COURSE GUIDE

PED 236: ELEMENTARY MATHEMATICS

COURSE DEVELOPER- PROF G. A. BADMUS
SCHOOL OF EDUCATION
NATIONAL OPEN UNIVERSITY OF NIGERIA
VICTORIA ISLAND, LAGOS

COURSE WRITTER **DR LUCY ERAKHUEME**
FACULTY OF EDUCATION
UNIVERSITY OF BENIN

COURSE EDITOR **PROF G. A. BADMUS**
SCHOOL OF EDUCATION
NATIONAL OPEN UNIVERSITY OF NIGERIA
VICTORIA ISLAND, LAGOS

OFFICER IN CHARGE **DR DOROTHY OFOHA**
SCHOOL OF EDUCATION
NATIONAL OPEN UNIVERSITY OF NIGERIA
VICTORIA ISLAND, LAGOS

COURSE COORDINATOR **DR C. A. OKONKWO**
SCHOOL OF EDUCATION
NATIONAL OPEN UNIVERSITY OF NIGERIA
VICTORIA ISLAND, LAGOS

TABLE OF CONTENT

Page

Introduction

PED 236: Elementary Mathematics is a one semester, 2 credit course. The course consists of sixteen units which includes basic arithmetic operations in integers, indices, logarithm and surd, fractions, number bases, approximation, rate, proportion and ratio, factorization, linear equation, quadratic equation, simultaneous equation, algebraic graphs, change of subject of formulae, linear inequalities, length, perimeter, circles, sector, triangles, area and volume of geometrical solids and shapes.

Course Aim

This course is developed for students of the National Open University of Nigeria (NOUN) who have poor background in Mathematics. It is intended to refresh and strengthen the students' knowledge of ordinary level (O/L) Mathematics.

Course Objectives

To achieve the aim spelt out above, the course is structured into modules with a module comprising of at least 3 units. Each unit has specific objectives to be achieve and these objectives are spelt out in section 2.0 of every unit. You are encouraged to refer to these objectives during your study of the course in order for your to monitor your own progress. After completing a unit, you should endeavour to look at the unit objectives to ascertain that you have done what was required of you in the unit.

Working Through this Course

To complete this course, you are required to read the study units, reference books and other materials provided by NOUN. Each unit contains Self Assessment Exercises and Tutor Marked Assignments. At points in the course you will be required to submit assignments for assessment purposes.

Course Materials

The major components of this course are as follows:

1. Course Guide
2. Study Units
3. Tutor Marked Assignment

Assessment

There are two aspects of the assessment of this course. The first is the Tutor Marked Assignments (TMAs) while the second is the end of the semester written examination. You are expected to apply the information, knowledge and techniques gathered during the course during these assessments.

There are sixteen Tutor-Marked Assignments in the course. Your tutor will tell you which ones to submit to account for the 30% for continuous assessment. The end of semester examination will contribute the remaining 70%.

How to Get the Most from this Course

One great advantage of distance learning is that study units replace the university lecturer. You are expected to read and work through the specially designed study materials at your own pace and at a time and place that suits you best. The study units have a common format. The first item is an introduction to the subject matter of the unit and how a particular unit is related to the other units and the course as a whole. The introduction is followed by learning objectives that guides you on what you should be able to do by the time you complete the study of the unit. These objectives should serve as a guide in your studying of the units.

Working through these units will help you achieve the objectives of the units and prepare you for the assignments and examination. Every unit has a number of examples. You are encouraged to study the examples carefully and do each self assessment exercise as you come to it in the study unit.

The following is a practical strategy for working through the course.

1. Read this course guide thoroughly.
2. Organize a study schedule.
3. Stick to your study schedule strictly.
4. Adhere with unit 1 and read the introduction and objectives for the unit before reading the main body.
5. Assemble all study materials.
6. Work through the unit carefully.
7. Do the exercises and assignment and ensure that you have mastered the unit.
8. Then, move to the next unit.
9. Go on like this until you get to the last unit.

Study Units

There are sixteen study units in this course PED 236: Elementary Mathematics. They are as follows:

Module 1: Number and Numeration

Unit 1: Basic Arithmetic Operations in Integers

Unit 2: Indices, Logarithm and surds

Unit 3: Meaning and types of Fractions

Unit 4: Number Bases

Unit 5: Approximation, Rate, Proportion and Ratio

Module 2: Algebraic Processes

Unit 1: Factorization and Linear Equations

Unit 2: Quadratic and Simultaneous Equations

Unit 3: Algebraic Graphs

Unit 4: Change of Subject of Formulae

Unit 5: Linear Inequalities

Module 3: Measurement

Module 4: Geometrical Solids and Shapes

Unit 1: Triangles

Unit 2: Circle

Unit 3: Area and Volume of Various Geometrical Solids and Shapes

Assignments are compulsory and it is a unique feature of your course delivery system. They are called Tutor Marked Assignment (TMA). You must ensure that the assignments are answered and returned to the study center on dates as agreed between you and the course tutor. There are also some assignments, which are called Self Assessment Exercises. It is advisable that you attempt all the questions yourself. They are meant to probe your understanding of the concepts in the units.

You are going to be evaluated in this course by the Tutor Marked Assignments which will contribute to final grade. It is compulsory for you to and submit all the assignment and they will be graded. The grades for the best ones (the number to be decided by your tutor) will be used in your assessment.

Tutors and Tutorials

Your tutor will mark and comment on your assignments, your progress in the course will be closely monitored by the course tutor. You are to report in difficulties encountered to the course tutor who will give you necessary assistance in the course. You must mail your tutor marked assignment to your tutor well before the due date.

Attendance at Tutorials

Tutorials is the only opportunity you have for a face-to-face contact with your Tutor. You need to attend tutorials regularly sot that you can ask your Tutor necessary questions and get clarifications. You will learn a lot from participating in class discussions.

I wish you success with the course and hope you will find it interesting, useful and rewarding.

PED 236: ELEMENTARY MATHEMATICS

TABLE OF CONTENT

Module 3: Geometrical Solids and Shapes

Module 4: Measurement

MODULE 1: NUMBER AND NUMERATION

Unit 1: Basic Arithmetic Operations in Integers

Unit 2: Indices, Logarithms and Surds

Unit 3: Meaning and types of Fractions

Unit: 4: Number Bases

Unit 5: Approximations, Rate, Proportion and Ratio

UNIT 1: Basic Arithmetic Operation in Integers

Content

1.0 Introduction

This unit is designed to give you an insight into positive and negative number system. This can be conceived as steps of equal sizes along a line as shown below, with positive numbers to the right of zero and negative numbers to the left of zero.

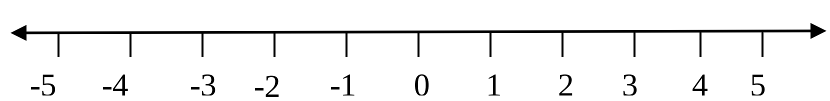


Figure 1: Number Line

This line is called a number line. We can use it to perform the operations of addition. A walk forward can be taken as positive numbers and a walk backward can be taken as negative numbers.

2.0 Objective

At the end of this unit, you should be able to:

i. correctly interpret positive and negative integer

ii. carry out the basic operations on integer.

ii. identify daily life examples of position and negative number.

3.0 Main body

3.1 Meaning of Integer and Basic Arithmetic Operation

An integer is any positive or negative whole number.

Basic arithmetic operation in integers are the addition, subtraction, multiplication and division of positive and negative numbers.

We have idea about counting numbers and all counting numbers are positive numbers. The opposite of positive is negative in the number line. The numbers to the right of zero are positive numbers and the numbers to the left of zero are negative numbers. The more a positive number move away from zero the bigger the number, but the more a negative number move away from zero the smaller the number i.e – 1 is greater than – 10.

3.2 Daily Life Example of Positive and Negative Numbers

1. Let us take the ground level as our zero point. A man that is tapping a palm tree is above the ground level, so the position of that man is positive. On the other hand if a man is digging a well, the man is below the ground level. The position of such a man is negative since he is under the ground. So it can be said that positive distance are above the ground and negative distance are below the ground.
2. If a man is said to moves ten (10) spaces to his right (forward) and then moves seven (7) spaces to the left (backward). What is the man position from the starting point. The man position forward is positive + 10 and the position backward is negative – 7. His position is 3 spaces to the right from the starting point. This can be written as +3.
3. If a woman that lives in story building walks six (6) steps upstairs, and walks fourteen (14) steps downstairs. What is her final position. Her final position is 8 steps downstairs. This can be written as -8.

3.3 Integers and Number Lines

(a) Draw a horizontal line and divide it into equal parts. Mark the starting point as zero.

(i) To the right of zero are positive numbers as shown below. The arrow shows that the line continues indefinitely.

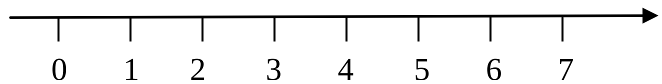


Figure 2: Positive Number Line

(ii) To the left of zero are negative numbers as shown below. The arrow shows that the line continue indefinitely.

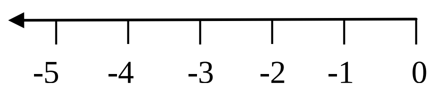


Figure 3: Negative number line

(iii) When the positive and negative lines are joined , it become a full number line.

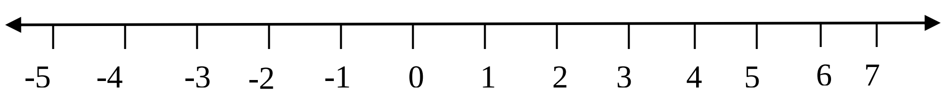


Figure 4: Full Number line

(iv) Similarly the number line can also be in a vertical form. With numbers on top of zero as positive and the ones below zero as negative.

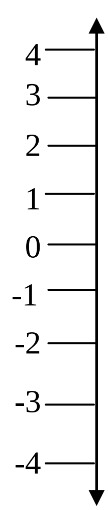


Figure 5: Verification Full Number line

Putting the horizontal and vertical number line together will give us the shape of a cardinal point. North, South, East and West.

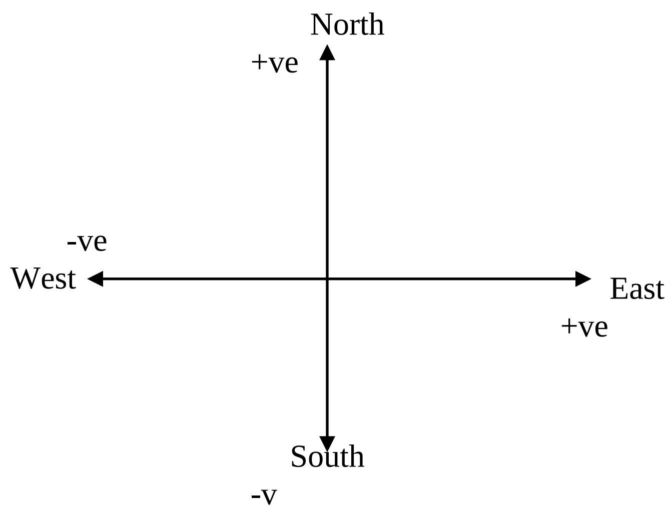


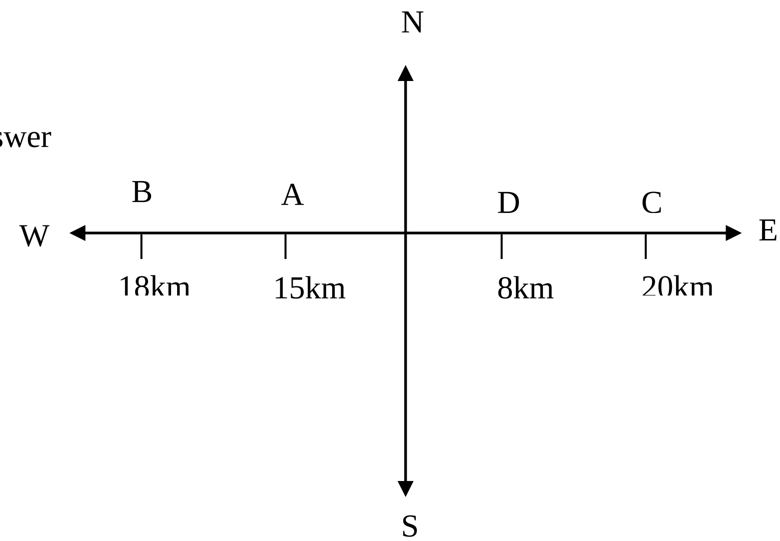
Fig. 6: The four cardinal point.

Examples

Location of points on the number line.

1. If a straight road runs east-west through point o. the distances east of 0 are positive and the distances west of 0 are negative.
2. If a straight road also runs from North-South thought point 0. The distance north of zero is positive and that of south of zero is negative.
3. Express the positions of the points below in terms of positive or negative distance.
   (i) A is 15km west of 0.
   (ii) B is 18km west of 0.
   (iii) C is 20km east of 0.
   (iv) D is 8km east of 0.

Answer



(i) The distance of A is – 15km

(ii) The distance of B is – 18km

(iii) The distance of D is + 8km

(iv) The distance of C is + 20km

(4) From the above question, what is the distance between

(i) A and D (ii) A & C (iii) B & D (iv) B & C (v) A & B (vi) C & D

Answer

(i) 15km + 8kn = 23km

(ii) 15km + 20km = 35km

(iii) 18km + 8km = 26km

(iv) 18km + 20km = 38km

(v) 18km - 15km = 3km

(vi) 20km – 8km – 12km

Self Assessment Exercise

(1) In the pairs of number below, put the correct sign (> or <) between them;

(a) -8, +6 (b) 0, -5 +4 (c) -2, +4 (d) -4, -8 (e) 10, -8

(2) Arrange the following numbers in orders, smallest first and represent it on a number line

-8, +8, 0, -1, -15, -6, + 18, +1.

(3) When you move 8 steps to the west and then 10 steps to the east. What is your present position from the starting point?

(4) A bucket of water was drawn from a well 18m deep and was taken to a story building 20m above the ground what is the final position of the bucket of water.

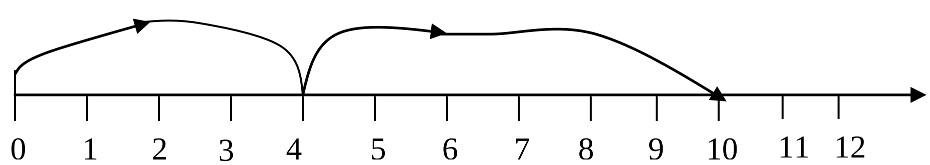
3.4 Addition of Integers

Positive and negative numbers are called directed numbers. When we move to the right hand or upwards we count positive numbers. We count negative, when we move to the left or downward, depending on whether we use the horizontal or vertical number line. We use the arrow signs to show the direction of movements.

3.4.1 Using Number Line

Example 1

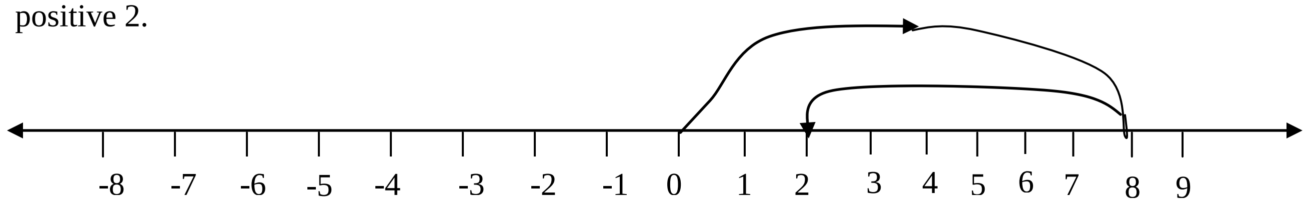
If I Count 4 steps to the right from the point zero, and from the present position count another 6 steps to the right. Where will my final stop be



Therefore 4 + 6 = 10

Example 2

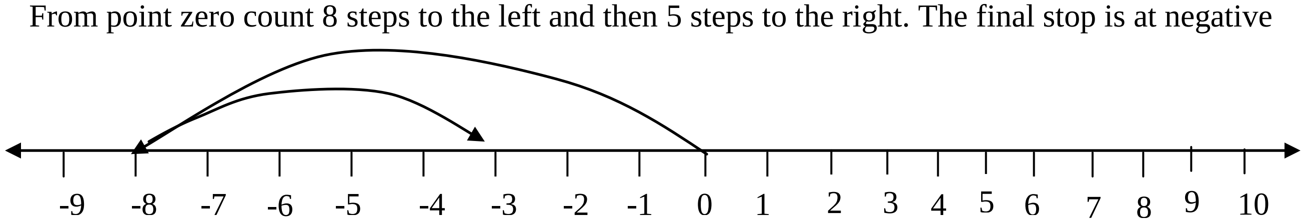
From point zero, count 8 steps to the right the count 6 steps to the left. The final stop is at positive 2.



Therefore +8 –(+6) = 2

Example 3

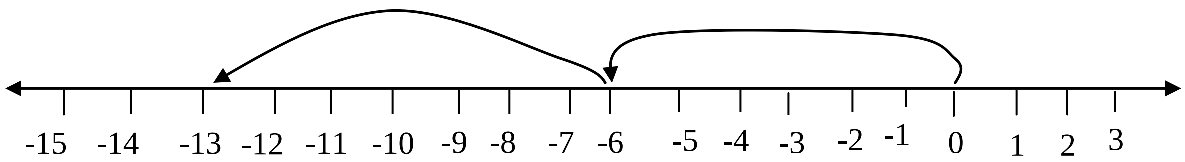
From point zero count 8 steps to the left and then 5 steps to the right. The final stop is at negative



$$(-8) + 5 = -3$$

Example 4

rom zero count 7 steps to the left and again another 6 steps to the left. The final stop is at negative 13.



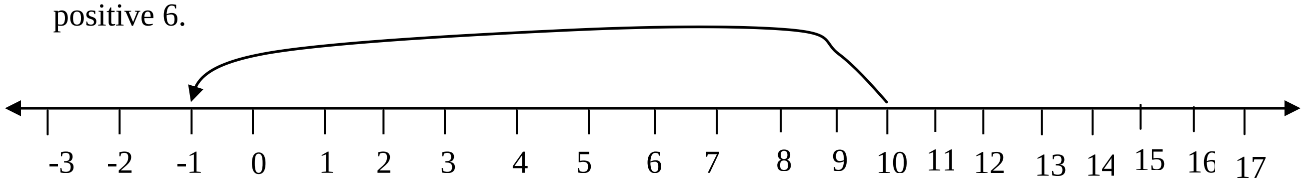
Therefore (-7) +(-6) = -13.

3.5 Subtraction of Integers

Subtraction is the opposite of addition.

Example 1

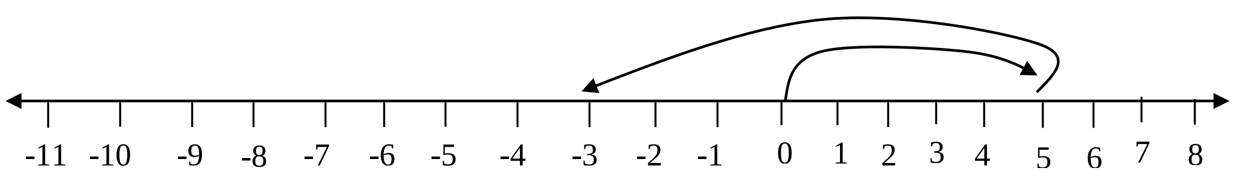
From the point zero, move 10 steps to the right and then 4 steps to the left. The final stop is at positive 6.



Therefore 10 – 4 = 6.

Example 2

From the point zero, move 5 steps to the right and 8 step to the left. The final stop is at negative 3.



Therefore 5 – 8 = – 3 .

Note:

In subtraction, if the negative number is greater than the positive number the normal addition should be done but the answer will taken a negative sign. i.e take the small number away from the big number, but the answer will have a negative sign since the bigger number has a negative sign.

(3) Evaluating without number line

(a) (-9) – (-3)

When there is a negative (minus) sign in between two numbers (each in a bracket), the negative sign will change whatever sign is in the following bracket. If it is positive sign, it will change to negative sign and if it is negative signs, it will change to positive sign i.e (-9) – (-3) will now read (-9) +3 which will be done as – (9-3) = -6.

(b) 29 – (-18) will now read

29 + 18 = 47

(c) (-17) – (-28) will now read

(-17) + 28 which will be done as

28 – 17 = 11

(d) -38 – (+ 5) will now read

- 38 – 5 which will be done as

- (38 + 5) = -43

4.0 Conclusion

This unit considered the basic operations on integer. The use of number lines in integer operations.

5.0 Summary

The highlights of the unit includes the following.

- When to use positive and negative sign in real life problems.
- How number lines are used in the addition and subtraction in integers.

6.0 Tutor Marked Assignment

1. Use number line to solve the following problems:

(a) -8 – 10 (b) 10 – 11 (c) 8 – 5 (d) -10 + 6

7.0 References/Further Reading

Channon, J.B. et. al (2002): New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

UPE Teacher Education Project (1977)

MODULE 1: UNIT 2

UNIT 2: Indices, Logarithms and Surds

Content

1.0 Introduction

The short form of writing a number which multiplies itself a given number of time is called ***Index Number***. The laws help to make calculations easy using indices (plural of index). Indices help to have a good understanding of logarithm. There is a relationship between the two. Surds are irrational number that can't be written in ratio form.

2.0 Objective

At the end of this unit you should be to: (i) solve problems whose indices are +ve/-ve whole number (ii) apply it to multiplication and division (iii) solve simple equations involving indices (iv) determine the logarithm of a number in relation to indices (v) use the four figure table to solve logarithm problems (vi) perform basic arithmetic operations in surd and (vii) rationalize surd operations.

3.0 Content

(1) Prime number are integer which is only divisible by 1 and itself; that is a prime number has only two factors which is 1 and itself.

Examples of primes numbers are 2, 3, 5, and 7.

What is an Index?

It is an integer placed above and to the right of a number. For example 5^3, 3 is the index of 5 and in 9^4, 4 is the index of 9.

In 5^3 above, 5 is the base number and 3 is the power. So it is in 18^6 and 9^4, 18 and 9 are the base while 6 and 4 are the powers respectively. The number 5^3 are read as "5 raised to power 3"

Examples if prime numbers

(1) Express 16 as the product of power of its primes

2	16
2	8
2	4
2	2
	1

16 = 2 x 2 x 2 x 2

That is 2 in 4 places which can also be written as 2^4

$16 = 2^4$

(2) Express 18 as a product of power of its prime

2	18
3	9
3	3
	1

18 = 2 x 3 x 3

i.e $18 = 2 \times 3^2$

Express the number below as product of power of its prime.

(a) 42 (b) 108 (c) 675

Solutions

2	42
3	21
7	7
	1

42 = 2 x 3 x 7

2	108
2	54
3	27
3	9
3	3
	1

108 = 2 x 2 x 3 x 3 x 3

i.e $108 = 2^2 \times 3^3$

3	675
3	225
3	75
5	25
5	5
	1

675 = 3 x 3 x 3 x 5 x 5

i.e $675 = 3^3 \times 5^2$

Self Assessment Test

Express the following numbers as a production of power of its prime

(a) 288

(b) 1404

3.1 Fundamental Laws of Indices

(1) $a^m \times a^n = a^{m+n}$

(2) $a^m \div a^n = a^{m-n}$

(3) $a^{-n} = \frac{1}{a^n}$

(4) $a^{\frac{1}{n}} = \sqrt[n]{a}$

(5) $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

(6) $a^o = 1$

(7) $\left(a^m\right)^n = a^{mn}$

Applying the law of multiplication

Example

(1) Simplify the following

(a) $5^2 \times 5^3$ (b) $6^4 \times 6^2$ (b) $a^o \times b^2 \times b^3$

(a) $5^2 = 5 \times 5$

$5^3 = 5 \times 5 \times 5$

$5^2 \times 5^3 = (5 \times 5) \times (5 \times 5 \times 5) = 5^5$

$= 5^{2+3} = 5^5$

(b) $6^4 = 6 \times 6 \times 6 \times 6$

$6^2 = 6 \times 6$

$6^4 \times 6^2 = (6 \times 6 \times 6 \times 6) \times (6 \times 6) = 6^6$

$= 6^{4+2} = 6^6$

(c) $a^o = 1$

$a^o \times b^2 \times b^3 = b^{2+3} = b^5$

In general, if you are ask to multiply two or more members having the same base, we add the power (indices) and raise the base member to the sum of the indices i.e.

$a^8 \times a^6 \times a^b \times a^c = a^{8+6+b+c}$

(2) Simplify …

(a) $8a^2 \times 6a^8$ (b) $7a^5 \times 6a^5 \times 6a^4 \times 5a^o$

(a) $8a^2 \times 6a^8 = 8 \times 6 \times a^8 = 48a^{2+8} = 48a^{10}$

(b) $7a^5\ 6a^4 \times 5a^o = 7 \times 6 \times 5\ a^{5+4} = 42a^9$

Self Assessment Test

Simplify the following

(a) $4^3 \times 4^5$

(b) $15M^2 \times 2M^4 \times M^6$

3.2 Applying the Law of Division

Evaluate the following

(a) $5^6 \div 5^4$ (b) $6^9 \div 6^6$ (c) $a^8 \div a^2$ (d) $a^5 \div a^5$ (e) $5p^8 \div 5p^6$

Solution

(a) $$5^6 \div 5^4 = \frac{5x5x5x5x5x5}{5x5x5x5} = 5x5 = 5^2$$

i.e $5^{6-4} = 5^2$

(b) $$6^9 \div 6^6 = \frac{6x6x6x6x6x6x6x6x6}{6x6x6x6x6x6} = 6x6\,x6 = 6^3$$

i.e $6^{9-6} = 6^3$

(c) $$a^8 \div a^5 = \frac{axaxaxaxaxaxaxa}{axaxaxaxa} = axa\,xa = a^3$$

i.e $a^{8-5} = a^3$

(d) $$a^5 \div a^5 = \frac{axaxaxaxa}{axaxaxaxa} = \frac{1}{1} = 1$$

i.e $a^{5-5} = a^0 = 1$

(e) $$15P^8 \div 5P^6 = \frac{\overset{3}{15}xPxPxPxPxPxPxPxP}{5xPxPxPxPxPxP} = 3xP\,xP = 3P^2$$

$$i.e\ \frac{15}{5}\ P^{8-6} = 3P^2$$

Self Assessment Test

Evaluate the following

(a) $18R^6 \div 3R^2$

(b) $45P^{10} \div 15P^6$

3.3 Multiple Indices

An indexed number a^4 raised to a given index say 2 is able to have multiple indices thus $\left(a^4\right)^2$

(1) Simplify $\left(a^4\right)^2$

$\left(a^4\right)^2 = a^4 \times a^4$

$= (axaxaxa) \times (axaxaxa)$

$= axaxaxaxaxaxaxa$

$= a^8$

That is $\left(a^4\right)^2 = a^{4x2} = a^8$

Note: When a number a^m is raised to another index n, we multiply the indices thus.

$\left(a^m\right)^n = a^{mxn} = a^{mn}$

Example

(1) $\left(8^2\right)^3$ (2) $\left(6b^4\right)^2$ (3) $10\left(M^2\right)^3$

Solutions

(1) $\left(8^2\right)^3 = 8^{2x3} = 8^6$

(2) $\left(6b^4\right)^2 = 6^2 b^{4x2} = 36b^8$

(3) $10\left(M^2\right)^3 = 10M^{2x3} = 10M^6$

3.4 Rules for Fractional, Negative and Zero Indices

Fractional Index

$$a^{\frac{m}{n}} = \left(a^{\frac{1}{n}}\right)^m = \left(\sqrt[n]{a}\right)^m$$

$$i.e\ a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$$

Note: In case of the fractional index $\frac{m}{n}$, n represents the root of the number and M represents the power.

Simplify the following

(1) $(16)^{\frac{3}{2}}$ (2) $(81)^{\frac{3}{4}}$ (3) $(64)^{\frac{3}{5}}$ (4) $(625)^{\frac{5}{4}}$

(1) $(16)^{\frac{3}{2}} = \left(\sqrt{16}\right)^2 = 4^3 = 64$

(2) $(81)^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = 3^3 = 27$

(3) $(64)^{\frac{3}{5}} = \left(\sqrt[5]{64}\right)^3 = 2^3 = 8$

(4) $(625)^{\frac{5}{4}} = \left(\sqrt[4]{625}\right)^5 = 5^4 = 625$

Self Assessment Test

Evaluate the following

(a) $(100)^{\frac{1}{2}}$ (b) $(81)^{\frac{2}{4}}$

Negative Index

$$a^{-n} = \frac{1}{a^n}$$

Write the number as the denominator of a fraction with the index positive and with 1 as numerator.

Examples

(1) Simplify (a) 2^{-5} (b) 6^{-2} (c) $6b^{-3}$ (d) $(5a)^{-2}$

Solutions

(a) $2^{-5} = \frac{1}{2^5} = \frac{1}{32}$

(b) $6^{-5} = \frac{1}{6^2} = \frac{1}{36}$

(c) $6b^{-3} = 6\frac{1}{b^3} = \frac{6}{b^3}$

(d) $(5a)^{-2} = \frac{1}{(5a)^2} = \frac{1}{5^2 a^2} = \frac{1}{25a^2}$

Zero Index

$a^0 = 1$

Any number raised to the power of zero (0) is always equal to 1.

Examples

(1) Simplify (a) a^0 (b) $(4a)^0$ (c) $(100)^0$ (d) $\left(\frac{81}{19}\right)^0$

Solutions

(a) $a^0 = 1$ $(4a)^0 = 4^0 \times a^0 = 1 \times 1 = 1$

(c) $(700)^0 = 1$ (d) $\frac{81^0}{9^0} = \frac{1}{1} = 1$

Self Assessment Test

(a) $(20)^{-2}$ (b) $(5a)^0 (121)^{-1/2}$

Note: Indices multiplication is simplified by addition. While that of division is simply done by subtraction. The indices to be multiplied or divided must be of the base.

3.5 Logarithm

Using logarithms of numbers, multiplication problems can be done (solved) by simple addition while division problem can be solved by simply subtraction. It can also be used in the problem and roots of number.

Recall

(i) $\text{Log}_{10} = 10^1$ (ii) $\text{Log}_{10} 100 = 2$

(iii) $\text{Log}_{10} 1000 = 3$

Comparing $100 = 10^2$ and $\text{Log}_{10} 100 = 2$ it can be see that the index in the first equation which is 2 is the same as the logarithm of the second equation which is also 2. It is clear that the logarithm of the number 100 to base 10 gives the power to which the number 10 must be raised to get 100.

Examples

$10,000 = 10^4$, $\text{Log}_{10} 10,000 = 4$

$10,000 = 10^5$, $\text{Log}_{10} 10,0000 = 5$

Note:

The representation of Log_{10} 100 reads logarithm of 100 to base 10.

Looking for the logarithm of a positive number of base 10 is the power to which 10 must be raised to obtain that particular member.

That is $\text{Log}_{10} P = R$ which mean

$10^R = P$

Now express the numbers below in form of logarithm.

(i) $1000 = 10^3$ which is $Log_{10}1000 = 3$

(ii) $1 = 10^0$ which is $Log_{10} 1 = 0$

(iii) $10 = 10^1$ which is $Log_{10} 10 = 1$

(iv) $M = 10^n$ which is $Log_{10} M = n$

In most cases the base number is omitted. When the base is not written it is assumed to be in base 10. The numbers are not always in tens. It can be any number. The logarithm of a number are of two parts. The whole number part and decimal parts. The whole number part is called the characteristics and the decimal part is called the mantissa.

The relationship between indices and logarithm.

Express the equations below in index form.

(i) $Log_{10} 1000 = 3$ (ii) $Log_{10}^{x} = C$

(iii) $Log_{10}M = \sqrt{P}$

Self Assessment Test

1. How many parts is the logarithm of a number made up of and what are the parts.
2. Express in the form of logarithm

   (i) $a^m = n$ (ii) $M^x = P$

3. Express in index form

   (i) $Log_{10} 1.4 = 0.1461$ (ii) $Log_{10} 5.02 = 0.7160$

3.5.1 Calculation of Logarithm to Base 10

(1) Number Greater than 1

Any number of the form $M \times 10^n$ is said to be in standard form. So a number can be expressed in standard form.

Examples

Express in standard form

(i) 576.23 (ii) 0.05247 (iii) 0.000689

Solution

(i) $575.23 = 5.763 \times 10^2$

(ii) $0.05237 = 5.237 \times 10^{-2}$

(iii) $0.000689 = 6.89 \times 10^{-4}$

The logarithm of a number is made up of the characteristics and the materials. In finding the logarithm of a number the calculator can be used. In some calculator, the number is entered before pressing the log button. In others you press the log button before you enter the number.

3.5.2 Using the Four Figure Table

First look for the characteristic of the number and then the mantissa.

Characteristics

The characteristics is always one less than the digits in a whole number (i.e the numbers before the decimal point where there is a decimal number). Count these digits then subtract 1.

Examples

(1) Find the characteristics of the numbers below

(i) 5678 (ii) 678.902 (iii) 12 3 (iv) 93281.1

Solution

(i) 5678 has 4 digits

Therefore characteristic is 4 – 1 = 3

(ii) 678.902 has 3 digits before the decimal point

Therefore characteristics is 3 – 1 = 2

(iii) 93281.1 has 5 digits before the pt

Therefore characteristic is 5 – 1 = 4

Mantissa

This is read from the four figure table (logarithm table). Now get your logarithm table and let start. We shall use the sample of the logarithm table below.

Consider *Log 5678*

Logarithms of Numbers

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
10	0000	0043	0086	0128	0170	0212	0253	0294	0334	0374	4	8	12	17	21	25	29	33	37
11	0414	0453	0492	0531	0569	0607	0645	0682	0719	0755	4	8	11	15	19	23	26	30	34
12	0792	0828	0864	0899	0934	0969	1004	1038	1072	1106	3	7	10	14	17	21	24	28	31
13	1139	1173	1206	1239	1271	1303	1335	1367	1399	1430	3	6	10	13	16	19	23	26	29
14	1461	1492	1523	1553	1584	1614	1644	1673	1703	1732	3	6	9	12	15	18	21	24	27
15	1761	1790	1818	1847	1875	1903	1931	1959	1987	2014	3	6	8	11	14	17	20	22	25
16	2041	2068	2095	2122	2148	2175	2201	2227	2253	2279	3	5	8	11	13	16	18	21	24
17	2304	2330	2355	2380	2405	2430	2455	2480	2504	2529	2	5	7	10	12	15	17	20	22
18	2553	2577	2601	2625	2648	2672	2695	2718	2742	2765	2	5	7	9	12	14	16	19	21
19	2788	2810	2833	2856	2878	2900	2923	2945	2967	2989	2	4	7	9	11	13	16	18	20
20	3010	3032	3054	3075	3096	3118	3139	3160	3181	3201	2	4	6	8	11	13	15	17	19
21	3222	3243	3263	3284	3304	3324	3345	3365	3385	3404	2	4	6	8	10	12	14	16	18
22	3424	3444	3464	3483	3502	3522	3541	3560	3579	3598	2	4	6	8	10	12	14	15	17
23	3617	3636	3655	3674	3692	3711	3729	3747	3766	3784	2	4	6	7	9	11	13	15	17
24	3802	3820	3838	3856	3874	3892	3909	3927	3945	3962	2	4	5	7	9	11	12	14	16
25	3979	3997	4014	4031	4048	4065	4082	4099	4116	4133	2	3	5	7	9	10	12	14	15
26	4150	4166	4183	4200	4216	4232	4249	4265	4281	4298	2	3	5	7	8	10	11	13	15
27	4314	4330	4346	4362	4378	4393	4409	4425	4440	4456	2	3	5	6	8	9	11	13	14
28	4472	4487	4502	4518	4533	4548	4564	4579	4594	4609	2	3	5	6	8	9	11	12	14
29	4624	4639	4654	4669	4683	4698	4713	4728	4742	4757	1	3	4	6	7	9	10	12	13
30	4771	4786	4800	4814	4829	4843	4857	4871	4886	4900	1	3	4	6	7	9	10	11	13
31	4914	4928	4942	4955	4969	4983	4997	5011	5024	5038	1	3	4	6	7	8	10	11	12
32	5051	5065	5079	5092	5105	5119	5132	5145	5159	5172	1	3	4	5	7	8	9	11	12
33	5185	5198	5211	5224	5237	5250	5263	5276	5289	5302	1	3	4	5	6	8	9	10	12
34	5315	5328	5340	5353	5366	5378	5391	5403	5416	5428	1	3	4	5	6	8	9	10	11
35	5441	5453	5465	5478	5490	5502	5514	5527	5539	5551	1	2	4	5	6	7	9	10	11
36	5563	5575	5587	5599	5611	5623	5635	5647	5658	5670	1	2	4	5	6	7	8	10	11
37	5682	5694	5705	5717	5729	5740	5752	5763	5775	5786	1	2	3	5	6	7	8	9	10
38	5798	5809	5821	5832	5843	5855	5866	5877	5888	5899	1	2	3	5	6	7	8	9	10
39	5911	5922	5933	5944	5955	5966	5977	5988	5999	6010	1	2	3	4	5	7	8	9	10
40	6021	6031	6042	6053	6064	6075	6085	6096	6107	6117	1	2	3	4	5	6	8	9	10
41	6128	6138	6149	6160	6170	6180	6191	6201	6212	6222	1	2	3	4	5	6	7	8	9
42	6232	6243	6253	6263	6274	6284	6294	6304	6314	6325	1	2	3	4	5	6	7	8	9
43	6335	6345	6355	6365	6375	6385	6395	6405	6415	6425	1	2	3	4	5	6	7	8	9
44	6435	6444	6454	6464	6474	6484	6493	6503	6513	6522	1	2	3	4	5	6	7	8	9
45	6532	6542	6551	6561	6571	6580	6590	6599	6609	6618	1	2	3	4	5	6	7	8	9
46	6628	6637	6646	6656	6665	6675	6684	6693	6702	6712	1	2	3	4	5	6	7	7	8
47	6721	6730	6739	6749	6758	6767	6776	6785	6794	6803	1	2	3	4	5	5	6	7	8
48	6812	6821	6830	6839	6848	6857	6866	6875	6884	6893	1	2	3	4	4	5	6	7	8
49	6902	6911	6920	6928	6937	6946	6955	6964	6972	6981	1	2	3	4	4	5	6	7	8
50	6990	6998	7007	7016	7024	7033	7042	7050	7059	7067	1	2	3	3	4	5	6	7	8
51	7076	7084	7093	7101	7110	7118	7126	7135	7143	7152	1	2	3	3	4	5	6	7	8
52	7160	7168	7177	7185	7193	7202	7210	7218	7226	7235	1	2	2	3	4	5	6	7	7
53	7243	7251	7259	7267	7275	7284	7292	7300	7308	7316	1	2	2	3	4	5	6	6	7
54	7324	7332	7340	7348	7356	7364	7372	7380	7388	7396	1	2	2	3	4	5	6	6	7
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
55	7404	7412	7419	7427	7435	7443	7451	7459	7466	7474	1	2	2	3	4	5	5	6	7
56	7482	7490	7497	7505	7513	7520	7528	7536	7543	7551	1	2	2	3	4	5	5	6	7
57	7559	7566	7574	7582	7589	7597	7604	7612	7619	7627	1	2	2	3	4	5	5	6	7
58	7634	7642	7649	7657	7664	7672	7679	7686	7694	7701	1	1	2	3	4	4	5	6	7
59	7709	7716	7723	7731	7738	7745	7752	7760	7767	7774	1	1	2	3	4	4	5	6	7
60	7782	7789	7796	7803	7810	7818	7825	7832	7839	7846	1	1	2	3	4	4	5	6	6
61	7853	7860	7868	7875	7882	7889	7896	7903	7910	7917	1	1	2	3	4	4	5	6	6
62	7924	7931	7938	7945	7952	7959	7966	7973	7980	7987	1	1	2	3	3	4	5	6	6
63	7993	8000	8007	8014	8021	8028	8035	8041	8048	8055	1	1	2	3	3	4	5	5	6
64	8062	8069	8075	8082	8089	8096	8102	8109	8116	8122	1	1	2	3	3	4	5	5	6
65	8129	8136	8142	8149	8156	8162	8169	8176	8182	8189	1	1	2	3	3	4	5	5	6
66	8195	8202	8209	8215	8222	8228	8235	8241	8248	8254	1	1	2	3	3	4	5	5	6
67	8261	8267	8274	8280	8287	8293	8299	8306	8312	8319	1	1	2	3	3	4	5	5	6
68	8325	8331	8338	8344	8351	8357	8363	8370	8376	8382	1	1	2	3	3	4	4	5	6
69	8388	8395	8401	8407	8414	8420	8426	8432	8439	8445	1	1	2	2	3	4	4	5	6
70	8451	8457	8463	8470	8476	8482	8488	8494	8500	8506	1	1	2	2	3	4	4	5	6
71	8513	8519	8525	8531	8537	8543	8549	8555	8561	8567	1	1	2	2	3	4	4	5	5
72	8573	8579	8585	8591	8597	8603	8609	8615	8621	8627	1	1	2	2	3	4	4	5	5
73	8633	8639	8645	8651	8657	8663	8669	8675	8681	8686	1	1	2	2	3	4	4	5	5
74	8692	8698	8704	8710	8716	8722	8727	8733	8739	8745	1	1	2	2	3	4	4	5	5
75	8751	8756	8762	8768	8774	8779	8785	8791	8797	8802	1	1	2	2	3	3	4	5	5
76	8808	8814	8820	8825	8831	8837	8842	8848	8854	8859	1	1	2	2	3	3	4	5	5
77	8865	8871	8876	8882	8887	8893	8899	8904	8910	8915	1	1	2	2	3	3	4	4	5
78	8921	8927	8932	8938	8943	8949	8954	8960	8965	8971	1	1	2	2	3	3	4	4	5
79	8976	8982	8987	8993	8998	9004	9009	9015	9020	9025	1	1	2	2	3	3	4	4	5
80	9031	9036	9042	9047	9053	9058	9063	9069	9074	9079	1	1	2	2	3	3	4	4	5
81	9085	9090	9096	9101	9106	9112	9117	9122	9128	9133	1	1	2	2	3	3	4	4	5
82	9138	9143	9149	9154	9159	9165	9170	9175	9180	9186	1	1	2	2	3	3	4	4	5
83	9191	9196	9201	9206	9212	9217	9222	9227	9232	9238	1	1	2	2	3	3	4	4	5
84	9243	9248	9253	9258	9263	9269	9274	9279	9284	9289	1	1	2	2	3	3	4	4	5
85	9294	9299	9304	9309	9315	9320	9325	9330	9335	9340	1	1	2	2	3	3	4	4	5
86	9345	9350	9355	9360	9365	9370	9375	9380	9385	9390	1	1	2	2	3	3	4	4	5
87	9395	9400	9405	9410	9415	9420	9425	9430	9435	9440	0	1	1	2	2	3	3	4	4
88	9445	9450	9455	9460	9465	9469	9474	9479	9484	9489	0	1	1	2	2	3	3	4	4
89	9494	9499	9504	9509	9513	9518	9523	9528	9533	9538	0	1	1	2	2	3	3	4	4
90	9542	9547	9552	9557	9562	9566	9571	9576	9581	9586	0	1	1	2	2	3	3	4	4
91	9590	9595	9600	9605	9609	9614	9619	9624	9628	9633	0	1	1	2	2	3	3	4	4
92	9638	9643	9647	9652	9657	9661	9666	9671	9675	9680	0	1	1	2	2	3	3	4	4
93	9685	9689	9694	9699	9703	9708	9713	9717	9722	9727	0	1	1	2	2	3	3	4	4
94	9731	9736	9741	9745	9750	9754	9759	9763	9768	9773	0	1	1	2	2	3	3	4	4
95	9777	9782	9786	9791	9795	9800	9805	9809	9814	9818	0	1	1	2	2	3	3	4	4
96	9823	9827	9832	9836	9841	9845	9850	9854	9859	9863	0	1	1	2	2	3	3	4	4
97	9868	9872	9877	9881	9886	9890	9894	9899	9903	9908	0	1	1	2	2	3	3	4	4
98	9912	9917	9921	9926	9930	9934	9939	9943	9948	9952	0	1	1	2	2	3	3	4	4
99	9956	9961	9965	9969	9974	9978	9983	9987	9991	9996	0	1	1	2	2	3	3	3	4
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Antilogarithms of Numbers

