutor's concern for his learning, thus heightening or at least sustaining the learner's desire to learn. The source of all these advantages of the tutorial method stems from the very nature of its teaching method, i.e. the one-to-one relationship. However, it is this unique trait of the tutorial method that concurrently gives rise to some drawbacks for this teaching device.

i) It is an extremely expensive method in terms of money, time and effort. One tutor cannot ideally handle more than 5 students at a time. The quality of teaching of this instructional device inversely relates to the quantity of its output. In the contemporary countries where primary education is extended to as many of their citizens, the classroom situation cannot afford such an expensive teaching method. At the end of the tutorial session, both the tutor and the learner may be utterly exhausted.

ii) It may unnecessarily foster unhealthy identification of the learner with the tutor. The tutorial sessions provide a permanent structure where the tutor and the learner meet in privacy at weekly or fortnightly intervals. Soon the tutor and the learner get to know each other very well. Often out of this intensity of relationship, the learner becomes identified with his tutor for an array of reasons, from hero-worshipping of the charismatic idol to psychological mimicking of the oppressor out of the need for self-defence and survival. Thus, the learner acts, thinks and behaves like his tutor, imitating all his mannerisms and life style. However, the extent to which the tutor consciously or unconsciously creates situations which are favourable for converting the student into his own image, depends on the maturity of the tutor's personality, emotional make-up and also on his perception of his role in such educational endeavour.

iii) It is liable to provoke an enormous amount of anxiety in the learner; thereby retarding the latter's learning progress. The tutorial session is conducted under a one-to-one relationship imbued with an intense emotional overtone. The student, normally in an inferior intellectual position, lays bare his weaknesses and biases for the tutor to criticise and submits himself rather helplessly to the tutor's whims and personality idiosyncrasies. Under the pangs of anxiety, the learner develops various patterns of adaptation out of the need to survive the trauma. He learns quickly about the tutor's preferred school of thought, prejudices and likes, and attempts to shape his own thinking accordingly. Or the student may retreat to submissiveness and passivity, agreeing with all the

viewpoints
and arguments of the tutor. Some learners' level of anxiety is so high in the tutorial sessions that they are literally paralysed,

losing their capacity for oral and rhetoric expression. In short, the emotional tension and nervousness on the part of the learner inhibits the spirit of active, independent learning, the capacity for critical and creative thinking, and the ability of oral and debating facility, all of which the tutorial method, as a teaching device, is meant to develop in the learner.

In view of the impracticability and the demerits of the tutorial method, some modified forms of the tutorial system have been developed. None of these new innovations resembles the traditional tutorial in structural appearance. The one-to-one relationship has expanded to cover a group of students; but the original emphasis on independent and active learning, critical and analytical thinking, oral and written proficiency, and a deep and broad approach to a topic is all retained in varying degrees in these new teaching innovations. Some of these modified instructional activities are:

- small group tutorial, seminar, colloquium, panel presentation, and debate discussion.

a) Small Group Tutorial: this bears the closest structural resemblance to the traditional tutorial. In this method, the tutor takes a small group of students, about five to seven, in his periodic tutorial sessions, where each student takes turn to prepare and to present his written report orally and the other students, led by the tutor, will fire questions, comments and queries towards the student charged with the presentation

b) Seminar: this form of teaching-learning activity is usually restricted to the graduate students or to the seniors in the college. In the seminar, a student will take turn to deliver a paper on a particular topic, carefully written after much thorough reading and critical reflections. The other students will listen to the presentation, make notes of his references and follow up by further reading. The role of the instructor is to direct the discussion, in which all students take part. In both the small-group tutorial and the seminar, the instructor's presence and guidance are essential. However, in the colloquium, panel presentation and debate-discussion, the instructor's role is becoming less and less in significance and in defectiveness in nature, while the students are expected to take on more and more active and independent learning.

c) Colloquium: the students meet in a group and talk about their reading, confronting each other with challenges and questions, and probing into the multifarious facets of an issue with intent to seek depth and breadth in knowledge. The end product of such training may result that realisation of the art of conversation, a sense of style in speaking, and the interplay of poise, gravity, humour and controversy will be achieved by the students.

d) Panel-Presentation: a group of students, typically from 3 to 6, form a panel. Each panelist prepares a sub-part in a subject in some depth and then presents it before the class. The panel chairman participates to keep the discussion to the point, to invite silent class members to talk or to give an occasional summary to suggest how the discussion has progressed. The purpose is to provide an opportunity for the panelists to learn rhetoric speaking in front of a group and to practice intellectual defence.

e) Debate-discussion: debate-discussion is related to the panel-presentation, but with this method, two or more speakers usually take definite points of view, present their opinions and facts, and participate later by responding to questions and comments from others in the class. Quickness and sharpness in thinking and the art of exposition and discourse are the fruits of such learning activity.

3.4 Cooperative Group Learning

Cooperative group learning allows studentsto work through mathematics equations in a small group setting. To implement this teaching method, arrange students in groups of four to six individuals. After the basic information and assignment has been presented to the class, allow students to break off into their groups and work through the assigned equations. Move about the classroom to interact with each group as they work on their projects and hold a presentation period at the end of the lesson for each group to present its conclusions.

3.5 Laboratory Method

Kinesthetic learners excel in environments where they can physically manipulate objects. Incorporate kinesthetic learning into mathematics instruction by holding mathematics labs. Within the lab setting, give students tangible objects to work through mathematics equations and test theories. Teacher creativity is vital to teaching mathematics via the laboratory method, as you will be required to develop projects for the students to take on. Some examples may include using tiles for basic addition or using toothpicks for examining the principles of geometry.