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.00	1000	1002	1005	1007	1009	1012	1014	1016	1019	1021	0	0	1	1	1	1	2	2	2
.01	1023	1026	1028	1030	1033	1035	1038	1040	1042	1045	0	0	1	1	1	1	2	2	2
.02	1047	1050	1052	1054	1057	1059	1062	1064	1067	1069	0	0	1	1	1	1	2	2	2
.03	1072	1074	1076	1079	1081	1084	1086	1089	1091	1094	0	0	1	1	1	1	2	2	2
.04	1096	1099	1102	1104	1107	1109	1112	1114	1117	1119	0	1	1	1	1	2	2	2	2
.05	1122	1125	1127	1130	1132	1135	1138	1140	1143	1146	0	1	1	1	1	2	2	2	2
.06	1148	1151	1153	1156	1159	1161	1164	1167	1169	1172	0	1	1	1	1	2	2	2	2
.07	1175	1178	1180	1183	1186	1189	1191	1194	1197	1199	0	1	1	1	1	2	2	2	2
.08	1202	1205	1208	1211	1213	1216	1219	1222	1225	1227	0	1	1	1	1	2	2	2	3
.09	1230	1233	1236	1239	1242	1245	1247	1250	1253	1256	0	1	1	1	1	2	2	2	3
.10	1259	1262	1265	1268	1271	1274	1276	1279	1282	1285	0	1	1	1	1	2	2	2	3
.11	1288	1291	1294	1297	1300	1303	1306	1309	1312	1315	0	1	1	1	1	2	2	2	3
.12	1318	1321	1324	1327	1330	1334	1337	1340	1343	1346	0	1	1	1	1	2	2	2	3
.13	1349	1352	1355	1358	1361	1365	1368	1371	1374	1377	0	1	1	1	2	2	2	3	3
.14	1380	1384	1387	1390	1393	1396	1400	1403	1406	1409	0	1	1	1	2	2	2	3	3
.15	1413	1416	1419	1422	1426	1429	1432	1435	1439	1442	0	1	1	1	2	2	2	3	3
.16	1445	1449	1452	1455	1459	1462	1466	1469	1472	1476	0	1	1	1	2	2	2	3	3
.17	1479	1483	1486	1489	1493	1496	1500	1503	1507	1510	0	1	1	1	2	2	2	3	3
.18	1514	1517	1521	1524	1528	1531	1535	1538	1542	1545	0	1	1	1	2	2	2	3	3
.19	1549	1552	1556	1560	1563	1567	1570	1574	1578	1581	0	1	1	1	2	2	3	3	3
.20	1585	1589	1592	1596	1600	1603	1607	1611	1614	1618	0	1	1	1	2	2	3	3	3
.21	1622	1626	1629	1633	1637	1641	1644	1648	1652	1656	0	1	1	2	2	2	3	3	3
.22	1660	1663	1667	1671	1675	1679	1683	1687	1690	1694	0	1	1	2	2	2	3	3	3
.23	1698	1702	1706	1710	1714	1718	1722	1726	1730	1734	0	1	1	2	2	2	3	3	4
.24	1738	1742	1746	1750	1754	1758	1762	1766	1770	1774	0	1	1	2	2	2	3	3	4
.25	1778	1782	1786	1791	1795	1799	1803	1807	1811	1816	0	1	1	2	2	2	3	3	4
.26	1820	1824	1828	1832	1837	1841	1845	1849	1854	1858	0	1	1	2	2	3	3	3	4
.27	1862	1866	1871	1875	1879	1884	1888	1892	1897	1901	0	1	1	2	2	3	3	3	4
.28	1905	1910	1914	1919	1923	1928	1932	1936	1941	1945	0	1	1	2	2	3	3	4	4
.29	1950	1954	1959	1963	1968	1972	1977	1982	1986	1991	0	1	1	2	2	3	3	4	4
.30	1995	2000	2004	2009	2014	2018	2023	2028	2032	2037	0	1	1	2	2	3	3	4	4
.31	2042	2046	2051	2056	2061	2065	2070	2075	2080	2084	0	1	1	2	2	3	3	4	4
.32	2089	2094	2099	2104	2109	2113	2118	2123	2128	2133	0	1	1	2	2	3	3	4	4
.33	2138	2143	2148	2153	2158	2163	2168	2173	2178	2183	0	1	1	2	2	3	3	4	4
.34	2188	2193	2198	2203	2208	2213	2218	2223	2228	2234	1	1	2	2	3	3	4	4	5
.35	2239	2244	2249	2254	2259	2265	2270	2275	2280	2286	1	1	2	2	3	3	4	4	5
.36	2291	2296	2301	2307	2312	2317	2323	2328	2333	2339	1	1	2	2	3	3	4	4	5
.37	2344	2350	2355	2360	2366	2371	2377	2382	2388	2393	1	1	2	2	3	3	4	4	5
.38	2399	2404	2410	2415	2421	2427	2432	2438	2443	2449	1	1	2	2	3	3	4	4	5
.39	2455	2460	2466	2472	2477	2483	2489	2495	2500	2506	1	1	2	2	3	3	4	5	5
.40	2512	2518	2523	2529	2535	2541	2547	2553	2559	2564	1	1	2	2	3	4	4	5	5
.41	2570	2576	2582	2588	2594	2600	2606	2612	2618	2624	1	1	2	2	3	4	4	5	5
.42	2630	2636	2642	2649	2655	2661	2667	2673	2679	2685	1	1	2	2	3	4	4	5	6
.43	2692	2698	2704	2710	2716	2723	2729	2735	2742	2748	1	1	2	3	3	4	4	5	6
.44	2754	2761	2767	2773	2780	2786	2793	2799	2805	2812	1	1	2	3	3	4	4	5	6
.45	2818	2825	2831	2838	2844	2851	2858	2864	2871	2877	1	1	2	3	3	4	5	5	6
.46	2884	2891	2897	2904	2911	2917	2924	2931	2938	2944	1	1	2	3	3	4	5	5	6
.47	2951	2958	2965	2972	2979	2985	2992	2999	3006	3013	1	1	2	3	3	4	5	5	6
.48	3020	3027	3034	3041	3048	3055	3062	3069	3076	3083	1	1	2	3	4	4	5	6	6
.49	3090	3097	3105	3112	3119	3126	3133	3141	3148	3155	1	1	2	3	4	4	5	6	6
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

											Differences								
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9
.50	3162	3170	3177	3184	3192	3199	3206	3214	3221	3228	1	1	2	3	4	4	5	6	7
.51	3236	3243	3251	3258	3266	3273	3281	3289	3296	3304	1	2	2	3	4	5	5	6	7
.52	3311	3319	3327	3334	3342	3350	3357	3365	3373	3381	1	2	2	3	4	5	5	6	7
.53	3388	3396	3404	3412	3420	3428	3436	3443	3451	3459	1	2	2	3	4	5	5	6	7
.54	3467	3475	3483	3491	3499	3508	3516	3524	3532	3540	1	2	2	3	4	5	6	6	7
.55	3548	3556	3565	3573	3581	3589	3597	3606	3614	3622	1	2	2	3	4	5	6	6	7
.56	3631	3639	3648	3656	3664	3673	3681	3690	3698	3707	1	2	3	3	4	5	6	7	8
.57	3715	3724	3733	3741	3750	3758	3767	3776	3784	3793	1	2	3	3	4	5	6	7	8
.58	3802	3811	3819	3828	3837	3846	3855	3864	3873	3882	1	2	3	4	4	5	6	7	8
.59	3890	3899	3908	3917	3926	3936	3945	3954	3963	3972	1	2	3	4	5	5	6	7	8
.60	3981	3990	3999	4009	4018	4027	4036	4046	4055	4064	1	2	3	4	5	6	6	7	8
.61	4074	4083	4093	4102	4111	4121	4130	4140	4150	4159	1	2	3	4	5	6	7	8	9
.62	4169	4178	4188	4198	4207	4217	4227	4236	4246	4256	1	2	3	4	5	6	7	8	9
.63	4266	4276	4285	4295	4305	4315	4325	4335	4345	4355	1	2	3	4	5	6	7	8	9
.64	4365	4375	4385	4395	4406	4416	4426	4436	4446	4457	1	2	3	4	5	6	7	8	9
.65	4467	4477	4487	4498	4508	4519	4529	4539	4550	4560	1	2	3	4	5	6	7	8	9
.66	4571	4581	4592	4603	4613	4624	4634	4645	4656	4667	1	2	3	4	5	6	7	8	10
.67	4677	4688	4699	4710	4721	4732	4742	4753	4764	4775	1	2	3	4	5	7	8	9	10
.68	4786	4797	4808	4819	4831	4842	4853	4864	4875	4887	1	2	3	4	6	7	8	9	10
.69	4898	4909	4920	4932	4943	4955	4966	4977	4989	5000	1	2	3	5	6	7	8	9	10
.70	5012	5023	5035	5047	5058	5070	5082	5093	5105	5117	1	2	4	5	6	7	8	9	11
.71	5129	5140	5152	5164	5176	5188	5200	5212	5224	5236	1	2	4	5	6	7	8	10	11
.72	5248	5260	5272	5284	5297	5309	5321	5333	5346	5358	1	2	4	5	6	7	9	10	11
.73	5370	5383	5395	5408	5420	5433	5445	5458	5470	5483	1	3	4	5	6	8	9	10	11
.74	5495	5508	5521	5534	5546	5559	5572	5585	5598	5610	1	3	4	5	6	8	9	10	12
.75	5623	5636	5649	5662	5675	5689	5702	5715	5728	5741	1	3	4	5	7	8	9	10	12
.76	5754	5768	5781	5794	5808	5821	5834	5848	5861	5875	1	3	4	5	7	8	9	11	12
.77	5888	5902	5916	5929	5943	5957	5970	5984	5998	6012	1	3	4	5	7	8	10	11	12
.78	6026	6039	6053	6067	6081	6095	6109	6124	6138	6152	1	3	4	6	7	8	10	11	13
.79	6166	6180	6194	6209	6223	6237	6252	6266	6281	6295	1	3	4	6	7	9	10	11	13
.80	6310	6324	6339	6353	6368	6383	6397	6412	6427	6442	1	3	4	6	7	9	10	12	13
.81	6457	6471	6486	6501	6516	6531	6546	6561	6577	6592	2	3	5	6	8	9	11	12	14
.82	6607	6622	6637	6653	6668	6683	6699	6714	6730	6745	2	3	5	6	8	9	11	12	14
.83	6761	6776	6792	6808	6823	6839	6855	6871	6887	6902	2	3	5	6	8	9	11	13	14
.84	6918	6934	6950	6966	6982	6998	7015	7031	7047	7063	2	3	5	6	8	10	11	13	15
.85	7079	7096	7112	7129	7145	7161	7178	7194	7211	7228	2	3	5	7	8	10	12	13	15
.86	7244	7261	7278	7295	7311	7328	7345	7362	7379	7396	2	3	5	7	8	10	12	13	15
.87	7413	7430	7447	7464	7482	7499	7516	7534	7551	7568	2	4	5	7	9	10	12	14	16
.88	7586	7603	7621	7638	7656	7674	7691	7709	7727	7745	2	4	5	7	9	11	12	14	16
.89	7762	7780	7798	7816	7834	7852	7870	7889	7907	7925	2	4	5	7	9	11	13	14	16
.90	7943	7962	7980	7998	8017	8035	8054	8072	8091	8110	2	4	6	7	9	11	13	15	17
.91	8128	8147	8166	8185	8204	8222	8241	8260	8279	8299	2	4	6	8	9	11	13	15	17
.92	8318	8337	8356	8375	8395	8414	8433	8453	8472	8492	2	4	6	8	10	12	14	15	17
.93	8511	8531	8551	8570	8590	8610	8630	8650	8670	8690	2	4	6	8	10	12	14	16	18
.94	8710	8730	8750	8770	8790	8810	8831	8851	8872	8892	2	4	6	8	10	12	14	16	18
.95	8913	8933	8954	8974	8995	9016	9036	9057	9078	9099	2	4	6	8	10	12	14	17	19
.96	9120	9141	9162	9183	9204	9226	9247	9268	9290	9311	2	4	6	8	11	13	15	17	19
.97	9333	9354	9376	9397	9419	9441	9462	9484	9506	9528	2	4	7	9	11	13	15	17	20
.98	9550	9572	9594	9616	9638	9661	9683	9705	9727	9750	2	4	7	9	11	13	16	18	20
.99	9772	9795	9817	9840	9863	9886	9908	9931	9954	9977	2	5	7	9	11	14	16	18	20
x	0	1	2	3	4	5	6	7	8	9	1	2	3	4	5	6	7	8	9

Using the table to look at the mantissa of 5678 it is read like this; 56 under 7 difference 8. that is look for the number 56 under the column x. When you get to 56 move along the row of 56 and stop in column 7 of that row and the number corresponding to 56 index 7 is your answer, keep it. This is 7536. Then go the differences section (column) index column under column 8 we have 6 which must be added to 7536 to give the final mantissa i.e 7536 + 6 = 7542.

What is the character i.e of 5678? 4 digits, character i.e. is 3.

Hence $Log\ 5678 = 3.7542$

Examples

Find the logarithm of

(1) 3875 (2) 14.32 (3) 432.1

Solutions

(1) Character i.e = 3

Mantissa – Read off 38 under 7

Differences 5 from the log logarithm table you will get 5883

$Log3875 = 3.5883$

(2) $Log14.32 = 1.498$

(3) $Log432.1 = 2.6356$

Number less than 1

These characteristic are usually less than 1 (that is negative). It is 1 greater than the number of zero immediately in front of the decimal point. Write the number in standard form and the index will be the characteristic.

Examples

(1) Write down the character of the following.

(i) 0.00234 (ii) 0.0456 (iii) 0.687.

Solutions

(i) $0.00234 = 2.34 \times 10^{-3}$; characteristics = $\bar{3}$

(ii) $0.0456 = 4.56 \times 10^{-2}$

Characteristic = $\bar{2}$

(iii) $0.687 = 6.87 \times 10^{-4}$

Characteristic = $\bar{1}$

(2) Find the logarithm of the following

(i) 0.000678 (ii) 0.003256

Solutions

(i) *Log0.0001678*

0.000678 = 6.78 x 10^{-4}

Check 67 index 8 in this question no difference

Log0.00678 = $\bar{4}.8312$

(ii) *Log0.003256*

0.003256 = 3.256 x 10^{-3}

Log0.003256 - $\bar{3}.5127$

Self Assessment Test

(1) Find the logarithm of the numbers below.

(a) 4568 (b) 0.00254 (c) 0.000765 (d) 0.01371

(2) 2268

(3) 1256

The Anti-Logarithm Table

The table is used to get the original number whose logarithm is given. To get the anti-logarithm, you will read off the Mantissa only from the anti-logarithm table. That is helps us to find the decimal point in the number gotten from the anti-logarithm table. That is done after adding 1 to the characteristic. Your count should be from the left hand side of the answer.

Examples

Find the anti-logarithm of the numbers below.

(1) 2.7658 (2) 0.3254 (3) $\bar{2}.6541$

Solution

(1) In the anti-logarithm table look for .76 under 5 difference 8. You will get 5832. Add 1 to the characteristic i.e 2 + 1 = 3. Count 3 decimal places from the left and place the point in the appropriate place. 583.2

(2) In the same way, check .32 under 5 differences 4. It gives 2115, characteristic 0 + 1 =1. Final answer 2.115.

(3) Check .65 under 4 differences 1. Gives 4509; characteristic $\bar{2}$ + 1 = -2 + 1 +1. That is move backward is 4 by 1 place and place the decimal point i.e 0.4509. (which is the same as 4.509×10^{-2}).

Self Assessment Questions

(1) Find the anti-logarithm of the numbers below.

(a) 3.1285 (b) 0.2356 (c) $\bar{3}.6214$

3.5.3 The Laws of Logarithm

There are three basic laws which must be obeyed whenever you solve problems using the logarithm table. They are known as the fundamental laws of logarithm. They are stated below.

Given A and B and C.

(1) $Log(AxB) = LogA + LogB$

(2) $Log\left(\frac{A}{B}\right) = LogA - LogB$

(3) $LogA^c = CLogA$

or

$LogB^c = CLogB$

Multiplication

Apply Law (1)

Examples

(1) Evaluate 32.56 by 9.563

Solution

No	Log
32.56	1.5127
9.563	0.9806
	2.4933

Find the anti-log of the number 4933. That is .49 under 3 difference 3, we have 3114, add 1 to the characteristic which will now be 1 + 2 = 3. Starting the count from the left, the decimal point will now be between 1 and 4. The answer is 311.4.

(2) Evaluate

0.06525 x 21.36

No	Log
0.0652	$\bar{2}.8144$
21.36	1.3296
	0.1440

On anti-log table find 14 under 4 i.e 1393 1 + 0 = 1 final answer = 1.393

Division

Apply low (2) $Log\left(\frac{A}{B}\right) = LogA - LogB$

Examples

(1) Divide 76.783 by 0.6421

No	Log
6.783	0.8315
0.6421	$\bar{1}.8076$
	1.0239

Anti-log

0.02 under 3 difference 9.

= 1056 . The decimal place 1+1 = 2. Final answer = 10.56

Note: 0-(T) ≅ 0-(-1) = 1

Self Assessment Test

Evaluate the following

(1) 22.65 x 81.11

(2) 0.7641 x 1.786

(3) 1.7815 ÷ 8.326

(4) 0.01561 ÷ 1.321

Power/Roots of Numbers

Use Law(3): $\log A^c = C \log A$

Find the log. of the number and multiply by the power. As for roots, write the number in index form, find the log and multiply by the index which is a fraction.

Note:

$\sqrt{P} = P^{\frac{1}{2}}$, $\sqrt[3]{a} = a^{\frac{1}{3}}$, $\sqrt[n]{M} = M^{\frac{1}{n}}$

Examples

(1) Evaluate (a) $(6.623)^4$ (b) $(0.0781)^3$ (c) $\sqrt{56.72}$ (d) $\sqrt[5]{0.6423}$

Solutions

No	Log
6.623	0.8211

→

No	Log
$(6.623)^4$	0.8211 x 4
	3.2844

Anti-log of .2844

= 1925

(b)

No	Log
0.0781	$\bar{2}.8927$

→

No	Log
$(0.0781)^3$	$(\bar{2}.8927)\ x\ 3$
	$\bar{4}.6781$

Anti-log of .67814

= 4764

= 0.0004764

= 0.0005

(c) $\sqrt{56.72} = (56.72)^{\frac{1}{2}}$

No	Log
56.72	1.7538

→

No	Log
$(56.72)^{\frac{1}{2}}$.	1.7538 x $\frac{1}{2}$
0.8769	$\frac{1.7538}{2}$

Anti-log .8769

= 7.732

Self Assessment Test

(1) Evaluate the following

(a) $\left(\frac{78.68}{91.23}\right)^3$ (b) $\sqrt{237.1 x 67.89}$ (c) $\frac{(90.77)^2 \ x \, 45.61}{(16.74)^3 \ x \ \sqrt{66.45}}$

3.6 SURDS

There are two types of numbers. They are rational and irrational (non-rational) numbers. Rational numbers are numbers that can be expressed as exact fractions or ratios; such as 2, $3\frac{1}{2}$, 0.8, $\sqrt{64}$ which will give $\frac{2}{1}, \frac{7}{2}, \frac{4}{5}$, ± 8.

Irrational numbers are numbers that cannot be written as ratios, such as π'.

π' = 3.141592....., the decimal containing without recurring. Other examples $\sqrt{3}$=1.752050..., $\sqrt{28}$= 5.291502.... etc

Irrational numbers of this types are referred to as surds.

Surds that have number under the square root sign as the same are called like surds, for example $\sqrt{5}$, $\frac{\sqrt{5}}{4}$ are like surds.

3.6.1 Basic Facts of Surds

(1) $\sqrt{Mn} = \sqrt{M} \ x \sqrt{n}$

(2) $\sqrt{M+n} = \sqrt{M} + \sqrt{n}$

(3) $\sqrt{\frac{M}{n}} = \frac{\sqrt{M}}{\sqrt{M}}$

(4) $\sqrt{M - n} = \sqrt{M} - \sqrt{n}$

(5) $\sqrt[n]{M} = \sqrt{(n^2)M}$

These factors are useful when simplifying surds.

Simplification of Surds

As much as possible, express the number under the square root sign as product of two factors, one such be a perfect square. Then simplify the surd by taking the square root of the perfect square.

Examples

(1) Simplify (a) $\sqrt{8},$ (b) $\sqrt{a^2b},$ (c) $\sqrt{128},$ (d) $\sqrt{75}$

Solution

(a) $8 = 2x2x2 = 4x2, \sqrt{8} = \sqrt{4x2} = \sqrt{4}x\sqrt{2} = 2x\sqrt{2} = 2\sqrt{2}$

(b) $\sqrt{a^2b} = \sqrt{a^2}x\sqrt{b} = \sqrt{axa}x\sqrt{b} = a\sqrt{b}$

(c) $\sqrt{128}\ (= \sqrt{64x2})$

$\sqrt{128} = \sqrt{64}\,x\sqrt{2} = 8\,x\ \sqrt{2} = 8\sqrt{2}$

(d) $2\sqrt{75} = 2\sqrt{25\,x\ 3} = 2\sqrt{25}x\sqrt{3} = 2x5x\sqrt{3} = 10\sqrt{3}$

There can also be a reverse process of the last example.

Examples

Express the following as a square root of a single number

(a) $4\sqrt{3},$ (b) $2\sqrt{5}$ (c) $8\sqrt{4}$

Solution

(a) $4\sqrt{3} = \sqrt{(4)^2\ x\ \sqrt{3}} = \sqrt{16}\,x\sqrt{3} = \sqrt{16\,x3} = \sqrt{48}$

(b) $2\sqrt{5} = \sqrt{(2)^2\ x\ \sqrt{5}} = \sqrt{4}\,x\sqrt{5} = \sqrt{4\,x5} = \sqrt{20}$

(c) $8\sqrt{4} = \sqrt{(8)^2\ x\ \sqrt{4}} = \sqrt{64}\,x\sqrt{4} = \sqrt{64\,x4} = \sqrt{256}$

3.6.2 Basic Arithmetic Operation on Surds

3.6.2.1 Addition and Subtraction of Surds

We can add two or more of such surds. Simplification of such surds must first be done before any addition or subtraction.

Examples

Simplifying the following

(1) $3\sqrt{2} + 8\sqrt{2}$ (2) $7\sqrt{6} + 5\sqrt{6}$

(3) $4\sqrt{5} - 2\sqrt{5}$ (4) $3\sqrt{18} + \sqrt{50} - 8\sqrt{8}$

Solutions

(1) $3\sqrt{2} + 8\sqrt{2} = (3+8)\sqrt{2} = 11\sqrt{2}$

(2) $7\sqrt{6} + 5\sqrt{6} = (7+5)\sqrt{6} = 12\sqrt{6}$

(3) $4\sqrt{5} - 2\sqrt{5} = (4-2)\sqrt{6} = 2\sqrt{5}$

(4) $3\sqrt{18} + \sqrt{50} - 8\sqrt{8}$

$= 3\sqrt{9x2} + \sqrt{25x2} - 8\sqrt{4x2}$

$= 3\sqrt{9}\ x\sqrt{2} + \sqrt{25}\ \ x\sqrt{2} - 4\sqrt{4}x\sqrt{2}$

$= 3x3\sqrt{2} + 5\ \sqrt{2} - 4x2\sqrt{2}$

$= 9\sqrt{2} + 5\sqrt{2} - 8\sqrt{2}$

$= (9+5)\sqrt{2} - 8\sqrt{2} = 14\sqrt{2} - 8\sqrt{2}$

$= (14-8)\sqrt{2} = 6\sqrt{2}$

Self Assessment Test

(1) $2\sqrt{2} - 3\sqrt{8} + 4\sqrt{32}$

(2) $\sqrt{5} + \sqrt{45} - \sqrt{80} + 2$

(3) $3\sqrt{27} - \sqrt{12} + \left(\sqrt{3}\right)^2$

(4) $\sqrt{49} - \sqrt{28} + \sqrt{63}$

3.6.2.2 Multiplication and Division of Surds

Multiplication of Surds

In multiplying surds, simplification must first take place as much as possible. Then the whole numbers will be taken with whole numbers and surds will be taken with surds.

Examples:

(1) Simplifying the following:

(a) $\sqrt{50}\ x\sqrt{75}$ (b) $3\sqrt{32}\,x\ 5\sqrt{8}\ x\,3\sqrt{18}$ (c) $\left(5\sqrt{5}\right)^2$

Solutions

(a) $\sqrt{50}\ x\sqrt{72} = \sqrt{25x2}\,x\sqrt{36x2}$

$= 5\sqrt{2}\ x6\sqrt{2} = 30\sqrt{2x2} = 30\sqrt{4} = 30x2 = 60$

(b) $3\sqrt{32}\ x5\sqrt{8}\ x3\sqrt{18}$

$= 3\sqrt{16x2}\ x5\sqrt{4x2}\ x3\sqrt{9x2}$

$= 3x4\sqrt{2}\ x5x2\sqrt{2}\ x3x3\sqrt{2}$ s

$= 12\sqrt{2}\ x\ 10\sqrt{2}\ x9\sqrt{2} = (12x10\ x9)\sqrt{2x2\ x2}$

$= (12x\ 10\ x9)\sqrt{8} = 1080x\ 2\sqrt{2}$

$= 2160\sqrt{2}$

(c) $\left(5\sqrt{5}\right)^2 = 5\sqrt{5}\ x5\sqrt{5}$

$= (5x\ 5)\left(\sqrt{5}\ x\ \sqrt{5}\right)$

$= 25\sqrt{25} = 25x5 = 125$

Division of Surds

If the denominator of a fraction is a surd, it is usually best to rationalize the denominator. To rationalize the denominator means to make the denominator into a rational number, usually a whole number. To do this, you have a multiply the numerator and the denominator of the fraction by a surd that makes the denominator rational which will not in any way affect the original numbers.

Example 1

Rationalize the denominators of the following.

(a) $\frac{12}{\sqrt{3}}$ (b) $\frac{8}{\sqrt{12}}$ (c) $\frac{40}{\sqrt{5}}$ (d) $\frac{9}{\sqrt{12}}$

Solutions

(a) $\frac{12}{\sqrt{3}} = \frac{12}{\sqrt{3}}\ x\ \frac{\sqrt{3}}{\sqrt{3}}$

Multiplying $\frac{12}{\sqrt{3}}$ by $\frac{\sqrt{3}}{\sqrt{3}}$ is like multiplying it by 1 since $\frac{\sqrt{3}}{\sqrt{3}}$ is the same as 1.

$$\therefore \frac{12}{\sqrt{3}}\ x\ \frac{\sqrt{3}}{\sqrt{3}} = \frac{12\sqrt{3}}{\sqrt{3x3}} = \frac{12\sqrt{3}}{\sqrt{9}} = \frac{12\sqrt{3}}{\sqrt{9}} = \frac{12\sqrt{3}}{3}$$
$$= 4\sqrt{3}$$

(b) $\frac{8}{\sqrt{12}} = \frac{8}{\sqrt{4x3}} = \frac{8}{2\sqrt{3}} = \frac{8}{2\sqrt{3}}\ x\frac{\sqrt{3}}{\sqrt{3}} = \frac{8\sqrt{3}}{2\sqrt{9}}$

$$= \frac{8\sqrt{3}}{2x3} = \frac{8\sqrt{3}}{6} = \frac{4\sqrt{3}}{3}$$

(c) $\frac{40}{\sqrt{5}} = \frac{40}{\sqrt{5}}\ x\ \frac{\sqrt{5}}{\sqrt{5}} = \frac{40\sqrt{5}}{\sqrt{5x5}} = \frac{40\sqrt{3}}{\sqrt{25}}$

$$= \frac{40\sqrt{5}}{5} = 8\sqrt{5}$$

(d) $\frac{9}{\sqrt{18}} = \frac{9}{\sqrt{2x9}} = \frac{9}{3\sqrt{2}}\ x\ \frac{\sqrt{2}}{\sqrt{2}} = \frac{9\sqrt{2}}{3\sqrt{2x2}}$

$$= \frac{9\sqrt{2}}{3x2} = \frac{3\sqrt{2}}{2}$$

Example 2

Simplify the following

(a) $\frac{\sqrt{16}}{2}$ (b) $\frac{\sqrt{18}}{6}$ (c) $\sqrt{\frac{12}{7}}$ (d) $\frac{3\sqrt{6}\ x\ 2\sqrt{7}}{2\sqrt{24}\ x\ \sqrt{28}}$

Solutions

(a) $\frac{\sqrt{16}}{\sqrt{2}} = \frac{\sqrt{16}}{\sqrt{2}}\ x\ \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{16x2}}{\sqrt{2x2}} = \frac{\sqrt{32}}{\sqrt{4}} = \frac{\sqrt{2\ x16}}{2}$

$$\frac{4\sqrt{2}}{\sqrt{2}} = 2\sqrt{2}$$

(b) $\frac{\sqrt{18}}{6} = \frac{\sqrt{18}}{6} \; x \; \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{18x6}}{\sqrt{6x6}} = \frac{\sqrt{108}}{\sqrt{36}} = \frac{\sqrt{36x3}}{6}$

$\frac{6\sqrt{3}}{6} = \sqrt{3}$

(c) $\sqrt{\frac{12}{7}} = \frac{\sqrt{12}}{\sqrt{7}} = \frac{\sqrt{12}}{\sqrt{7}} \; x \; \frac{\sqrt{7}}{\sqrt{7}} = \frac{\sqrt{12x7}}{\sqrt{7x7}}$

$= \frac{\sqrt{84}}{\sqrt{49}} = \frac{\sqrt{4x21}}{7} = \frac{2\sqrt{21}}{7}$

(d) $\frac{3\sqrt{6} \; x \; 2\sqrt{7}}{2\sqrt{24} \, x \sqrt{28}} = \frac{3\sqrt{6} \; x \, 2\sqrt{7}}{2\sqrt{4x6} x \sqrt{4x7}}$

$\frac{3\sqrt{6} \; x \; 2\sqrt{7}}{4\sqrt{6} \, x \, 2\sqrt{7}} = \frac{3\sqrt{6}}{4\sqrt{6}} = 3/4$

Further examples

(1) Simplify the following

(a) $\left(3\sqrt{2} + 5\right)\left(3 + 6\right)$

(b) $\left(2\sqrt{3} + \sqrt{5}\right)\left(4\sqrt{3} - 4\sqrt{5}\right)$

(c) $\left(\sqrt{6} - 5\sqrt{10}\right)^2$

Solution

(a) $\left(3\sqrt{2} + 5\right)\left(\sqrt{2} + 6\right)$

$= \left(3\sqrt{2}\right)\left(\sqrt{2}\right) + 6 \, x \, 3\sqrt{2} + 5\sqrt{2} + 5 \, x \, 6$

$= 3\sqrt{4} + 18\sqrt{2} + 5\sqrt{2} + 30$

$= 3 \, x \, 2 + 8\sqrt{2} + 5\sqrt{2} + 30$

$= 6 + 30 + 8\sqrt{2} + 5\sqrt{2} = 36 + 8\sqrt{2} + 5\sqrt{2} = 36 \neq 13\sqrt{12}$ s

(b) $\left(2\sqrt{3} + \sqrt{5}\right)\left(4\sqrt{3} - + 4\sqrt{5}\right)$

$= \left(2\sqrt{3}\right)\left(4\sqrt{3}\right) - \left(2\sqrt{3}\right)\left(4\sqrt{5}\right) + \left(4\sqrt{3}\right)\left(\sqrt{5}\right) - \left(4\sqrt{5}\right)\sqrt{5}$

$= 8\sqrt{9} - 8\sqrt{15} + 4\sqrt{15} - 4\sqrt{25}$

$= 8 \; x \, 3 - 4\sqrt{15} - 4 \; x \, 5$

$24 - 4\sqrt{15} - 20$

$= 4 - 4\sqrt{15} = 4(1 - \sqrt{15})$

(c) $\left(\sqrt{6} - 5\sqrt{10}\right)^2$

$\left(\sqrt{6} - 5\sqrt{10}\right)\left(\sqrt{6} - 5\sqrt{10}\right) = \left(\sqrt{6}\right)^2 - \sqrt{6}.x5\sqrt{10} - 5\sqrt{10} = \sqrt{6} + 5\sqrt{10}\,5\sqrt{10}$

$= 6 - 5\sqrt{60} - 5\sqrt{60} + 25\sqrt{100}$

$= 6 - 10\sqrt{4x15} + 25\sqrt{10x10}$

$= 6 + 250 - 20\sqrt{15}$

Conjugate of a Binomial Surd

Any expression that contain two terms in which one or both of the terms are surd is call a binomial surd expression. To rationalize this surd, we use the idea of the difference of two squares. For example, to rationalize $\sqrt{M} - n$ we multiply it by $\sqrt{M} + n$ to give $\left(\sqrt{M}\right)^2 - n$, which is a rational number $\sqrt{M} - n$ and $\sqrt{M} + n$ are said to be conjugate of another.

Examples

(1) Evaluate the following

(a) $\left(3\sqrt{2} - 6\right)\left(3\sqrt{2} + 6\right)$

(b) $\frac{8}{2\sqrt{3} - 1}$ (c) $\frac{5\sqrt{5} + 3}{4\sqrt{5 - 3}}$ (d) $\frac{8}{5 - \sqrt{2}}$

Solutions

(a) $\left(3\sqrt{2} - 6\right)\left(3\sqrt{2} + 6\right) = 9\sqrt{4} + 18\sqrt{2} - 18\sqrt{2} - 36$

$9\sqrt{4} - 36 = 18 - 36$

$= -18$

Recall $a^2 - b^2 = (a + b)(a - b)$

(b) $\frac{8}{2\sqrt{3} - 1} = \frac{8}{\left(2\sqrt{3} - 1\right)} x \frac{\left(2\sqrt{3} + 1\right)}{\left(2\sqrt{3} + 1\right)} = \frac{16\sqrt{3} + 8}{\left(2\sqrt{3}\right)^2 - (1)^2}$

$= \frac{16\sqrt{3} + 8}{12 - 1} = \frac{16\sqrt{3} + 8}{11}$

(c) $\frac{(5\sqrt{5}+3)}{(4\sqrt{5}+3)} x \frac{(4\sqrt{5}-3)}{(5\sqrt{5}+3)}$

$$= \frac{20\,x\,5+15\sqrt{5}\ +12\sqrt{5}+9}{16\,x\,5\ -12\sqrt{5}\ +12\sqrt{5}-9}$$

$$\frac{100+27\sqrt{5}+9}{80-9} = \frac{109+27\sqrt{5}}{71}$$

(d) $\frac{6}{5-\sqrt{2}} x \frac{5\sqrt{2}}{5+\sqrt{2}}$

$$\frac{6(5+\sqrt{2})}{5^2-(\sqrt{2})} = \frac{30+\ 6\sqrt{2}}{25-2} = \frac{30+6\sqrt{2}}{23}$$

Self Assessment Test

Simplify the following

(1) $\sqrt{2}\ x\sqrt{3}\ x\sqrt{5}\ x\ \sqrt{6}\ x\ \sqrt{20}$

(2) $\sqrt{50}\ x\sqrt{72}$

(3) $(6\sqrt{7})^2$

(4) $\frac{\sqrt{32}}{\sqrt{6}}$ (b) $\sqrt{\frac{128}{8}}$

(5) $\frac{10}{\sqrt{72}}$ (6) $\frac{10}{\sqrt{5}} x \frac{5}{\sqrt{50}}$

(7) $\sqrt{3}(\sqrt{6}+\sqrt{24})$

(8) $\frac{\sqrt{2}+\sqrt{5}}{3\sqrt{5}-2}$ (9) $\frac{2-3\sqrt{3}}{2+3\sqrt{3}}$

(10) $\left(\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)^2$ (11) $\frac{\sqrt{6}+2}{5-\sqrt{7}}$

4.0 Conclusion

This unit considered prime numbers and its relationship with indices. The law of indices and how it is used in the basic operation. The relationship between indices and logarithm and the use of four figure table. Surds and the basic arithmetic operation were also studied.

The unit considered how surds operations, addition, subtraction, multiplication and division are carried out. The conjugate of surd was finally considered.

5.0 Summary

The highlights of the unit includes the following.

- How numbers are express as product of its prime.
- How the fundamental laws of indices are used in the solving of basic problems.
- The basic operation about surds was treated.
- The rationalization of surds was also treated.
- The conjugate of surds.
- The relationship between indices and logarithm.
- How to use the four figure table for numbers greater than one and numbers less than one.
- Using logarithm table for the production, quotient, powers and roots of number.

6.0 Tutor Marked Assignment

(1) Express the following as a product of power of prime.

(a) 15288 (b) 5184

(2) Evaluate (a) $81a^6 \div 3a^3$ (b) $\left(\frac{81}{16}\right)^{-\frac{3}{2}}$

(3) Simplify

(a) $\sqrt{\frac{84a^5b0^{-3}}{4a^8 5^6}}$ (b) $\frac{\left(\frac{27^{1/3}}{16^{\frac{1}{4}}}\right)}{27^{1/2}}$

(c) $\sqrt{\left(125^2\right)^{-\frac{1}{2}}}$ (d) $2^{\frac{2}{3}} - \left(2^{2\frac{1}{2}} - 1^{\frac{4}{3}}\right)$

(e) $4^{-\frac{1}{2}} x\ 16^{\frac{3}{4}} \div 4^{\frac{1}{2}}$

(5) Evaluate

(a) $34.83\ x\ 42.87$

(b) $\frac{17.83\,x\ 246.9}{256.2\,x\,3.28}$

(6) Evaluate

$$\frac{(36.12)^4 \ x \ \sqrt{92.5}}{218 \, x \, 3.12}$$

(7) Simplify the following

(a) $\frac{\sqrt{36}}{\sqrt{6}}$ (b) $\sqrt{\frac{81}{5}}$

(c) $\sqrt{5}+\sqrt{12}-\sqrt{27}$

(d) $3\sqrt{8}-3\sqrt{2} + 4\sqrt{18}$

(e) $\left(\sqrt{3}+\frac{1}{\sqrt{3}}\right)^2$ (f) $\sqrt{2\left(3\sqrt{89}+\sqrt{6}\right)}$

(g) $\left(\sqrt{3}-\sqrt{2}\right)\left(2\sqrt{5}+\sqrt{6}\right)$

(h) $\left(1-4\sqrt{5}\right)^2$ (i) $\left(\frac{\sqrt{2} + \sqrt{5}}{\sqrt{2-\sqrt{5}}}\right)$

(j) $\frac{2}{6 \, x \ \sqrt{8}}$ (k) $\frac{3 - 3\sqrt{5}}{3\sqrt{3}-5\sqrt{5}}$

7.0 References/Further Reading

Channon, J.B. et. al (2002): New General Mathematics for Junior Secondary Schools.1,2,&3 Lagos: Group (FE) Ltd.

Channon, J.B. et. al (2002): New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 1: UNIT 3

Unit 3: Meaning and Types of Fraction

Content

1.0 Introduction

In this unit we will examine fractions as a class of numbers. We will be able to add, subtract, multiply and divide fractions, decimals and percentages. We will also consider, the use of fractions in our daily life and human activities and how fractions can be changed to decimals and percentages and vice versa.

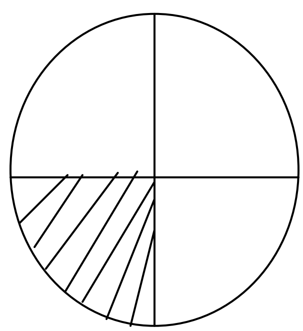
2.0 Objectives

By the end of the unit, you will be able to:

1. describe a fraction
2. demonstrate a fractional part of a whole
3. find an equivalent fraction to a given fraction and determine whether the two given fractions are equivalent or not.
4. state what proper and improper fractions are.
5. represent a named fraction in symbol.
6. reduce a given proper fraction to its lowest term or lower an equivalent fraction to the least denomination.
7. carry out basic operation in decimals.
8. change decimals to percentages and percentages to decimals.

3.0 Main Body

3.1 The Meaning of Fractions



The above circle is divided into 4 equal parts. One (1) of the parts is shaded. The shaded portion can be represent in a fraction form. This fraction tells how the shaded part is related to the whole. The shaded part can be written as $^1/_4$ with one as numerator and four (4) the denominator.

Note: The number above the line is called the numerator, while the one below is called the denominator. The numerator represents the shaded part of the whole and the denominator also represent the equal parts into which the whole has been divided.

In the same way, if a whole is divided into ten (10) equal parts and three (3) parts are shaded, the faction representing this is written as $^3/_{10}$. The three is number while ten (10) is the denominator.

Note: A common fraction can be seen as a short form of writing a division problem. If you are asked to share 5 mangos between 8 people which can be written as 5÷8 or $\frac{5}{8}$. The fraction line indicates division. The numerator is called the dividend and the denominator is called the divisor.

Note: A fraction represents a whole (number) and not two numbers.

Take a look at the figure above again the shaded portion is one of the four equal parts. It is one quarters, it is $\frac{1}{4}$. The three unshaped parts called three quarter, is written as $^3/_4$.

Naming and representation of fractions go on as illustrated above. The table below extends the system.

Description	Name	Symbol
One of two equal parts	A half	½
One of ten equal parts	One tenth	$\frac{1}{10}$
One of five equal parts	One fifth	$\frac{1}{5}$
One of four equal parts	One quarter	$\frac{1}{4}$
Two of three equal parts	Two thirds	$\frac{2}{3}$

Self Assessment

Complete the table below

Description	Name	Symbol
One of six equal parts	-	-
Seven of ten equal parts	-	-
-	Four ninths	-
-	Eight fifteenths	-
-	-	$\frac{5}{17}$
-	-	$\frac{11}{20}$

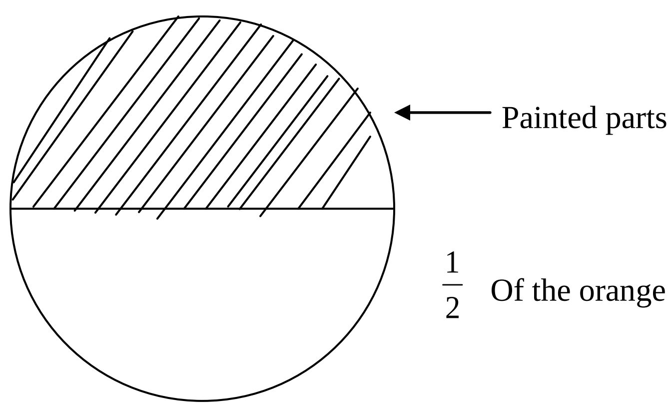
3.2 Equivalent Fractions

Two or more fractions are said to be equivalent or `same' if they represent the same value.

Example

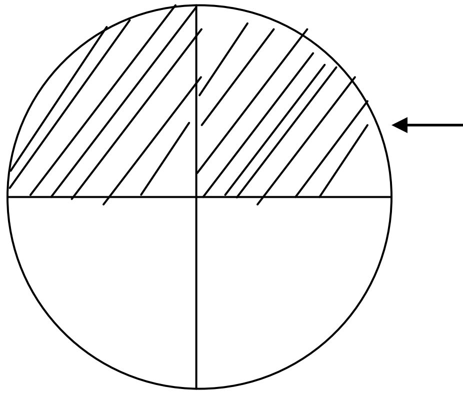
Take four orange of equal sizes

(1) Take the first one and cut into two equal parts. Paint one part, that is half of the orange and write down the result.



$\frac{1}{2}$ Of the orange is painted

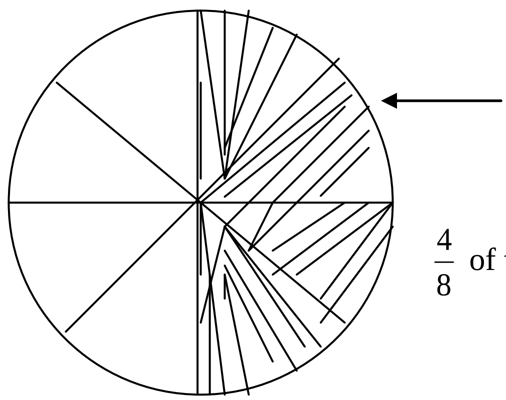
(2) Take the second orange and cut into four equal parts paint two out of the four parts.



Painted parts

$\frac{2}{4}$ of the orange part is painted

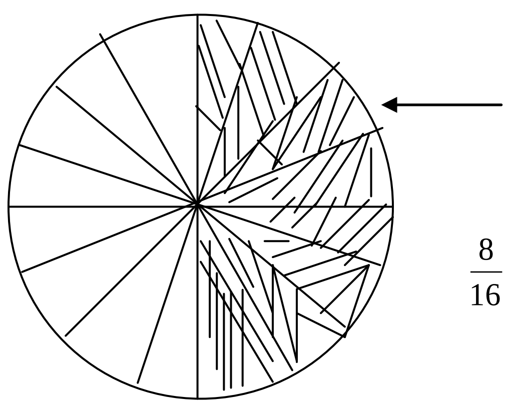
(3) Take the third orange and cut into eight equal parts. Paint four out of the eight parts.



Painted parts

$\frac{4}{8}$ of the orange parts is painted

(4) Take the last orange and cut into 16 equal parts. Paint eight out of the sixteen parts.



Painted parts

$\frac{8}{16}$ 0f the orange parts is pointed

If you look at the four diagrams carefully, you ca observed that in all it is half of the orange that is painted.

The painted parts are:

$$\frac{1}{2}, \frac{2}{4}, \frac{4}{8} \& \frac{8}{16}$$

$$\therefore \frac{1}{2} = \frac{2}{4} = \frac{4}{8} = \frac{8}{16}$$

The fraction $\frac{1}{2}, \frac{2}{4}, \frac{4}{8} \frac{8}{16}$ represent the same area and they are therefore called equivalent fraction.

Note: that :

$$\frac{1}{5} = \frac{2}{10} = \frac{3}{15}$$

$$\frac{2}{10} = \frac{1}{5} x \frac{2}{2}$$

$$\frac{3}{15} = \frac{1}{5} x \frac{3}{3}$$

In each case, the numerator and the denominator have been multiplied by the same number to get equivalent fraction.

Examples

1. The equivalent fractions of

(i) $\frac{1}{3}$ (ii) $\frac{3}{5}$ (iii) $\frac{2}{7}$ (iv) $\frac{3}{8}$ & (v) $\frac{2}{9}$

(i) $\frac{1}{3} x \frac{2}{2} = \frac{2}{6}, \frac{1}{3} x \frac{3}{3} = \frac{3}{9}, \frac{1}{3} x \frac{4}{4} = \frac{4}{12}$ etc

(ii) $\frac{2}{5} x \frac{2}{2} = \frac{6}{10}, \frac{3}{5} x \frac{3}{3} = \frac{9}{15}, \frac{3}{5} x \frac{4}{4} = \frac{13}{20}$ etc

(iii) $\frac{2}{7} x \frac{2}{2} = \frac{4}{14}, \frac{2}{7} x \frac{3}{3} = \frac{6}{21}, \frac{2}{7} x \frac{4}{4} = \frac{8}{28}$ etc

(iv) $\frac{3}{8} x \frac{2}{2} = \frac{6}{16}, \frac{3}{8} x \frac{3}{3} = \frac{9}{24}, \frac{3}{8} x \frac{4}{4} = \frac{12}{32}$ etc

(v) $\frac{2}{9} x \frac{2}{2} = \frac{4}{18}, \frac{2}{9} x \frac{3}{3} = \frac{6}{27}, \frac{2}{9} x \frac{4}{4} = \frac{8}{36}$ etc

3.3 Ordering Factions in Terms of Magnitude

In ordering fractions, we have to find the greatest or the least of the given fractions then compare their results. To find out if two given fractions are equivalent, reduce each fraction to lowest term and compared the result.

Examples

(1) Express $\frac{24}{48}$ in its lowest term

Solution

24 = 2 x 2 x 2 x 3

48 = 2 x 2 x 2 x 2 x 3

By multiplication

$$\frac{24}{48} = \frac{12x2}{24x2} = \frac{6x2}{12x2} = \frac{3x2}{6x2} = \frac{3x1}{3x2} = \frac{1}{2}$$

Or

By division

$$\frac{24}{48} = \frac{24}{48} \div \frac{2}{2} = \frac{12}{24} \div \frac{2}{2} = \frac{6}{12} \div \frac{2}{2} = \frac{3}{6} \div \frac{3}{3} = \frac{1}{2}$$

Prime Factors

$$\frac{24}{48} = \frac{2x2x2x3}{2x2x2x2x3} = \frac{1x1x1x1}{1x1x1x2x1} = \frac{1}{2}$$

Example 2

(a) Is $\frac{7}{28}$ equivalent to $\frac{9}{36}$?

(b) Is $\frac{6}{18}$ equivalent to $\frac{8}{24}$?

(c) Is $\frac{8}{64}$ equivalent to $\frac{9}{81}$?

Solutions

(a) Is $\frac{7}{28}$ equivalent to $\frac{9}{36}$?

$$\frac{7}{28} = \frac{1}{4} x \frac{7}{7} = \frac{1}{4}$$

$$\frac{9}{36} = \frac{1}{4} x \frac{9}{9} = \frac{1}{4}$$

$$\therefore \frac{7}{28} = \frac{1}{4} = \frac{9}{36}$$

(b) Is $\frac{6}{18}$ equivalent to $\frac{8}{24}$?

$$\frac{6}{18}=\frac{1}{3}x\ \frac{6}{6}=\frac{1}{3}$$

$$\frac{8}{24}=\frac{1}{3}x\ \frac{8}{8}=\frac{1}{3}$$

$$\therefore\frac{6}{18}=\frac{1}{3}=\frac{8}{24}$$

(c) Is $\frac{8}{64}$ equivalent to $\frac{9}{81}$?

$$\frac{8}{64}=\frac{1}{8}x\ \frac{8}{8}=\frac{1}{8}$$

$$\frac{9}{81}=\frac{1}{9}x\ \frac{9}{9}=\frac{1}{9}$$

$\frac{1}{8}$ is not equal to $\frac{1}{9}$

So, $\frac{8}{64}$ is not equivalent to $\frac{9}{81}$

Examples 3

Mr. Musa is to send money to John and Mary. If John is to take three quarters $\left(\frac{3}{4}th\right)$ of the amount and Mary is to receives two fifths $\left(\frac{2}{5}th\right)$. Who receives the bigger amount?

Solution

To answer this question, we must make the denominator of the two fractions equal. This will enable us to know which of the two fractions is bigger.

$$\frac{3}{4}=\frac{3}{4}x\ \frac{5}{5}=\frac{15}{20}$$

$$\frac{2}{5}=\frac{2}{5}x\ \frac{4}{4}=\frac{8}{20}$$

Since the two fraction, $\frac{15}{20},\frac{8}{20}$ have the same denominator, we check the numerator to see which is bigger 15 is bigger than 8.

So, $\frac{15}{20} > \frac{8}{20}$ *or* $\frac{3}{4} > \frac{2}{5}$

So, John gets more money than Mary

Example 4

Which fraction is bigger among the following pairs.

(a) $\frac{3}{7}, \frac{2}{8}$ (b) $\frac{2}{7}, \frac{3}{9}$

Solution

(a) $\frac{3}{4} = \frac{3}{7} x \frac{8}{8} = \frac{24}{56}$

$\frac{2}{8} = \frac{2}{8} x \frac{7}{7} = \frac{14}{56}$

Since 24 > 14 $\therefore \frac{3}{7} > \frac{2}{8}$

Note: Look for the L.C.M. of 7 & 8 which is 56. Which means the denominator of the two fractions must be 56.

(b) $\frac{2}{7} \& \frac{3}{9}$

LCM of 7 & 9 = 63

$\frac{2}{7} = \frac{2}{7} x \frac{9}{9} = \frac{18}{63}$

$\frac{3}{9} = \frac{3}{9} x \frac{7}{7} = \frac{21}{63}$

Since 21 > 18, $\therefore \frac{3}{9} > \frac{2}{7}$

Self Assessment Question

1. Which of the following pairs of fractions are equivalent or not.

(a) $\frac{15}{25}, \frac{21}{35}$ (b) $\frac{24}{120}, \frac{18}{54}$ (c) $\frac{50}{150}, \frac{16}{48}$ (d) $\frac{75}{100}, \frac{60}{75}$ (e) $\frac{2}{22}, \frac{3}{33}$

2. Which fraction is greater?

(a) $\frac{7}{10}, \frac{5}{6}$ (b) $\frac{12}{15}, \frac{3}{5}$ (c) $\frac{3}{9}, \frac{2}{5}$ (d) $\frac{2}{5}, \frac{2}{6}$

3. Reduce the fractions to their lowest terms

(a) $\frac{100}{120}$ (b) $\frac{75}{150}$ (c) $\frac{648}{972}$ (d) $\frac{675}{1125}$

4. Arrange the fraction in ascending order of magnitude

(a) $\frac{2}{3}, \frac{5}{8}, \frac{6}{11}$ (b) $\frac{2}{5}, \frac{3}{7}, \frac{5}{9}$

3.4 Improper Fractions and Mixed Numbers

Fractions in which the numerators are less than their denominator is called proper fraction, otherwise they are improper fraction. For example if you are ask to divide 20 by 4 & 35 by 3. Both can be written as

$$20 \div 4 = \frac{20}{4} = 5$$

$$35 \div 3 = \frac{35}{3} = 11 \text{ remainder } 2$$

The fraction $\frac{20}{4}$ and $\frac{35}{3}$ are called improper fraction; because the numerators are not less than their denominator i.e the denominators are less than the numerators.

Note: When we divide 35 by 3 it will give $11\frac{2}{3}$ *i.e* $\frac{35}{3} = 11\frac{2}{3}$

The fraction $11\frac{2}{3}$ is called a mixed number. This is because it has a whole number as well as a fraction part. All numbers that have a whole as well as a fraction are called mixed numbers e.g 1½ , $3\frac{5}{6}$, $3\frac{1}{8}$ etc

3.4.1 Changing Mixed Numbers to Improper Fractions

Mixed number can be express as a single fraction. Recall that a whole number can be express as a fraction with a denominator 1. That is, we can express 5 as 5/1 or 5 ÷ 1 = 5.

$$5\frac{1}{3} = \frac{5}{1} + \frac{1}{3}$$

L.C.M of 1 and 3 is 3

$$\frac{5}{1} + \frac{1}{3} = \frac{15+1}{3} = \frac{16}{3}$$

Method I

Converting the whole number into the given fraction.

That is $\frac{5}{1} = \frac{5}{1} \ x \ \frac{3}{3} = \frac{15}{3}$

Then $\frac{15}{3} + \frac{1}{3} = \frac{15+1}{3} = \frac{16}{3}$

Method II

Multiplying the denominator of the fraction by the whole number and add it to the numerator of the fraction and base the final result on the original denominator. $5\frac{1}{4}$ i.e

5 x 3 = 15 + 1 = 16. $5\frac{1}{3}=\frac{16}{3}$

3.4.2 Changing Improper Fractions to Mixed Numbers

You will recall that improper fraction has its numerator greater than the denominator. In this operation, we divide the numerator by the denominator and write the result as a whole number and a remainder. This remainder is now expressed as a fraction.

Examples

1. Change $\frac{17}{3}$ to mixed member

$\frac{17}{3} = 17 \div 3 = 5$ remainder 2

$2 \div 3 = \frac{2}{3}$

$\therefore \frac{17}{3} = 5\frac{2}{3}$

2. Change (a) $\frac{63}{8}$ (b) $\frac{34}{15}$ to mixed number

(a) $\frac{63}{8} = 63 \div 8 = 7$ remaining 7

$7 \div 8 = \frac{7}{8} \therefore \frac{63}{8} = 7\frac{7}{8}$

(b) $\frac{34}{15} = 34 \div 15 = 2$ remaining 4

$4 \div 15 = \frac{4}{15} \therefore \frac{34}{15} = 2\frac{4}{15}$

3.5 Operations of Addition and Subtraction on Fractions including word Problems

In this section, we are going to learn how to carry out addition and subtraction on fractions. First the addition, followed by the subtraction then the addition and subtraction simultaneously. Finally, we will solve word problems related to addition and subtraction of fractions.

3.5.1 Addition of Fractions

We can add two or more fractions if the fractions have the same denominators.

Examples

(1) Add (a) $\frac{2}{8}$ *to* $\frac{5}{8}$ (b) $\frac{1}{5}$ *to* $\frac{3}{5}$ (c) $\frac{5}{9}$ *to* $\frac{3}{9}$ (d) $\frac{2}{7}$ *to* $\frac{1}{7}$ (e) $\frac{3}{4}$ *to* $\frac{1}{4}$

Solution

Since both have the same denominator add the numerators and retain the denominator.

(a) $\frac{2}{8}+\frac{5}{8}=\frac{2+5}{8}=\frac{7}{8}$

(b) $\frac{1}{5}+\frac{3}{5}=\frac{1+3}{5}=\frac{4}{5}$

(c) $\frac{5}{9}+\frac{3}{9}=\frac{5+3}{9}=\frac{8}{9}$

(d) $\frac{2}{7}+\frac{1}{7}=\frac{2+1}{7}=\frac{3}{7}$

(e) $\frac{3}{4}+\frac{1}{4}=\frac{3+1}{4}=\frac{4}{4}=1$

(2) Add (a) $\frac{3}{11}$, $\frac{6}{11}$ & $\frac{5}{11}$ (d) $\frac{3}{7}$, $\frac{6}{7}$ & $\frac{5}{11}$ (c) $\frac{4}{5}$, $\frac{2}{5}$ & $\frac{1}{5}$ (b) $\frac{2}{15}$, $\frac{4}{15}$, $\frac{9}{15}$ & $\frac{8}{115}$

Solution

Method I

$\frac{3}{8}=\frac{3}{8}\ x\ \frac{9}{9}=\frac{27}{72}$

$\frac{2}{9}=\frac{2}{9}\ x\ \frac{8}{8}=\frac{16}{72}$

$\frac{5}{12}=\frac{5}{12}\ x\ \frac{6}{6}=\frac{30}{72}$

$\frac{27}{72}+\frac{16}{72}+\frac{30}{72}=\frac{27+16+30}{72}=\frac{73}{72}=1\frac{1}{72}$

Method II

Step 1: Get the LCM of the denominators of the fractions.

$\frac{3}{8}+\frac{2}{9}+\frac{5}{12}$

LCM = 72

$\frac{3}{8}+\frac{2}{9}+\frac{5}{12}=72$

Step 2: Divide the LCM by the various denominators and multiply each by its various numerator

For 3/8, divide 72 by 8 = 9

3 x 9 = 27

for 2/9, divide 72 by 9 = 8 i.e 2 x 8 = 16

for 5/12, divide 72 by 12 = 6

5 x 6 = 30

step 3: Add the answers in step 2

$$i.e \quad \frac{3}{8}+\frac{2}{9}+\frac{5}{12}=\frac{27+16+30}{72}$$

$$=\frac{73}{72}=1\frac{1}{72}$$

Example (b)

(a) $\frac{3}{11}+\frac{6}{11}+\frac{5}{11}=\frac{3+6+5}{11}=1\frac{5}{11}$

(b) $\frac{3}{7}+\frac{6}{7}+\frac{5}{7}=\frac{3+6+5}{7}=\frac{4}{7}=2$

(c) $\frac{4}{5}+\frac{2}{5}+\frac{1}{5}=\frac{4+2+1}{5}=\frac{7}{5}=1\frac{2}{5}$

(d) $\frac{2}{15}+\frac{4}{15}+\frac{9}{15}+\frac{8}{15}=\frac{2+4+9+8}{15}=\frac{23}{5}=1\frac{8}{15}$

(3) Simplify (a) $\frac{3}{8}+\frac{2}{9}+\frac{5}{12}$ (b) $\frac{1}{4}+\frac{2}{5}+\frac{1}{8}+\frac{1}{2}$ (c) $\frac{3}{7}+\frac{4}{5}+\frac{1}{3}$

Solution

(a) $\frac{3}{8}+\frac{2}{9}+\frac{5}{12}$

The L.C.M. of 8, 9, & 12 is

2	8, 9, 12
2	4, 9, 6
2	2, 9, 3
3	1, 9, 3
3	1, 3, 1
	1, 1, 1

L.C.M. *2 x 2 x 2 x 3 x 3*

8 x 9 = 72

2	4, 5, 6, 2
2	2, 5, 3, 1
3	1, 5, 3, 1
5	1, 5, 1, 1
	1, 1, 1, 1

2 x 2 x 2 x 3 x 5 = 60

Method I

$$\frac{1}{4} = \frac{1}{4}\ x\ \frac{25}{25} = \frac{25}{60}$$

$$\frac{2}{5} = \frac{2}{5}\ x\ \frac{12}{12} = \frac{24}{60}$$

$$\frac{1}{6} = \frac{1}{6}\ x\ \frac{10}{10} = \frac{10}{60}$$

$$\frac{1}{2} = \frac{1}{2}\ x\ \frac{30}{30} = \frac{30}{60}$$

$$\frac{25}{60} + \frac{24}{60} + \frac{10}{60} + \frac{30}{60} = \frac{25+24+10+30}{60} = \frac{89}{60} = 1\frac{29}{60}$$

Method II

$$\frac{1}{4} + \frac{2}{5} + \frac{1}{6} + \frac{1}{6} = \frac{\quad}{60}$$

Divide 60 by 4 = 25

1 x 25 = 25

$60 \div 5 = 12\,x\,2 = 24$

$60 \div 6 = 10\,x\,1 = 10$

$60 \div 2 = 30\,x1 = 30$

$$\frac{25}{60}+\frac{24}{60}+\frac{10}{60}+\frac{30}{60}=\frac{25+24+10+30}{60}=\frac{89}{60}=1\frac{29}{60}$$

Example (c): $\frac{3}{7}+\frac{4}{5}+\frac{1}{3}$

LCM of 7, 5 & 3

3	7, 5, 3
5	7, 5, 1
7	7, 1, 1
	1, 1, 1

LCM = 3 x 5 x 7 = 105

Method I

$$\frac{3}{7}=\frac{3}{7}x\,\frac{15}{15}=\frac{45}{105}$$

$$\frac{4}{5}=\frac{4}{5}x\,\frac{21}{21}=\frac{84}{105}$$

$$\frac{1}{3}=\frac{1}{3}x\,\frac{35}{35}=\frac{35}{105}$$

$$\frac{45}{105}+\frac{84}{105}+\frac{35}{105}=\frac{45+84+35}{105}=\frac{164}{105}=1\frac{59}{105}$$

Method II

$$\frac{3}{7}+\frac{4}{5}+x\,\frac{1}{3}=\frac{\quad}{105}$$

$(105 \div 7)x\,3 = 45$

$(105 \div 5)x\,3 = 84$

$(105 \div 3)x\,1 = 35$

$$\frac{45}{105}+\frac{84}{105}+\frac{35}{105}=\frac{45+84+35}{105}=\frac{164}{105}=1\frac{59}{105}$$

Examples 3

(a) Add $5\frac{3}{5}+7\frac{1}{5}$

(b) Add $3\frac{7}{12}+6\frac{5}{12}+7\frac{9}{12}$

(c) Add $3\frac{16}{18}+2\frac{4}{9}+6\frac{1}{3}$

Solutions

In this type of fractions, add the whole number parts of the mixed numbers first, then add the fractional parts.

(a) $5\frac{3}{5}+7\frac{1}{5}$

$5+7+\frac{3}{5}+\frac{1}{5}= 12+\frac{3}{5}+\frac{1}{5}=12\ \frac{3+1}{5}$

$= 12\frac{4}{5}$

(b) $7\frac{7}{12}+6\frac{5}{12}+7\frac{9}{12} = 3+6+7+\frac{7}{12}+\frac{5}{12}+\frac{9}{12}$

$16+\frac{7}{12}+\frac{5}{12}+\frac{9}{12}= 16\ \frac{7+5+9}{12}$

$=16\frac{21}{12}=16+1\frac{9}{12}$

$=17\frac{9}{12}$

$= 17\frac{3}{4}$

(c) $3\frac{16}{18}+2\frac{4}{9}+6\frac{1}{3}$

$= 3+2+6+\frac{16}{18}+\frac{4}{9}+\frac{1}{3}$

$= 11\frac{16}{18}+\frac{4}{9}+\frac{1}{3}$

LCM of 18, 9, 3 is 18.

$=11\frac{16+8+6}{18}$

$i.e\left(18\div18\right)x16=16$

$\left(18\div9\right)x\,4=8$

$\left(18\div3\right)x1=6$

$= 11\frac{30}{18}=11+1\frac{12}{18}=12\frac{12}{18}$

$$= 12\frac{12}{18} = 12\frac{2}{3}$$

3.5.2 Subtraction of Fractions

The procedure for subtraction of fractions is similar to that of addition, but instead of adding the numbers, we subtract.

Examples

(1) Simplify (a) $\frac{4}{5} - \frac{1}{5}$ (b) $\frac{1}{2} - \frac{1}{4}$ (c) $\frac{8}{9} - \frac{1}{3} - \frac{1}{2}$

Solution

(a) $\frac{4}{5} - \frac{1}{5} = \frac{4-1}{5} = \frac{3}{5}$

(b) $\frac{1}{2} - \frac{1}{4}$ LCM = 4

$$\frac{1}{2} = \frac{1}{2} x \frac{2}{2} = \frac{2}{4}$$

$$\frac{1}{4} = \frac{1}{4} x 1 = \frac{1}{4}$$

$$\frac{2}{4} - \frac{1}{4} = \frac{2-1}{4} = \frac{1}{4}$$

(c) $\frac{8}{9} - \frac{1}{3} - \frac{1}{2}$

LCM of 9, 3 & 2 = 18

$$\frac{8}{9} = \frac{8}{9} x \frac{2}{2} = \frac{16}{18}$$

$$\frac{1}{3} = \frac{1}{3} x \frac{6}{6} = \frac{6}{18}$$

$$\frac{1}{2} = \frac{1}{2} x \frac{9}{9} = \frac{9}{18}$$

$$\frac{16}{18} - \frac{6}{18} - \frac{9}{18} = \frac{16-6-9}{18} = \frac{16-15}{18} = \frac{1}{18}$$

Evaluating fractions involving mixed numbers

Simplify

(a) $6\frac{4}{11}-1\frac{2}{11}$ (b) $5\frac{3}{4}-3\frac{1}{2}$ (c) $3\frac{2}{5}-\frac{1}{3}$ (d) $9\frac{6}{14}-2\frac{3}{42}$

Solutions

(a) $6\frac{4}{11}-1\frac{2}{11} = (6-1)+\left(\frac{4}{11}-\frac{2}{11}\right)= 5+\frac{4-2}{11}$

$= 5+\frac{2}{11}=5\frac{2}{11}$

(b) $5\frac{3}{4}-3\frac{1}{2} = (5-3)+\left(\frac{3}{4}-\frac{1}{2}\right)$

$= 2+\left(\frac{3}{4}-\frac{1}{2}\right)= 2\frac{3-2}{4}$

$= 2+\frac{1}{4}=2\frac{1}{4}$

(c) $3\frac{2}{5}-\frac{1}{3} = (3-0)+\left(\frac{2}{5}-\frac{1}{3}\right)$

$= 3+\left(\frac{2}{5}-\frac{1}{3}\right)= 3+\frac{6-5}{15}$

$= 3+\frac{1}{15}=3\frac{1}{15}$

(d) $9\frac{6}{14}-3\frac{2}{7}-2\frac{3}{42}$

$= (9-3-2)+\left(\frac{6}{14}-\frac{2}{7}-\frac{3}{42}\right)$

$= (4)+\left(\frac{6}{14}-\frac{2}{7}-\frac{3}{42}\right)$

The LCM of 14, 7 & 42 is 42

$4+\frac{\quad}{42}$

i.e $(42\div 14)x\,6=18$

$(42\div 17)x\,2=12$

$(42\div 42)x\,3=3$

$$4+\frac{18-12-3}{42}=4+\frac{18-15}{42}$$

$$=4\left(\frac{3}{42}\div\frac{3}{3}\right)$$

$$=4\frac{1}{42}=4\frac{1}{14}$$

Evaluating fractions where the fraction you are to subtract is great than the other.

Examples

(1) Simplify (a) $1\frac{3}{5}-\frac{4}{5}$ (b) $6\frac{5}{9}-1\frac{7}{9}$ (c) $3\frac{2}{5}-2\frac{3}{5}$ (d) $5\frac{5}{9}-3\frac{4}{9}\ -\frac{1}{3}$

(a) $1\frac{3}{5}-\frac{4}{5}=1\frac{3-4}{5}=1\frac{3-4}{5}$

$$1\frac{3-4}{5}$$

4 is greater than 3 and since there is one whole number yet used, change it to improper fraction. i.e.

i.e $1=\frac{5}{5},\frac{5}{5}+\frac{3}{5}=\frac{8}{5}$

Now it will be $\frac{8}{5},\frac{4}{5}=\frac{8-4}{5}=\frac{4}{5}$

(b) $1\frac{5}{9}-1\frac{7}{9}=(6-1)+\left(\frac{5}{9}-\frac{7}{9}\right)=5\frac{5}{9}-\frac{7}{9}$

This can't be done since 7>5. What we need to do is to borrow (take) 1 from 5 and convert it to improper fraction. Then you can work it out.

$$=4+1\frac{5}{9}-\frac{7}{9}$$

$$=4\frac{14}{9}-\frac{7}{9}=4\frac{14-7}{9}=4\frac{7}{9}$$

(c) $3\frac{2}{5}-2\frac{3}{5}-\frac{1}{5}=(3-2-0)+\left(\frac{2}{5}-\frac{3}{5}-\frac{1}{5}\right)$

$$=1\frac{2}{5}-\frac{3}{5}-\frac{1}{5}$$

Convert $1\frac{2}{5}$ to improper fraction before working it out.

$$1\frac{2}{5} = \frac{7}{5}$$

The question will now look like this

$$\frac{7}{5}-\frac{3}{5}-\frac{1}{5} = \frac{7-3-1}{5} = \frac{7-(3-1)}{5}$$

$$\frac{7-4}{5} = \frac{3}{5}$$

(d) $$5\frac{5}{9}-3\frac{4}{9}-\frac{1}{3} = (5-3-0)+\left(\frac{5}{9}-\frac{4}{9}-\frac{1}{3}\right)$$

$$= 2\frac{5}{9}-\frac{4}{9}-\frac{1}{3}$$

First convert the fractions to their equivalents that have the same denominator

$$5\frac{5}{9}-3\frac{4}{9}-\frac{1}{3} = 2\frac{5}{9}-\frac{4}{9}-\frac{3}{9}$$

Since the sum of 4 & 3 is greater than 5, do not attempt to subtract 4 and 3 from 5. Borrow (take) 1 from 2, remaining 1. Convert the 1 borrowed (taken) and $\frac{5}{9}$ to improper fraction.

$$2\frac{5}{9}-\frac{4}{9}-\frac{3}{9} = 1+1\frac{5}{9}-\frac{4}{9}-\frac{3}{9}$$

$$= 1\frac{14-4-3}{9} = 1\frac{14-(4-3)}{9}$$

$$1\frac{14-7}{9} = 1\frac{7}{9}$$

3.5.3 Addition and Subtraction (together) of Fractions

(1) Evaluate (a) $\frac{3}{6}-\frac{6}{9}+\frac{4}{18}$ (b) $\frac{9}{16}-\frac{3}{8}+\frac{5}{6}-\frac{3}{4}$ (c) $\frac{5}{12}+\frac{4}{5}-\frac{7}{8}-\frac{13}{30}+\frac{5}{6}$

These problems are solved by converting the fractions to equivalent fractions having same denominator.

Solutions

(a) $\frac{3}{6}-\frac{6}{9}+\frac{4}{18}$, LCM = 18

$\frac{3}{6}=\frac{9}{18}, \frac{6}{9}=\frac{12}{18} \& \frac{4}{18}=\frac{4}{18}$

$\frac{3}{6}=\frac{6}{9}+\frac{4}{18}=\frac{9}{18}-\frac{12}{18}+\frac{4}{18}=\frac{9-12+4}{18}$

$=\frac{9+4-12}{18}=\frac{13-12}{18}=\frac{1}{18}$

(b) $\frac{9}{16}-\frac{3}{8}+\frac{5}{6}-\frac{3}{4}$, LCM = 48

$\frac{9}{16}=\frac{27}{48}, \frac{3}{8}=\frac{18}{48}, \frac{5}{6}=\frac{40}{48}, \frac{3}{4}=\frac{36}{48}$

$\frac{9}{16}-\frac{3}{8}+\frac{5}{6}-\frac{3}{4}=\frac{27}{48}-\frac{18}{48}+\frac{40}{48}-\frac{36}{48}$

$=\frac{27-18+40-36}{48}=\frac{27+40-18-36}{48}$

$\frac{(27+40)-(18+36)}{48}=\frac{67-54}{48}=\frac{13}{48}$

(c) $\frac{5}{12}+\frac{4}{5}-\frac{7}{8}-\frac{13}{30}+\frac{5}{6}$, LCM 120

$\frac{5}{12}=\frac{50}{120}; \frac{4}{5}=\frac{96}{120}; \frac{7}{8}=\frac{105}{120}$

$\frac{13}{30}=\frac{50}{120}; \frac{5}{6}=\frac{100}{120}$

$=\frac{50}{120}+\frac{96}{120}-\frac{105}{120}-\frac{52}{120}+\frac{100}{120}$

$=\frac{50+96+100-105-52}{120}$

$=\frac{246-157}{120}=\frac{89}{120}$

3.5.4 Addition and Subtraction of Mixed Number

(1) $6\frac{5}{7}-4\frac{4}{7}+\frac{1}{7}-\frac{6}{7}$

(2) $11\frac{15}{18}-8\frac{7}{9}+\frac{17}{18}-2\frac{5}{6}$

(3) $14\frac{2}{5}-5\frac{7}{10}-4\frac{1}{2}-\frac{2}{3}$

Solution

(1) $6\frac{5}{7}-4\frac{4}{7}+\frac{1}{7}-\frac{6}{7}=(6-4)+\left(\frac{5}{7}-\frac{4}{7}+\frac{1}{7}-\frac{6}{7}\right)$

$=2\frac{5}{7}-\frac{4}{7}+\frac{1}{7}-\frac{6}{7}$

$=\left(1\frac{7}{7}+\frac{5}{7}\right)-\frac{4}{7}+\frac{1}{7}-\frac{6}{7}$

$=1\frac{7+5-4+1-6}{7}$

$=1\frac{7+5+1-4-1}{7}=1\frac{13-10}{7}=1\frac{3}{7}$

(2) $=11\frac{15}{18}-8\frac{7}{9}+\frac{17}{18}-2\frac{5}{6}$

$=(11-8-2)\left(\frac{15}{18}-\frac{7}{9}+\frac{17}{18}-\frac{5}{6}\right)$

LCM = 18, $1\frac{15}{18}-\frac{7}{9}+\frac{17}{18}-\frac{5}{6}$

$\frac{7}{9}=\frac{14}{18},\frac{5}{6}=\frac{15}{18}$

$\therefore 1\frac{15}{18}-\frac{7}{9}+\frac{17}{18}-\frac{5}{6}=1\frac{15}{18}-\frac{14}{18}+\frac{17}{18}-\frac{15}{18}$

$1\frac{15-14+17-15}{18}=1\frac{(15+17)-(14+15)}{18}$

$=1\frac{32-29}{18}=1\frac{3}{18}\div\frac{3}{3}$

$= 1\frac{1}{6}$

(3) $14\frac{2}{5} - 5\frac{7}{10} - 4\frac{1}{2} - \frac{2}{3}$

$= (14 - 5 - 4) + \left(\frac{2}{5} - \frac{7}{10} - \frac{1}{2} - \frac{2}{3}\right)$

$= 5 + \frac{2}{5} - \frac{7}{10} - \frac{1}{2} - \frac{2}{3}$; LCM =30

$\frac{2}{5} = \frac{12}{30}, \frac{7}{10} = \frac{21}{30}, \frac{1}{2} = \frac{15}{30} - \frac{21}{30} - \frac{15}{30} - \frac{20}{30}$

$= 5\frac{12 - 21 - 15 - 20}{30} = 5\frac{12 - (21 + 15 + 20)}{30}$

$= 5\frac{12 - 56}{30}$

In this case we can not continue the way it is. We have to borrow (take) two (2) from five (5) and convert it and add it to 12 and then conclude the work. The 12 at the pin side should read $\frac{12}{30}$

$2 = \frac{60}{30}$, then $\frac{60}{30} + \frac{12}{30} = \frac{72}{30}$

$= 3\frac{72 - 56}{30} = 3\frac{16}{30} \div \frac{2}{2} = 3\frac{8}{15}$

3.5.5 Word Problems Leading to Addition and Subtraction of Fractions

Word problems should be thoroughly read and translated into addition or subtraction of fractions before you proceed as explained before to solve.

Lets start with simple examples

(1) What is the sum of 2 divided by 5 and three divided by seven?

Solution

The main terms in the question are the sum of 2 divided by $5 = 2 \div 5 = \frac{2}{5}$, and 3 divided by $7 = 3 \div 7 = \frac{3}{7}$

The sum $\frac{2}{5} + \frac{3}{7} = \frac{14 + 15}{45} = \frac{29}{35}$

(2) What is the difference between four fifth of one object and three quarters of the same object?

Solution

Four fifth = $\frac{4}{5}$ & three quarters = $\frac{3}{4}$

Differences is given by subtraction

$$=\frac{4}{5}-\frac{3}{4}=\frac{16-15}{20}=\frac{1}{20}$$

(3) If I walked one quarter of a journey, ran one fifty of it and rode on a motorcycle for the rest of the journey. By what fraction of the journey is the distance 1 rode, greater than the distances 1 walked and ran put together?

Solution

I walked $\frac{1}{4}$, & ran $\frac{1}{5}$ of the journey put together my walk and ran will give $\frac{1}{4}+\frac{1}{5}$ of the journey

$$\frac{1}{4}+\frac{1}{5}=\frac{5+4}{20}=\frac{9}{20}$$

The journey is an entity (whole) i.e 1. The rest of the journey is $1-\frac{9}{20}=\frac{11}{20}$

$$\frac{11}{20}-\frac{9}{20}=\frac{11-9}{20}=\frac{2}{20}\div\frac{2}{2}=\frac{1}{10}$$

(4) John is a student of the University of Benin. At a given day, he plays football for one and three quarter hours, then watches home video for one and half hours, and does his assignment for one and three fifth hours. How long did he taken in doing the activities?

Solution

Total time taken = (Time for football + the time for home video time + for assignment)

Football = $1\frac{3}{4}$ hrs, home video = $1\frac{1}{2}$ hrs.

Assignment = $1\frac{3}{5}$ hrs

Total time taken = $\left(1\frac{3}{4}+1\frac{1}{2}+1\frac{3}{5}\right)$hrs

$$=3\frac{15+10+12}{20}$$

$$=3\frac{37}{20}\text{ hrs }=\left(3+1\frac{17}{20}\right)\text{hrs}$$

$$=4\frac{17}{20}\text{ hrs}$$

(5) In an election there were three candidates, $\frac{3}{5}$ of the electors voted for the winner. The rumber-up received $\frac{5}{7}$ of the remaining votes (a) what fraction of the electors voted for the third candidate (b) if the winner received 6321 votes how many electors voted. (c) What did runner-up received and the least.