3.6 Exposition Method

Use this method to quickly and effectively expose students to new mathematical information. Begin by presenting students with a clearly defined explanation of the concept you are teaching,working sequentially through the steps required to complete an equation.

Facilitate discussion with the class by posing questions based on information the students have already learned and build on their prior knowledge. Although this method is focused primarily on the instructor rather than student participation, you can help engage students by presenting the information in a lively manner.

3.7 Guided Discovery

Guided discovery is the opposite of the exposition method, as it requires students to recognise principles independently. To begin a lesson using guided discovery, present students with a series of equations or scenarios that are similar to one another. Ask students to identify how to best solve the equations or to identify which mathematical concepts apply to each scenario. This method acts as an effective review for material that has already been taught. However, some students may become frustrated if they are unable to discover the answers on their own. Be sure to make yourself available during independent working time to help students on an individual basis.

4.0 CONCLUSION

When measured against the two criteria, namely : (i) the objectives of teaching, and (ii) the common factors influencing learning, each of the three widely used teaching methods - lecture, discussion and tutorial - is able to meet part, not the whole, of the two criteria. One method may satisfy one of the criteria more than the other method. It is also apparent from experience that none of the three teaching methods is used in its pure form. Each is supplemented by newer instructional innovations.

For instance, formal lecturing is not predominant in primary school. It is augmented by discussion periods from the class, buzz groups and brainstorming sessions. Discussion method is made more productive by introducing some forms of lecture-presentation which precedes the discussion itself. Likewise, the tutorial has transformed into a variety of activities. In some of them, the teacher retains his classical role as the "charismatic leader" and "intellectual inspirer", while in others, his role has the "peer" reduced to one in the discussion. The supplementation by new auxiliary methods bespeaks that each of the three traditional teaching methods is not perfect intrinsically and that each is good for certain purpose.

5.0 SUMMARY

These three instructional devices are all equally good for different purposes, and a good education exposes the pupil to them all. Each of them has its difficulties and its defects; each of them contains unique advantages. A teacher who uses only one method is in danger of developing one group of skills in his pupils and only part of his own powers as an educator. A pupil who knows only one way of learning will find it hard to conceive what rich possibilities lie unused in his own mind. All three are useful for some purposes, bad for others; all are valuable.

There is no "one most preferred method of teaching". Whether a particular instructional medium is efficient or productive has to be assessed in the light of (a) the student's needs, (b) the teacher's personality and skills as an educator, and (c) the administration and the broad purpose of the classroom where the method is being employed.

The students are human beings, with different socio-cultural background, intellectual and emotional endowments, educational aspirations and experience. All these will predetermine, in great measure, the individual student's reaction to and perception of the teaching-learning activity.

The teacher is also a human being, unique in physical and psychological constitution, personality characteristics and social experiences. One teacher may be talented in utilising one particular group of teaching techniques and methods, but very inadequate in others. Therefore, it is express that the effectiveness of a particular instructional device will be greatly affected by the teacher's skills and personality. Teachers teach well by many different methods. No one method is clearly superior in all situations. Elements of course content, background and group make-up of the students and the teacher, and relationship with the administration, bear significance to which teaching method is to choose.

6.0 TUTOR-MARKED ASSIGNMENT

i. Identify three main teaching methods and distinguish among them.
ii. Discuss teaching methods which can serve as adjunct to other teaching methods and explain the ways by which they can be employed together.

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UNIT 3 FACILITATING DEVELOPMENT OF MATHEMATICAL KNOWLEDGE FOR TEACHING

CONTENTS

1.0 INTRODUCTION

Metacognition is one of the latest buzz words in educational psychology. We engage in metacognitive activities every day. Metacognition enables us to be successful learners, and has been associated with intelligence (e.g., Borkowski, Carr, & Pressley, 1987; Sternberg, 1984, 1986a, 1986b). Metacognition refers to a higher order thinking which involves active control over the cognitive processes engaged in learning. Activities such as planning how to approach a given learning task, monitoring comprehension, and evaluating progress toward the completion of a task are metacognitive in nature. Because metacognition plays a critical role in successful learning, it is important to study metacognitive activity and development to determine how students can be taught to better apply their cognitive resources through metacognitive control.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain what metacognitive is all about
- state the critical elements of metacognitive
- implement the strategy in mathematics classroom
- apply the strategy to help pupils' with learning difficulties.

3.0 MAIN CONTENT

3.1 Metacognitive Strategies

The purpose of teaching metacognitive strategies is to provide students explicit teacher instruction for a specific metacognitive (learning) strategy.

3.1.1 What are they?

- First, a metacognitive strategy is a memorable "plan of action" that provides students an easy to follow procedure for solving a particular mathematics problem.
- Second, metacognitive strategies are taught using explicit teaching methods.
- Metacognitive strategies include the student's thinking as well as their physical actions.
- Some of the most common metacognitive strategies come in the form of mnemonics which are meaningful words where the letters in the word each stand for a step in a problem-solving process or for important pieces of information about a particular topic of interest.
- Metacognitive strategies are memorable and it must accurately represent the learning task.

3.1.2 What are the Critical Elements of this Strategy?

The following list includes critical elements of teaching metacognitive strategies:

- metacognitive strategies are taught using explicit teaching methods (see Explicit Teacher Modeling)
- metacognitive strategies are accurate and efficient procedures for specific mathematics problem-solving situations.
- metacognitive strategies are memorable
- metacognitive strategies incorporate both student thinking and student actions necessary for performing target mathematics skill
- students need ample practice opportunities to master use of a metacognitive strategy.
- students' memory of a metacognitive strategy is enhanced when students are provided with individual strategy cue sheets and/or when the metacognitive strategy is posted in the classroom
- monitor students' use of strategies and reinforce their appropriate use of strategies.