Solution

Three candidates: A, B & C

Let the total votes cast be x

A received $=\frac{3}{5}$ of x $=\frac{3x}{5}$

The remaining votes is $x-\frac{3x}{5}=\frac{5x-3x}{5}$

$$=\frac{2x}{5}$$

B Received $\frac{5}{7}$ of $\frac{2x}{5}=\frac{5}{7}x\frac{2x}{5}=\frac{10x}{35}$

C. vote = $x-\left(\frac{2x}{5}+\frac{10x}{35}\right)$

$$x-\left(\frac{14x+10x}{35}\right)$$

$$=x-\frac{24x}{35}$$

$$=\frac{35x-24x}{35}=\frac{11x}{35}$$

(a) $\frac{3x}{5} = 6321$

$\therefore x = \frac{6321x5}{3} = \frac{31605}{3} = 10{,}535$

(b) To know the remaining votes is 10,535 – 6324 = 4,214

Note the runner-up receive $\frac{5}{7}$ of the remaining votes.

$\frac{5}{7} x \frac{4214}{1} = 3{,}010$

(c) Those that voted for the last candidate = 10,535 – (6321+3010)

(10,535-9331)

= 1204

Assignment

(1) Add (a) $\frac{1}{2}, \frac{1}{3} \,\&\, \frac{1}{5}$ (b) $\frac{1}{4}, \frac{2}{8}, \frac{4}{16} \,\&\, \frac{16}{32}$ (c) $1\frac{2}{3}, 3\frac{1}{6} - 2\frac{5}{6}$ (d) $\frac{3}{13}, 2\frac{2}{7} \,\&\, 3\frac{6}{9}$

(2) Simplify (a) $\frac{3}{7} - \frac{4}{5}$ (b) $4\frac{2}{9} - 3\frac{1}{5}$ (c) $2\frac{3}{4} - 1\frac{4}{5}$ (d) $15\frac{4}{9} - 4\frac{1}{8} - 3\frac{4}{6}$

(3) Evaluate (a) $\frac{1}{2} - \frac{2}{3} + \frac{2}{9}$ (b) $\frac{10}{15} - \frac{4}{30} + \frac{1}{5} - \frac{3}{10}$ (c) $16\frac{13}{18} - 9\frac{4}{9} - 3\frac{1}{6} + \frac{15}{18}$

(d) $16\frac{3}{5} - 6\frac{4}{10} - 3\frac{1}{3} - \frac{5}{6}$

(4) In a mixed school with 840 students $\frac{3}{4}$ of them are boys. How many girls are there.

3.6 Multiplication of Fractions

We have learn how to carry out addition and subtraction on fractions. In this section, we are going to learn about the multiplication of fractions. This we will do in four ways:

(1) multiplication of fraction

(2) division of fraction

(3) simultaneous multiplication and division of fractions and

(4) word problems leading to multiplication or/and division of fractions

Objective

At the end of this section, you should be able to:

i. multiply two or more fractions together and express the product in its lowest terms

ii. divide a fraction by another fraction and express the results in lowest terms.

iii. carry out multiplication and division simultaneously

iv. solve some word problems that involves multiplication or division over fraction.

If we have fractions of the form

$$\frac{a}{b} x \frac{c}{d} = \frac{ac}{bd}$$

If can be seen that in multiplication of fraction we multiply the numerators together and the denominators together. The product numerators becomes the numerator of the answer and the product of the denominators becomes the denominator of the answer. If the numerator and denominator have common factor(s), you reduce the answer to its lowest term.

Example

(1) Evaluate the following

(a) $\frac{2}{5} x \frac{3}{7}$ (b) $\frac{5}{9} x \frac{3}{4}$ (c) $\frac{4}{7} x \frac{3}{8}$ (d) $\frac{5}{6} x \frac{3}{15}$

Solutions

(a) $\frac{2}{5} x \frac{3}{7} = \frac{2}{5} x \frac{3}{7} = \frac{6}{35}$

(b) $\frac{5}{9} x \frac{3}{4} = \frac{5}{9} x \frac{3}{4} = \frac{15}{36}$

(c) $\frac{4}{7} x \frac{3}{8} = \frac{4}{7} x \frac{3}{8} = \frac{12}{56} \div \frac{4}{4} = \frac{3}{14}$

(d) $\frac{5}{6} x \frac{3}{15} = \frac{5x3}{6x15} = \frac{15}{90} \div \frac{15}{15} = \frac{1}{6}$

Even if mixed fractions are involved the procedure is the same.

2. Evaluate (a) $\frac{1}{2} x \frac{3}{5} x \frac{6}{11} x \frac{16}{18}$ (b) $\frac{3}{7} x \frac{6}{11} x \frac{3}{4} x \frac{1}{2}$

Solutions

(a) $\frac{1}{2} x \frac{\not{3}}{\not{5}} x \frac{\not{6}}{11} x \frac{1\not{5}^{3}}{1\not{8}} = \frac{1x1x1x3}{2x1x11x1} = \frac{3}{22}$

(b) $\frac{3}{7} x \frac{\not{6}^{3}}{11} x \frac{3}{4} x \frac{1^{3}}{\not{2}} = \frac{3x3x3x1}{7x11x4x1} = \frac{27}{308}$

3.7 Division of Fraction

The reciprocal of $\frac{1}{n}$ is n, the reciprocal of *m* is $\frac{1}{m}$. So to divide 1.5 by 3, we multiply 15 by the reciprocal of 3 which is $\frac{1}{3}$.

This is $15 \div 3 = 15x\frac{1}{3} = \frac{15}{3} = 5$

Also

$40 \div 8 = 40\,x\frac{1}{8} = \frac{40}{8} = 5$

The reciprocal of $\frac{2}{7}$ is $\frac{7}{2}$. A number multiply by its reciprocal is one.

$\frac{2}{7}x\frac{7}{2} = \frac{14}{14} = 1$

The parts of divisions are identified as follows. Dividend

$\frac{3}{7}x\frac{7}{13} = \frac{3}{13}$

When you interchange the positions of the dividend and the divisor, it affect the quotient. So in the division of one fraction by another fraction. We multiply the dividend by the reciprocal of the divisor.

Examples

(1) Evaluate the following

(a) divide $\frac{2}{5}$ by 4 (b) divide $\frac{6}{15}$ by $\frac{3}{10}$ (c) divide 50 by $\frac{1}{20}$

Solutions

(a) $\frac{2}{5} \div 4 = \frac{2}{5} \div \frac{4}{1}x\frac{2x1}{5x4} = \frac{2}{20}$

$\frac{2}{20} \div \frac{2}{2} = \frac{1}{10}$

(b) $\frac{6}{15} \div \frac{3}{10} = \frac{6}{15}x\frac{10}{3} = \frac{6x10}{15x3} = \frac{60}{45} \div \frac{15}{15} = \frac{4}{3} = 1\frac{1}{3}$

OR

$\frac{6}{15} \div \frac{3}{10} = \frac{6}{15}x\frac{10}{3} = \frac{6x10}{15x3} = \frac{2x2}{3x1} = \frac{4}{3} = 1\frac{1}{3}$

(c) $50 \div \frac{1}{20} = \frac{50}{1} \div \frac{1}{20} = \frac{50}{1} = \frac{20}{1} = \frac{50x20}{1x1} = \frac{1000}{1} = 1000$

If a mixed number is involved change it to an improper fraction and proceed.

Example

(1) Evaluate

(a) $2\frac{1}{2}x4\frac{1}{4} \div 6\frac{1}{4}$ (b) $\frac{7}{20} \div 6\frac{1}{3}x\frac{3}{5} \div \frac{1}{40}$

Solution

(a) $2\frac{1}{2}x4\frac{1}{3} \div 6\frac{1}{4}$

Change to improper fraction first

$$= \frac{5}{2}x\frac{13}{3} \div \frac{25}{4}$$

$$= \frac{5}{2}x\frac{13}{3}x\frac{4}{25} = \frac{5x13x4}{2x3x25} = \frac{1x13x2}{1x3x5} = \frac{26}{15} = 1\frac{11}{15}$$

(b) $\frac{7}{20} \div 6\frac{2}{3}x\frac{3}{5} \div \frac{1}{40}$

$$\frac{7}{20} \div 6\frac{2}{3}x\frac{3}{5} \div \frac{1}{40}$$

$$\frac{7}{20}x\frac{3}{20}x\frac{3}{5}x\frac{40}{1}$$

$$= \frac{7x3x3x1}{1x10x5x1} = \frac{63}{50} = 1\frac{13}{50}$$

Assignment

1. Evaluate the following

(a) $7\frac{2}{7}x\frac{6}{21}$ (b) $7\frac{7}{9} \div \frac{4}{5}$ (c) $3\frac{1}{2}x6\frac{1}{4} \div \frac{7}{16}$

3.8 Decimals (Addition and Subtraction)

In decimal, the work must be set out correctly. That is units must be under units, the decimal points under the decimal points…. and so on. For example, if you are ask to add 75.32 and 7.5.

The work must be set out as:

75.32

+5.5

After the work have be set out in the right order as above, addition and subtraction are done ine the same way as you do for whole numbers, but always remember to write down the decimal point when you come to it.

Examples

1. Add 25.36 and 2.68
2. Add 81.02 and 61.2
3. Add 1.23 and 31.01
4. Subtract 1.23 from 3.72
5. Subtract 7.05 from 9.14
6. A string is 82.61m long when new. Lengths of 12.36M and 28.51M and 16.76M are cut off.

   (a) What length of string is cut off altogether (b) How much string is left.
7. Find the difference between 91.21 and the sum of 36.82 and 45.78.

(1) 25. 36

+2. 68

28. 04

(2) 81. 02

+61. 20

142.22

(3) 1.23

+31.01

32.24

(4) 3.72

-1.23

2.49

(5) 9.14

-7.05

2.09

6(a) Length cut off

$$\begin{array}{r} 12.36m \\ +28.51m \\ 16.76m \\ \hline 57.63m \\ \hline \end{array}$$

(b)

$$\begin{array}{r} 82.61m \\ -57.63m \\ \hline 24.98m \\ \hline \end{array}$$

7(a) $(91.21)-(36.82+45.78)$

$$\begin{array}{r} 36.82 \\ +\ 45.78 \\ \hline 82.60 \\ \hline \end{array}$$

(b)

$$\begin{array}{r} 91.21 \\ -\ 82.60 \\ \hline 8.61 \\ \hline \end{array}$$

3.9 Changing Fractions to Decimals

It is usually quicker to divide the numerator of the fraction by its denominator, taking care to write down the decimals point as it raises.

Examples

(1) Express the following as a decimal

(a) $\frac{1}{2}$ (b) $\frac{4}{5}$ (c) $\frac{3}{4}$ (d) $1\frac{1}{8}$ (e) $\frac{1}{3}$ (f) $\frac{7}{9}$

(a) $\frac{1}{2} =$

$$\begin{array}{r|l} & 0.5 \\ \hline 2 & 10 \\ \hline & 10 \end{array}$$

Two into one cannot, write down zero (0) and put a decimal in front of the zero. Add zero to the one (1). Next two into ten, that will give five. Write the five down in fount of the decimal point. So

$\frac{1}{2} = 0.5$

(b) $\frac{4}{5}$

$$\begin{array}{r|l} & 0.8 \\ \hline 5 & 40 \\ & 40 \end{array}$$

i.e $\frac{4}{5} = 0.8$

(c) $\frac{3}{4}$

$$\begin{array}{r|l} & 0.75 \\ \hline 4 & 30 \\ & 28 \\ \hline & 20 \\ & 20 \end{array}$$

i.e $\frac{3}{4} = 0.75$

(d) $1\frac{1}{8} = \frac{9}{8}$

$$\begin{array}{r|l} & 1.125 \\ \hline 8 & 9 \\ & 8 \\ \hline & 10 \\ & 8 \\ \hline & 20 \\ & 16 \\ \hline & 40 \\ & 40 \end{array}$$

Question (a) to (d) are terminating decimal

(e) $\frac{1}{3}$

$$\begin{array}{r|l} & 0.3333 \\ \hline 3 & 10 \\ & 9 \\ \hline & 10 \\ & 9 \\ \hline & 10 \\ & 9 \\ \hline & 1 \end{array}$$

(f) $\frac{7}{9}$

```
        0.777…
     ┌────────
   9 │ 70
       63
     ────────
       70
       63
     ────────
       70
        63
     ────────
         7
```

$\frac{1}{3}$ and $\frac{7}{9}$ are never-ending decimal fraction. $\frac{1}{3}$ = 0.333… & $\frac{7}{9}$ = 0.777….

The digits 3 and 7 are respected as often as we like. We say they are recurring decimal.

0.333… is zero point three recurring and is written as 0.3.

0.777… is zero point seven recurring and is written as 0.7.

Many fractions give rise to recurring decimals. For example

$\frac{4}{11}$ = 0.363636 …. 0.36.

$\frac{1}{6}$ = 0.166666… = 0.16

Self Assessment Questions

(1) Express the following as decimals

(a) $\frac{1}{4}$ (b) $\frac{3}{5}$ (c) $\frac{1}{25}$ (d) $\frac{5}{8}$ (e) $1\frac{1}{8}$ (f) $\frac{7}{16}$ (g) $\frac{8}{9}$ (h) $\frac{5}{11}$ (i) $\frac{1}{7}$ (k) $\frac{7}{12}$

3.10 Changing Decimals to Fraction

Any decimal can be expressed as a fraction with denominator which is a power of ten.

For example

0.6 = $\frac{1}{10}$, 0.73 = $\frac{73}{100}$

0.125 = $\frac{125}{1000}$

Examples

Express the following as fractions

(a) 0.8 (b) 0.75 (c) 0.135 (d) 1.25

Solutions

(a) $0.8 \frac{0.8}{10} = \frac{8}{10} = \frac{4}{5}$

After writing the decimal number, put one under the decimal point and put zero under every other number in fount of the decimal point.

(b) $0.75 \frac{0.75}{100} = \frac{75}{100} = \frac{15}{20} = \frac{3}{4}$

(c) $0.135 \frac{0.135}{1000} = \frac{27}{200}$

(d) $1.25 = \frac{1.25}{100} = \frac{125}{100} = \frac{25}{20} = \frac{5}{4} - 1\frac{1}{4}$

3.11 Multiplication of Decimals

In multiplication, multiply the given numbers without decimal points. Then count the digits after the decimal points in the numbers being multiplied. Place the decimal point so that the product has the same number of digits after the point.

Examples

(1) Find the products of the numbers below:

(a) 0.61 x 2 (b) 0.78 x 0.4 (c) 0.113 x 0.31

(d) 0.003 x 0.15 (e) 2.013 x 0.55

(2) A car need 3.5 litres of petrol of a journey. If a litre of petrol cost 62 kobo, low much will the journey cost.

Solutions

(a)
```
   0. 6 1
 x    2
 --------
   1. 2 2
 --------
```

(b)
```
   0. 7 8
 x  0. 4
 --------
   0.3 1 2
 --------
```

(c) 0. 1 1 3
x 0.31

3 3 9

1 1 3

0.03503

(d) 0. 0 0 3

x 0. 1 5

0 0 3

0 0 1 5

0.0 0 0 4 5

(e) 2. 0 1 3

x 0 5 5

1 0 0 6 5

1 0 0 6 5

1. 1 0 7 1 5

(2) 3 . 5

x 0. 6 2

2 1 0

7 0

2. 1 7 0

OR

3. 5

x 6 2

2 1 0

7 0

2 1 7. 0 kobo

2 1 7 kobo ₦2.17

3.12 Division of Decimals

In division of decimal numbers, if the divisor is a whole number, divide in the usual way care must be taken to include the decimal point in the correct place. But if the divisor contains a decimal fraction, make an equivalent division such that the divisor is a whole number. To do that:

(a) count how many places to the right the digits in the divisor must move to make it a whole number.

(b) move the digits of both numbers to the right by this number of places.

(c) divide as usual.

Examples

(1) $10.71 \div 63$ (2) $1.305 \div 0.29$

(3) $40.2 \div 0.006$ (4) $1.936 \div 3.4$

Solutions

(1) $10.71 \div 63$

$\frac{1}{3}$

$$\begin{array}{r|l} & 0.17\ldots \\ \hline 63 & 10.71 \\ & 6.3 \\ \hline & 4.41 \\ & 4.41 \\ \hline \end{array}$$

$10.71 \div 63 = 0.17$

(2) $1.305 \div 0.29 = 130.5 \div 29$

$$\begin{array}{r|l} & 4.5 \\ \hline 29 & 130.5 \\ & 116 \\ \hline & 14.5 \\ & 14.5 \\ \hline \end{array}$$

$1.305 \div 0.29 = 130.5 \div 29 = 4.5$

(3) $40.2 \div 0.0.006 = 40200 \div 6$

$$
\begin{array}{r|l}
 & \ \ 6700 \\ \hline
6 & 40200 \\
 & 36 \\ \hline
 & \ \ 42 \\
 & \ \ 42 \\ \hline
 & \ \ \ 00 \\
 & \ \ \ 00 \\ \hline
 & \ \ \ \ 00
\end{array}
$$

$40.2 \div 0.006 = 40200 \div 6 = 6700$

(4) $1.938 \div 3.4 = 19.38 \div 34$

$$
\begin{array}{r|l}
 & \ 0.57 \\ \hline
34 & 19.38 \\
 & 17.0 \\ \hline
 & \ 2.38 \\
 & \ 2.38
\end{array}
$$

$1.938 \div 3,4 = 19.38 \div 34 = 0.57$

Self Assessment Question

1. Find the product of
   - (a) 0.251 and 6.63
   - (b) 6.83 & 0.63
   - (c) 54.3 & 0.056
2. A ream of paper contains 510 sheets. Each sheet is 0.016cm thick. Find the thickness of the ream of paper.
3. Find the value of the following division
   - (a) $1800 \div 0.06$
   - (b) $\frac{30}{0.5}$ (c) $\frac{0.3}{0.05}$
4. A test car travels 98.6km on 7.25 litres of petrol. How many km does it travel on 1 litre of petrol?

3.13 Percentages

Remember that 15% means $\frac{15}{100}$. Thus 15% is same as 0.15.

$$15\% = \frac{15}{100} = 0.15$$

This shows that to change a percentage to a decimal fraction, divide the percentage by 100. To change a decimal fraction to a percentage, multiply by 100.

For example,

0.653 = (0.653 x 100)% = 65.3%

Examples

(1) Express the following percentages as decimals.

(a) 10% (b) 80% (c) 75% (d) 24%

(2) Express the following decimals as percentages

(a) 0.35 (b) 0.12 (c) 0.22 (d) 0.05

Solution

(1) (a) $10\% = \frac{10}{100} = \frac{1}{10} = 0.1$

(b) $80\% = \frac{80}{100} = \frac{8}{10} = 0.8$

(c) $75\% = \frac{75}{100} = 0.75$

(d) $24\% = \frac{24}{100} = 0.24$

(2) (a) $0.35 = \frac{0.35}{100} = 35\%$

(b) $0.12 = \frac{0.12}{100} = \frac{2}{100} = 12\%$

(c) $0.22 = \frac{0.22}{100} = \frac{22}{100} = 22\%$

(d) $0.05 = \frac{0.05}{100} = \frac{5}{100} = 5\%$

Self Assessment of Questions

(1) Express (a) 55% (b) 68% (c) 75% as decimals

(2) Express (a) 0.11 (b) 0.62 (c) 0.73 as percentages

4.0 Conclusion

The unit considered fractions and its various form and the basic operations on fractions. Decimal was also considered and how to change from fraction to decimal and decimal to fraction. The basic operation in decimal was also carried out. The unit also considered percentages and it various form.

5.0 Summary

The highlights of the unit include the following;

- Fractions and its various form.
- Basic operation on fractions.
- Decimals and its basic operation.
- Changing from fractions to decimal and decimals to fractions.
- Percentages.

6.0 Tutors Marked Questions

(1) What is the sum of $2\frac{7}{12}$ and $4\frac{5}{8}$.

(2) Find the sum of $8\frac{3}{5}$ and $6\frac{2}{3}$. Find the difference between this sum and $18\frac{1}{3}$

(3) By how much is the sum of $2\frac{3}{5}$ and $14\frac{1}{2}$ less than $25\frac{2}{5}$

(4) A tank holds 25 litres of water. The capacity of a cup is $\frac{3}{10}$ of a litre. How many cups of water does the tank hold?

(5) A farmer uses $\frac{5}{16}$ of a field for growing cassava. He uses $\frac{3}{4}$ of the remainder for growing corn, what is left to grow yam?

(6) In a box of 400 oranges, 28 are bad. What percentage is bad and what percentage is good.

(7) The distance between two towns is 18km. A man walks from one town to the other. He walks most of the way runs the last 480 metres what percentage of the journey did he run?

7.0 References/Further Reading

New General Math J.S.S.S. 1, 2, & 3.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary School 1,2,&3 (3rd ed.) Ibadan: University Press Plc.

MODULE 1 UNIT 4

Unit 4: Number Bases

Content

1.0 Introduction

In this unit, we will learn how to express numbers in different bases. We have learnt how to expressed numbers in the power of prices. There will be three sections. Section 1 will deal with numbers in base ten, section 2 will be number in other bases and section 3 will be conversion from one base to another.

2.0 Objective

At the end of this unit, you should be able to:

(1) Write numbers in base ten

(2) Write numbers in other bases

(3) Convert number from one base to another

3.0 Main Body

3.1 Numbers in Base Ten (Denary)

Some people traditionally count in 5s or use market days. When counting week days we count in 7s. When counting seconds or minutes will count in 60s. In the present days it is general for people to count in 10s.

The digits that are used to represent the numbers are 0,1,2, 3,4,5,7,8,9. The value of the digit is known by the place it occupied in the number. For example 6825 is the same as

$$\begin{array}{rr} & 6000 \\ + & 800 \\ + & 20 \\ + & 5 \\ \hline & 6825 \\ \hline \end{array}$$

In standard form we have

$6000 = 6 \times 10^3$

$800 = 8 \times 10^2$

$20 = 2 \times 10^1$

$5 = 5 \times 10^0$

When, we put all together, we have :

$$\begin{aligned} 6835 &= 6 \times 1000 + 8 \times 100 + 2 \times 10 + 5 \\ &= 6 \times 10^3 + 8 \times 10^2 + 2 \times 10^1 + 5 \times 10^0 \end{aligned}$$

Note:

Since it is based on the power of ten it is called the base ten system.

Examples

(1) Write the expansion form of the following base ten numbers.

(i) 34568 (ii) 132571

Solution

(i) 34568 =

$$\begin{array}{rl} & 30,000 = 3 \times 10^4 \\ + & 4000 = 4 \times 10^3 \\ + & 500 = 5 \times 10^2 \\ + & 60 = 6 \times 10^1 \\ + & 8 = 8 \times 10^0 \\ \hline & 34568 \\ \hline \end{array}$$

$\therefore 34,568 = 3 \times 10^4 + 4 \times 10^2 + 5 \times 10^2 + 6 \times 10^1 + 8 \times 10^0$

(ii) 132571 =

$$
\begin{array}{rrcl}
 & 100{,}000 & = & 1 \times 10^5 \\
+ & 30{,}000 & = & 3 \times 10^4 \\
+ & 2{,}000 & = & 2 \times 10^3 \\
+ & 500 & = & 5 \times 10^2 \\
+ & 70 & = & 7 \times 10^1 \\
+ & 1 & = & 1 \times 10^0 \\
\hline
 & 132{,}571 & & \\
\hline
\end{array}
$$

$\therefore$ 132,571 = $1 \times 10^5 + 3 \times 10^4 + 2 \times 10^3 + 5 \times 10^2 + 7 \times 10^1 + 1 \times 10^0$

(2) Write the base ten numbers expanded numbers in their usual form.

(i) $7x10^4 + 6x10^3 + 9+10^2 + 4x10^1 + 2x10^0$

(ii) $9 \times 10^5 + 7 \times 10^2 + 2 \times 10^1 + 3 \times 10^0$

Solutions

(i)
$$
\begin{array}{lcl}
7 \times 10^4 = 7 \times 1000^0 & = & 70{,}000 \\
6 \times 10^3 = 6 \times 1000 & = & 6{,}000 \\
9 \times 10^2 = 9 \times 100 & = & 900 \\
4 \times 10^1 = 4 \times 10 & = & 40 \\
2 \times 10^0 = 2 \times 1 & = & 2 \\
\hline
 & & 76942 \\
\hline
\end{array}
$$

(ii)
$$
\begin{array}{lcr}
9 \times 10^5 = 9 \times 100{,}000 & = & 900{,}000 \\
7 \times 10^2 = 7 \times 100 & = & 700 \\
2 \times 10^1 = 2 \times 10 & = & 20 \\
3 \times 10^0 = 3 \times 1 & = & 3 \\
\hline
 & & 900{,}723 \\
\hline
\end{array}
$$

Self Assessment Test

(1) Write the base 10 number below in expanded form

(a) 32576 (b) 2005 (c) 3456 (d) 13902

(2) Write down the following base 20 expanded numbers in their usual form.

(a) $6 \times 10^5 + 7 \times 10^3 + 6 \times 10^2 + 4 \times 10^1 +1 \times 10^0$

(b) $8 \times 10^6 + 5 \times 10^5 + 5 \times 10^2 + 2 \times 10^1 + 8 \times 10^0$

(c) $9 \times 10^4 + 8 \times 10^3 + 7 \times 10^2 + 5 \times 10^1 + 5 \times 10^0$

Section 2

3.2 Numbers in Other Bases

The binary (base two) system is of the number two. The binary system is one of the most importance system after the base ten system. Its importance is because it is used all over the world by all counters for numerical calculations. The binary system are made up of, only two number (digits), that is 1 and 0. Computer contains a large number of switches. There are two switches, one of the switch represent 1 which is `on' and the other represent 0 which is `off'. The diagram below shows the first ten binary number in respect to binary number (base ten).

Binary	Value in Powers of 2	Binary Numbers (base ten)
1	1 x1	1
10	$1 \times 2^1 + 0 \times 1$	2
11	$1 \times 2^1 + 1 \times 1$	3
100	$1 \times 2^2 + 0 \times 2^1 + 0 \times 1$	4
101	$1 \times 2^2 + 0 \times 2^1 + 1 \times 1$	5
110	$1 \times 2^2 + 1 \times 2^1 + 0 \times 1$	6
111	$1 \times 2^2 + 1 \times 2^1 + 1 \times 1$	7
1000	$1 \times 2^5 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 1$	8
1001	$1 \times 2^3 + 0 \times 2^2 + 1 \times 1$	9
1010	$1 \times 2^3 + 0 \times 2^2 + 0 \times 2^1 + 0 \times 1$	10

Numbers can also be written in other base numbers such as base eight which is also very important, base three, base four, base five, and so on.

To write a base ten number in other bases, we have to divide the base ten number by the lower base numbers.

Examples

(1) Write the following base ten numbers in base 2, 5, and 8

(i) 156 (ii) 345

Solution

Read the answer in the direction of the arrow.
From the bottom to the top

2	156_{10}
2	78R0
2	39R0
2	19R1
2	9R1
2	4R1
2	2R0
	1R0

$156_{10} = 10011100_2$

We can also convert 156_{10} to base 5

5	156
5	31R1
5	6R1
	1R1

$156_{10} = 1111_5$

Conversion of 156_{10} to base 8

8	156
8	19R4
	2R3

$156_{10} = 234_8$

Examples

Conversion of 345_{10} to base 2

2	345
2	172R1
2	86R0
2	43R0
2	21R1
2	10R1
2	5R0
2	2R1
	1R0

$345_{10} = 101011001_2$

Conversion of 345_{10} to base 5

5	345_{10}
5	69R0
5	13R4
	2R3

$345_{10} = 2340_5$

Conversion of 345_{10} to base 8

8	345_{10}
8	43R1
	5R3

$3450_{10} = 531_8$

Self Assessment Test

(1) Name the three most important base numbers.

(2) How does the computer use base two?

(3) Convert the following base ten numbers to base 2, 7 and 9

(i) 132 (ii) 716 (iii) 431

3.3 Conversion Of Numbers From One Base To Another

In converting a number from a given base to a number of another base, first convert such number to base 10 and then convert the new member to the new base.

Examples

(1) Express 682_9 to base 4

Solution

First convert 682_9 to base ten

$6 \times 9^2 + 8 \times 9^1 + 2 \times 9^0 = 6 \times 81 + 8 \times 9 + 2 \times 1$

$= 486 + 72 + 2$

$$\begin{array}{r} 486 \\ +\ 72 \\ +\ 2 \\ \hline 560_{10} \\ \hline \end{array}$$

$682_9 = 560_{10}$

Now convert 560_{10} to base 4.

4	560
4	140R0
4	35R0
4	8R03
4	2R0
	20300_4

$682_9 = 20300_4$

2. Express 10110011_2 to base 5

Solution

Convert 10110011_2 to base 10

$10110011_2 = 1 \times 2^7 + 0 \times 2^6 + 1 \times 2^5 + 1 \times 2^4 + 0 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0$

$= 1 \times 128 + 0 \times 64 + 1 \times 32 + 1 \times 16 + 0 \times 8 + 0 \times 4 + 1 \times 2 + 1 \times 1$

$= 128 + 0 + 32 + 16 + 0 + 0 + 2 + 1$

$= 128 + 32 + 16 + 3$

$= 179_{10}$

Then; convert 179_{10} to base 5

5	179
5	35R4
5	7R0
	1R2

i.e 12045_5

$= 10110011_2 = 12045_5$

Self Assessment Test

(1) Express (a) 1356_7 to base 3 & 7

(b) 1423_5 to base 9 & 6

3.4 Basic Operation in Number Bases

The basic operations of addition, subtraction, multiplication and division can also be carried out in number bases.

3.4.1 Addition and Subtraction

You can only add or subtract numbers of the same bases.

Examples

(1) Add 213_4 & 321_4

$$\begin{array}{r} 2\,1\,3_4 \\ +\ \ 3\,2\,1_4 \\ \hline 1200 \\ \hline \end{array}$$

Our addition is in base 4, so when we add and get a 4 or a number greater than 4, we divide such number by 4 and write the remainder down under the digits being added and carry the quotient to the higher digit.

(2) Add 1021_3 and 11011_2 write the answer in base 2

Solution

Since the final answer is going to in base 2, we have to convert 1021 base 3 to base 2 first before the addition can take place.

1021_3 to base 10 before to base 2

$1021_3 = 1 \text{ x } 3^3 + 0 \text{ x } 3^2 + 2 \text{ x } 3^1 + 1 \text{ x } 3^0$

$= 1 \text{ x } 27 + 0 \text{ x } 9 + 2 \text{ x } 3 + 1 \text{ x } 1$

$= 27 + 0 + 6 + 1 = 34_{10}$

2	34	
2	17R0	
2	8R1	
2	4R0	
2	2R0 ↑	10010_2
	1R0	

You then add both number since they are in base 2

$100010_2 + 11011_2$

$$\begin{array}{r} 100010_2 \\ +\ \ 11011_2 \\ \hline 111101_2 \\ \hline \end{array}$$

(3) Subtract

(a) 32421_5 from 43210_5

(b) 1011_2 from 11011_2

(c) 2345_6 from 1654_8 answer in base 8

Solutions

(a) 43210_5

$- \ 32421_5$

10234_5

In this case when you carry 1 it will be called 5 since our operation is in base five and add to the number before we subtract.

(b) $1\ 1\ 0\ 1\ 1_2$

$- \ 1\ 0\ 1\ 1_2$

$1\ 1\ 0_2$

(c) To be able to carry out the operation of subtraction, the two number must be in the same base number. Since the answer will be in base 8, we have to convert to 2345 base 6 to base 8.

2345 base 6 to base 10 then to base 8

$2345_6 = 2 \times 6^3 + 3 \times 6^2 + 4 \times 6^1 + 5 \times 6^0$

$= 2 \times 216 + 3 \times 38 + 4 \times 6 + 5 \times 1$

$= 432 + 108 + 24 + 5$

$= 569_{10}$

$2345_6 = 569_{10}$ to base 8

8 | 569

8 | 71R1

8 | 8R6

  | 1R0 $\qquad 1061_8$

$= 569_{10} = 1061_8$

$2345_6 = 1061_8$

Finally $1645_8 - 1061_8$

$$\begin{array}{r} 1\,6\,5\,4_8 \\ -\ 1\,0\,6\,1_8 \\ \hline 5\,7\,3_8 \\ \hline \end{array}$$

Self Assessment Question

(1) Evaluate the following

(i) $1011_2 + 10111_2$

(ii) $2312_4 + 1322_4$

(iii) $324_4 + 132_4$ answer in base 5

(iv) $12132_4 - 3221_4$

(v) $2134_5 - 1343_5$

(vi) $3456_7 - 5432_6$ answer in base 7

3.4.2 Multiplication and Division in Different Number Bases

In this operation, it may be only one of the number that will be of number base. The other will be ordinary number.

(1) Multiply (a) 1234_5 by 4 (b) 561_7 by 5_1 (c) 321_6 by 35_6

Solution

(a)
$$\begin{array}{r} 1\,2\,3\,4_5 \\ \times\quad 4 \\ \hline 11101_5 \\ \hline \end{array}$$

Multiply 4 by 4 to get 16 divide 16 by 5 it will give 3 remainder 1. Write down 1 under the unit section and carry 3. Multiply 3 by 4 and add the previous 3. i.e 12 + 3 = 15 ÷ 5 = 3 R0. Write down 0 carry 3. Continue in this way till the last number.

(b)
$$\begin{array}{r} 5\,6\,1_7 \\ \times\quad 5 \\ \hline 4125_7 \\ \hline \end{array}$$

(c)
$$\begin{array}{r} 3\,2\,1_6 \\ \times\ 35_6 \\ \hline 2445 \\ +\ 1403 \\ \hline 20515_6 \\ \hline \end{array}$$

Examples

(1) Divide (a) 21101_3 by 2 (b) 4570_8 by 6

(c) 2302_4 by 2

Solution

(a)

2	21111_3
	10202_3

The division is a digit in base 3, we find how many 2 are in 2. This gives 1 with no remainder. Next digit, since 2 is greater than 1, 2 in 1 is 0 remainder 1 write down 0 under 1. We now convert the 1 to 3 since the base of the divided is 3, and add it to the next digit to give 3 + 1 = 4. How many 2 in 4. There are two 2 remainder 0. Write down the 2 under the next 1. Next 2 divide 1 is 0 remainder 1, write down under 1. We now convert the 1 to 3 since the base of the dividend is 3 and add it to the next digit which is 1 to give 3 + 1 = 4. How many 2 in 4. There are two 2 in 4 remainder 0. Write down the 2 under the last 1. The division is over when you multiply the answer by 2, you will get the divisor.

(2) 7570_8

6	7570_8
	1224

6 into 7 gives 1 remainder 1. Write down the 1 under 1. Since it is of base 8, the one remaining will be call 8 and added to 5 i.e 8 + 5 = 13.6 into 13 will give 2 remainder 1. Write down the 2 under 5 and carry 1. Call the one 8 and added to 7 i.e 8 + 7 = 15, 6 into 15 will give 2 remainder 3 write down the 2 under 7 and convert the 3 to base 8 i.e 3 x 8 = 24 add the 24 to the next digit which is 0 i.e 24 + 0 = 24, 6 into 24 will give 4 without remainder.

(3)

2	2302_4
	1121_2

Divide as in examples 1 and 2 above.

Note that whenever you have a remainder after dividing by the divided you convert the remainder using base 2.

Self Assessment Test

(1) 1202_3 x 2 (2) 2341_5 x 3 (3) 346_1 x 15_1

(4) 4321_5 x 312_5 (5) 423_6 x 35_6

(6) $3234_5 \div 3$ (7) $6621_7 \div 6$ (8) $322_4 \div 3$

4.0 Conclusion

This unit considered number bases. The conversion from one base to another. Finally, basic operations using base numbers were treated.

5.0 Summary

The highlights of the unit are as follows:

- How the counting system is carried out.
- What to do when converting from one base to another.
- The basic operations in base numbers.

6.0 Tutor Marked Assignment

Solve the problems below:

(1) $3215_6 \times 5$ (2) $123_4 \times 3$ (3) $687_9 \times 8$ (4) $2331_4 \div 3$

(5) $231_6 \div 4$ (6) $1046367_7 \div 267$

7.0 References/Further Reading

Channon, J.B. et. al (2002): New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

National Teacher's Institute, Kaduna (2002).

Pivotal Teacher Training Programme (PTTP) (2000).

MODULE 1: UNIT 5

Unit 5: Approximations, Rate, Proportion & Ratio

Content

1.0 Introduction

Approximation is writing a number to a desired degree of accuracy, either rounding up or down. When we give the distance and the time that cars, planes or trains use to cover distances, it is always written in approximate, when we talk of populations of a place or state it is always the estimated one. The population of Benin is estimated to be 8.1 million.

In this unit, you will be taught various forms of approximations and how they are computed.

You will also learn how to deal with problems involving sharing commodities, contributions, and consumptions and making provisions in our world of inequalities.

2.0 Objectives

At the end of this unit you should able to:

(1) Round off number

a. to the nearest whole number

b. to a given number of decimal places

c. to a given number of significant

2. Solve problems involving:

a. Rate of doing things when the right information is given

b. proportions when enough information is given and

c. ratios when necessary and sufficient information are given

3.0 Main Body

3.1 Rounding off Numbers

When we round off numbers, the digits 1, 2, 3, 4 are rounded down, while the digit 5, 6, 7, 8, 9 are rounded up. When we round down, the digits from 1 to 4 are call zero and while when rounding up, the digits from 5 to 9 is call 1 and added to the next digit.

We can also round off numbers to the nearest tens, hundreds, thousands or millions.

Examples

1. Approximate each of the following to the nearest tens, hundreds and thousands.

(a) 12567 (b) 66532 (c) 78537

Solutions

(a)

TH	H	T	U
12	5	6	7

Since we are stopping at the tens column, so round up 7 units to gives one ten i.e 6 tens + 1 ten = 7 tens.

(i) 12567 becomes 12570 to the nearest ten

(ii) In case of the hundred, we will stop at the column of hundred, so round up 6 tens to give 1 hundred, i.e. 5 hundreds + 1 hundred = 6 hundreds.

$\therefore$ 12567 becomes 12600 to the nearest hundred

(iii) In the same way, in thousand 12567 = 13000 i.e round off 500 to give 1 thousand and add it to 12 to give 13,000.

(b)

TH	H	T	U
66	5	3	2

(i) 66 3 3 2 = 66530 to nearest 10 round down

(ii) 66 3 3 2 = 66500 to nearest 100 round down

(iii) 66 3 3 2 = 67000 to nearest 1000 round up

(b)	TH	H	T	U
	78	5	3	7

(i) 78537 = 78540 to nearest 10

(ii) 78537 = 78500 to nearest 100

(iii) 78537 = 79000 to nearest 1000

Self Assessment Test

Approximate the numbers below to the nearest (a) ten (b) hundred (c) thousand (1) 56781 (2) 8236 (3) 95788

3.2 Decimal Point and Significant Figure

Numbers can also be round off to decimal places. Numbers can also be approximated to various significant figures. The number zero is only significant if it is situated after any none zero real number in the whole number part e.g. in 2508, the zero is significant, but in 0.054, 12.30 and 0.00048 the zeros are not significant.

Examples

(1) Round off each of the numbers below to:

(i) one decimal place

(ii) two decimal places

(iii) three decimal places

(a) 0.006 (c) 7.7010

(b) 4.715 (d) 16.8077

Solutions

(a) 0.0066 = 0.0 to 1d.p

= 0.00 to 2d.ps

= 0.007 to 3d.ps

(b) 4.2715 = 4.3 to 1d.p

= 4.27 to 2d.ps

= 4.272 to 3d.ps

(c) 16.8077= 16.8 to 1d.ps

= 16.81 to 1d.ps

= 16.808 to 1d.ps

(d) 7.7010 = 7.7 to 1d.p

= 7.70 to 2d.ps

= 7.701 to 3d.ps

(2) Approximate 186.75 to

(a) 1s.f. (b) 2s.f. (c) 3s.f. (d) 4s.f.

Solutions

(a) 186.75 = 200 to 1 sf.

(b) 186.75 = 190 to 2 sf.

(c) 186.75 = 187 to 3 sf.

(d) 186.75 = 186.8 to 4 sf.

Here, the rounding off is carried after the number of significant figures are indicated.

(3) Approximate 0.08567 to

(a) 1 st. (b) 2 sf. (c) 3 sf.

Solutions

(a) 0.08567 = 0.09 to 1 st.

The zeros before the first real number is not a significant figure. The rounding off is done after the first real number.

(b) 0.08567 = 0.086 2 sf.

(c) 0.08567 = 0.0857 3 sf.

Self Assessment Test

(1) Round of the each of the following numbers to

(a) 1 dp (b) 2 dps (c) 3 dps (d) 1 sf (e) 2 sf. (f) 3 sf.

(1) 0.006875 (2) 0.06759

(3) 7.5426 (4) 10.678

3.3 Estimations

This is rounding off of values before or after calculation has taken place. It means given a round idea of the answers.

Examples

(1) A school with 52 class has 41 students in one of the SS 1 class. What is the estimated populations of students in the school.

Solution

1. Since we are estimating the student population, we round off the number to 1 sf i.e

52 = 50 & 41 = 40

No of Class = 50

No of students per class = 40 population of students in school = 50 x 40 = 200 or

(2) Assume all class has the same number of students

= 51 x 41 = 2019

= 2000 students to 1 sf.

Assignment

(1) Approximate each of the following to 2 and 3 sf.