3.1.3 How do I implement the Strategy?

1. Choose an appropriate metacognitive strategy for the mathematics skill.
2. Describe and model the strategy at least three times. Use those instructional components emphasised in explicit teacher modeling (see the instructional strategy Explicit Teacher Modeling.)
3. Check student understanding. Ensure they understand both the strategy and how to use it.
4. Provide ample opportunities for students to practice using the strategy.
5. Provide timely corrective feedback and remodel use of strategy as needed.
6. Provide students with strategy cue sheets (or post the strategy in the classroom) as students begin independently using the strategy. Fade the use of cues as students demonstrate they have memorised the strategy and how (as well as when) to use it. (*some students will benefit from a "strategy notebook" in which they keep both the strategies they have learned and the corresponding mathematics skill they can use each strategy for.)
7. Make a point of reinforcing students for using the strategy appropriately.
8. Implicitly model using the strategy when performing the corresponding mathematics skill in class.

3.2 How Does Metacognitive Instructional Strategy Positively Impact Students who have Learning Problems?

- It provides students an efficient way to acquire, store, and express mathematics-related information and skills.
- It provides students who have memory problems an efficient way to retrieve from memory information they have learned.
- It facilitates independence by those learners who are typically dependent on high levels of teacher support.
- It helps students move from concrete and representational understanding to abstract understanding.

We turn our attention to studies that not only identify deficiencies in teacher knowledge but also carry out interventions to remedy them. Jaworski (2001) details the nature of the teacher educator action as facilitating the connection between theory and practice by developing effective activities that, in turn, promote teachers' ability to create effective mathematical activities for their own students. In a more specific way, Cooney and Wiegel (2003) propose three principles for teaching teachers mathematics, addressing the kinds of mathematical experiences that promote an open and process-oriented approach to teaching, suggesting that pre-service teachers should: (i) experience mathematics as a pluralistic subject; (ii) explicitly study and reflect on school mathematics and (iii) experience mathematics in ways that foster the development of process-oriented teaching styles. Cramer (2004), influenced by NCTM standards, also provides a pedagogical model to frame mathematics courses for teachers which consists of:

- mathematics content is embedded in problem settings; learners collect data, generate hypotheses, and verify conjectures
- learners work in small groups to optimise the opportunity for discourse
- questions are posed to help learners construct mathematical knowledge
- learners' language (oral and written) plays an important role in facilitating the transition from problem solving and exploration to formal mathematical abstractions
- connections within and among mathematical topics are emphasised
- technology use is integrated into the daily activities of the course.

These three examples of how we can approach pre-service teachers' learning of mathematics for teaching support the view that there is no clear-cut approach to addressing the concerns about pre-service teachers' mathematics knowledge.

4.0 CONCLUSION

Metacognition enables students to benefit from instruction and influences the use and maintenance of cognitive strategies. While there are several approaches to metacognitive instruction, the most effective involve providing the learner with both knowledge of cognitive processesand strategies (to be used as metacognitive knowledge), and experience or practice in using both cognitive and metacognitive strategies and evaluating the outcomes of their efforts (develops metacognitive regulation). Simply providing knowledge without experience or vice versa does not seem to be sufficient for the development of metacognitive control. The study of metacognition has provided educational psychologists with insight about the cognitive processes involved in learning and what differentiates successful students from their less successful peers. It also holds several implications for instructional interventions, such as teaching students how to be more aware of their learning processes and products as well as how to regulate those processes for more effective learning.

5.0 SUMMARY

Stated very briefly, knowledge of person variables refers to general knowledge about how human beings learn and process information, as well as individual knowledge of one's own learning processes. Metacognitive experiences involve the use of metacognitive strategies or metacognitive regulation. Metacognitive strategies are sequential processes that one uses to control cognitive activities, and to ensure that a cognitive goal (e.g., understanding a text) has been met. These processes help to regulate and oversee learning, and consist of planning and monitoring cognitive activities, as well as checking the outcomes of those activities

6.0 TUTOR-MARKED ASSIGNMENT

i. Explain the concept "metacognitive strategy"
ii. What are the critical elements of metacognitive strategy?
iii. Pick a mathematics topic and explain how you will use metacognitive strategy to teach it.

7.0 REFERENCES/FURTHER READING

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MODULE 4 ASSESSMENT IN MATHEMATICS EDUCATION AND BASIC MATHEMATICS PROPERTIES

UNIT 1 CLASSROOM ASSESSMENT TECHNIQUE

CONTENTS

1.0 INTRODUCTION

Assessment is important because of the decisions you will make about children when teaching and caring for them. You will be called upon every day to make decisions before, during, and after your teaching. Whereas some of these decisions will seem small and inconsequential, others will be "high stakes," influencing the life course of children. All of your assessment decisions taken as a whole will direct and alter children's learning outcomes. Assessment can enhance your teaching and students' learning and if you use assessment procedures appropriately, you will help all children learn well.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state the characteristics of classroom assessment
- mention the assumptions on which classroom assessment is based upon and distinguish between them.