(a) 0.02356 (b) 3.456 (c) 40582

(2) Approximate each of the following to 2 and 3 decimal places

(a) 45.325 (b) 3.8136 (c) 40582

(3) A school has 9 classes. One of the classes has 43 students. Estimate the numbers of students in the school.

3.4 Rate at which things Happens

The expressions 60km/h, ₦100/day 10km/litre are all examples of rates. The first rate tells the distance done in 1 hour. The second tells how much money is made per/day. While the third tells the distance traveled on 1 litre.

Examples

1. A car goes 180km in 3 hours. What is its rate in km/h?

   In 3 hours the car travels 180km.

   In 1 hour the car will travel $\frac{180}{3}km = 60km$

   The rate of the car is 60km/h.

2. Mr. John gets 18,00 for 6 days work what is the rate of pay per day? In 6 days Mr. John gets N18,000

   In 1 day Mr. John will get $\frac{N8000}{6} = N3000$

   Mr. John rate of pay is N3000/day

3. A car uses 50 litres of petrol to travel 850km. Express its petrol consumption as a rate in km per litre.

On 50 litres the car travels 850km

On 1 litre the car will trave $\frac{180}{50}km = 17km$

The petrol consumption is 17km/litre

4. Calculate the tax on N150,000.0, if the rate of tax is 3 kobo in (a) a naira (b) N50.00

(a) In N1.00, tax is 3 kobo = ₦ $\frac{3}{100}$

$\therefore$ in ₦ 150,000.00, the tax = ₦$\left(\frac{3}{100} x \frac{150,000}{1}\right)$

₦ 4500

(b) In N50.00, tax is 3 kobo = ₦ $\frac{3}{100}$

In ₦ 150,000.00, the tax = ₦$\left(\frac{3}{500} x \frac{150,000}{1}\right)$

₦ 90

Assignment

1. A boy cycles at a rate of 10km/h for 3½ hours. How far does he travel?
2. A trader reduces all his prices by 20kobo in the naira. Find the new price of a shirt which originally cost N800.
3. A farmer needs 10 men to clear his farm in 5 days. How many men will he need, if he must finish clearing the farm in 3 days.

3.5 Proportion

This is a relation between two variable in which one of the variable is as multiple of the other.

Example

1. A man gets N1000 for 10 days of work. Find the amount for (a) 5 days (b) 30 days (c) y days

Solution

For 10 days the man gets N1000

For 1 day the man will get N1000 ÷ 10 = N100

(a) For 5 days the man will get 5 x ₦100

= ₦500

(b) For 30 days the man will get 30 x ₦100

= ₦ 3000

(c) For y days the man will get y x ₦100

= ₦100y

The above method is call the unitary method. By unitary method, we first find the rate for one unit before answering the set question.

Examples

Mr. Ojo bought 20 birds for N600.00

(a) How much will buy 13 birds?

(b) How much will buy 8 birds?

(c) How many birds can N4,620.00 buy?

Solution

(a) 20 birds cost N6600.00

1 bird will cost N6600 ÷20 = N330

For 13 birds = ₦ $\frac{6600}{20} x \frac{13}{1} = 4290$ (or N330 x 13)

(b) For 8 birds = = ₦ $\frac{6600}{20} x \frac{8}{1} = 2640$ (or N330 x 8)

(c) If N330 can buy 1 bird.

How man birds can N4,600.00 buy

$= \frac{4020}{30} =$ 14 birds

3.5.1 Direct Proportion

A direct proportion is a proportion in which one variable increases as a result of the other variables increase.

Example 3

The cost of 5 books is N480.00. Find the cost of 8 books.

Here as the books increase in number, the money to buy it also increases.

5 books cost N480.00

1 book cost ₦ $\frac{480.00}{50} = 96.00$

2 books will cost N96.00 x 2

3 books will cost N96.00 x 3 and so on

$\therefore$ 10 books will cost N96.00 x 10 = N9600.00

Such proportion are called direct proportion. There are some proportion that are not direct.

Example 4

Six man can weed a plot in 18 days. How many days will it take 9 men to weed the same plot?

6 men weed the plot for 18 days

1 man will weed the plot for 18 x 6 day

$\therefore$ 9 men will weed the plot for ₦ $\frac{18x16}{9} =$ 12 days

In the above example, the number of days decreases as the number of men increases. Proportion of this natural is called inverse proportion.

3.5.2 Ratio

This is a way of comparison, it allows us to compare the magnitude of two or more things that are related.

Both parts of a ratio may be multiplied or divided by the same number. It is usual to express ratios as whole numbers in their lowest terms.

Notation

By way of notation, ratio can be written as *(i) atob* (ii) $a{:}b$ (iii) $\frac{a}{b}$

In (i) & (ii), the number corresponding to the first number of the ratio is usually written first while the number corresponding to the second number of the ratio comes next.

In (ii) ratio is expected as a fraction. The first number of the ratio becomes the numerator while the second number of the ratio becomes the denominator. Since ratio can be express as fraction it behaves like fraction. But ratio are usually reduced to the lowest form. Note that in ratio;

(a) the quantities being compared must be in the same unit.

(b) Ratio has no unit

Expressing One Quantity as a Ratio of Another Quantity

Examples

(1) Express 60 kobo as a

(a) ratio to ₦1.00

(b) ratio to ₦1.80

Solution

(a) 60 kobo: ₦1.00

60 kobo: 100 kobo

= 60:100

= 6:10 = 3; 5

(b) 60 kobo: ₦1.80

60 kobo: 100 kobo

60:180

6:18

1:3

(c) Express 10mm as a ratio of 7cm

3. Express 3 days as a ratio of 3 weeks

4. Express 15mins as a ratio of 1 hr

Solutions

= 10mm: 7cm

10mm:70mm

(Since 10mm = 1cm)

= 10:70

= 1:7

3. 3 days: 3 weeks

3 days: 21 days

(Since 7 days = 1 weeks)

3:21

1:7

Sharing Quantities in a given Ratio

The sharing of quantities is usually best done through the use of ratio, in our every day activities. The Federal Government use ratio in the sharing of nation resources to states based on their population. In financial partnerships, profits are usually shared in the ratio of the amount contributed by the partners. Sharing are done in our homes based on needs or age of the children.

Examples

1. Share the sum of N50,000.00 between John and Joseph in the ratio of 6:2. Calculate John's and Joseph's shares.

Solution

Quantity to share = N50,000.00

Sum of ratio = 6 + 2 = 8

$\therefore$ value of a share ₦ $\frac{50{,}000}{8} = 6250$

John's share to Joseph's share 6:2

$\therefore$ John's share = 6 x N6250.00

=N37,500.00

Joseph's share = 2 x N6250.00

= N12,500.00

(2) Share 120 bags of rice between two villages in the ratio of 5:3.

Solution

Quantity to share 120 days of rice sum of ratio = 5 + 3 = 8

$\therefore$ one village share $\frac{120}{8} = 15$ bags

The first village will get 15 x 5 = 75 bags

The second village will get 15 x 3 = 45 bags

(3) Three sons are aged 16 years, 13 and 11 years. Two hundred and forty (240) sheep are shared between them in the ratio of their ages. How many sheep does each get?

Solution

Ratio of son's ages = 16:13:11

Sum of ratio = 16 + 13 + 11 = 40

1 share = $\frac{240}{40}$ sheep

= 6 sheep

Age 16 = 16 x 6 sheep = 96 sheep

Age 13 = 13 x 6 sheep = 78 sheep

Age 11 = 11 x 6 sheep = 66 sheep

They will get 96, 78 and 66 sheep respectively

4.0 Conclusion

The unit considered rounding off numbers, decimal point and significant figure, estimations and rate at which thing happen

5.0 Summary

The highlights of the unit are as following:

- How decimal point and significant figure are carried out.
- How to estimate.
- Rate of which things happens.

6.0 Tutor Marked Assignment

Solve the problems below;

(1) Approximate (a) 23467 (b) 556782 to the nearest ten, hundred and thousands.

(2) Round off (a) 8.7658 (b) 6.6815 to (i) 2dp (ii) 3dp

(3) Approximate (a) 0.0056 (b) 10.678 to (i) 2dp (ii) 3dp

(4) Share the sum of ₦100,000.00 between Mary and John in the ratio of 5:2. Calculate their shares .

7.0 References/Further Reading

Channon, J.B. et. al (2002): New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

National Teacher's Institute, Kaduna (2002).

Pivotal Teacher Training Programme (PTTP) (2000).

MODULE 2: Algebraic Process

Unit 1: Factorisation and Linear Equations

Unit 2: Quadratic and Simultaneous Equations

Unit 3: Algebraic Graphs

Unit 4: Change of Subject of Formulae

Unit 5: Linear Inequalities

UNIT 1: Factorisation and Linear Equations

Content

1.0 Introduction

This unit is designed to give you an understanding of the nature of algebraic expression and linear equations. It will acquaint you with the techniques of factorizing and simplifying algebraic expressions and solving linear equations. You are therefore encouraged to study the unit carefully.

2.0 Objectives

By the end of this unit, you should be able to:

(1) expand algebraic expressions by removing brackets;

(2) factorise algebraic expressions by taking common factors;

(3) factorise algebraic expressions involving fractions;

(4) simplify algebraic expressions involving fractions;

(5) solve linear equations;

(6) solve linear equations involving fractions;

(7) solve linear equations involving word problems.

3.0 Main Body

3.1 Expansion by Removing Brackets

An algebraic expression may be of the form a(b+c). In order to expand this expression, you remove brackets. In removing the brackets, you must observe certain rules. These includes:

1. if there is a positive (+) sign in front of the bracket, any sign connecting the terms inside the bracket must remain unchanged. For example, a(b+c) = ab+ac.
2. if there is a negative (-) sign in front of the bracket, any sign connecting the terms inside the bracket must change. For example, -a(b+c) = -ab-ac.

Examples

Remove brackets from the following expressions.

(1) $P(q-2)$ (2) $5(P+b)$ (3) $r(7s-15)$ (4) $-3t(2s-t)$

(5) $-3(a^2-2)$ (6) $6(a+1)-5(a-3)$

Solutions

1. $P(q-2)=Pq-2P$
2. $5(P+6)=5P+30$
3. $r(7s-15)=7rs-15r$
4. $-3t(2s-t)=-6st+3t^2$
5. $-3(a^2-2)=-3a^2+6$
6. $6(a+1)-5(a-3)=6a+6-5a+15=a+21$

Self Assessment Exercise

Remove brackets from the following expressions.

(1) $-a(a+1)$ (2) $-a(a-1)$ (3) $3(a-3)-b$

3.2 Factorisation by Taking Common Factors

When factorizing an algebraic expression involving two or more terms, you look for the common factor among the terms and take it out. A common factor is number of letter you can find in all the terms of the expression. Taking out the common factor will imply introducing brackets. The common factors(s) could be a letter, a number or both. Consider the expression, $rp+rs+rt.$ You can observe that r is common to the three terms in the expression. If you take out this common factor r you will have:

rp + rs + rt = r (p + s + t). Consider another expression; $5r^2+10rt+15rs.$ You will observe that 5 is a common factor and r is also a common factor. If you take out this common factor (5r), you will have $5r^2+10rt+15rs=5r(r+2t+3s)$.

Examples

Factorise the following expressions.

1. $5r+10rt+15r^2s$
2. $10r^2t-20r^2t^2+30r^2ts$
3. $14rst+21r^2s^2t^2$
4. $16ab^2-24a^2b+4ab$
5. $3xy+2abx+5rsx$
6. $(x+y)(r+t)+(x+y)(r-3t)$
7. $(2r+3y)^2+5y(2r+3y)$

Solution

1. $5r+10rt+15r^2s=5r(1+2t+3rs)$
2. $10r^2t-20r^2t^2+30r^2ts=10r^2t(1+2t+3S)$
3. $14rst+21r^2s^2t^2=7rst(2+3rst)$
4. $16ab^2-24a^2b+4ab=4ab(4b-6a+1)$
5. $3xy+2abx+5rsx=x(3y+2ab+5rs)$

6. $(x+y)(r+t)+(x+y)(r-3t)=(x+y)[(r+t)+(r-3t)]$

$=(x+y)[r+t+r-3t]=(x+y)\,(2r-2t)=2(x+y)(r-t)$

7. $(2r+3y)^2+5y(2r+3y)=(2r+3y)[(2r+3y)+5y]=(2r+3y)[2r+3y+5y]$

$=(2r+3y)(2r+8y)=(2r+3y)2(r+4y)=2(2r+3y)(r+4y)$

Self Assessment Exercise 2

Factorise the following expression.

1. $ab+2a^2b^2+3ab$

2. $6rt-3t+12r^2t$

3. $(r+t)(3p+q)-(r+t)(p+3q)$

3.3 Factorising Algebraic Expression Involving Fractions

When factorizing algebraic expressions involving fractions, the first step is to equalized all the denominators. This is done by finding the lowest common multiple (L.C.M.) of the denominators. For examples, in factorizing the expression;

$$\frac{a}{2x}+\frac{b}{4y}$$

You find the L.C.M of 2x and 4y which is 8xy. Therefore,

$$\frac{a}{2x}+\frac{b}{4y}=\frac{4ay+2bx}{8xy}=\frac{2(2ay+bx)}{8xy}$$

$$=\frac{2ay+bx}{4xy}$$

Also, consider the expression:

$$\frac{a-3}{4}+\frac{a+1}{5}$$

The L.C.M. of 4 and 5 is 20. Therefore,

$$\frac{a-3}{4}-\frac{a+1}{5}=\frac{5(a-3)-4(a+1)}{20}$$

$$=\frac{5a-15-4a-4}{20}$$

$$=\frac{a-19}{20}$$

3.4 Simplifying Algebraic Expressions Involving Fractions

To simplify a fractional algebraic expression, first factorise the expression then reduce the resulting fraction to it's lowest term.

Examples

Simplify the following expressions.

(1) $\frac{a-2}{2}+\frac{a+2}{3}$ (4) $\frac{2}{a+1}+\frac{3}{a-1}$

(2) $\frac{(a+b)}{9}+b$ (5) $\frac{n}{2n+1}+\frac{m}{2m-1}$

(3) $5a - -\frac{3(2a+b)}{11}$ (6) $\frac{x-1}{x+3}-\frac{x+1}{x-3}$

Solutions

1. $\frac{a-2}{2}+\frac{a+2}{3}$

The L.C.M. of 2 and 3 is 6. Therefore

$$\frac{a-2}{2}+\frac{a+2}{3}=\frac{3(a-2)+2(a+2)}{6}$$

$$= \frac{3a-6+2a+4}{6}=\frac{5a-2}{6}$$

2. $\frac{(a+b)}{9}+b=\frac{(a+b)}{9}+\frac{b}{9}$

The L.C.M. of 9 and 1 is 9. Therefore

$$\frac{(a+b)}{9}+\frac{b}{1}=\frac{(a+b)+9b}{9}=\frac{a+b+9b}{9}$$

$$= \frac{a+10b}{9}$$

3. $5a - \frac{3(2a+b)}{11}=\frac{5a}{1}-\frac{3(2a+b)}{11}$

$$= \frac{5a}{1}-\frac{(6a+3b)}{11}$$

The L.C.M of 1 and 11 is 11.

$$\therefore \frac{5a}{1}-\frac{(6a+3b)}{11}=\frac{55a-6a-3b}{11}=\frac{49a-3b}{11}$$

4. $\frac{2}{a+1}+\frac{3}{a-1}$

L.C.M of a + 1 and a – 1 = (a + 1)(a – 1)

$$\frac{2}{a+1}+\frac{3}{a-1}=\frac{2(a-1)+3(a+1)}{(a+1)(a-1)}=\frac{2a-2+3a+3}{a^2+a-a-1}$$

$$= \frac{5a+1}{a^2-1}$$

5. $\frac{n}{2n+1}-\frac{m}{2m-1}$

L.C.M of 2n + 1 and 2m-1 is (2n+1) (2m-1)

$$\frac{n}{2n+1}-\frac{m}{2m-1}=\frac{n(2m-1)-m(2n+1)}{(2n+1)(2m-1)}=\frac{2mn-n-2mn-m}{4mn+2m-2n-1}$$

$$= \frac{-n-m}{4mn+2m-2n-1}$$

6. $\frac{x-1}{x+3}-\frac{x+1}{x-3}$

L.C.M of x + 3 and x – 3 is (x + 3)(x – 3)

$$\frac{x-1}{x+3}-\frac{x+1}{x-3}=\frac{(x-3)(x-1)-(x+3)(x+1)}{(x+3)(x-3)}$$

$$= \frac{x^2-3x-x+3-(x^2+3x+x+3)}{x^2+3x-3x-9}$$

$$= \frac{x^2-4x+3-x^2-3x-x-3}{x^2+3x-3x-9}$$

$$= \frac{-8x}{x^2-9}=\frac{8x}{9-x^2}$$

Self Assessment Exercise 3

Simplify the following

(1) $\frac{4(x+3)}{5}+3x$ (2) $\frac{1}{a+b}-\frac{1}{a-b}$

(3) $\frac{5}{x-2}-\frac{6}{x+3}$

3.5 Solving Linear Equations

An equation is a mathematical statement that shows two algebraic expressions to be equal in value. For example, 4P + 40 = 10P + 12 is an equation. The unknown in this equation is P. Since the highest power of P in the equation is 1, the equation is called a linear equation. The above equation can only be true if p has a particular numerical value. To solve an equation means to find the particular numerical value of the unknown, in this case (P).

Examples:

Solve the following equations.

1. 4P + 42 = 10P + 12
2. 4s – 6 = 22
3. 11 = 9t – 16
4. 16 + 8p – 14 = 8 – p
5. 2t + 19 – 5t = t – 5
6. 3 – (3a – 7) = 43
7. 6(10r -2) = 8 (6r + 4)
8. 7(5p -4) – 10(3p – 2) = 0
9. 3(6 + 7y) + 2(1 – 5y) = 42
10. 3x – [3(1 + x) – 2x] = 3

Solutions

1. 4P + 42 = 10P + 12

Collect like terms by bringing the numbers together and the terms with P together. Remember that when a term crosses an equality (=) sign, the sign of the term is changed.

42 – 12 = 10P – 4P

simplifying, we have

30 = 6P

Since 6 is a common factor of 30 and 6, we can divide both sides of the equation by 6. this gives:

$$\frac{30}{6} = \frac{6P}{6}$$

5 = P

or

P = 5

2. $4s – 6 = 22$

Collect like terms

$4s = 22 + 6$

$4s = 28$

Divide both side by 4

$$\frac{4s}{4} = \frac{28}{4}$$

$s = 7$

3. $11 = 9t – 16$

$27 = 9t$

Collect like terms

$11 + 16 = 9t$

Divide both sides by 9

$$\frac{27}{9} = \frac{9t}{9}$$

$3 = t$

or

$t = 3$

4. $16 + 8p – 14 = 8 – p$

Collect like terms

$8p + p = 8 + 14 – 16$

$9p = 22 – 16$

$9p = 6$

Divide both sides by 9

$$\frac{9p}{9} = \frac{6}{9}$$

$$p = \frac{2}{3}$$

5. $2t + 19 – 5t = t – 5$

Collect like terms

$2t – 5t – t = -5 – 19$

$2t – 6t = -24$

-4t = -24

Divide both sides by -4

$$\frac{-4t}{-4} = \frac{-24}{-4}$$

t = 6

6. 3 – (3a – 7) = 43

Remove brackets

3 – 3a + 7 = 43

Collect like terms

-3a = 43 – 7 – 3

-3a + 43 - 10

-3a = 33

Divide both sides by -3

$$\frac{-3a}{3} = \frac{33}{-3}$$

a – -11

7. 6(10r – 2) = 8(6r + 4)

Remove brackets

60r – 12 = 48r + 32

Collect like terms

60r – 48r = 32 + 12

$\therefore$ 12r = 44

Divide both sides by 12

$$\frac{12r}{12} = \frac{44}{12}$$

$$r = \frac{11}{3} = 3\frac{2}{3}$$

8. 7(5p – 4) – 10(3p-2) = 0

Remove brackets

35p – 28 – 30p + 20 = 0

Simplify

35p – 30p – 28 + 20 = 0

5p – 8 = 0

5p = 8

Divide both sides by 5

$\frac{5p}{5} = \frac{8}{5}$

p = $\frac{8}{5} = 1\frac{3}{5}$

9. $3(6+7y)+2(1-5y)=42$

Remove brackets

18 + 21y + 2 – 10y = 42

Collect like terms

21y – 10y = 42 – 18 – 2

11y = 42 – 20

11y = 22

Divide both sides by 11

$\frac{11y}{11} = \frac{22}{11}$

y = 2

10. 3x - $[3(1+x)-2x]=3$

Remove the inner brackets

$3x-[3+3x-2x]=3$

$\therefore 3x-3-3x+2x=3$

Simplify

- 3 + 2x = 3

Collect like terms

$2x=3+3$

$2x=6$

Divide both side by 2

$\frac{2x}{2}=\frac{6}{2}$

$x=3$

Self Assessment Exercise 4

Solving the following equations.

1. $12a+(1-7a)=31$

2. $4(3n-1)=11n-3(n-4)$

3. $2-5(5t-2)-9(3t-2)$

3.6 Solving Linear Equations Involving Fractions

To solve a linear equation involving fractions, the first step you should take is to clear the fractions. This you can do by multiplying every term in the equation by the L.C.M. of the denominators of the fractions. Consider the equation.

$$\frac{a}{2}=\frac{a}{3}+\frac{1}{2}$$

The L.C.M of 2 and 3 and 2 is 6.

Multiply all the terms in the equation by 6. It gives;

$$6x\ \frac{a}{2}=6x\frac{a}{3}+\frac{6\,x1}{2}$$

$$\frac{6a}{2}=\frac{6a}{3}+\frac{6}{2}$$

Simplifying further, we have

$3a-2a+3$

Collecting like terms gives

$3a=2a=3$

a = 3

More examples are hereby solved to make it clearer.

Examples

Solve the following equations

(1) $\frac{4n}{3}-\frac{17}{21}=\frac{6n-1}{7}$

(2) $\frac{54}{2p+7}=\frac{52}{3p-4}$

(3) $\frac{pS}{10}+\frac{3}{5}=\frac{2S}{5}+\frac{7}{10}$

(4) $\frac{6a+3}{7} = \frac{2a-1}{3}$

(5) $\frac{3x+2}{5} - \frac{2x+3}{3} = 3$

Solutions

1. $\frac{4n}{3} - \frac{17}{21} = \frac{6n-1}{7}$

L.C.M. of 3, 21 and 7 is 21.

Multiply all the terms of the equation by 21.

$$\frac{21(4n)}{3} - \frac{21(17)}{21} = \frac{21(6n-1)}{7}$$

This is equal to:

7(4n) – 17 = 3(6n-1)

Remove brackets

28n – 17 = 18n – 3

Collect like terms

28n – 18n = -3 + 17

Simplify

10n = 14

Divide through by 10

$$\frac{10n}{10} = \frac{14}{10}$$

$n = 7/5$

2. $\frac{54}{2p+7} = \frac{52}{3p-4}$

L.C.M of the denominators is (2p + 7)(3p-4). Multiply each term in the equation by the L.C.M.

$$\frac{54(2p+7)(3p-4)}{2p+7} = \frac{52(2p+7)(3p-4)}{3p-4}$$

This gives:

54(3p – 4) = 52(2p + 7)

Remove brackets

162p – 216 = 104p + 364

Collect like terms:

162p – 104p = 364 + 216

58p = 580

Divide both sides by 58

$$\frac{58p}{58} = \frac{580}{58}$$

p = 10

3. $$\frac{9S}{10} + \frac{3}{5} = \frac{2S}{5} + \frac{7}{10}$$

L.C.M. of the denominators is 10

Multiply though by 10.

$$10\frac{(9S)}{10} + 10\frac{(3)}{5} = 10\frac{(2S)}{5} + 10\frac{(7)}{10}$$

This gives

9S + 2(3) = 2(2S) + (7)

Remove bracket:

9S + 6 = 4S + 7

Collect like terms:

9S – 4S = 7 – 6

Simplify

5S = 1

Divide both sides by 5

$$\frac{5S}{5} = \frac{1}{5}$$

S = 1/5

(4) $$\frac{6a+3}{7} = \frac{2a-1}{3}$$

L.C.M of 7 and 3 is 21

Multiply through by 21

$$\frac{21(6a+3)}{7} = \frac{21(2a-1)}{3}$$

This gives:

3(6a + 3) = 7(2a – 1). That is

18a + 9 = 14a – 7

Collect like terms:

18a – 14a = -7 – 9

4a = -16

Divide through by 4.

$$\frac{4a}{4} = \frac{-16}{4}$$

$$a = -4$$

5. $$\frac{3x+2}{5} - \frac{2x+3}{3} = 3$$

L.C.M. of denominators is 15

Multiply through by 15, we have

$$\frac{15(3x+2)}{5} - \frac{15(2x+3)}{3} = 15(3)$$

This gives:

3(3x + 2) -5 (2x + 3) = 15(3)

Remove brackets:

9x + 6 – 10x – 15 = 45

Collect like terms

9x – 10x = 45 + 15 – 6

Simplify

-x = 60 – 6

-x = 54

x = -54

Self Assessment Exercise 5

Solve the following equations.

1. $$\frac{4x+2}{4} + \frac{6x+2}{6} = 0$$

2. $$\frac{1}{x+1} = \frac{1}{2x+4}$$

3. $\frac{5x+16}{4} + \frac{x}{2} = \frac{4x+2}{3}$

3.7 Solving Linear Equations Involving Word Problems

Word problems are problems stated in words rather than being expressed with numbers and letter. In this case, you are expected to read the sentences carefully and based on your understanding, formulate an equation from it. You can then solve the equation using the procedures you have learnt. Consider the following examples.

(1) There exist a certain number. When the number is multiplied by 4 the result is the same as adding 15 to the number. Find the number.

Solution

The first step is to carefully read through the question and understand it. Now, what is this certain number? Make a representation for it. Let the certain number be x. multiply x by 4. This gives 4 x. The question say 4 x is the same as adding 15 to x. That is:

$4x = x + 15$

Collecting like terms gives:

$4x - x = 15$

Simplify

$3x = 15$.

Divide both sides by 3

$\frac{3x}{3} = \frac{15}{3}$

$x = 5$

2. The sum of 6 and one third of n is one more than twice n. Find the value of n.

Solution

The sum of 6 and one third of n can be expressed as:

$6 + \frac{1}{3}n$. That is

$6 + \frac{n}{3}$. Twice n can be expressed as 2n

One more than twice n can be expressed as:

$2n + 1$.

Therefore, the resulting equation is:

$6+\frac{n}{3}=2n+1$

Multiply all terms in the equation by 3. This is to eliminate the denominator 3. This give:

$3(6)+3\frac{(n)}{3}=3(2n+1)$

Remove brackets and simplify

18 + n = 6n + 3

Collect like terms.

18 - 3 = 6n – n

15 = 5n

Divide both sides by 5.

3 = n

or

n = 3

3. The result of adding 15 to a certain number and dividing the answer by 4 is the same as taking that number from 90 and dividing the answer by 3.

(a) Express this statement as an algebraic equation.

(b) Find the value of the number.

Solution

Let the number be *x*.

Adding 15 to the number and dividing by 4 can be expressed as

$\frac{x+15}{4}$

Taking that number from 90 and dividing by 3 can be express as.

$\frac{90-x}{3}$

Therefore,

$\frac{x+15}{4}=\frac{90-x}{3}$

L.C.M. of 4 and 3 is 12. Multiply through by 12

$\frac{12(x+15)}{4}=\frac{12(90-x)}{3}$

$3(x + 15) = 4(90 – x)$

Remove brackets

$3x + 45 = 360 - 4x$

Collect like terms: $3x + 4x = 360 – 45$. Simplifying, we have:

$7x = 360 – 45$

$7x = 315$

$x = 45$

(4) Esther and Andrew shared ₦800.00 such that Esther get ₦25.00 more than Andrew. Calculate how much each gets.

Solution

Let Andrew's share be *x*.

∴ Esther's share will be $x + 25$

The total amount shared = N800.00

That is

$x + x + 25 = 800$

$2x + 25 = 800$

$2x = 800 – 25$

$2x = 775$

$x =$ ₦387.50

Andrew got N38.50

Esther's share = 387.50 + 25 = N412.50

(5) A rectangle is 4 times as long as it is wide. If the perimeter of the rectangle is 50cm, find (i) the wide of the rectangle (ii) the length of the rectangle

Solution

Let the width of the rectangle be *x*.

∴ Length (L) = $4x$

Perimeter of rectangle = L + W + L + W

That is:

$4x + x + 4x + x = 50cm$

$10x = 50cm$

$x = 5cm$

(i) The width of the rectangle = $5cm$

(ii) The length of the rectangle = $4(5) = 20cm$

Self Assessment Exercise 6

A rectangle is 2 times as long as it is wide. If its perimeter is 48cm. Find the length of the rectangle.

4.0 Conclusion

This unit considered factorization and linear equations. Procedures for expanding and factorizing algebraic expressions were discussed. Solutions of linear equations including linear equations from word problems and fractional linear equations were also discussed.

5.0 Summary

The highlights of the unit includes the following:

- When removing brackets in an algebraic expression, if there is a positive (+) sign in front of the bracket, any sign connecting the terms inside the bracket must remain unchanged. If there is a negative (-) sign in front of the bracket, any sign connecting the terms inside the bracket must change.
- When factorizing an algebraic expression involving two or more terms, look for the common factors among the terms and take it out.
- When factorizing a fractional algebraic expression, look for the L.C.M of the denominators and multiply each term in the expression by it.
- To simplify a fractional algebraic expression, factorise the expression, then reduce the resulting fractions to their lowest terms.
- To solve an equation means to find the particular numerical value of the unknown.
- An equation is linear if the highest power of the unknown is 1.
- The first step in solving fractional linear equations is to clear the fractions.

6.0 Tutor Marked Assignment

1 Remove brackets from the following expressions.

(a) $3(S+1)+7(S+2)$ (b) $6(S+6)-5s-10$

(c) $2(P+1)-3(2p-12)-3(3p-1)$

2. Fractorise the following expressions.

(a) $6st + 2s^2t^2 + 4st$

(b) $(2n+3m)(m-n)+(m-n)(6n-9m)$

3. Simplify the following

(a) $\frac{4}{x-2}-\frac{5}{x-3}$

(b) $\frac{(x+3)}{5}+3x$

4. Solve the following equations.

(a) $6a - 2(1-9a) = 58$

(b) $5(10t - 2) - 9(3t + 2) - 12 = 0$

(c) $\frac{1}{a+1} = \frac{1}{2a+4}$

(d) $\frac{3-+5}{2}+\frac{p}{3}=\frac{7p-+1}{3}$

5. A certain number is multiplied by 7 the result is the same as adding one third of the number to 20. Find the number.

6. The length of a rectangular table top is 3½ times its width. If the perimeter of the table top is 414cm, find (i) the width (ii) the length of the table top.

7.0 References/Further Reading

Channon, J.B. et. al (2002). New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

Galadima, I (2004). Teaching Secondary School Algebraic World Problem: A Heuristic Approach. Lagos: Biga educational service.

Mathematical Association of Nigeria (2008). MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 2: UNITS 2

Unit 2: Quadratic and Simultaneously Equations

Content

1.0 Introduction

This unit is designed to give you insight into quadratic and simultaneous equations. It will help you to understand the nature of these equations and the different methods of finding solutions to them. In addition to the methods that will be discussed in this unit, we also employ the use of graphs in finding solution to quadratic and simultaneous equations. This aspect (graphical solution) will be considered in unit 3. You are encourage to study the unit carefully. Make sure your understand one step before moving to the next.

2.0 Objectives

By the end of this unit, you should be able to do the following:

1. factorise quadratic expressions
2. solve quadratic equations
3. solve simultaneous equations using elimination method
4. solve simultaneous equations using substitution method

3.0 Main body

3.1 Factorisation of Quadratic Expressions

A quadratic expression is one in the form $ax^2 + bx + c$ where a, b, and c are constants and a is not equal to zero. For example $2x^2 – 5x + 3$ is a quadratic expressions. It is also possible to have more than one unknown in a quadratic expression. For example $3x^2 + 5xy + 2y^2$ is a quadratic expression with two unknown x and y. Note that the highest power of the unknown is always two. There are three methods we can employ in factorizing a quadratic expression.

Method One: Difference of Two Squares

Note: The difference of the square of two quantities is equal to the product of their sun and their difference. Consider the expression $(a + b)(a - b)$.

Removing brackets we have

$a^2 – ab + ab – b^2$

Simplify, gives

$a^2 – b^2$. That is

$(a + b)(a – b) = a^2 – b^2$

Therefore when factorizing a term of difference of two squares, first write each term as a square. Consider the following.

1. $P^2 – 4$

   $P^2 = (P)^2$

   $- 4 = -(2)^2$

   $P^2 – 4 = P^2 – 2^2$

   $P^2 – 4 = (P + 2)(P-2)$

2. $4a^2 – 9$

   $=(2a)^2 – 3^2$

   $= (2a + 3)(2a – 3)$

3. $5P^2 – 45q^2$

Neither 5 nor 45 is a perfect square. But 5 is a factor of 45. So divide both terms by 5. Which give $P^2 – 9q^2$

Write each term as a square.

   $(P)^2 – (3q)^2$

   $= (P + 3q)(P – 3q)$

4. $16h^2 - 25$

   $= (4h)^2 – (5)^2$

   $= (4h + 5)(4h – 5)$

Method Two: Factorization by inspection

Suppose we want to factorise the expression $a^2 + 6a + 8$.

Our goal is to write $a^2 + 6a + 8$ in the form ( ) ( ) and fill the brackets. We know that a will appear first in the brackets and $a^2 = a.a$. Then, $a^2 + 6a + 8 = (a+...)(a+...)$.

The next step is to look for two numbers such that their product is + 8 and their sum is + 6. the number pairs which have their product as + 8 are:

+8 and +1

-8 and -1

+4 and +2

-4 and -2.

In order to know the right pair to pick, check which of the pairs will have their sum to be +6.

(+8) and (+1) = +9

(-8) and (-1) = -9

(+4) and (+2) = +6

(-4) and (-2) = -6

Since +4 and +2 gives the correct sum, we conclude that

$a^2 + 6a + 8 = (a + 4)(a + 2)$

Examples:

Factorise the following expressions.

i. $x^2 - 10x + 16$

look for two numbers whose products is + 16 and sum is – 10.

$16 = -8x - 2$

$- 10 = x -8$ *and* -2

$\therefore x^2 - 10x + 16 (x - 8)(x - 2)$

ii. $m^2 - 11m + 28$

look for two numbers whose product is +28 and sum is -11. These are –7 and -4

$m^2 - 11m + 28 = (m - 7)(m - 4)$

Method Three: Factorisation by Grouping

To factorise by grouping we must have a number of terms that we can put into groups. For instance, if the quadratic expression has three terms you must find a way to make it four terms so that you can group in twos. Consider the following examples.

1. $x^2 + 7x + 10$

$+ 7x$ can have be express as $5x + 2x$. We have choosen $+ 5x$ *and* $+ 2x$ because the product of 5 and 2 is 10 and the sum of 5 and 2 is 7. That is

$x^2 + 7x + 10 = x^2 + 5x + 2x + 10$

$= (x^2 + 5x) + (2x + 10)$

$= x(x + 5) + 2(x + 5)$

$= (x + 5)(x + 2)$

2. $x^2 – 10x + 21$

$-10x = -7x – 3x$

Note: that it is only -7 and -3 that have a product of + 21 and a sum of -10

$x^2 – 10x + 21 = x^2 - 7x - 3x + 21$

$= (x^2 - 7x) + (3x - 21)$

$= x(x - 7) - 3(x - 7)$

$= (x + 7)(x - 3)$

Alternatively

Find the product of the 1st and last terms.

x^2 x $21 = 21x^2$

Look for two numbers, such that their product is $21x^2$ and their sum is – $10x$ (the middle terms).

Factors of $21x^2$	sum of factors
$21x$ and x	$22x$
$-21x$ and $– x$	$-22x$
$-7x$ and $– 3x$	$-10x$
$+7x$ and $+ 3x$	$+ 10x$

The right factors are - $7x$ and – $3x$. So we substitute these for – $10x$ in the expression.

$x^2 - 10x + 21 = x^2 - 7x - 3x - 21$

$= (x^2 - 7x) - (3x - 21)$

$= x(x - 7) - 3(x - 7)$

$= (x - 7)(x - 3)$

3. $4x^2 – 37xy + 9y^2$

$-37xy$ can be express as - $36xy – xy$

This is because $4x^2$ x $9y^2 = 36x^2y^2$. Two factors whose product is $36x^2y^2$ and sum is $-37xy$ (the middle term) are – $36xy$ and $–xy$.

$=(4x^2 – 36xy) – (xy – 9y^2)$

$=4x(x – 9y) – y(x – 9y)$

$(x – 9y)(4x – y)$

Self Assessment Exercise 1

Factorise the following quadratic expressions

(i) Using inspection method (ii) Using grouping method

(a) $t^2 + 9t – 22$

(b) $6p^2 – 13p – 7$

3.2 Quadratic Equations

Consider the equation

$x^2 – 3x + 2 = 0$

The 1st step in solving this equation is to factorise the expression on the left hand side. You can employ any of the methods of factorization discussed in section 3.1 that is most appropriate.

$x^2 – 3x + 2 = (x – 2)(x -1)$ that is $(x – 2)(x – 1) = 0.$

When the product of two terms is zero, is either of the terms is zero or both terms are zero. So if

$(x – 2)(x -1) = 0.$

either

$x – 2 = 0$ or $x – 1 = 0$

if $x – 2 = 0$, *then* $x = 2$

if $x – 1 = 0$ *then* $x = 1$

$\therefore x = 2$ *or* 1

These values (2 and 1) are called the roots of the quadratic equation. This implies that solving a quadratic equation is the same as finding the roots of a quadratic equation.

Examples

Solve the following quadratic equations.

(1) $x^2 – 7x + 10 = 0$

(2) $2P^2 – 5P – 3 = 0$

(3) $16n^2 + 8n + 1 = 0$

(4) $4t^2 – 49 = 0$

(5) $6m^2 = m + 1$

Solutions

1 $x^2 – 7x + 10 = 0$

This can be factorised by inspection

$(x - 5)(x - 2) = 0$

Either

$(x - 5) = 0$ or $x - 2 = 0$

if $x - 5 = 0$ if $x - 2 = 02$

Then $x = 5$ then x = 2

That is x = 5 or 2

You can check whether your answer is correct by substituting the answers into the equation.

$x^2 - 7x + 10 = 0$

When $x = 5$, $x^2 - 7x + 10$

$5^2 - 7 \times 5 + 10 = 0$

$25 - 35 + 10 = 0$

$-10 + 10 = 0$

When $x = 2$,

$x^2 - 7x + 10 = 2^2 - 7 \times 2 + 10$

$= 4 - 14 + 10 = -10 + 10 = 0$

2. This shows that the answers x = 5 or 2 are correct

$2p^2 - 5p - 3 = 0$

This can be factorised by grouping,

$2P^2 \times -3 = -6p^2$

Two factors whose product is $6p^2$ and sum is $-5p$ are $-6p$ and p.

Substitute these for $-5p$ in the equation. That is

$2p^2 - 6p + p - 3 = 0$

$(2p^2 - 6p) + (p - 3) = 0$

$2p(p - 3) + (p - 3) = 0$

$(p - 3)(2p + 1) = 0$

Either p – 3 = 0 or 2p + 1 = 0

If P – 3 = 0 then, P = 3

If 2P + 1 = 0, then, 2P = -1

$\therefore$ P = $-\frac{1}{2}$

3. $16n^2 + 8n + 1 = 0$

This can also be factorised by grouping.

$16n^2 \times 1 = 16n^2$

Two factors whose product is $16n^2$ and sum equal to 8n are 4n and 4.. Substitute these for 8n.

That is

$16n^2 + 4n + 4n + 1 = 0$

$(16n^2 + 4n) + (4n + 1) = 0$

$4n(4n + 1) + (4n + 1) = 0$

$(4n + 1)(4n + 1) = 0$

Either

$4n + 1 = 0$

$n = -\frac{1}{4}$ twice

4. $4t^2 - 49 = 0$

Note that the left hand side is a difference of two squares. So express each term as a square.

$(2t)^2 - (7)^2 = 0$

$(2t + 7)(2t - 7) = 0$

Either

$2t + 7 = 0$ or $2t - 7 = 0$

$t = \underline{+} \frac{7}{2}$,

5. $6m^2 = m + 1$

Rearrange the equation to give a quadratic expression on the left hand side and zero on the right hand side. That gives

$6m^2 - m - 1 = 0$

This can be factorised by grouping

$6m^2 \times -1 = -6m^2$

two factors whose product is $-6m^2$ and sum equals $-m$ are $-3m$ and $2m$. substituting these for –m in the equation gives

$6m^2 - 3m + 2m - 1 = 0$

$(6m^2 - 3m) + (2m - 1) = 0$

$3m(2m - 1) + (2m - 1) = 0$

$(2m - 1)(3m - 1)$

Either

$2m - 1 = 0$ or $3m + 1 = 0$

If $2m - 0 = 0$ *then* $2m = 1$ *and* $m = \frac{1}{2}$

If $3m - 1 = 0$ *then* $3m = 1$ *and* $m = \frac{1}{2}$

$m = \frac{1}{2}$ *or* $\frac{1}{3}$

Self Assessment Exercise 2

Solving the following equations

1. $q^2 + 7q + 12 = 0$
2. $n^2 - 4n = 0$
3. $25t^2 = 9$

3.3 Simultaneous Linear Equations

Simultaneous Linear Equations are a pair of linear equations in two variables say x and y. The pair of values of x and y are such that they satisfy the equations simultaneously (at the same time). To solve a pair of simultaneous equations, you can use graphical method, elimination method or substitution method. Elimination and substitution methods will be discussed in this unit, while graphical method will be treated in unit 3.