3.0 MAIN CONTENT

3.1 Classroom Assessment

In the 1990's, educational reformers are seeking answers to two fundamental questions: (1) how well are students learning? (2) how effectively are teachers teaching? Classroom research and classroom assessment respond directly to concerns about better learning and more effective teaching. Classroom research was developed to encourage teachers to become more systematic and sensitive observers of learning as it takes place every day in their classrooms. Teachers have an exceptional opportunity to use their classrooms as laboratories for the study of learning and through such study to develop a better understanding of the learning process and the impact of their teaching upon it. Classroom assessment, a major component of classroom research, involves student and teachers in the continuous monitoring of students' learning. It provides teacher with feedback about their effectiveness, and it gives students a measure of their progress as learners. Most important, because classroom assessments are created, administered, and analysed by teachers themselves on questions of teaching and learning that are important to them, the likelihood that instructors will apply the results of the assessment to their own teaching is greatly enhanced.

Through close observation of students in the process of learning, the collection of frequent feedback on students' learning, and the design of modest classroom experiments, teachers can learn much about how students learn and, more specifically, how students respond to particular teaching approaches. Classroom assessment helps individual teachers obtain useful feedback on what, how much, and how well their students are learning. Teachers can then use this information to refocus their teaching to help students make their learning more efficient and more effective.

Teachers, who usually assume that their students are learning what they are trying to teach them, are regularly faced with disappointing evidence to the contrary when they mark tests and at the end of the term. Too often, students have not learned as much or as well as was expected. There are gaps, sometimes considerable ones, between what was taught and what has been learned. By the time teachers notice these gaps in knowledge or understanding, it is frequently too late to remedy the problems.

To avoid such unhappy surprises, teachers and students need better ways to monitor learning throughout the term. Specifically, teachers need a continuous flow of accurate information on students' learning. For example, if a teacher's goal is to help students learn points "A" through "Z" during the course, then that teacher needs first to know whether all students are really starting at point "A" and, as the teaching proceeds, whether they have reached intermediate points "B," "G," "L," "R," "W," and so on. To ensure high-quality learning, it is not enough to test students when the syllabus has arrived at points "M" and "Z." Classroom assessment is particularly useful for checking how well students are learning at those initial and intermediate points, and for providing information for improvement when learning is less than satisfactory.

Through practice in classroom assessment, teachers become better able to understand and promote learning, and increase their ability to help the students themselves become more effective, self-assessing and self-directed learners. Simply put, the central purpose of classroom assessment is to empower both teachers and their students to improve the quality of learning in the classroom.

Classroom assessment is an approach designed to help teachers find out what students are learning in the classroom and how well they are learning it. This approach has the following characteristics:

- **Learner-Centred**

Classroom assessment focuses the primary attention of teachers and students on observing and improving learning, rather than on observing and improving teaching. Classroom assessment can provide information to guide teachers and students in making adjustments to improve learning.

- **Teacher-Directed**

Classroom assessment respects the autonomy, academic freedom, and professional judgement of teachers. The individual teacher decides what to assess, how to assess, and how to respond to the information gained through the assessment. Also, the teacher is not obliged to share the result of classroom assessment with anyone outside the classroom.

- **Mutually Beneficial**

Because it is focused on learning, classroom assessment requires the active participation of students. By cooperating in assessment, students reinforce their grasp of the course content and strengthen their own skills at self-assessment. Their motivation is increased when they realise that teachers are interested and invested in their success as learners. Teachers also sharpen their teaching focus by continually asking themselves three questions: "what are the essential skills and knowledge I am trying to teach?" "How can I find out whether students are learning them?" "How can I help students learn better?"

As teachers work closely with students to answer these questions, they improve their teaching skills and gain new insights.

- **Formative**

The purpose of classroom assessment is to improve the quality of student learning, not to provide evidence for evaluating or grading students. The assessments are almost never graded and are almost always anonymous.

Context-Specific

Classroom assessments have to respond to the particular needs and characteristics of the teachers, students, and disciplines to which they are applied. What works well in one class will not necessary work in another?

Ongoing

Classroom assessment is an ongoing process, best thought of as the creating and maintenance of a classroom "feedback loop." By using a number of simple classroom assessment techniques that are quick and easy to use, teachers get feedback from students on their learning. Teachers then complete the loop by providing students with feedback on the results of the assessment and suggestions for improving learning. To check on the usefulness of their suggestions, teachers use classroom assessment again, continuing the "feedback loop." As the approach becomes integrated into everyday classroom activities, the communications loop connecting teachers and students-and teaching and learning- becomes more efficient and more effective.

3.2 Classroom Assessment and Good Teaching Practice

Classroom assessment is an attempt to build on existing good practice by making feedback on students' learning more systematic, more flexible, and more effective. Teachers already ask questions, react to students' questions, monitor body language and facial expressions, read homework and tests, and so on. Classroom assessment provides a way to integrate assessment systematically and seamlessly into the traditional classroom teaching and learning process

As they are teaching, teachers monitor and react to student questions, comments, body language, and facial expressions in an almost automatic fashion. This "automatic" information gathering and impression formation is a subconscious and implicit process. Teachers depend heavily on their impressions of student learning and make important judgments based on them, butthey rarely makethose informal assessments explicit or check them against the students' own impressions or ability to perform. In the course of teaching, teachers assume a great deal about their students' learning, but most of their assumptions remain untested.

Even when teachers routinely gather potentially useful information on students' learning through questions, quizzes, homework, and exams, it is often collected too late-at least from the students' perspective-to affect their learning. In practice, it is very difficult to "de-program" students who are used to thinking of anything they have been tested and graded on as being "over and done with." Consequently, the most effective times to assess and provide feedback are before starting a new topic or the midterm and final examinations. Classroom assessment aims at providing that early feedback. Classroom assessment is based on seven assumptions:

1. the quality of students' learning is directly, although not exclusively, related to the quality of teaching. Therefore, one of the most promising ways to improve learning is to improve teaching
2. to improve their effectiveness, teachers need first to make their goals and objectives explicit and then to get specific, comprehensible feedback on the extent to which they are achieving those goals and objectives
3. to improve their learning, students need to receive appropriate and focused feedback early and often; they also need to learn how to assess their own learning
4. the type of assessment most likely to improve teaching and learning is that conducted by teachers to answer questions they themselves have formulated in response to issues or problems in their own teaching
5. systematic inquiry and intellectual challenge are powerful sources of motivation, growth, and renewal for teachers, and classroom assessment can provide such challenge
6. classroom assessment does not require specialised training; it can be carried out by dedicated teachers from all disciplines
7. by collaborating with colleagues and actively involving students in classroom assessment efforts, faculty (and students) enhance learning and personal satisfaction.

To begin classroom assessment, it is recommended that only one or two of the simplest classroom assessment techniques are tried in only one class. In this way, very little planning or preparation time and energy of the teacher and students is risked. In most cases, trying out a simple classroom assessment technique will require only five to ten minutes of class time and less than an hour of time out of class. After trying one or two quick assessments, the decision as to whether this approach is worth further investments of time and energy can be made. This process of starting small involves three steps.

Step 1: Planning

Select one, and only one, of your classes in which to try out the classroom assessment. Decide on the class meeting and select a classroom assessment technique. Choose a simple and quick one.

Step 2: Implementing

Make sure the students know what you are doing and that they clearly understand the procedure. Collect the responses and analyse them as soon as possible.

Step 3: Responding

To capitalise on time spent assessing, and to motivate students to become actively involved, "close the feedback loop" by letting them know what you learned from the assessments and what difference that information will make.

Five Suggestions for a Successful Start:

1. if a classroom assessment techniques does not appeal to your intuition and professional judgement as a teacher, do not use it
2. do not make classroom assessment into a self-inflicted chore or burden
3. do not ask your students to use any classroom assessment technique you have not previously tried on yourself
4. allow for more time than you think you will need to carry out and respond to the assessment
5. make sure to "close the loop." Let students know what you learn from their feedback and how you and they can use that information to improve learning.

4.0 CONCLUSION

The aim of assessment is to improve students' performance and not merely to audit it. Assessment should be learner-centered and focused on students' achievement in relation to the goals of a course. Rather than being separate from learning, assessment plays a central role in the instructional process.

5.0 SUMMARY

Assessment helps teachers develop more complex relationships with their students by providing concrete pieces of work for students and teachers to discuss, as well as opportunities for formal and informal conversations about the work. Similarly, students work closely with each other providing and receiving feedback on their work.

Assessment helps students answer the questions "Am I getting it?" and "How am I doing?" Early and frequent feedback from the teacher, peers, and mentors will also provide students with the practice and the knowledge to better assess themselves and find answers to these questions.

Assessment can help make content connections clear. Teachers prompt students to make connections between their work and other subject matter.

Assessment also sheds light on which methods of instruction are most effective. Through assessment, a teacher gains the requisite information for choosing and utilising those teaching strategies that best help a learner progress towards the goals of a course.

6.0 TUTOR-MARKED ASSIGNMENT

i. Why is classroom assessment important?
ii. Discuss the assumptions on which classroom assessment is based.

7.0 REFERENCE/FURTHER READING

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UNIT 2 PURPOSE AND TOOLS OF ASSESSMENT

CONTENTS

1.0 INTRODUCTION

The popular conception of assessment is restricted to evaluating individual student's performance by tests designed to determine, at the end of a unit of time or instruction, what the student has already learned. But assessment should also be used during the learning process to enable teachers to monitor students' understanding and to modify curriculum and instruction, as well as to assess the effectiveness of school programs. Assessment of an individual student's performance should be a continuous process that involves many types of assessment activity. Students should play active roles in assessment so that each assessment experience is also an educational experience.

2.0 OBJECTIVES

At the end of the unit, you should be able to:

- state the purpose of assessment
- distinguish between the content principle, the learning principle and the equity principle as they relate to mathematics teaching and learning.

3.0 MAIN CONTENT

3.1 Purpose of Assessment

It is important to establish at the outset that the major purpose of assessment is to promote learning. The assessment is not the goal, but a means to achieve a goal. Three fundamental educational principles which form the foundation of all assessment that supports effective education (measuring what counts) are:

- **The Content Principle:** this suggests that assessment should reflect the mathematics that is most important for students to learn.
- **The Learning Principle**: this suggests that assessment should enhance mathematics learning and support good instructional practice.
- **The Equity Principle**: this suggests that assessment should support every student's opportunity to learn important mathematics.

New Jersey's mathematics standard states that experiences will be such that all students:

(i) are engaged in assessment activities that function primarily to improve learning
(ii) are engaged in assessment activities based upon rich, challenging problems from mathematics and other disciplines
(iii) are engaged in assessment activities that address the content of the curriculum

The content principle, the learning principle, and the equity principle were incorporated into the first three of the six assessment standards in the NCTM *Assessment Standards for School Mathematics* (2000).

(i) Assessment should reflect the mathematics that all students need to know and be able to do.
(ii) Assessment should enhance mathematics learning.

Assessments should be learning opportunities as well as opportunities for students to demonstrate what they know and can do. Although assessment is done for a variety of reasons, its main goal is to improve students' learning and inform teachers as they make instructional decisions. As such, it should be a routine part of ongoing classroom activity rather than an interruption.