Elimination Method

In this method of solving simultaneous linear equations, you eliminate one of the unknowns and solve for the other. Then, the solved one is substituted into any of the equations to find the value of the unknown earlier eliminated. Consider the following examples.

Examples

Using the method of elimination solve the following simultaneous equations.

1. $a + b = 7; a - b = 3$
2. $5p - 3q = 8; 2p - 5q = 7$
3. $15s + 2t = 7; 10s + \frac{1}{2} = 3$

Solutions

$a + b = 7; a - b = 3$

1. Write down the equations and label them.

   $a + b = 7$ - - - - - - *(i)*

$a - b = 3$ - - - - - - *(ii)*

Think of how to eliminate any of the unknowns, either a or b. Since the coefficients of b in both equations are numerically the same, both having opposite signs, b will be eliminated if both equations are added.

Equation(i) + Equation (ii) gives;

$a + b + a - b = 7 + 3$

$a + a = 7 + 3$

$2a = 10$

$a = \frac{10}{2} = 5$

substitute 5 for a in equation I (you can also substitute for a in equation ii).

This is gives

$5 + b = 7$

$b = 7 - 5$

$b = 2$

That is $a = 5$ & $b = 2$

(2) $5p - 3q = 8; 2p - 5q = 7$

Let $5p - 3q = 8$ - - - - - *(i)*

$2p - 5q = 7$ - - - - - *(ii)*

Find a way to equalize the coefficient of any of the unknowns in both equations. Multiply equation 1 by 2 and equation ii by 5 and label them equation 3 and 4 respectively.

Equation (i) x 2 and equation (ii) x 5.

$10p - 6q = 16$ - - - - - *(iii)*

$10p - 25q = 35$ - - - - *(iv)*

Subtract equation (iii) from (iv)

$10p = 25q - 10p - (-6q) = 35 - 16$

$-25q + 6q = 19$

$-19q = 19$

$q = \frac{19}{-19} = -1$

Substitute -1 for q in equation (i).

$5p - 3(-1) = 8$

$5p + 3 = 8$

$5p = 8 - 3p = 5$

$\therefore P = {}^{5}\!/_{5} = 1$

(3) The solution is $q = -1$ & $p = 1$

$15s + 2t = 7; 10s + \frac{t}{2} = 3$. Let

$15s + 2t = 7$ - - - - (i)

$10s + \frac{t}{2} = 3$ - - - - (ii)

Multiply (i) by 1 and (ii) by 4

$(i)\ x1 = 15s + 2t = 7$ - - - - (iii)

$(ii)\ x4 = 40s + 2t = 12$ - - - (iv)

Subtract (iii) from (iv)

$40s + 2t - 15s - 2t = 12 - 7$

$40s - 15s = 12 - 7$

$25s = 5$

$s = \frac{5}{25}$

$s = {}^{1}\!/_{5}$

Substitute ${}^{1}\!/_{5}$ for S in (i)

$15\ x\ \frac{1}{5} + 2t = 7$

$3 + 2t = 7$

$2t = 4$

$t = {}^{4}\!/_{2} = 2$

That is, $S = \frac{1}{5}$ and $t = 2$.

Substitution Method

In this method of solving simultaneous linear equations, you write one of the unknown in terms of the other unknown and substitute this expression into the second equation. Let us now solve the examples above using substitution method.

(1) $a + b = 7$ - - - - - - (i)

$a - b = 3$ - - - - - - (ii)

From equation (i)

$a = 7 - b$

Substitute $7 - b$ for a in equation 2.

$7 - b - b = 3$

$7 - 2b = 3$

$-2b = 3 - 7$

$-2b = -4$

$b = \frac{-4}{-2} = 2$

Substitute 2 for b in equation (i)

$a + 2 = 7$

$a = 7 - 2$

$a = 5$

That is $a = 5$ and $b = 2$

(2) $5p - 3q = 8$ - - - - - (i)

$2p - 5q = 7$ - - - - - (ii)

From equation (ii)

$p = \frac{7+5q}{2}$

Substitute for P in equation (i)

$5\left(\frac{7+5q}{2}\right) - 3q = 8$ - - - (iii)

Multiply equation (iii) by 2 to clear the fraction.

$5(7 + 5q) - 6q = 16$

$35 + 25q - 6q = 16$

$25q - 6q = 16 - 35$

$19q = -19$

$q = \frac{-19}{19} = -1$

Substitute – 1 for q in equation (i).

$5p - 3(-1) = 8$

$5p + 3 = 8$

$5p = 8 - 3$

$5p = 5$

$p = \frac{5}{5}$

$p = 1$

That is q = -1 and p = 1

(3) $15s + 2t = 7$ - - - - - *(i)*

$10s + \frac{t}{2} = 3$ - - - - - *(ii)*

Multiply equation (ii) by 2 to clear fraction. That gives: $20s + t = 6$ - - - *(iii)*

For equation (i)

$t = \frac{7-15s}{2} = 7$

Substitute $\frac{7-15s}{2}$ for t in - - - - - - *(iii)*

We have:

$20s + \frac{7-15s}{2} = 6$ - - - - - - - - *(iv)*

Multiply (iv) by 2 to clear the faction.

$40s + 7 - 15s = 12$

$40s - 15s = 12 - 7$

$25s = 5$

$s = \frac{5}{25} = 1/5$

Substituting $1/5$ for s in *(i) gives:*

$15 \times 1/5 + 2t = 7$

$3 + 2t = 7$

$2t = 4$

$t = \frac{4}{2} = 2$

That is $s = {}^1\!/\!_5$ & $t = 2$.

Self Assessment Exercise 3

Solve the following equations

(i) by elimination method

(ii) by substitution method

(1) $2x + 3y = 24; 3x + 2y = 26$

(2) $6a – 5b – 14 = 0; 3a + 2b – 16 = 0$

3.4 Simultaneous Linear and Quadratic Equations

It is impossible to have a simultaneous equations in which one of the equations is linear and the other quadratic. The equations can also be solved by adopting the methods discussed earlier in this unit. All you need to do is to study the question carefully to find out whether elimination or substitution method will be most appropriate at any point in time.

Examples

Solve the following equations.

(1) $3x^2 – 4y = -1; 2x – y = 1$

(2) $x^2 – y^2 = 27; x + y = 3$

(3) $4x^2 – y^2 = 15$

$2x – y = 5$

(4) $2x – 5y = 7$

$xy = 6$

Solution

1. $3x^2 – 4y = -1$ - - - - - (1)

$2x – y = 1$ - - - - - - (2)

From equation (2)

$y = 2x – 1$ - - - - - - (3)

Substituting for y in equation 1.

$3x^2 -4(2x – 1) = -1$

$3x^2 - 8x + 4 = -1$

That is

$3x^2 - 8x + 5 = 0$ – - - - - - (4)

Recall that $5 \times 3x^2 = 15x^2$

Two factors whose product is $+ 15x^2$ and sum is $-8x$ are $-5x$ and $-3x$ substitute $-5x$ and $-3x$ for $-8x$ in equation (4). That gives

$3x^2 - 5x - 3x + 5 = 0$

Factorise the expression on the left hand side.

$(3x^2 - 3x) + (5 - 5x) = 0$

That is:

$3x(x - 1) + 5(1 - x) = 0$

That is

$-3x(1 - x) + 5(1 - x) = 0$

$(1 - x)(5 - 3x) = 0$

either

$1 - x = 0$ or $5 - 3x = 0$

Then, $1 = x$ or $5 = 3x$

and $\frac{5}{3} = x$

That is

$x = 1$ or $5/3$

Substitute for x in equation 3.

When $x = 1$

$y = 2(1) - 1$

$y = 2 - 1$

$y = 1$

When x *is* $5/3$

$$y = 2\left(\frac{5}{3}\right) - = \frac{10}{3} - 1$$

$$y = 7/3$$

The solution of the simultaneous equations are:

$$(1,1), \left(5/3, 7/3\right)$$

In each bracket, the 1st number is the value of *x* while the other is the value of *y*.

2. $x^2 - y^2 = 27$ - - - - - - (1)

$x + y = 3$ - - - - - - - - (2)

The left hand side of equation 1 is a difference of two squares. Then,

$$(x + y)(x - y) = 27 ----(3)$$

From equation 2,

$x + y = 3$

Therefore,

$$3(x - y) = 27$$

$$x - y = 9 -----(4)$$

Add equation 2 and 4 to eliminate one of the unknown.

$x + y + x - y = 3 + 9$

$2x = 12$

$x = 6$

Substitute 6 for *x* in 2.

$6 + y = 3$

$y = 3 - 6$

$y = -3$

Solution = (6, 3)

3. $4x^2 - y^2 = 15$ - - - - - - (1)

$2x - y = 5$ - - - - - - - - (2)

The expression on the left hand side of equation 1, is a difference of two squares. So express each term as a square.

$(2x)^2 (y)^2 = 15.$ That is,

$$(2x + y)(2x - y) = 15 -----(3)$$

But $2x – y = 5$ from equation 2. Substitute 5 for $2x – y$ in equation 3.

$5(2x + y) = 15$

$10x + 5y = 15$ - - - - - - (4)

Multiply equation 2 by 5. Which gives

$10x – 5y = 25$ - - - - - - (5)

Subtract equation 4 from 5.

$10x – 5y – 10x – 5y = 25 – 15$

$-10y = 10$

$y = 10/-10 = -1$

Substitute -1 for y in equation 2.

$2x – -1 = 5$

$2x = 4$

$x = 4/2 = 2$

Solution

$x = 2, y = -1$ $(2,-1)$

(4) $2x – 5y = 7$ - - - - - (1)

$xy = 6$ - - - - - - - - - (2)

From 2,

$$y = \frac{6}{x}$$

Substitute for y in 1.

$$2x - 5\ x\ \frac{6}{x} = 7$$

Multiply through by x to clear the fraction. It gives

$2x^2 – 30 = 7x$

$2x^2 - 7x – 30 = 0$ - - - - - - (3)

$30 \text{ x } 20x^2 = 60x^2$

Two factors whose product is $60x^2$ and sum $– 7x$ are $– 12x$ and $+ 5x$

Substitute $-12x$ and $5x$ for $7x$ in equation 3. That gives

$2x^2 - 12x + 5x - 30 = 0$

$(2x^2 - 12x) + (5x - 30) = 0$

$2x(x - 6) + 5(x - 6) = 0$

$(x - 6)(2x + 5) = 0$

either

$x - 6 = 0$ or $2x + 5 = 0$

then, *if* $x - 6 = 0$ if $2x + 5 = 0$, then

$2x = -5$ and

$x = 6$ $x = \frac{-5}{2}$

That is, $x = 6$ or $\frac{-5}{2}$. Substitute for x in equation 2.

When $x = 6$

$$y = \frac{6}{6} = 1$$

When $x = \frac{-5}{2}$

$$y = 6\ x\left(\frac{-5}{2}\right) = 6\,x\,\frac{2}{5} = \frac{12}{-5}$$

Solution

$$(6,1),\ \left(\frac{-5}{2}, \frac{12}{5}\right).$$

Self Assessment Exercise 4

Solve the following equations

(1) $x^2 + y^2 = 34; x + y = 2$

(2) $x^2 - 4y^2 = 9; x + 2y = 1$

4.0 Conclusion

This unit x-rayed quadratic expressions, quadratic equations, simultaneous linear equations and simultaneous linear and quadratic equations. Different methods of factorizing quadratic expressions as well as methods of solving quadratic equations were discussed. Also, different methods of solving simultaneous linear equations and simultaneous linear and quadratic equations were considered.

5.0 Summary

The following are the highlights of the unit.

- A quadratic expression is one in the form $ax^2 + bx + c$, where *a, b* and *c* are constants and *a* is not equal to zero.
- A quadratic expression can be factorized by inspection, by grouping and by difference of two squares.
- The difference of two squares of two quantities is equal to the product of their sum and their difference.
- A simultaneous equation can be solved by elimination method or substitution method.
- Quadratic equation can be solved by factorization, by completing the squares and by formular which is not discussed in this unit.

6.0 Tutor Marked Assignment

1. Factorise the following

(a) $ab + bd^2 + b^2 c + bf$

(b) $12x^2y + 24x^3y^2 + 30x^4y$

(c) $3ac + 3ad + bc + bd$

2. Solve the following equation

$x^3 – 10x + 25 = 0$ by factorization

3. Solve the equation $U^2 – 14U – 3 = 0$ by completing the square

4. Solve the following pairs of simultaneous equations

(a) $2x – y = 8; 3x + y = 17$

(b) $\frac{5x}{8} - \frac{y}{20} = \frac{1}{4}; \frac{2x}{3} - \frac{3y}{3} = \frac{2}{15}$

(c) $4x^2 - 9y^2 = 19; 2x + 3y = 1$

(d) $3x + y = 25; xy = 8$

7.0 References/Further Readings

Channon, J.B. et al (2002). New General Mathematic for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

Galadima, I. (2004). Teaching Secondary School Algebraic World Problems: A Heuristic, Approach. Lagos: Biga education services.

Mathematical Association of Nigeria (2008). MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 2: UNIT 3

UNIT 3: Algebraic Graphs

Content

1.0 Introduction

As mentioned in unit 2, equations (linear, quadratic, simultaneous) can be solved using graphs. These graphs are called algebraic graphs. Solving equations graphically involves calculating a range of values of the unknown for which the equation is true or are true simultaneously. These points are then plotted on a Cartesian plane. Using a graph paper. The vertical and horizontal lines drawn on the plane are called axes, y is used to label the vertical axis while x is used to label the horizontal axis. This unit is designed to give an understanding of how to solve different types of equations using graphs.

2.0 Objectives

By the end of this unit, you should be able to pilot:

1. graphs of linear equations
2. graphs of quadratic equations
3. graphs of simultaneous equation

3.0 Main Body

3.1 Graphs of Linear Equations

Consider the equation,

$y - x = 3$

This is a linear equation. Note that the highest power of the unknown is one.

From that equation, we have:

$y = 3 - x$.

Pick a range of values of x and calculate the corresponding value of y.

Suppose $x = -1,0,1,2,3,4$.

When $x = (-1)$

$y = 3 - (-1)$

$y = 4$

When $x = 0$

$y = 3 - 0 = 3$

When $x = 1$

$Y = 3 - 1 = 2$

When $x = 2$

$y = 3 - 2 = 1$

When $x = 3$

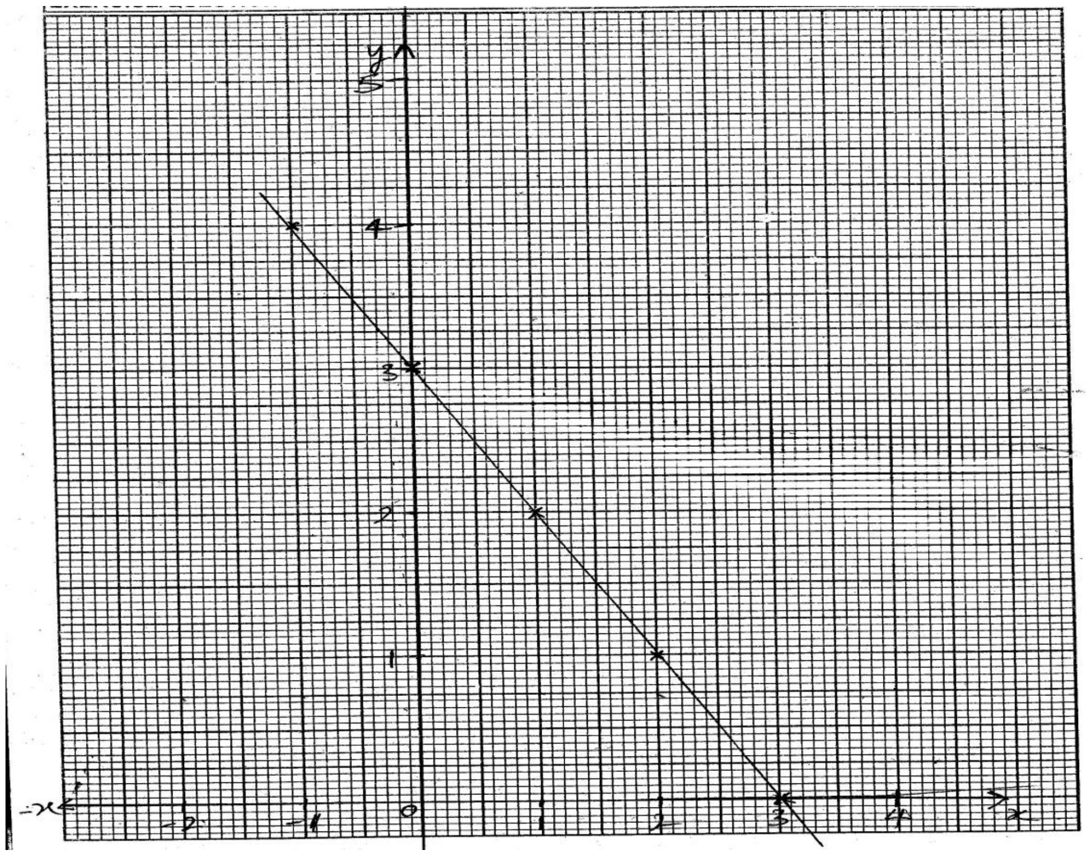
$y = 3 -3 = 0$

Representing this in a table gives

x	-1	0	1	2	3
y	4	3	2	1	0

The figure below is the graph of ($x + y = 3$)

The scale is 1cm to 1 unit. If scale is not given, you can choose your own scale.



Note that the graph is a straight line graph. Linear equations always give a straight line graph.

Consider another example.

Using a scale of 2cm to 1 unit on the *x* axis and 1cm to 1 unit on the y axis, plot the graph of

$y = 3x + 5$ with values of *x* ranging from -2 to +2.

Solution

$y = 3x + 5$. When $x = -2. y = 3(-2) + 5 = -6 + 5 = -1$

When $x = -1$

$y = 3(-1) + 5 = -3 + 5 = 2$

When $x = 0$

$y = 3(0) + 5 = 0 + 5 = 5$

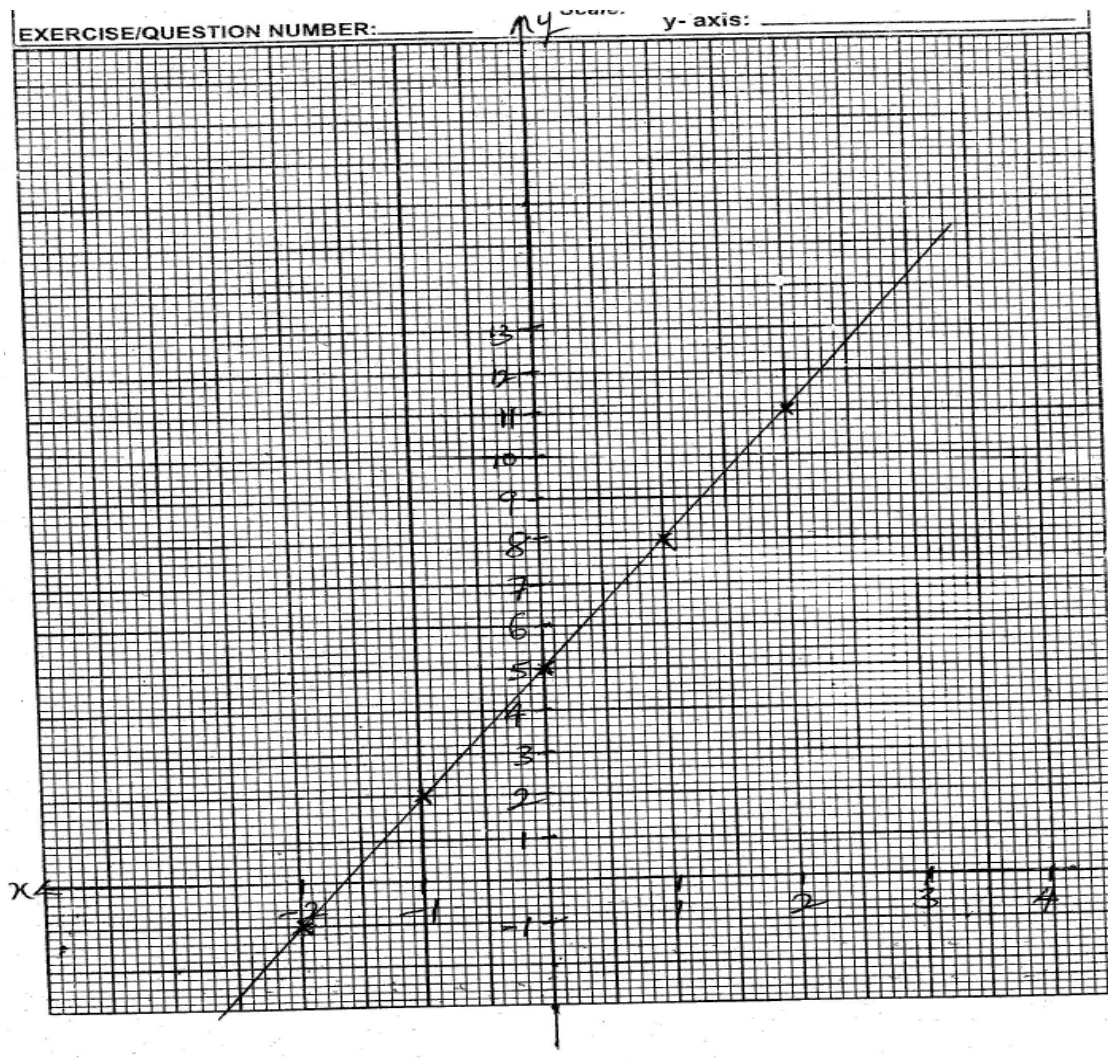
When $x = 1$

$y = 3(1) + 5 = 3 + 5 = 8$

When $x = 2$

$y = 3(2) + 5 = 6 + 5 = 11$

x	-2	-1	0	1	2
y	-1	2	5	8	11



The scale of the vertical and horizontal axes can be the same and they can also differ.

Self Assessment Exercise 1

Plot the graphs (1) $y = 2x + 5$ *for values of x ranging from 0 to 5* (2) $2x + y = 1$ *for values of x ranging from 8 to 8.*

3.2 Graphs of Quadratic Equation

Recall that in a quadratic equation, the highest power of the unknowns is two. Graphical solution of quadratic equations follows the same procedure as graphical solution of linear equations. A table of values of *x* and *y* is developed by calculating corresponding values of *y* for a range of values of *x*. Consider the following examples.

(1) Plot the graph of the quadratic equation:

$y = 2x^2 - 2$. That is, values of x from – 3 to 3

Solution

$y = 2x^2 - 2$

When $x = -3$

$y = 2(-3)^2 - 2 = 2(9) - 2 = 18 - 2 = 16$

When $x = -2$

$y = 2(-2)^2 -2 = 2(4) -2 = 8 - 2 = 6$

When x = 1

$y = 2(-1)^2 - 2 = 2(1) - 2 = 2 - 2 = 0$

When $x = 0$

$y = 2(0)^2 - 2 = 2(0) - 2 = 0 - 2 = -2$

When $x = 1$

$y = 2(1)^2 - 2 = 2(1) -2 = 2 - 2 = 0$

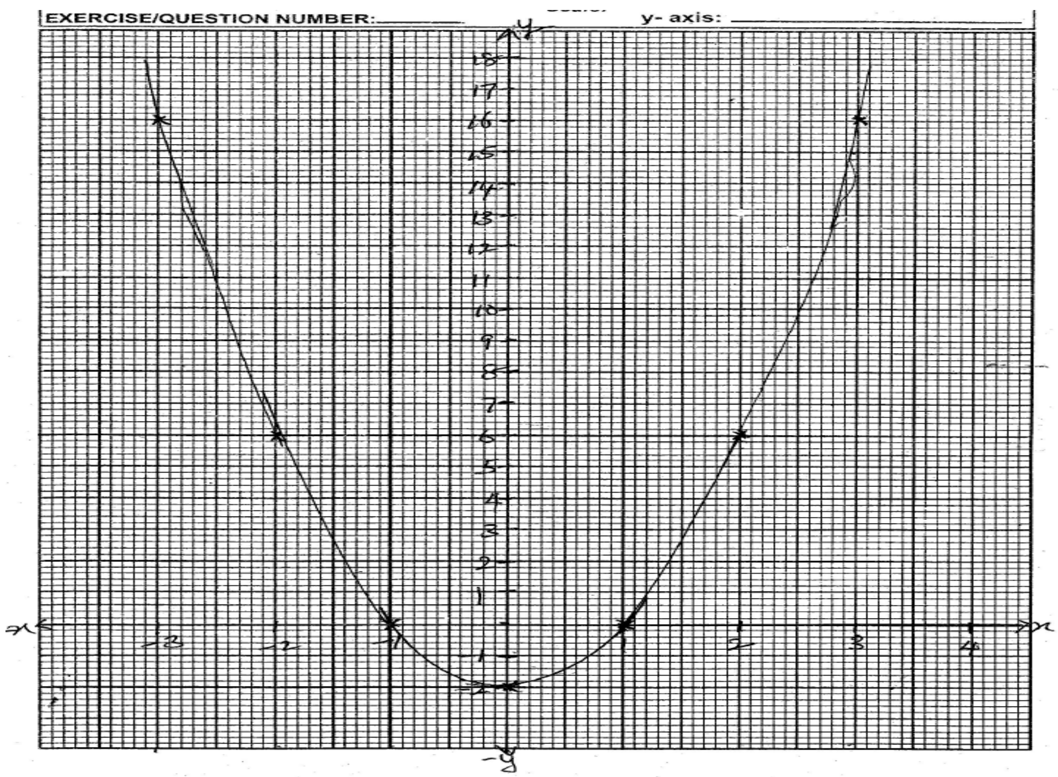
When $x = 2$

$y = 2(2)^2 - 2 = 2(4) -2 = 8 - 2 = 6$

When $x = 3$

$y = 2(3)^2 - 2 = 2(9) - 2 = 18 - 2 = 16$

x	-3	-2	-1	0	1	2	3
y	16	6	0	-2	0	6	16



Scale: 1cm to 2 units on y axis. 2cm to 1 unit on x axis. Note that the graphs cut the x = -1 and +1. the points x = -1, +1 are called the roots of the equation.

(2) Plot the graph of the following quadratic equation and find its' roots.

$y = x^2 – 3x – 2$ *for* $0 \leq x \leq 4$

When $x = 0$

$y = 0^2 – 3(0) – 2 = 0 – 0 – 2 = -2$

When $x = 1$

$y = (1)^2 – 3(1) – 2 = 1 – 3 - 2 = -1 – 5 = -4$

When $x = 2$

$y = 2^2 – 3(2) -2 = 4 – 6 – 2 = -4$

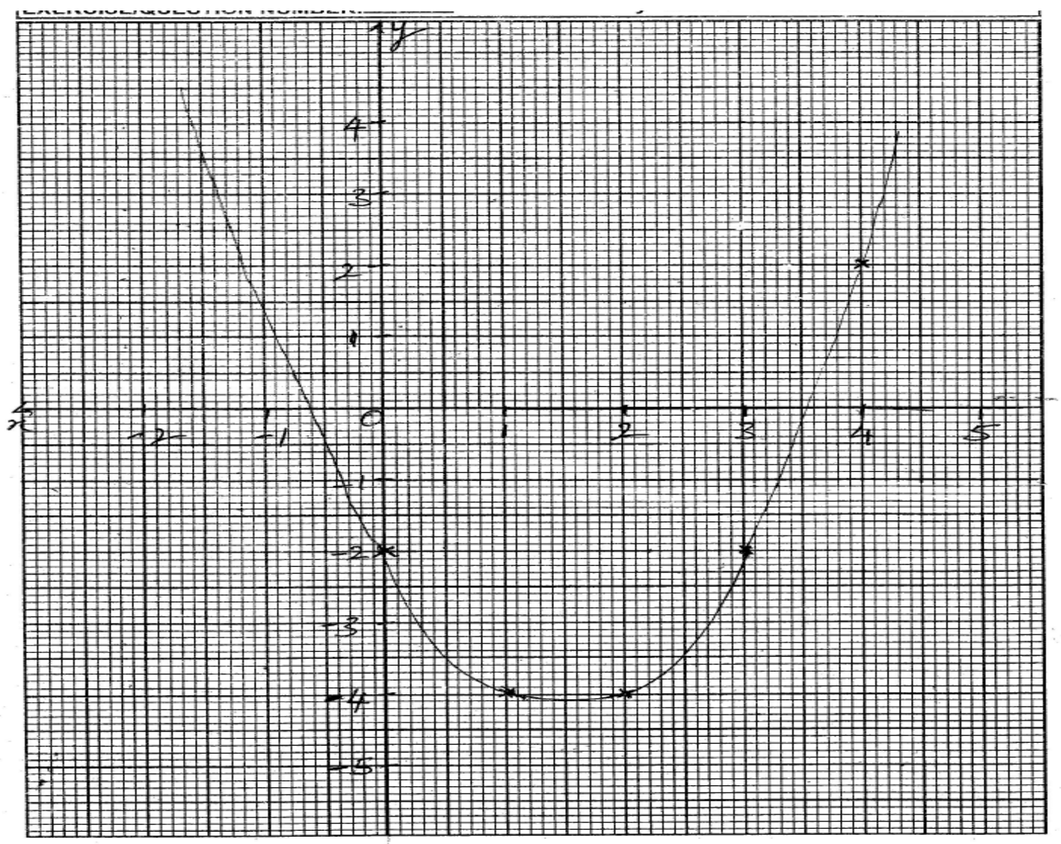
When $x = 3$

$y = 3^2 – 3(3) – 2 = 9 – 9 – 2 = -2$

When $x = 4$

$y = 4^2 - 3(4) - 2 = 16 - 12 - 2 = 2$

x	0	1	2	3	4
Y	-2	-4	-4	-2	2



The roots of the graph are the points at which the graph cuts the x axis;

Roots is $x = -0.6, 3.5$.

Note that the graph of a quadratic equation is always a curve. When the power of the unknown is positive, it will have a minimum value at the burning point. But it is negative the graph will have a maximum value at the burning point.

Self Assessment Exercise 2

Plot the graphs of the following quadratic equations and find their roots.

1. $y = x^2 - 4x - 2$ for $-3 \leq x \leq 3$.
2. $y = (x + 1)(x - 2)$ for $-2 \leq x \leq 3$

3.3 Graphs of Simultaneous Linear Equations

Simultaneous equations are a pair of equations having a range of values of x and y for which the equations are true at the same time. To solve simultaneous equations graphically, develop a table of values for each equation and plot the two sets of points on the same axes. Consider the following examples.

1. Plot the graph of the following simultaneous equations.

$x + y = 4$

$2x – y = 6$ for $-1 \leq x \leq 6$

Solution

Let $x + y = 4$ - - - - - - - (1) and

$2x – y = 6$ - - - - - - - - - -- (2)

From (1)

$x + -y = 4$

$y = 4 – x$

When $x = 1$

$y = 4 – -1 = 4 + 1 =5$

When $x = 0$

$y = 4 – 0 = 4$

When $x = 1$

$y = 4 – 1 = 3$

When $x = 2$

$y = 4 – 2 = 2$

When $x = 3$

$y = 4 – 3 = 1$

When $x = 4$

$y = 4 – 4 = 0$

When $x = 5$

$y = 4 – 5 = -1$

When $x = 6$

$y = 4 -6 = -2$

The values are represented in the table below.

x	-1	0	1	2	3	4	5	6
y	5	4	3	2	1	0	-1	-2

From (2)

$2x - y - 6$

$y = 2x - 6$

When $x = -1$

$y = 2(-1) - 6 = -2\ 6 = -8$

When $x = 0$

$y = 2(0) -6 = 0 - 6 = -6$

When $x = 1$

$y = 2(1) - 6 = 2 - 6 = -4$

When $x = 2$

$y = 2(2) - 6 = 4 - 6 = -2$

When $x = 3$

$y = 2(3) -6 = 6 -6 = 0$

When $x = 4$

$y = 2(4) - 6 = 8 - 6 = 2$

When $x = 5$

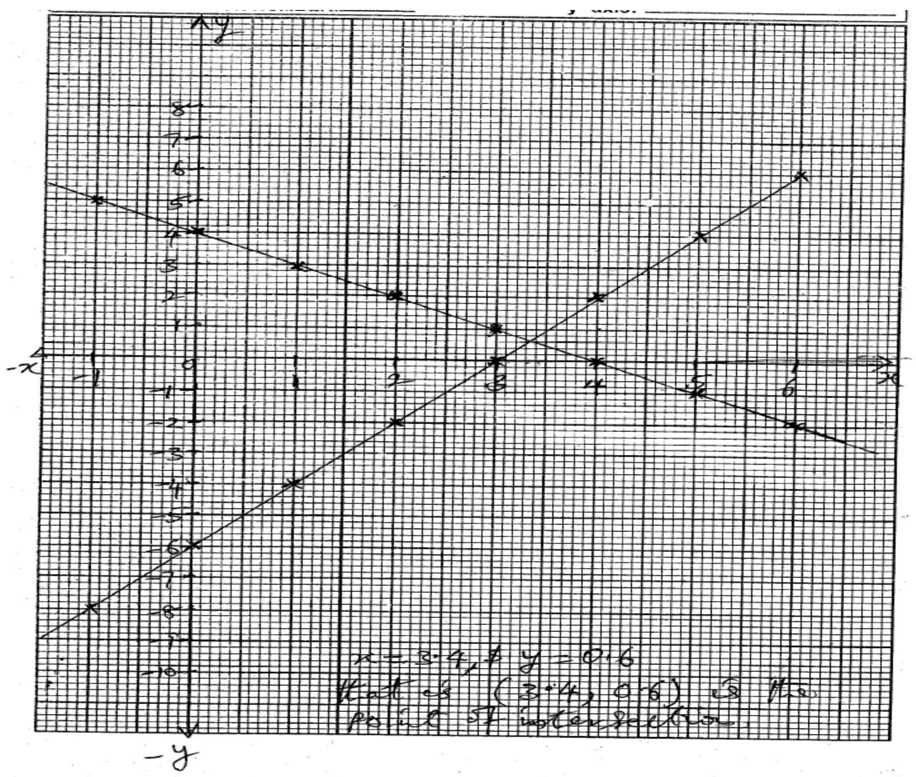
$y = 2(5) - 6 = 10 - 6 = 4$

When $x = 6$

$y = 2(6) -6 = 12 - 6 = 6$

x	-1	0	1	2	3	4	5	6
y	-8	-6	-4	-2	0	2	4	6

Plot the two sets of points on the same axis.



Intersection and it represent the solution of the simultaneous equations.

(2) Using graphical method, find the solution of the simultaneous equations.

$2x + 3y = 6$

$x + y = 5$ *for* $-2 \leq x \leq 5$

Solution

Let

$2x + 3y = 6$ - - - - - - - 1

$x + y = 5$ - - - - - - - - - 2

From (1)

$$y = \frac{6-2x}{3}$$

When $x = -2$

$$y = \frac{6-2(-2)}{3} = \frac{6+4}{3} = \frac{10}{3} = 3.3$$

When $x = -1$

$$y = \frac{6-2(1)}{3} = \frac{6+2}{3} = \frac{8}{3} = 2.7$$

When $x = 0$

$$y = \frac{6-2(0)}{3} = \frac{6+0}{3} = \frac{6}{3} = 2$$

When $x = 1$

$$y = \frac{6-2(1)}{3} = \frac{6-2}{3} = \frac{4}{3} = 1.3$$

When $x = 2$

$$y = \frac{6-2(2}{3} = \frac{6-4}{3} = \frac{2}{3} = 0.7$$

When $x = 3$

$$y = \frac{6-2(3)}{3} = \frac{6-6}{3} = \frac{0}{3} = 0$$

When $x = 4$

$$y = \frac{6-2(4)}{3} = \frac{6-8}{3} - \frac{2}{3} = -0.7$$

When $x = 5$

$$y = \frac{6-2(5)}{3} = \frac{6-10}{3} = \frac{-4}{3} = -1.3$$

When $x = 6$

$$y = \frac{6-2(6)}{3} = \frac{6-12}{3} = \frac{-6}{3} = -2$$

x	-2	-1	0	1	2	3	4	5	6
y	3.3	2.7	2	1.3	0.7	0	-0.7	-1.3	-2

From 2.

$y = 5 – x$

When $x = -2$

$y = 5 – -2 = 5 + 2 = 7$

When $x = -1$

$y = 5 - -1 = 5 + 1 = 6$

When $x = 0$

$y = 5 - 0 = 5$

When $x = 1$

$y = 5 - 1 = 4$

When $x = 2$

$y = 5 - 2 = 3$

When $x = 3$

$y = 5 - 3 = 2$

When $x = 4$

$y = 5 - 4 = 1$

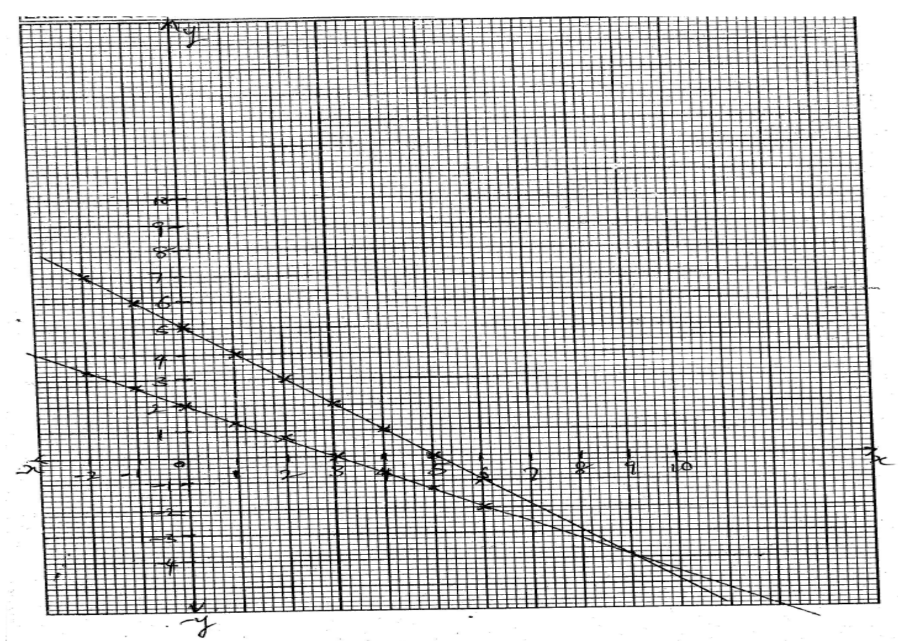
When $x = 5$

$y = 5 - 5 = 0$

When $x = 6$

$y = 5 - 6 = -1$

x	-2	-1	0	1	2	3	4	5	6
y	7	6	5	4	3	2	1	0	-1



Scale: 1cm represent 1 unit on both axis

Solution: $y = 9$ & $y = -4$

Self Assessment Exercise 3

Using graphical method, find the solution of the following simultaneous equations.

$3x + y = 1$

$2x – y = 4$ for $-3 \leq x \leq 5$

3.4 Graphs of Simultaneous Equations, One Linear And One Quadratic

Recall that the graph of a linear equation is always a straight line, while the graph of a quadratic equation is always a curve. To solve linear quadratic simultaneous equations, follow the procedure described in section 3.3. The result will be a curve and a straight line graph on the same axes with one or more points of intersection.