(i) Assessment should promote equity

Assessment should be a means of fostering growth toward high expectations rather than a filter used to deny students the opportunity to learn important mathematics. In an equitable assessment, each student has an opportunity to demonstrate his or her matheematical power; this can only be accomplished by providing multiple approaches to assessment, adaptations for bilingual and special education students, and other adaptations for students with special needs. Assessment is equitable when students have access to the same accommodations and modifications that they receive in instruction.

(ii) Assessment should be an open process

Three aspects of assessment are involved here. First, information about the assessment process should be available to those affected by it, the students. Second, teachers should be active participants in all phases of the assessment process. Finally, the assessment process should be open to scrutiny and modification.

(iii) Assessment should promote valid inferences about mathematics learning

A valid inference is based on evidence that is adequate and relevant. The amount and type of evidence that is needed depends upon the consequences of the inference. For example, a teacher may judge students' progress in understanding place value through informal interviews and use this information to plan future classroom activities.

(iv) Assessment should be a coherent process

Three types of coherence are involved in assessment. First, the phases of assessment must fit together. Second, the assessment must match the purpose for which it is being conducted.

Finally, the assessment must be aligned with the curriculum and with instruction.

These principles should be kept in mind as changes in assessment strategies are contemplated, developed, tested, and implemented. They should be kept in mind by classroom teachers and all others involved in assessment — for example, local education authority committees selecting a standardised norm-referenced test, local education inspectors or school headmasters analysing data from a collection of students' portfolios, and mathematics curriculum planners reviewing proposed test items for the state wide tests.

4.0 CONCLUSION

Assessment plays a crucial role in the education process. It determines much of the work students undertake (possibly all in the case of the most strategic student), and affects their approach to learning. It can be argued that assessment is an indication of which aspects of the course are valued most highly. The assessment of outcomes provides the feedback necessary to make sound educational decision.

5.0 SUMMARY

Assessment is supported by the content principle**,** the learning principle and equity principle. Assessment should reflect the mathematics which all students need to know and be able to do, should enhance mathematics learning, should be an open process, should promote valid inferences about mathematics learning and should be a coherent process.

6.0 TUTOR-MARKED ASSIGNMENT

i. Identify three fundamental educational principles which form the foundation of all assessment that supports effective education.
ii. List areas in mathematics that assessment should support.

7.0 REFERENCES/FURTHER READING

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UNIT 3 BASIC NUMBER PROPERTIES: ASSOCIATIVE, COMMUTATIVE AND DISTRIBUTIVE

CONTENTS

1.0 INTRODUCTION

There are three basic properties of numbers, and your textbook will probably have just a little section on these properties, somewhere near the beginning of the course, and then you will probably never see them again (until the beginning of the *next* course). My impression is that covering these properties is a holdover from the "New mathematics" fiasco of the 1960s. While the topic will become relevant in matrix algebra and calculus (and become amazingly important in advanced mathematics, a couple years after calculus), they really do not matter a whole lot now.

Why not? Because every mathematics system you have ever worked with has obeyed these properties! You have never dealt with a system where $a\times b$ did not in fact equal $b\times a$, for instance, or where $(a\times b)\times c$ did not equal $a\times(b\times c)$. That is why the properties probably seem somewhat pointless to you. Do not worry about their "relevance" for now; just make sure you can keep the properties straight so you can pass the next test. The lesson below explains how I kept track of the properties.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- state the distributive property of numbers
- apply the distributive property to solve arithmetic and simple
- algebraic problems
- state the associative property of numbers

- apply the associative property to solve arithmetic and simple algebraic problems
- state the commutative property of numbers
- apply the commutative property to solve arithmetic and simple algebraic problems.

3.0 MAIN CONTENT

3.1 Distributive Property

The distributive property is easy to remember, if you recall that "multiplication *distributes* over addition". Formally, they write this property as "$a(b + c) = ab + ac$". In numbers, this means, that $2(3 + 4) = 2\times3 + 2\times4$. Any time they refer in a problem to using the distributive property, they want you to take something through the parentheses *(or factor something out)*; any time a computation depends on multiplying through a parentheses *(or factoring something out)*, they want you to say that the computation used the distributive property.

Why is the following true? $2(x + y) = 2x + 2y$

Since they distributed through the parentheses, this is true **by the Distributive Property.**

Use the Distributive Property to rearrange: $4x – 8$

The distributive property either takes something through a parenthesis or else factors something out. Since there are not any parentheses to go into, you must need to factor out of. Then the answer is "**By the Distributive Property, $4x – 8 = 4(x – 2)$**".

"But wait!"you say "the distributive Property says multiplication distributes over *addition,* not *subtraction!* What gives?" You make a good point. This is one of those times when it is best to be flexible. You can either view the contents of the parentheses as the subtraction of a positive number ("$x – 2$") or else as the addition of a negative number ("$x + (–2)$"). In the latter case, it is easy to see that the distributive property applies, because you are still adding; you are just adding a negative.

The other two properties come in two versions each: one for addition and the other for multiplication. (Note that the distributive property refers to both addition and multiplication too, but to both within just one rule).

3.2 Associative Property

The word "associative" comes from "associate" or "group". The Associative Property is the rule that refers to grouping. For addition, the rule is "$a + (b + c) = (a + b) + c$"; in numbers, this means 2 + (3 + 4) = (2 + 3) + 4. For multiplication, the rule is "$a(bc) = (ab)c$"; in numbers, this means 2(3×4) = (2×3)4. Any time they refer to the associative property, they want you to regroup things; any time a computation depends on things being regrouped, they want you to say that the computation uses the associative property.

Rearrange, using the Associative Property: 2(3*x*)

They want you to regroup things, not simplify things. In other words, they do not want you to say "6*x*". They want to see the following regrouping: **(2×3)*x*.**

Simplify 2(3*x*), and justify your steps.

In this case, they *do* want you to simplify, but you have to tell why it is okay to do just exactly what you have *always* done. Here is how this works:

2(3*x*)	**original (given) statement**
(2×3)*x*	**by the Associative Property**
6*x*	**simplification (2×3 = 6)**

Why is it true that 2(3*x*) = (2×3)*x*?

Since all they did was regroup things, this is true by the Associative Property.

3.3 Commutative Property

The word "commutative" comes from "commute" or "move around", so the commutative property is the one that refers to moving stuff around. For addition, the rule is "$a + b = b + a$"; in numbers, this means 2 + 3 = 3 + 2. For multiplication, the rule is "$ab = ba$"; in numbers, this means 2×3 = 3×2. Any time they refer to the commutative property, they want you to move stuff around; any time a computation depends on moving stuff around, they want you to say that the computation uses the commutative property.

Use the Commutative Property to restate "3×4×*x*" in at least two ways.

They want you to move stuff around, not simplify. In other words, the answer is not "12*x*"; the answer is any two of the following:

4 × 3 × *x*, 4 × *x* × 3, 3 × *x* × 4, *x* × 3 × 4, and *x* × 4 × 3

Why is it true that 3(4*x*) = (4*x*)(3)?

Since all they did was move stuff around (they did not regroup), this is true by the Commutative Property.

3.4 Worked Examples

Simplify 3*a* – 5*b* + 7*a*. Justify your steps.

I am going to do the exact same algebra I have always done, but now I have to give the name of the property that says it is okay for me to take each step. The answer looks like this:

3*a* – 5*b* + 7*a*	**original (given) statement**
3*a* + 7*a* – 5*b*	**Commutative Property**
(3*a* + 7*a*) – 5*b*	**Associative Property**
a*(3 + 7) – 5*b	**Distributive Property**
a*(10) – 5*b	**simplification (3 + 7 = 10)**
10*a* – 5*b*	**Commutative Property**

The only fiddly part was moving the "– 5*b*" from the middle of the expression (in the first line of the table above) to the end of the expression (in the second line). If you need help keeping your negatives straight, convert the "– 5*b*" to "+ (–5b)". Just do not lose that minus sign!

Simplify 23 + 5*x* + 7*y* – *x* – *y* – 27. Justify your steps.

23 + 5*x* + 7*y* – *x* – *y* – 27	**original (given) statement**
23 – 27 + 5*x* – *x* + 7*y* – *y*	**Commutative Property**
(23 – 27) + (5*x* – *x*) + (7*y* – *y*)	**Associative Property**
(–4) + (5*x* – *x*) + (7*y* – *y*)	**simplification (23 – 27 = –4)**
(–4) + *x*(5 – 1) + *y*(7 – 1)	**Distributive Property**
–4 + *x*(4) + *y*(6)	**simplification**
–4 + 4*x* + 6*y*	**Commutative Property**

Simplify $3(x + 2) - 4x$. Justify your steps.

$3(x + 2) - 4x$	**original (given) statement**
$3x + 3\times2 - 4x$	**Distributive Property**
$3x + 6 - 4x$	**simplification ($3\times2 = 6$)**
$3x - 4x + 6$	**Commutative Property**
$(3x - 4x) + 6$	**Associative Property**
$x(3 - 4) + 6$	**Distributive Property**
$x(-1) + 6$	**simplification ($3 - 4 = -1$)**
$-x + 6$	**Commutative Property**

4.0 CONCLUSION

This unit has treated: the distributive property of numbers; how to apply the distributive property to solve arithmetic and simple algebraic problems, and how to apply the commutative property to solve arithmetic and simple algebraic problems.

5.0 SUMMARY

In this unit, you have learnt how to state the distributive property of numbers, how to apply the distributive property to solve arithmetic and simple algebraic problems, and how to apply associative property to solve arithmetic and simple algebraic problems, etc.

6.0 TUTOR-MARKED ASSIGNMENT

i. Why is it true that $3(4 + x) = 3(x + 4)$?
ii. Why is $3(4x) = (3\times4)x$?
iii. Why is $12 - 3x = 3(4 - x)$?

7.0 REFERENCES/FURTHER READING

Basic Number Properties: Associative, Commutative, and Distributive. http://www.purplemathematics.com/modules/numbprop.htm

Stapel, Elizabeth (2011). 'Basic Number Properties: Associative, Commutative, and Distributive. Purple Mathematics'. Availablefrom http://www.purplemathematics.com/modules/numbprop.htm.

UNIT 4 OTHER NUMBER PROPERTIES: IDENTITIES, INVERSES, SYMMETRY

CONTENTS

1.0 INTRODUCTION

We need to know that "the identity" is whatever does not change your number at all, and "the inverse" is whatever turns your number into the identity. For addition, "the identity" is zero, because adding zero to anything does not change anything. The "inverse" is the additive inverse: it is the same number, but with the opposite sign. For instance, suppose your number is –6, and you are adding. The identity is zero, and the inverse is 6, because –6 + 6 = 0.

For multiplication, "the identity" is one, because multiplying by one does not change anything. The "inverse" is the multiplicative inverse: the same number, but on the opposite side of the fraction line. For instance, suppose your number is –6, and you are multiplying. The identity is one, and the inverse is –1/6, because (–6)(–1/6) = 1.

You also know (if you have done any equation solving) that you can do anything you want to an equation, as long as you do the same thing to both sides. This is the "property of equality".