Examples

1. Solve graphically the pair of equations.

$y = 2x$

$y = 3x^2 – 5x + 3$ for $-2 \leq x \leq 4$

Solution

Let $y = 2x$ - - - - - - 1 & $y = 3x^2 – 5x + -3$ - - - - - - 2

From (1)

$y = 2x$

when $x = -2$

$y = 2(-2) = -4$

When $x = -1$

$y = 2(-1) = -2$

When $x = 0$

$y = 2(0) = 0$

When $x = 1$

$y = 2(1) = 2$

When $x = 2$

$y = 2(2) = 4$

When $x = 3$

$y = 2(3) = 6$

When $x = 4$

$y = 2(4) \quad = \quad 8$

x	-2	-1	0	1	2	3	4
Y	-4	-2	0	2	4	6	8

From (2)

$y = 3x^2 – 5x + 3$

When $x = -2$

$y = 3(-2)^2 – 5(-2) + 3 = 12 + 10 + 3 = 25$

When $x = -1$

$y = 3(-1)^2 -5(-1) + 3 = 3 + 5 + 3 = 11$

When $x = 0$

$y = 3(0)^2 – 5(0) + 3 = 0 – 0 + 3 = 3$

When $x = 1$

$y = 3(1)^2 -5(1) + 3 = 3 – 5 + 3 = 1$

When $x = 2$

$y = 3(2)^2 – 5(2) + 3 = 12 – 10 + 3 = 5$

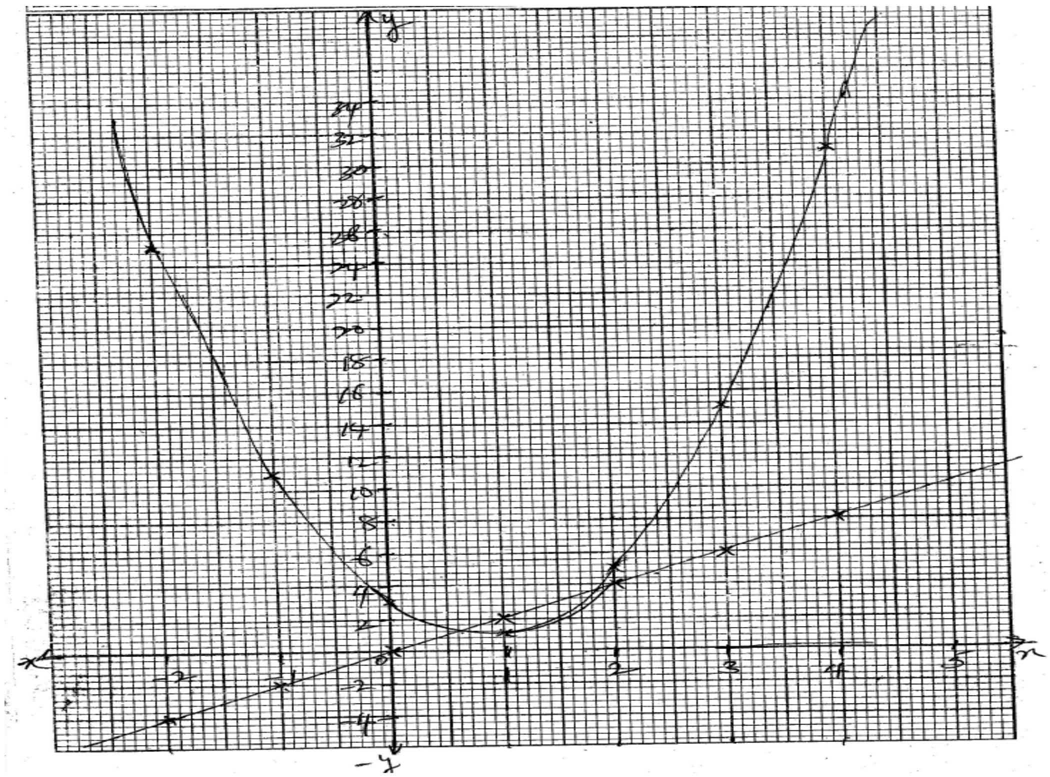
When $x = 3$

$y = 3(3)^2 – 5(3) + 3 = 27 + 15 + 3 = 15$

When $x = 4$

$Y = 3(4)^2 – 5(4) + 3 = 48 + 20 + 3 = 31$

x	-2	-1	0	1	2	3	4
y	25	11	3	1	5	15	31



Scale: 1cm represent 5 units on y axis 1cm represent 1 unit on x axis.

The graphs meets at the points $x = 0.6, y = 1$ and $x = 1.9$ and $y = 3.5$

These are the solutions of the equations.

2. Solve graphically the pair of equations

$y = 2x^2 - 3x - 6$ and

$y = 1 - 3x$ for $-3 \leq x \leq 4$

Hence find the solution of the equation

$2x^2 - 7 = 0$

(b) Draw a tangent to the cure at $x = -1$, and find the gradient of the tangent at that point.

Solution

Let

$y = 2x^2 - 3x - 6$ - - - - - - - *(1)* and

$y = 1\text{-}3x$ – - - - - - - - - - - *(2)*

From (1)

When $x = -3$

$y = 2(-3)^2 – 3(-3) – 6 = 18 + 9 – 6 = 21$

When $x = - 2$

$y = 2(-2)^2 – 3(-2) – 6 = 8 + 6 – 6 = 8$

When $x = -1$

$y = 2 (-1)^2 – 3 (-1) – 6$

$y = 2 + 3 -6 = -1$

When $x = 0$

$y = 2(-0)^2 – 3(-0) – 6 = 0 – 0 – 6 = -6$

When $x = 1$

$y = 2(1)^2 – 3(1) – 6 = 2 – 3 – 6 = - 7$

When $x = 2$

$y = 2(2)^2 – 3(2) – 6 = 8 – 6 – 6 = - 4$

When $x = 3$

$y = 2(3)^2 – 3(3) – 6 = 18 – 9 – 6 = -3$

When $x = 4$

$y = 2(4)^2 – 3(4) – 6 = 32 – 12 – 6 = 14$

x	-3	-2	-1	0	1	2	3	4
y	21	8	-1	-6	-7	4	-3	14

From (2)

When $x = -3$

$y = 1 – 3(-3) = 1 + 9 = 10$

When $x = -2$

$y = 1-3(-2) = 1 + 6 = 7$

When $x = -1$

$y = 1 – 3 (-1) = 1 + 3 = 4$

When $x = 0$

$y = 1 – 3(0) = 1 - 0 = 1$

When $x = 1$

$y = 1 – 3(1) = 1 – 3 = -2$

When $x = 2$

$y = 1 – 3(2) = 1 – 6 = -5$

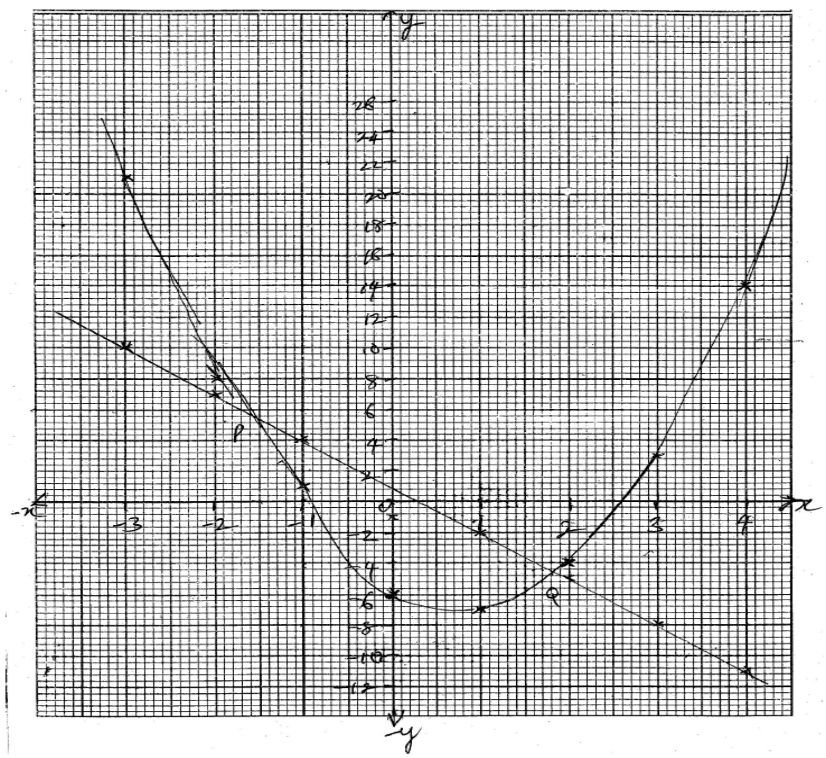
When $x = 3$

$y = 1 -3(3) = 1 – 9 = -8$

When $x = 4$

$y = 1 - 3(4) = 1 - 12 = -11$

x	-3	-2	-1	0	1	2	3	4
y	10	7	4	1	2	-5	-8	-11



Equate equations 1 and 2. That is

$2x^2 - 3x - 6 = 1 - 3x$. Then

$2x^2 - 3x + 3x - 6 - 1 = 0$

$2x^2 - 7 = 0$

The solution of $2x^2 - 7 = 0$ is given by points P and q on the graph.

P = (1.8, 4.5) & q = (-1.9, 7).

b. Gradient (AB) = $\dfrac{AC}{CB} = 8\,\text{cm}$

Self Assessment Exercise 4

On the same axis, draw the graph of $y + 2 = x(x+1)$ and $y = 2x$ for $-3 \leq x \leq 3$

(b) From your graph, find the solution of the equation $x^2 + x - 2 = 0$

4.0 Conclusion

In this unit, graphical solutions of different types of equations were discussed. This includes graphical solution of linear equations, quadratic equations, simultaneous linear equations and simultaneous equations, are linear are quadratic. Solution of gradient to a curve at a point was also discussed.

5.0 Summary

The following are the highlights of the unit.

- A linear equation always give a straight line graph
- The graph of a quadratic equation is always a curve
- The scale on the x and y axes may or may not be the same
- In a quadratic graph, the points at which the curve cuts the x axis are called the roots of the equation.
- The points of intersection of the graphs of simultaneous equations represents the solution of the equations.

A quadratic graph may have a maximum value or a minimum value at the turning point.

6.0 Tutor Marked Assignments

(1) Plot the graphs of the following equations.

(i) $y = 3x + 5$ for $-2 \leq x \leq 4$

(ii) $y = 2x^2 + 3x - 6$ for $-2 \leq x \leq 2$

(2) Using graphical method, find the solution of the following simultaneous equations.

$2x - y = 4$ and $3x - 2y = 6$

for $-3 \leq x \leq 6$

(3) Solve graphically, the pair of equations.

$y = 2 + 3x - x^2$ and $y = 3x + 2$

for $-4 \leq x \leq 4$

hence find the solution of the equation

$4 - x^2 = 0$

(b) Draw a tangent to the curve at $x = 4$, and find the gradient of the tangent at that point.

7.0 Reference/Further Reading

Mathematical Association of Nigeria (2008). MAN Mathematics for Senior Secondary Schools Books 3 (3rd ed). Ibadan: University Press Plc.

MODULE 2: UNIT 4

Unit 4: Change of Subject of Formulae

Content

1.0 Introduction

In this unit, you will be exposed to the topic, change of subject of formula. The process involved in changing the subject of a formula will be explicitly discussed. You are therefore encouraged to read through the unit carefully and get acquainted with the facts.

2.0 Objectives

At the end of this unit, you should be able to:

1. explain the meaning of subject of formulae
2. identify the subject of a formula
3. change the subject of a formula
4. substitute in formula to find the numerical value of a named variable

3.0 Main body

3.1 Subject of Formulae

A formula is an equation which contains two or more variables. The equation describes the relationship between the different variables. For example, the equation

$V = U + at$

is a formula which represents the velocity of a moving object where

V = Final Velocity

U = Initial Velocity

a = acceleration

t = time taken

Also, the equation

$$I = \frac{PTR}{100}$$

is the formula for simple interest

I = simple interest

P = Principle

T = Time

R = Rate

The first variable of the formula, that is, the variable which is expressed in terms of the other variables, is called the subject of the formula. In the examples above, V and I are the subjects of the first and second formulae respectively. In the same vain the formulae $y = MC^2$ *and* $2x^2 + 1 = y$ have *y* as subject in each case.

Self Assessment Exercise 1

Identify the subject in each of the following

1. $S = Ut + \frac{1}{2}at^2$ (2) $F = Ma$ (3) $\frac{2a+11}{2} = y$ (4) $\frac{1}{S} = \frac{1}{v} + \frac{1}{u}$

3.2 Changing Subject of Formulae

In a formula there are many variables. the variable which is expected in terms of the others is called, the subject of the formula. The subject of a formula can be changed by rearranging the formula to express another variable in term of others. The procedure is like that of solving equation, solving for the variable to be made the subject of the formula as solution of the equation. For example.

$V = U + at$

Subtract at from both sides.

$V - at = U$ *or* $U = V - at$

The subject of the formula $V = U + at$ has been changed from *V* to *U*

a or *t* can also be made the subject of the formula. Consider the following steps.

$V = U + at$

Subtract U from both sides.

$V - U = U + at - U$

Simplifying, we have

$V - U = at$

Divide both sides by t

$$\frac{V - U}{t} = \frac{at}{t}$$

This gives

$$\frac{V - U}{t} = a \; or \; a = \frac{V - U}{t}$$

Again,

$V = U + at$

Subtract U from both sides. It gives

$V - U = at$

Divide both sides by a. This gives

$$\frac{V - U}{a} = t \; or \; t = \frac{V - U}{a}$$

A question that is likely to be on your mind at this junction is how do I know the arithmetic operation to carry out? How do I know when to add, subtract, divide or multiply? The guiding rules are as follows.

The subject of a formula must stand alone on its side of the equation. Therefore, operation that will result in removing all the other variables from that side of the formula is what must be carried out.

Consider the following examples.

(1) Make x the subject of each of the following

(i) $n = 4ax$ (ii) $P = \frac{x}{2}$ (iii) $y = 3x + 5$ (iv) $y = \frac{1}{3}x^2a$

Solution

(i) $n = 4ax$

Divide both sides by $4a$

$$\frac{n}{4a} = \frac{4ax}{4a}$$

$$\frac{n}{4a} = x \text{ or } x = \frac{n}{4a}$$

(ii) $P = \frac{x}{2}$

Multiply both sides by 2

$$P \; x \; 2 = \frac{x}{2} \, x 2$$

2P = *x* or *x* = *2P*

(iii) *y* = *3x* + *5*

Subtract 5 from both sides

y – *5* = *3x* + *5* – *5*

y – *5* = *3x*

Divide both sides by 3

$$\frac{y-5}{3} = \frac{3x}{3}$$

$$\frac{y-5}{3} = x \text{ or } x = \frac{y-5}{3}$$

(iv) $y = \frac{1}{3} x^2 a$

Multiply both sides by 3

$$3 \; x \; y = \frac{1}{3} x^2 a \; x \; 3$$

$3y = x^2a$

Divide both sides by *a*

$$\frac{3y}{a} = \frac{x^2 a}{a}$$

$$\frac{3y}{a} = x^2$$

Take the square roof of both sides

$$\sqrt{\frac{3y}{a}} = \sqrt{x^2}$$

$$\sqrt{\frac{3y}{a}} = x \, or \; x = \sqrt{\frac{3y}{a}}$$

2. Give that $P - q = \sqrt{\left(\frac{qb}{a} + q^2\right)}$

Make *a* the subject of the formula.

Solution

$$P - q = \sqrt{\left(\frac{qb}{a} + q^2\right)}$$

Square both sides

$$(P-q)^2 = \left(\sqrt{\frac{qb}{a} + q^2}\right)^2$$

$$(P-q)^2 = \frac{qb}{a} + q^2$$

Multiply through by *a*

$$a(P-q)^2 - qb = q^2$$

Collect like terms.

$$a(P-q)^2 = qb^2 + aq^2$$

Factorise the left hand side

$$a\left[(P-q)^2 - q^2\right] = qb$$

Divide both sides by the coefficient of *a*

$$a = \frac{qb}{(P-q)^2 - q^2}$$

$$a = \frac{qb}{P^2 - 2pq + q^2 - q^2}$$

$$a = \frac{qb}{P^2 - 2pq}$$

$$a = \frac{qb}{P \; (P - 2pq)}$$

Self Assessment Exercise 2

(1) Make R the subject of the formula $I = \frac{PRT}{100}$

(2) Given that $y = \frac{1}{2}$ Mx. Make M the subject of the formula.

3.3 Substitution in Formula

It is important for you to be able to find the numerical value of a variable in a formula when the values of the other variables are given. The process is called substitution. It involves replacing the variables with the values given, then simplifying to get the numerical value of the variable whose value was not given. Consider the following examples.

(1) Given that $S = 5t + 3t^2$. Find S when t = 2, 4 and 6.

Solution

$S = 5t + 3t^2$

When $t = 2$

$S = 5(2) + 3(2)^2$

$S = 10 + 3(4)$

$S = 10 + 12$

$S = 22$

When $t = 4$

$S = 5(4) + 3(4)^2$

$S = 20 + 3(16)$

$S = 20 + 48$

$S = 68$

When $t = 6$

$S = 5(6) + 3(6)^2$

$S = 30 + 3(36)$

$S = 20 + 108$

$S = 138$

(2) Given that $V = 9x + 5$. Make a table of values of y when $x = 2, -1, 0, 1, 2,3,4.$

Solution

$y = 9x + 5$

When $x = -2$

$y = 9(-2) + 5 = -18 + 5 = -13$

When $x = -1$

$y = =9(-1) + 5 = 9 + 5 = -4$

When $x = 0$

$y = 9(0) + 5 = 0 + 5 = 5$

When $x = 1$

$y = 9(1) + 5 = 9 + 5 = 14$

When $x = 2$

$y = 9(2) + 5 = 18 + 5 = 23$

When $x = 3$

$y = 9(3) + 5 = 27 + 5 = 32$

When $x = 4$

$y = 9(4) + 5 = 36 + 5 = 41$

These values can be presented in a table as shown below.

x	-2	-1	0	1	2	3	4
y	-13	-4	5	14	23	32	41

(3) Given that $A = 5 + r^2$. Find the value of r when $A = 30$

Solution

$A = 5 + r^2$

When $A = 30$

$30 = 5 + r^2$

$30 – 5 = r^2$

$25 = r^2$

Take the square root of both sides.

$\sqrt{25} = \sqrt{r^2}$

$5 = r$ or $r = 5$

4. Given that $S = \sqrt{t-4},$ find the value of t when $S = 2$

Solution

$Z = \sqrt{t-4}$

When $S = 2$

$S = \sqrt{t-4}$

Square both sides

$2^2 = \left(\sqrt{t-4}\right)$

$4 = t - 4$

$4 + 4 = t$

$8 = t$

Self Assessment Exercise 3

1. Given that $y = 4x^2 + 1$, find the value of y when $x = 3$ and the value of x when $y = 195.$
2. If $2S^2 = 50t$ find the value of S when $t = 1$

4.0 Conclusion

This unit discussed change of subject of formulae. The meaning of subject of formulae as well procedure of changing the subject of a formula and substituting in a formula were explained.

5.0 Summary

The main points in this unit are summarized below.

1. A formula is an equation which contains two or more variables.
2. A formula describes the relationship between variables.
3. The subject of a formula is the variable that stands alone being expressed in terms of other variables.
4. The subject of a formula can be change by rearranging the formula.
5. The process of changing the subject of a formula is the same as solving for an unknown in an equation.
6. The process of finding the numerical value of a variable in a formula, when the values of the other variables are given is called substitution.

6.0 Tutor Marked Assignment

1. Identify the subject of the formula is each of the following.

(a) $2x + 6 = y$ (b) $F(x) = 3x^3 + 2x - 5x - 3$

2. In each of the following, make x the subject of the formula.

(a) $5x^2 + 4y - 3 = 0$ (b) $5x + 5y + 10 = -5$

3. Given that $A = \frac{1}{3}h(x + y)$. Express y in terms of A, h and x

4. Find the value of the following when $y = -4$

(a) $2y$ **(**b) $3y^2$ (c) $y - 4$ (d) $\sqrt{32 - y}$

7.0 References/Further Reading

Galadima, I. (2004). Teaching Secondary School Algebraic World Problem: A Heuristic Approach. Lagos: Biga Educational Services.

Channon, J.B. et al (2002). New General Mathematics for Senior Secondary Schools. United Kingdom: London Group (FE) Ltd.

Mathematical Association of Nigeria (2008). MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 2: UNIT 5

Unit 5: Linear Inequalities

Content

1.0 Introduction

This unit is designed to given an understanding of the topic linear inequalities. The form of linear inequalities is similar to that of linear equations except that instead of the symbol = (equality) in linear equations, some other symbols are used in linear inequalities. These symbols and their meanings includes the following:

> is greater than

< is less than

≥ is greater than or equal to

≤ is less than or equal to

≠ is not equal to

Consider a class in a school having *x* number of students. If *x* is equal to 50, then, it can be expressed as $x = 50$.

This is an equation. But if there are less than 50 students in the class, then, $x < 50$. This is an inequality. The procedure for solving inequality is the same as that for solving linear equations except that when an inequality is divided or multiplied by a negative number, the inequality sign is reversed.

2.0 Objectives

At the end of this unit, you should be able to;

1. Solve inequalities in one variable
2. Solve combined inequalities

3.0 Main Body

3.1 Inequalities in one Variable

Inequality is said to be in one variable if there is only one unknown in the inequality. That, is there is only one variable to be solved for.

Solve the following inequalities

(1) $5 - 2y < y - 4$ (2) $3n + 3 > 9$

Solution

1. $5 - 2y < y - 4$

Collect like terms

$5 + 4 < y + 2y$

$9 < 3y$

Divided both sides by 3

$3 < y$ or $y > 3$

2. $3n + 3 > 9$

Collect like terms

$3n > 9 - 3$

$3n > 6$

Divide both side by 3.

$n > 2$

The answer of an inequality can be represented on a number line. Consider the following examples.

3. $3x - 8 \leq 5x$

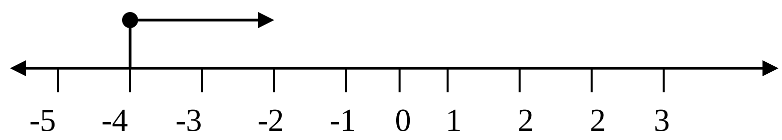
Collect like terms

$3x - 5x \leq 8$

$-2x \leq 8$

Divide both sides by -2

$x \geq -4$



The shaded circle (•) implies that the value – *4* is included in the range of values of *x*

4. $\frac{5x-1}{3}-\frac{1-2x}{5}<8+x$

Simplify the left hand side

$\frac{5(5x-1)-3(1-2x)}{15}<8+x$

$\frac{25x-5-3+6x}{15}<8+x$

$\frac{31x-8}{15}<8+x$

Multiply both sides by 15

$31x - 8 < 120 + 15x$

Collect like terms

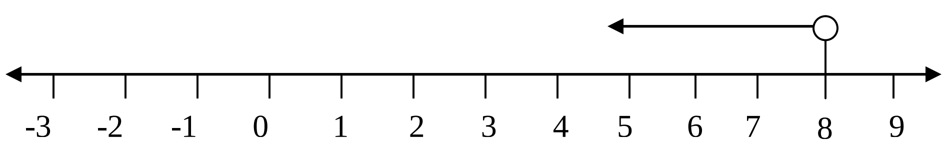
$31x - 15x < 120 + 8$

$16x < 128$

Divide both side by 16

$x<\frac{128}{16}$

$x < 8$



The unshared circle (0) implies that the value 8 in not included in the range of values of *x.*

Self Assessment Exercise 1

Solve the following in equalities

(1) $y \geq 3 - 7y$ (2) $-2(y-3) > -3(y+2)$

3.2 Combined Inequalities

It is also possible to solve a pair of inequalities simultaneously. Consider the following examples.

(1) Solve simultaneously the inequalities.

$3 + x \leq 5$ and $8 + x \geq 5$

Solution

$3 + x \leq 5$ *and* $8 + x \geq 5$

Collect like terms Collect like terms

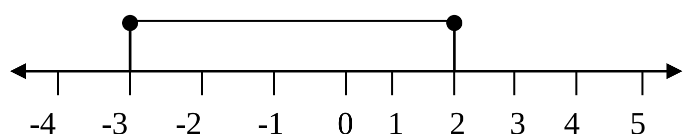
$x \leq 5 - 3$ $x \geq 5 - 8$

$x \leq 2$ $x \geq -3$

Combining the results, we have

$-3 \leq x \leq 2$

This answer can be represented on a number line as follows.



2. What is the range of values of y for which $2y - 1 > 3$ and $y - 3 < 5$ are simultaneously true?

Solution

$2y - 1 > 3$ *and* $y - 3 < 5$

Collect like terms Collect like terms

$2y > 3 + 1$ $y < 5 + 3$

$2y > 4$ $y < 8$

Divide both sides by 2

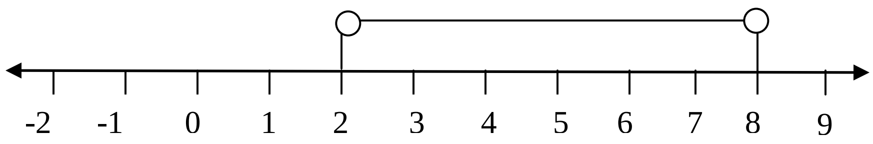
$$y > \frac{4}{2}$$

$y > 2$

Combing the results gives

$2 < y < 8$

This solution can also be represented on a number line as follows.



Self Assessment Exercise 2

1. Draw number line to show the following.

   (a) $0 < x < 5$ (b) $-3 \leq n < 2$

2. If $4x - 7 \leq 3x$ and $3x \leq 5x\ 8$, for what range of values of x are both inequalities simultaneously true?

4.0 Conclusion

This unit considered inequalities in one variable. The meaning of inequality and the symbols involved in inequalities were explained. The unit also considered solution of linear inequalities in one variable as well as solution of combined inequalities.

5.0 Summary

The following are the main points in the unit.

1. $>$ means, is greater than
2. $<$ means, is less than
3. $\geq$ means, is greater than or equal to
4. $\leq$ means, is less than or equal to
5. $\neq$ means is not equal to
6. Solution of linear inequalities adopt the same procedure as solution of linear equations.
7. When an inequality is multiplied or divided by a negative number the inequality sign is reversed.
8. To find solution of a pair of inequalities, they are solved separately and the answers are combined.

6.0 Tutor Marked Assignment

1 Illustrate the following inequalities in a number line.

(a) $x < 2$ (b) $x < 5$ (c) $x \leq -4$ (d) $x \geq 2$ (e) $-2 \leq x < 5$ (f) $0 < x \leq 4$

(g) $0 \leq x \leq$ (h) $-3 < x < 3$

2. If y is positive, for what range of values of y is $6 + 2y \leq 12$ true?

3. Solve simultaneously

   $6n < 27 - 3n$ and $n - 8 < 3n$

4. Given that m is an integer, find the three highest value of m in each case

   (a) $2m + 5 < 16$ (b) $10 - 3m > 8$

5. Given that s is an integer, find the three lowest values of s in each case.

   (a) $18 + s > 1$ (b) $7 - s < 9s$

7.0 Reference/Further Readings

Mame, H.F.; Kalejaiye, A.O.; Chima, Z.I.; Garba, G.U. and Ademosu, M.O. (2006). New General Mathematics for Senior Secondary School 2. (3rd ed). Malaysia: Congman Publishers.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 3: Geometrical Solids and Shapes

Unit 1: Triangles

Unit 2: Circle

Unit 3: Area and Volume of various Geometrical Solids and Shapes

Content

1.0 Introduction

A triangle is a three sided plane shape with three angles. They are of various types. It can be identify by the size of the lines and their angles.

2.0 Objectives

At the end of this unit, you should be able to:

1. Identify by name the various typed of triangles
2. Identify the general properties of triangles and specific properties of each type.
3. Solve problems on any basic type of triangles.
4. Know some of the theorems of triangles.

3.0 Main Body

3.1 Triangles

Definition: A triangle is a plane shape, bounded by three straight lines.

Types

(1) Scalene triangle

(2) Isosceles triangle

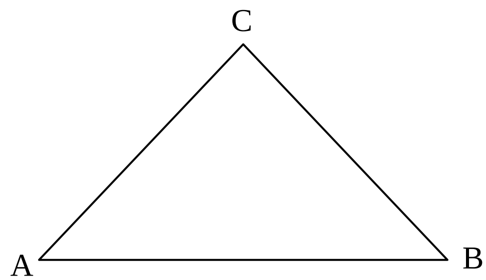
(3) Equilateral triangle

(4) An acute-angled triangle

(5) A right-angled triangle

(6) An isosceles right angled triangle

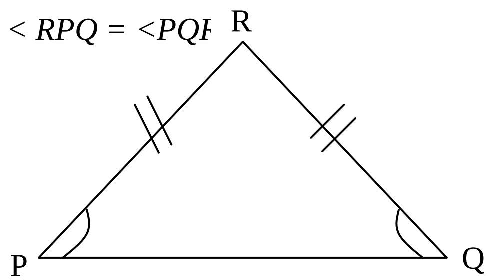
(1) ***Scale Triangle:*** This is a triangle in which no sides are equal and angles are also equal.



Scalene Triangle

(2) ***Isosceles Triangle:*** This is a triangle in which two of the three sides are equal. The two angles facing the equal sides (i.e. angels at the base) are also equal.

/PQ/=/QR/ & < RPQ = <PQR

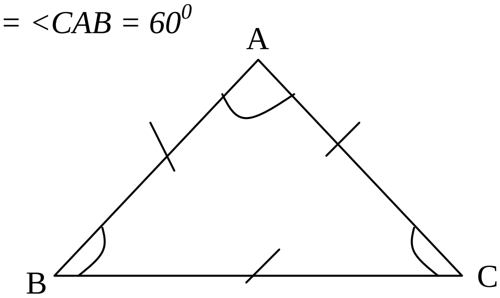


PQ is called the base of isosceles triangle RPQ

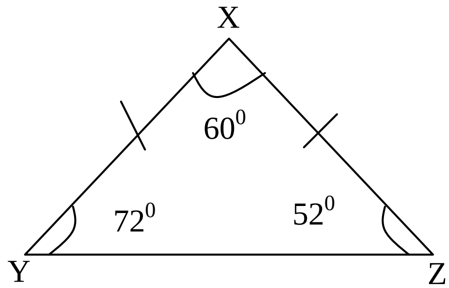
(3) ***Equilateral Triangle:*** In this type of triangle all three side are equal and the three angels are also equal (i.e 60^0).

/AB/=/BC/=/CA/

$<ABC = <BCA = <CAB = 60^0$

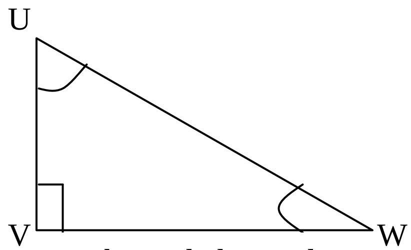


4. ***An acute-angled Triangle***: In this triangle the three angles are acute angles (all three are angles less than 90^0).



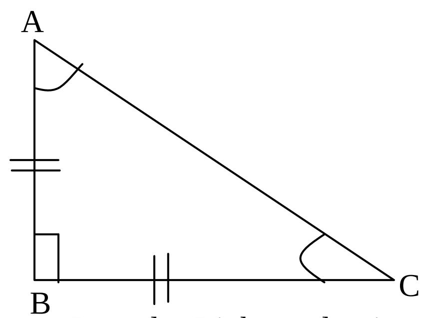
5. ***A right-angle Triangle:*** This is a triangle in which one of the angles is a right-angle (i.e 90^0).

<UVW = 90⁰



Right-angled triangle

6. ***An Isosceles Right-angled Triangle**:* This is a triangle in which the two sides that contain the right-angle are equal. Since the two sides are equal, the two angle are also equal.



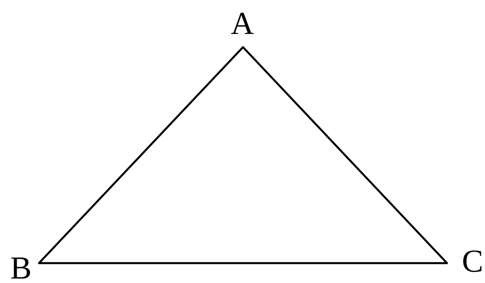
Isosceles Right-angle triangle

3.2 Properties of Triangle

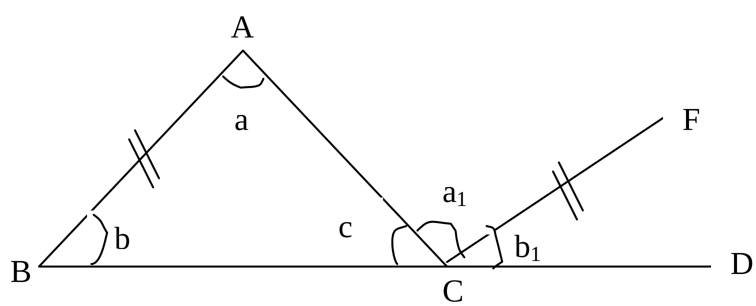
Theorem 1: The exteriors angle of a triangle is equal to the sum of two interior opposite angle.

Given: ΔABC

Prove: that *<ABC + <BAC = <ACD*



Construction: Through point C draw a line parallel to line AB (or AB) produce line BC to point D.



Proof: From the above diagram

$a = a_1$ (alternative angles AB//CF)

$b = b_1$ (corresponding angles AB//CF)

$\therefore\ a + b = a_1 + b_1$

<ACD = <ABC + <BAC

Theorem 2

The sum of the angles of a triangle is two right angles i.e. 180^0

Proof. Using the above diagram and from the proof above.

We have shown that $\therefore\ a + b = a_1 + b_1$ + c. Let us add angle c to both side

We have $a + b + c = a_1 + b_1 + c$.

But $a_1 + b_1 + c$ are adjacent angles on a straight line. (Angles on straight is 180^0)

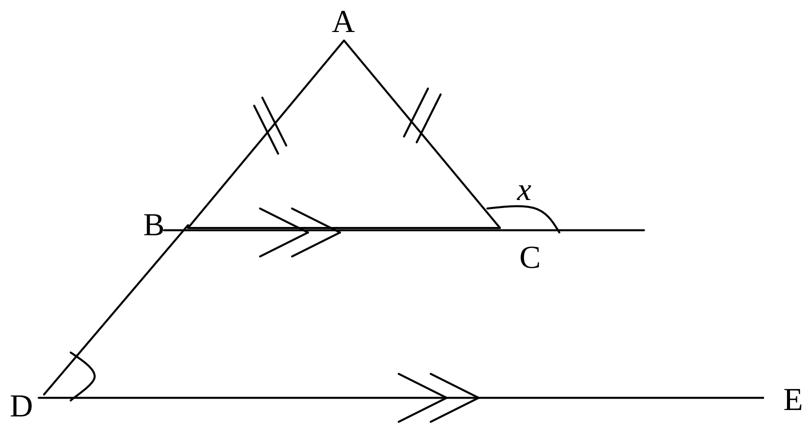
$\Rightarrow\ a_1 + b_1 + c\ 180^0$

$\therefore\ a + b = c = 180^0$

Hence the sum of angles of ΔABC = 2 right angles.

Examples

In the diagram below find < x given /AB/=/AC/, BC//DE & <ADE = 68^0



Solution

Given /AB = /AC/

BC/DE and < ADE = 68^0

<ADE = < ABC = 68^0 (corresponding angles since BC//DE)

Since /AB/ = /AC/, which means <ABC = <ACB = 68^0 (base angles of an isosceles triangle)

<ABC + x = 180^0 (adjacent angles on a straight line)

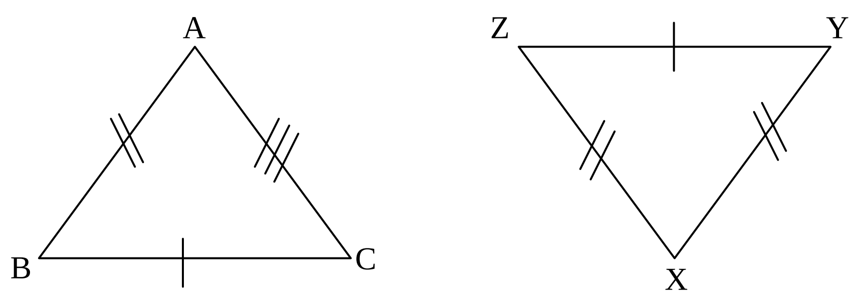
$\therefore x = 180^0$ - < ACB

$x = 180^0 - 68^0 = 112^0$

3.3 Conditions for Congruency

Two triangles are congruent if they are equal in all respect in terms of sides, angles and area.

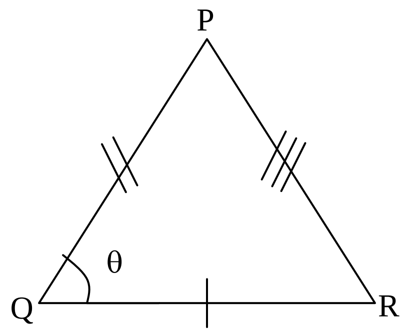
(1) Two triangles are congruent if each of their corresponding sides are equal. (i.e if the three are sides equal an abbreviation for reference is SSS).



/AB/=/XZ/,/BC/=/YZ/ and /AC/=/YX/

So ΔABC = ΔXYZ (SSS).

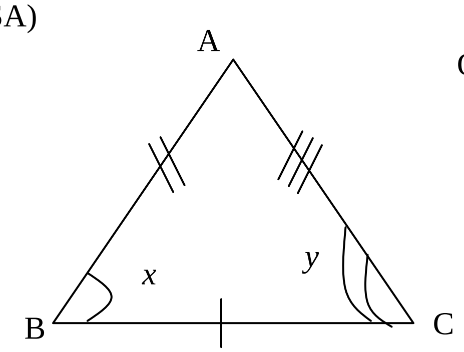
(2) There is said to be congruent in two triangles, if two sides and the included angle of one triangle are equal to the corresponding two side and the included angle of the other (abbreviation for reference is SAS).



/PQ/=/UV/, /QR/=/VW/

∴<PQR=<UVW and PQR=ΔUVW(SAS)

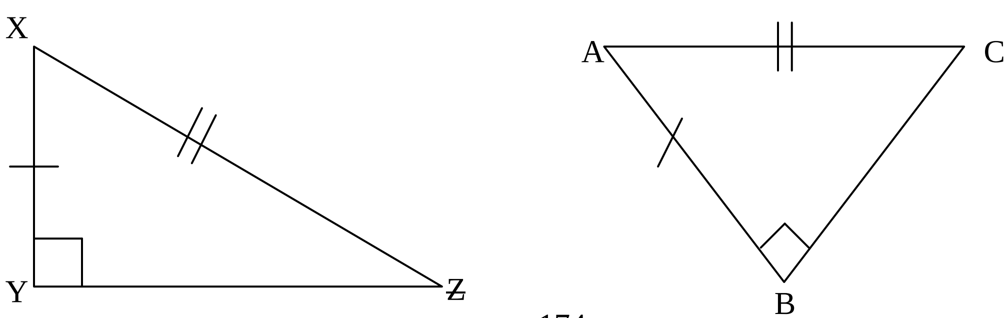
3. Two triangles are said to be congruent if two angels and one side of one triangle are equal to the corresponding angles and one side of the other triangle (abbreviation for reference is ASA)



/BC/=/PQ/, <ABC = <OPQ and <BCA = <PQO

∴ <ABC = ΔOPQ (AAS)

4. Two triangles are said to be congruent if the two triangles are right-angled such that, the hypotenuse and one side of the triangle are equal to the hypotenuse and corresponding side of the other triangle (abbreviation for reference is RHS).



$/XY/=/AB/$, $/XZ/=/AC/$, and $<XYZ = <ABC = 90^0$

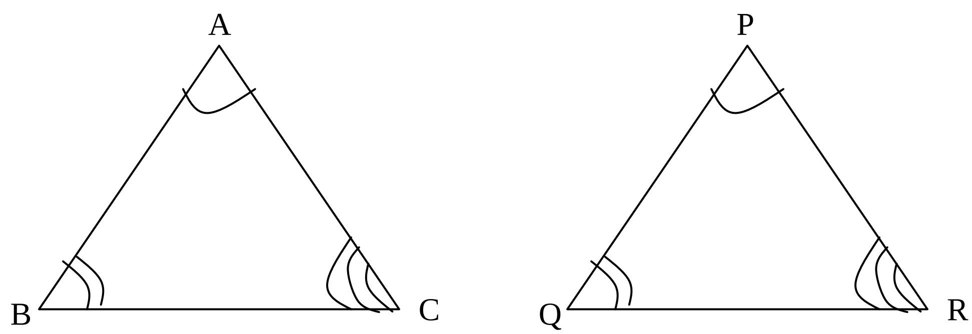
$\therefore \Delta XYZ = \Delta ABC$ *(RHS)*

3.4 Similar Triangles

(i) Definition

Two triangles are said to be equiangular to each other, if the angles of one triangle, taken in other, are respectively equal to the angles of the other in the same order.

1. Two triangle are said to be similar if they are equiangular to one another.



<A = <P, <B = <Q, <C = <R

$\therefore$ ΔABC and ΔPQR are similar.

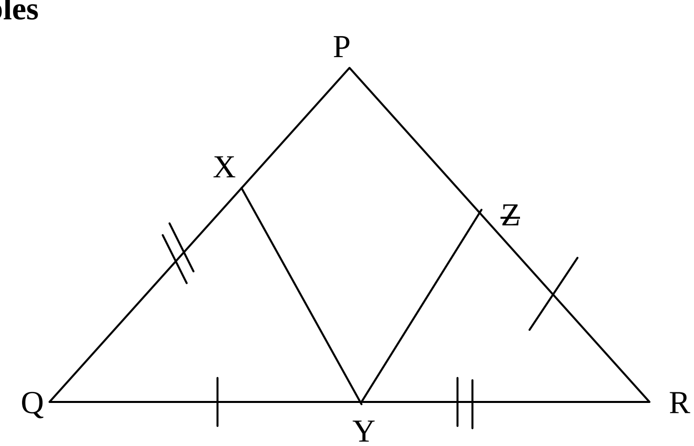
2. If two triangles are equiangular then their corresponding sides are proportional.
3. If two triangles are equiangular then they are similar and hence their corresponding sides are proportional (from the above diagram).

   Hence ΔABC, & ΔPQR are similar implies that

   $$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} \text{ or } \frac{AB}{BC} = \frac{PQ}{QR}$$

Examples

1



From the above diagram X, Y, Z are points on the sides of ∆PQR such that /QX/ = /RY/ & /RZ/ = /QY/, If /PQ/=/PR/, prove that

(i) /XY/ = /YƵ/

(ii) Angle XYƵ = Angle XQY or <XYZ = <XQY

Solution

Proof: From the figure above.