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- apply trichotomy law, transitive (moving across) property, the reflexive property, the symmetric property, the additive identity and the multiplicative inverse to solve mathematical problem.

The basic fact that you need for solving many equations, especially quadratics, is that, if $p \times q = 0$, then must have either $p = 0$ or else

$q =$
0. The only way you can multiply two things and end up with zero is if

one (or both) of those two things was zero to start with. This is the "zero-product property". And there are some properties that you use to solve word problems, especially where substitution is required. Anything equals itself: this is the "reflexive" (reflecting onto itself) property. Also, it does not matter which order the equality is in; if $x = y$, then $y = x$: this is the "symmetric" (they match) property. You can "cut out the middleman", so to speak; if $x = y$ and $y = z$, then you can say that $x = z$: this is the "transitive" (moving across) property. Two numbers are either equal to each other or unequal; this is the "trichotomy" law (so called because there are three cases for two given numbers, $a < b$, $a = b$, or $a > b$). And you can plug in for variables, so if $x = 3$, then $4x = 12$, because $4x = 4(3)$: this is the "substitution" property.

Here are some examples. Note: textbooks vary somewhat in the names they give these properties; you will need to refer to the examples in your book to know the exact format you should use.

3.0 MAIN CONTENT

3.1 Determine which Property was Used

$1 \times 7 = 7$

They multiplied, and they did not change anything: **the multiplicative identity.**

$-7y = -7y$

This is obvious: anything equals itself. They used **the reflexive property.**

If $10 = y$, then $y = 10$.

When solving an equation, I might rearrange things so I end up with the variable on the left. But I only switched sides; I did not actually change anything: **the symmetric property.**

$x + 0 = x$

They added, and they did not change anything: **the additive identity. If $2(a$**

$+ b) = 3c$, and $a + b = 9$, then $2(9) = 3c$.

You might be confused here between the transitive property and the substitution property. If you look closely, what they did was substitute "9" for "$a + b$", so they used **the substitution property.**

2 = *x*, so 2 + 5 = *x* + 5

They did the backwards of solving an equation, but the point is that they were working with an equation. They changed the equation by adding equal things to both sides: **the additive property of equality.**

If *x* + 2 = 10, then *x* + 2 + (–2) equals what, and why?

They solved the equation by getting rid of the 2 from both sides. Since they added the same thing to both sides, they got ***x* = 8 by the additive property of equality.**

(*x* – 3)(*x* + 4) = 0, so *x* = 3 or *x* = –4.

They set the quadratic equal to zero, factored, and then solved each factor: **the zero-product property.**

4*x* = 8, so *x* = 2

They solved the equation by dividing both sides by 4, or, which is the same thing, multiplying both sides by (1/4). In other words, they changed the equation by doing the same multiplying to both sides: **the multiplicative property of equality.**

If *x* is not equal to *y* and not less than *y*, what must be true of *x*, and why?

By the trichotomy law, there are only three possible relationships between *x* and *y*, and they have eliminated two of them. Then ***x* > *y*, by the trichotomy law.**

***x* + (–*x*) = 0**

They added, and they ended up with zero: **the additive inverse.**

$$\left(\frac{3}{3}\right)\left(\frac{2}{5}\right) + \left(\frac{5}{5}\right)\left(\frac{4}{3}\right) = \frac{6}{15} + \frac{20}{15}$$

They converted to a common denominator by multiplying both fractions by a useful form of 1; remember that $\frac{3}{3}$ and $\frac{5}{5}$ are just 1! So they used **the multiplicative identity.**

If 5*x* = 0, what is *x*, and why?

You can do this in either of two ways: multiply both sides by $\frac{1}{5}$ **(the multiplicative property of equality)** and then get ***x* = 0**, or you could say that, since 5 doesn't equal zero, then x must equal zero **(by the zero-product property)**.

$(^2/_3)(^3/_2) = 1$

They multiplied, and they ended up with one: **the multiplicative inverse.**

If $3x + 2 = y$ and $y = 8$, then $3x + 2 = 8$.

You might be confused here between the transitive property and the substitution property. What they did here was "cut out the middleman" by removing the "y" in the middle, so they used **the transitive property.**

If $–x = 14$, what does x equal, and why?

To solve this, you would multiply both sides by a negative one, to cancel off the minus sign. So:

$x = –14$, by the multiplicative property of equality.

If $x = 3$ and $y = –4$, then what does xy equal, and why?

By substitution (plugging in for the variables), you get (3) (–4). In other words:

$xy = –12$, by the substitution property**.**

Can $x < x$? Why or why not?

By the reflexive property, $x = x$. By the trichotomy law, if $a = b$ then a cannot be less than b. So the answer is "**no, by the reflexive property and the trichotomy law**"

4.0 CONCLUSION

Applying the trichotomy law, transitive (moving across) property, the reflexive property, the symmetric property, the additive identity and the multiplicative inverse to solve mathematics problem are the key basics that lead to the understanding of mathematics and teachers should stress the learning of these basic principles.

5.0 SUMMARY

Identities, inverses, and symmetry are the basic concepts needed to prove some basic properties of mathematics such as: the multiplicative identity; the reflexive properties; the symmetric property; the identity property and the substitution property.

6.0 TUTOR-MARKED ASSIGNMENT

i. List and define the basic concepts needed to prove some properties of mathematics.

7.0 REFERENCE/FURTHER READING

Stapel, Elizabeth. (2011). 'Other Number Properties: Identities, Inverses, Symmetry.'. http://www.purplemathematics.com/ modules/numbprop2.htm.