In ∆s XQY and YRƵ /XQ/=/YR/ (given)

<XQY = <YR-Ƶ (base of isosceles)

$\therefore$ ∆XQY = ∆YRƵ (SAS)

(i) Hence /XY/ = /Y-Ƶ)

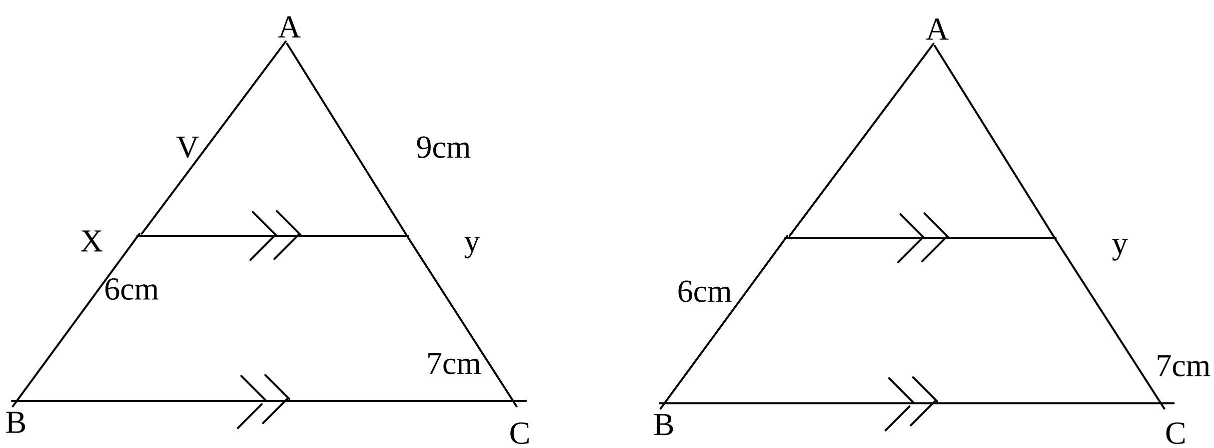
(ii) <XQY = 180^0 – (<QXY+ <XYQ) (Sum of angles in a ∆) and

<XYZ = 180^0 – (<XYQ+ <ƵYR) (Sum of angles on a straight line)

Since <QYX = <ƵYR = <XYƵ is common

$\therefore$ <XYƵ = <XQY

2.



Find the value of V in the above diagram

Solution

Consider ∆AXY and ∆ABC, <AXY = <ABC (corr. agnles XY//BC)

<AYX = <ACB (Corr. angles XY//BC, <A is common).

∆s ABC, AXY are similar and hence

$$\frac{AX}{AB} = \frac{AY}{AC,} \quad AX = V,, / AB / = V + 6CM$$

/AY/ = 9cm and /AC/ = 9cm + 7cm = 16cm

Hence $\frac{V}{V+6cm} = \frac{9cm}{16cm} \Rightarrow 16V = 9V + 54$

$\therefore$ *16V – 9v = 54*

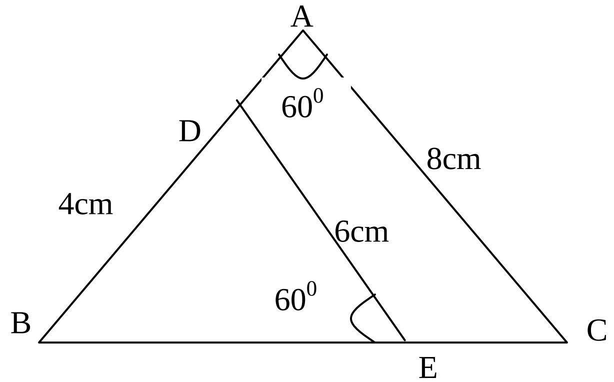
7V = 54

$V = \frac{54}{7} = 7.714$

$= 7.7cm$

3. In the figure below

/AD/ = 4cm, /DE/ = 6cm and /AC/ = 8cm find /BC/



Solution

In ΔABC and ΔEDB

<BAC = <BED (given 60^0)

<CBA = <EBD (same angle)

$\therefore$<EDB = <ABE (sum of angles of a Δ)

$\therefore$ΔEDB is similar to ΔABC

$\therefore \frac{/EB/}{/AB/} = \frac{/BD/}{/BC/} = \frac{/DE/}{/CA/}$

Substituting the given value

$\therefore \frac{/EB/}{AB/} = \frac{4}{/BC/} = \frac{6}{8}$

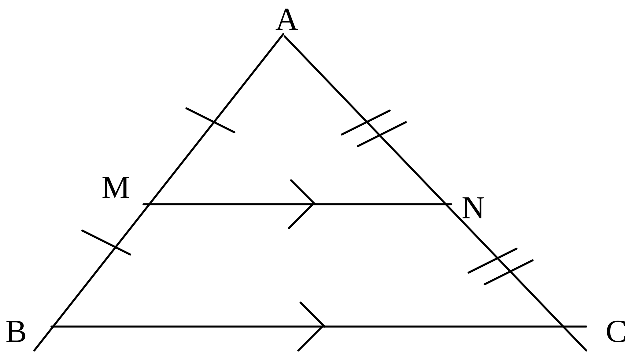
$\therefore \frac{4}{/BC/} = \frac{6}{8}$ $or \therefore \frac{/BC/}{4} = \frac{8}{6}$

$/BC/ = \frac{4x8}{6} = \frac{32}{6}$ $5\,{}^{3}\!/_{6} = 5\frac{1}{3}cm$

3.5 The Mid-Point and Intercept Theorem

1. **The Mid-Point Theorem**

The line joining the mid-points of two sides of a triangles is parallel to the third side and equal to half of it.



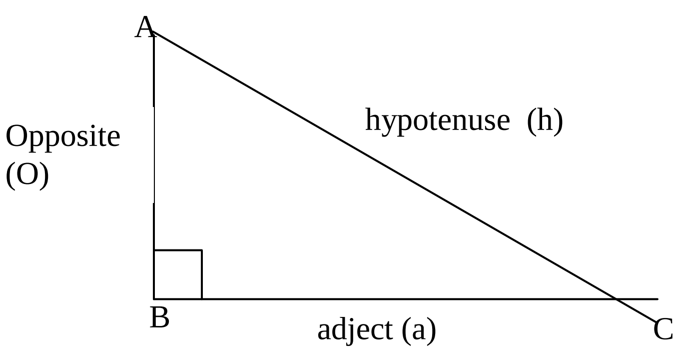
If M, N are the mid-points of AB and AC respectively. Than (i) /MN/ // $\frac{/BC/}{2}$

/MN// /BC/

(ii) MN/BC

3.6 Pythagoras' Theorem

It says that the square of the hypotenuse is equal to the sum of the square of the other two sides in a right-angled triangle. [The hypotenuse in the use side facing the right-angle (90^0)].



$(\text{Hypotenuse})^2 = (\text{opposite})^2 + (\text{adject})^2$

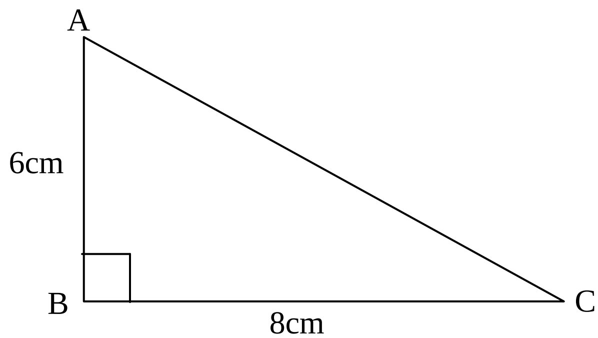
$(AC)^2 = (AB)^2 + (BC)^2$ i.e $h^2 = O^2 + a^2$

Applications of the Pythagoras Theorem

The Pythagoras, rule can help to solve many problem such as calculating lengths when two sides of a right angled triangle are known.

Examples

(1) Find the length of AC in the figure below



By Pythagoras rule

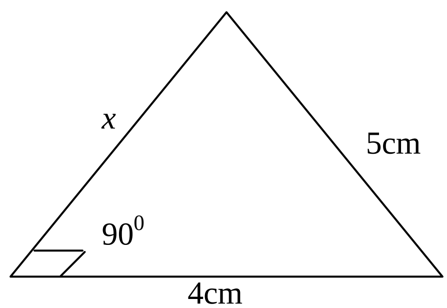
$AC^2 = AB^2 + BC^2$

$AC^2 = 6^2 + 8$

$= 36 + 64 = 100$

$AC = \sqrt{100} = 10cm$

(2) Find the value of x is the figure below.



$5^2 = x^2 + 4^2$

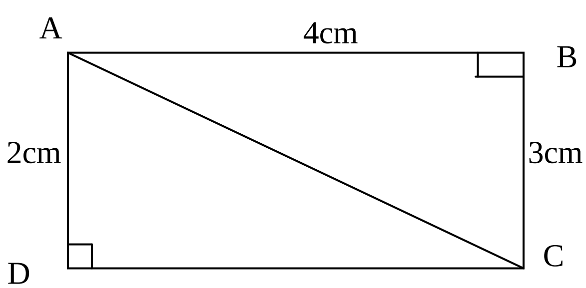
$25 = x^2 + 16$

$25 – 16 = x^2$

$9 = x^2$

$x = \sqrt{9} = 3$

Calculate the value of x in the quadrilateral ABCD.



Draw the diagonal AC in ΔABC, since $<ABC = 90^0$ & $<ADC = 90^0$

First calculate AC and then CD.

$AC^2 = AB^2 + BC^2$ (by Pythagoras' theorem)

$AC^2 = 4^2 + 3^2$

$= 16 + 9 = 25$

$AC = \sqrt{25} = 5$

$AC^2 = AD^2 + CD^2$

$5^2 = 2^2 + y^2$

$25 = 4 + y^2$

$y^2 = 25 - 4$

$y^2 = 21$

$y = \sqrt{21}$

$= 4.58cm$

4.0 Conclusion

The unit considered triangle and its properties. Three theorems were also discussed.

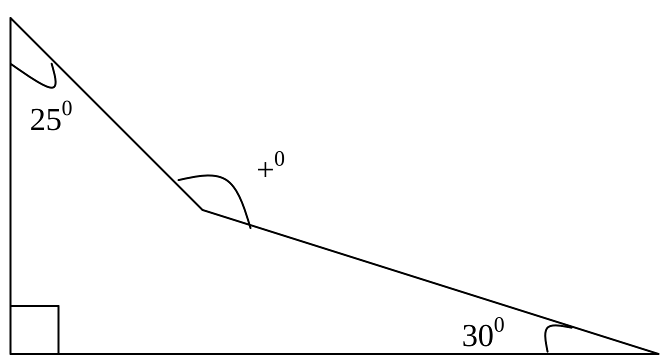
5.0 Summary

The highlights of the unit includes the following.

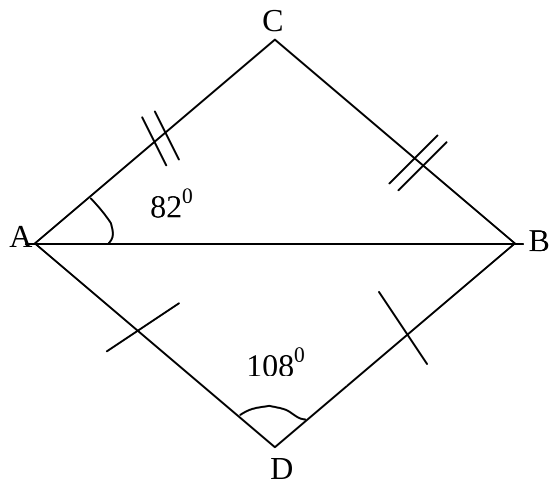
- Triangles and its various types
- Condition for congruency and similarity
- Mid-point, intercept and Pythagoras's theorems and their application

6.0 Tutor Marked Assignment

(1) What is the value of the angle marked *x* in the figure below.

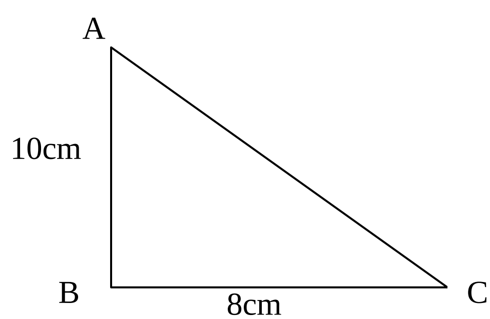


(2) Isosceles triangles ABC & ABC are drawn opposite sides of a common base AB if <ABC = 82^2 and <ADB = 180^0, calculate <ACB & <CBD. See the diagram below.

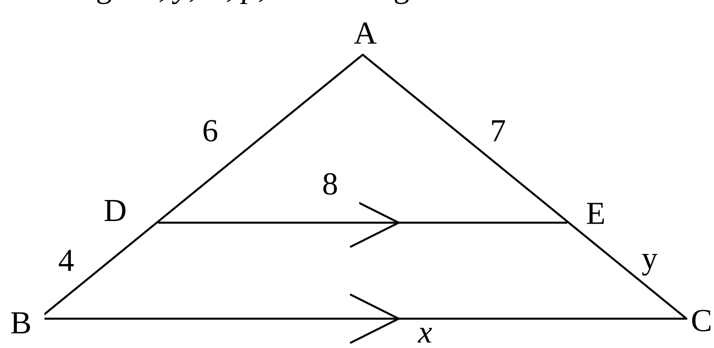


(3) A wire 16cm long goes from top to a 8 metre pole to a point on a vertical wall 20m above the ground. What is the angle between the wire and the wall? (assume that the wire is stretched tight).

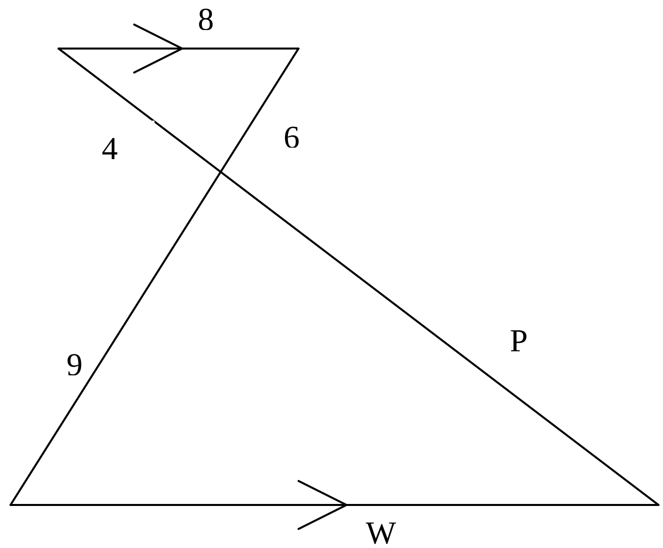
(4) Calculate the length of hypothesis of the figure below:



(5) Find the length *x, y, w, p,* in the diagrams below



(b)



7.0 References/Further Reading

Mame, H.F.; Kalejaiye, A.O.; Chima, Z.I.; Garba, G.U. and Ademosu, M.O. (2006). New General Mathematics for Senior Secondary School 2. (3rd ed). Malaysia: Congman Publishers.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 3: UNIT 2

Unit 2: Circles

Content

1.0 Introduction

A circle is a plane shape (figure) which is bounded by a closed curve which is called the circumference of the circle. The distance from the centre of the circle to any point on the circumference is equal.

2.0 Objective

At the end of this unit, you should be able to:

(i) identify and define the various parts of a circle as a plane shape

(ii) identify the chord properties and angle properties of a circle

(iii) identify the properties of a cyclic quadrilateral

(iv) state properties of tangent to the circle and angle to alternative segments of a circle.

(v) solve any problem involving properties of circle

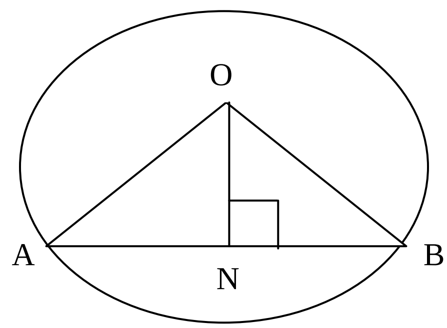
3.0 Main Body

3.1 Definition of Circle

A circle is a plane figure bounded by a closed curve. From the centre every point on the circumference of a circle is equidistance. It has a radius which is a straight line drawn from the centre of the circle to any point on the circumference. A chord of a circle is a straight line joining any two points on the circumference. A diameter of a circle is a chord which passes through the centre of the circle. The diameter (d) is twice the radius (r) of a circle i.e d = 2r.

3.1.1 **Theorem 1**

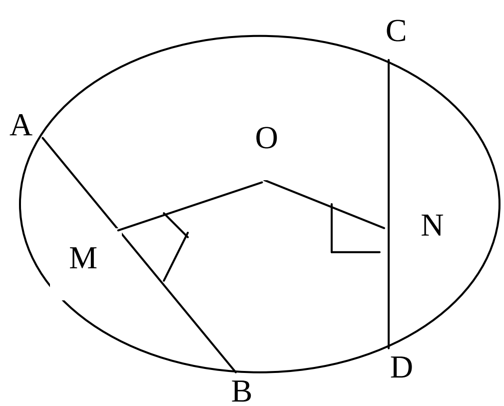
A line jointing the centre of a circle to the mid-point of a chord is perpendicular to the chord.



AB is a chord. The line ON joins the centre of circle to the mid-point of chord. AB line ON is said to be perpendicular to chord AB and angles ONA and ONB are 90^0 each.

3.1.2 **Theorem 2**

Equal chords are equidistance from the centre. In the figure below, if /AB/ = /CD/ then /OM/=/ON?

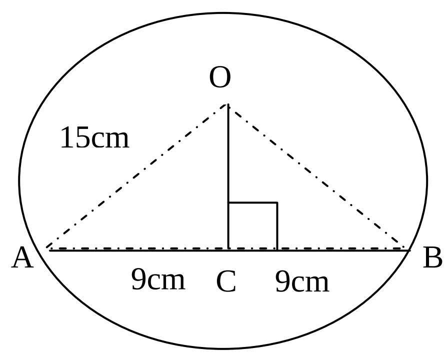


3.1.3 **Theorem 3**

Equal chords stand on equal arcs i.e from the figure above, if chord AB = chord CD then arc AB = arc CD.

Examples

1. A chord of a circle is 18cm long. The radius of the circle is 15cm. Calculate the distance of the mid-point of the chord from the centre of the circle.



From the figure O is the centre of circle /AB/= 18cm, /AO/ = 15cm = radius.

C is the mid-point of AB.

$\therefore$ */AC/=/CB/ = 9cm i.e. /AB/=/AC/ + /CB/.*

In ΔACO,

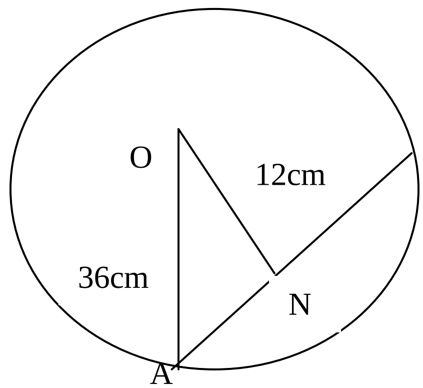
$/AO/^2 = /AC/^2/ + /CO/^2$ (Pythagoras rule)

$/CO/^2 = 225 - 81 = 144$

$/CO/ = \sqrt{144} = 12$

2. Find the length of a chord of a circle of radius 36cm if chord is 12cm from the centre of the circle.

Solution



From the diagram /AO/ = 36cm, /ON/ = 12cm

$\therefore /AO/^2 = /AN/^2 + /NO/^2$

$36^2 = /AN/^2 + 12^2$

$/AN/^2 = 36^2 - 12^2$

$/AN/^2 = 1296 - 144$

$AN = \sqrt{1152} = 33.94cm$

The length of chord AB = *2 x AN*

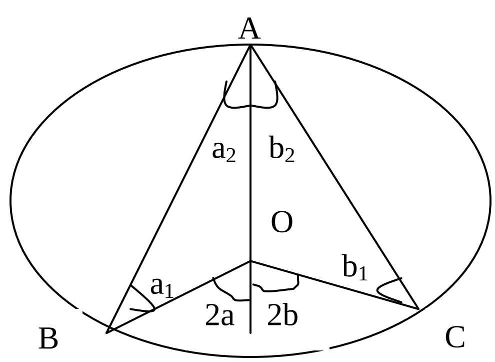
= *2 x 33.94*

= *67.88cm*

= *68cm*

3.2 Angle Properties of Circles

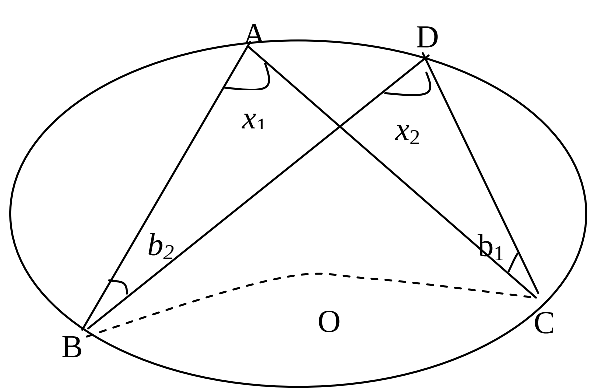
3.2.1 Theorem 1: The angle which an arc of a circle subtends at the centre of the circle is double that which it subtend at any point on the circumference.



A circle <ABC with centre O.

<BOC = <2BAC

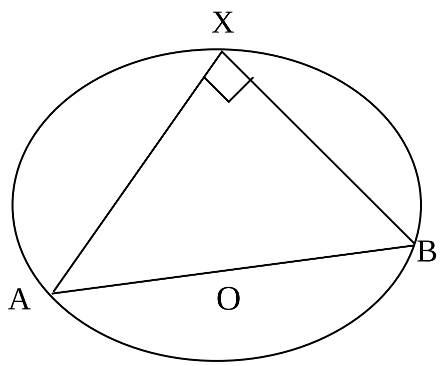
3.2.2 **Theorem:** Angles in the same segment of a circle are equal. (Angles subtends by the same chord or arch at the circumference are equal).



A and D are any points on the major of circle ABCD.

BAC = BDC i.e $x_1 = x_2$

3.2.3 **Theorem 3:** The angle in a semicircle is a right angle.



<AOB = 2 <AXB (angles at centre = 2X angle at circumference)

$A\hat{O}B = 180^0$ (angle on a straight line)

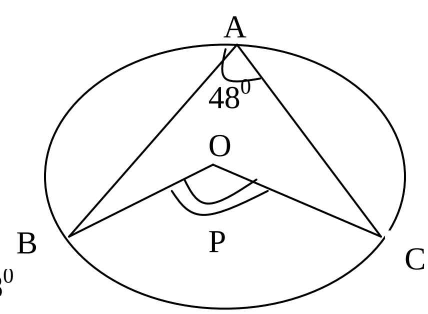
$\therefore 2A\hat{X}B = 180^0$

$A\hat{X}B = \frac{180^0}{2} = 90^0$

Examples

Find the lettered angle in the diagrams below. O is the centre of each circle.

a.

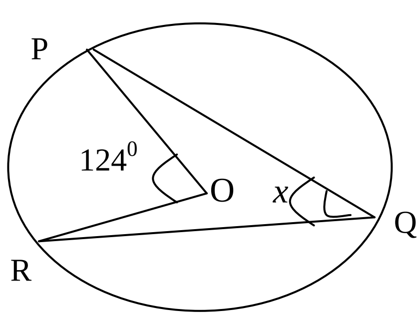


<BOC =

<P = 2 x 48^0

$\therefore$ P = 96^0

b.



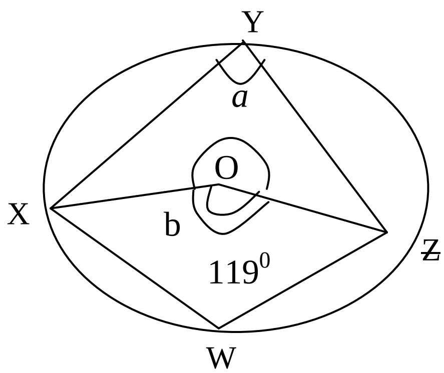
$P\hat{O}R = 2P\hat{Q}R$

$124^0 = 2 < x$

$\angle x = \frac{124^0}{2} = 62^0$

$\therefore x = 62^0$

c.



$\angle$O = 2 X$\hat{W}$Z

$\angle$O = 2 x 119^0

$\angle$O = 238^0

$\angle$b + $\angle$O = 360^0 (angles at a point)

$\angle$b = $360^0 – 238^0$ ie 360^o - $\angle$0

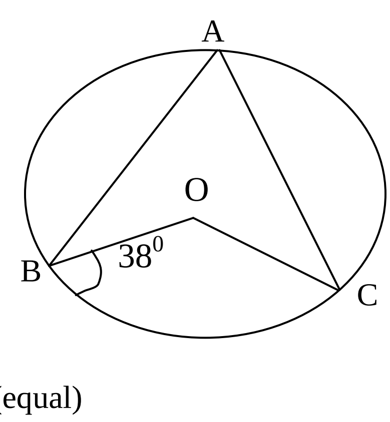
$\angle$b = 122^0

$122^0 = 2\angle$a

$\angle$a = $\frac{122}{2} = 61^0$

$\therefore$ a = 61^0

2. In the figure below ABC are points on a circle O. $\hat{O}B$C = 38^0, what is the size of $\angle$BAC.



/BO/ = /OC/ = radius (equal)

IF $\hat{O}B$C = $38^0 \Rightarrow$ O$\hat{C}$B = O$\hat{B}$C = 38^0 (sum of $\angle$s in a Δ)

$\angle BOC = 180^0 - (2 \ x \ 38^0)$

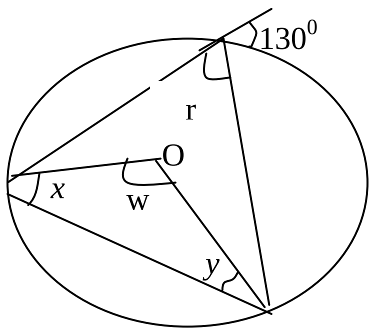
$= 180^0 - 76^0 = 104^0$

$BOC = 2B\hat{A}C$

$104 = 2B\hat{A}C$

$\therefore \ BAC = \frac{104^0}{2} = 52^0$

3. Find the lettered angles in the figures below, with the centre O.



Solution

a. $\angle r + 130^0 = 180^0$ (angles on a straight line)

$\angle r = 180^0 - 130^0 = 50^0$

$\angle \ w = 2 \ x \ 50^0 = 100^0$ (ie <w=<r)

$\angle w = 100^0$

$\angle x - \angle y + \angle w = 180^0$ (sum of angles in Δ)

$\angle x = \angle y$ (base angles of isosceles Δ)

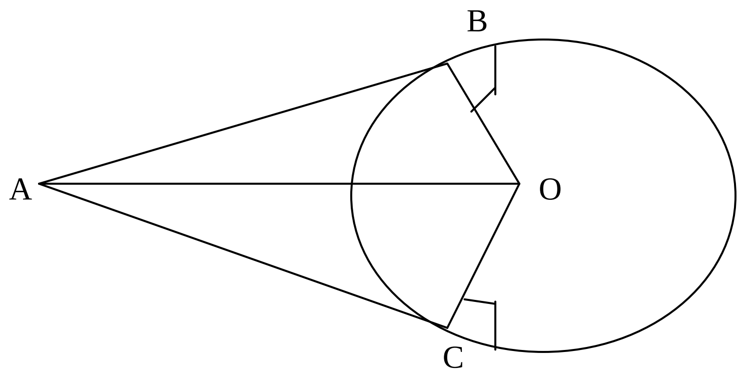
$\Rightarrow 2 \ \angle \ x = 180^0 - \angle w, \ 2\angle x = 180^0 - 100^0, \ 2\angle x = 80$

$\angle \ x = \frac{80^0}{2} = 40^0$

$x = 40^0$ *i.e* $\angle x = \angle y = 40^0$

3.3 Tangent Properties

3.3.1 **Theorem 1:** The tangent at any point of a circle is perpendicular to the radius.



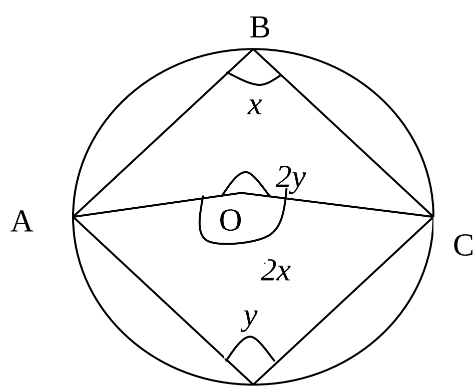
If /AB/ and /AC/ are tangents to the circle and touch the circle at B and C then $O\hat{B}A = O\hat{C}A = 90^0$.

3.3.2 **Theorem 2:** If two tangents AB and AC are drawn to touch the circle at B and C respectively from a point A outside the circle (see figure above) then

(i) /AB/ = /AC/, (ii) $B\hat{A}O = C\hat{A}O$, (iii) $A\hat{O}B = A\hat{O}C$

3.4 Cycle Quadrilaterals

3.4.1 **Theorem 1:** The opposite angles of a cyclic quadrilateral are supplementary.



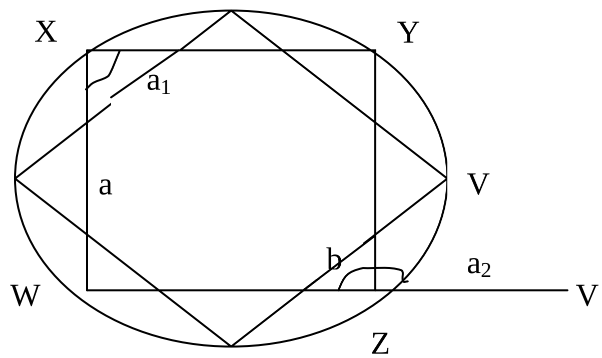
$2x + 2y = 360^0$ (angles at a point)

$2A\hat{B}C = A\hat{O}C = 2y$

$\therefore A\hat{B}C + A\hat{D}C = 180^0$

$\therefore x + y = 180^0$

3.4.2 **Theorem 2:** The exterior angle of a cyclic quadrilateral is equal to the interior opposite angle.



(angles on a straight line)

$b + a_1 = 180^0$ (opp. Angles of cycle guard).

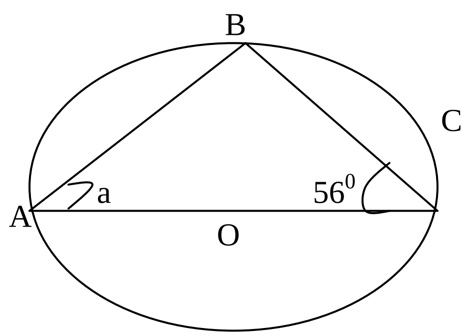
$b + a_2 = 180^0$ (angle on a *straight line)*

$\therefore W\hat{X}Y = Y\hat{Z}V \therefore a_1 = a_2 \left(= 180^0 - b\right)$

Examples

1. Find the marked angles in the figures below. Where O is the centre of the circle.

a

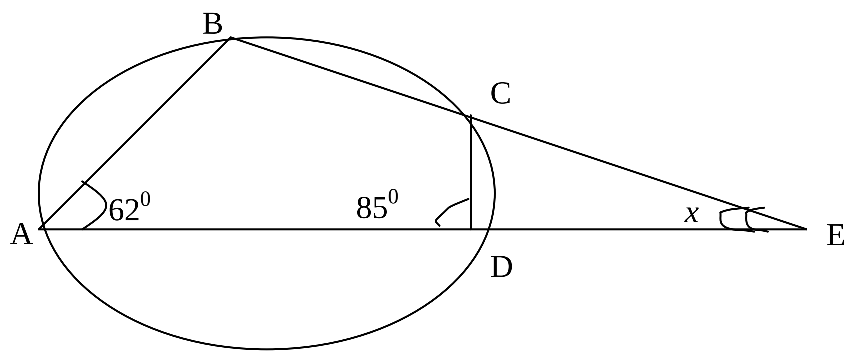


From theorem 3.3.2: $A\hat{B}C = 90^0$

$\angle a + \angle ABC + 56^0 = 180^0$ (sum of $\angle$s in a Δ)

$\angle a = 180^0 - \left(90^0 + 56^0\right)$

$a = 34^0$



From theorem 3.4.2

$\angle BAD \ \angle \ DAC = 62^0$

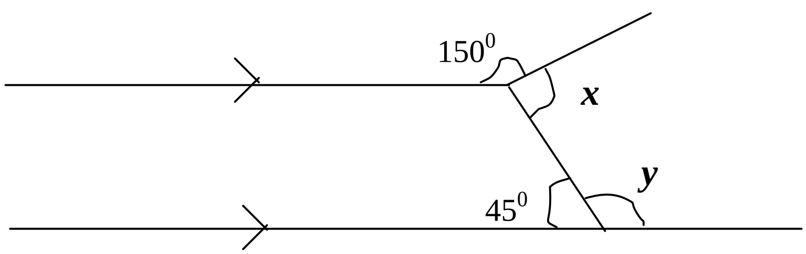
$\angle CDE = 180^0 - 85^0 = 95^0$ ($\angle$s on a straight line)

$62^{0} + 95^0 + x^0 = 180^0$ (sum of $\angle$s in a Δ)

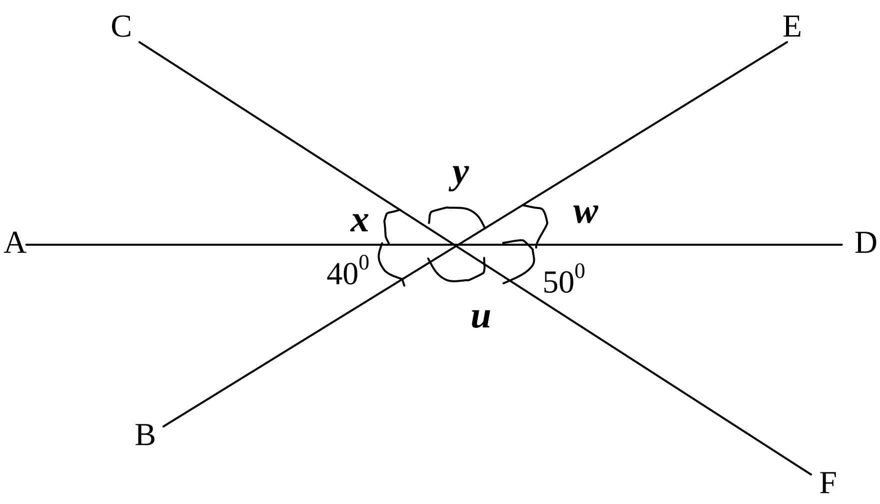
$x^0 = 180^0 - 157^0 = 23^0$

Self Assessment Questions

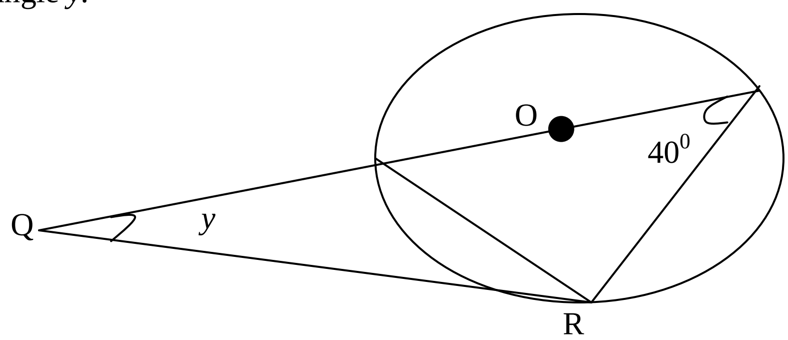
1. In the figures below find the values of the marked angels.



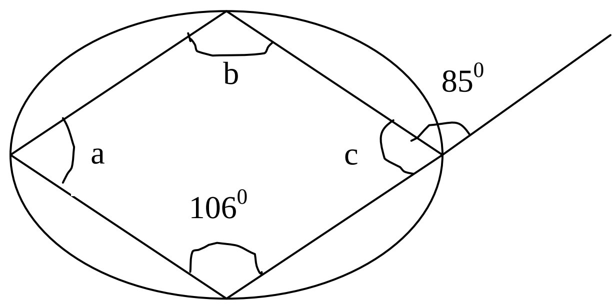
2. AD, BE, & CF are straight line intersecting at O find the marked angles.



3. In the figure below O is the centre of the circle, QR is a tangent to the circle. Calculate the angle *y*.



6. Solve for a, b, & c below.



4.0 Conclusion

The unit considered circle and its properties. Some theorem were also considered.

5.0 Summary

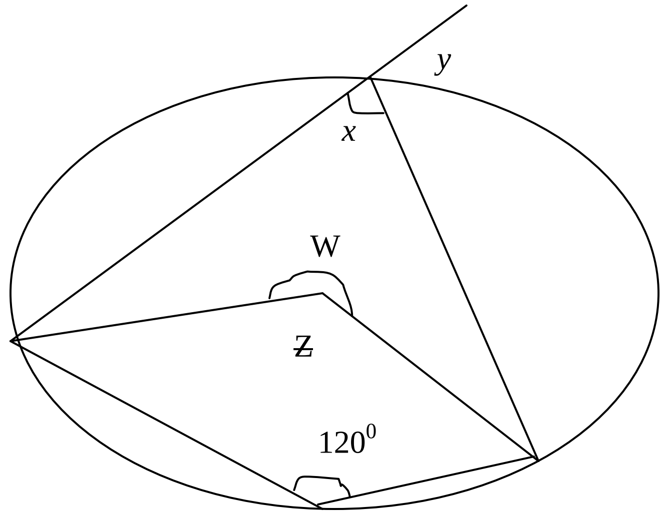
The highlight of the unit includes the following.

- Some circle theorem
- The angle properties
- The Tangent properties
- Cyclic Quadrilaterals

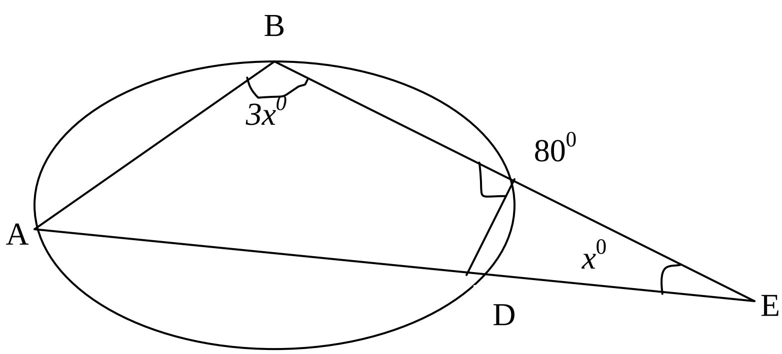
6.0 Tutor Marked Assignment

(1) An isosceles triangle of side 13cm, and 10cm is inscribed is a circle. What is the radius of the circle?

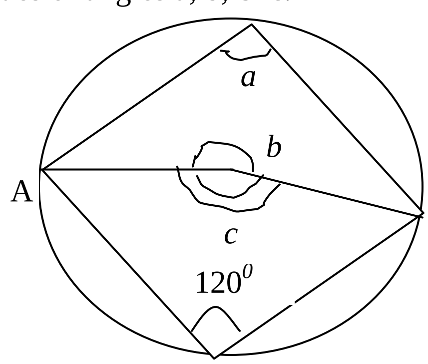
2. Find the lettered angles is the figure below. Where a point O is the centre of the circle.



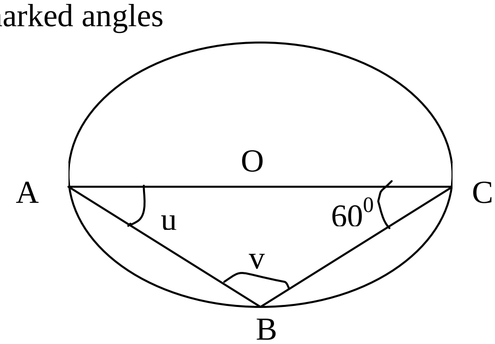
3. Calculate the value of x in the figure below.



4. Find the values of angles a, b, & c.



5. Solve for the marked angles



7.0 References/Tutor Reading

Galadima, I (2004): Teaching Secondary School Algebraic World Problem: A Heuristic, Approach. Lagos: Biga educational serviced.

Channon, J.B. et. al (2002): New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 3: UNIT 3

UNIT 3: Area and Volume of Various Geometrical Solids and SHAPES

Content

1.0 Introduction

Plane shapes are flat surface which extends to all directions without end. Any figure drawn on a plane is called a plane figure (shape).

These are shapes like rectangle, square, triangle parallelogram, trapezium and circle. Solid shape are those shape in solid form. Such as cube, cuboid, prism, cylinder, cone and spheres.

We can calculate the area of plane shapes the area as well as the volume of solid shape.

2.0 Objective

At the end of this unit, you should be able to:

(i) calculate the surface area of various geometrical solids and plane shapes.

(ii) calculate the volumes of various geometrical solids shapes.

3.0 Main Body

The outer view of solids and plane shapes are called its surface. A face is any of the pieces which make up a surface. Plane shape can be of various dimension regular and irregular shapes. Some of the regular shapes are rectangle, square triangle, circle parallelogram, trapezium while solid shapes are those that have surface as well as volume.

3.1 Formula for Area of Plane Shapes

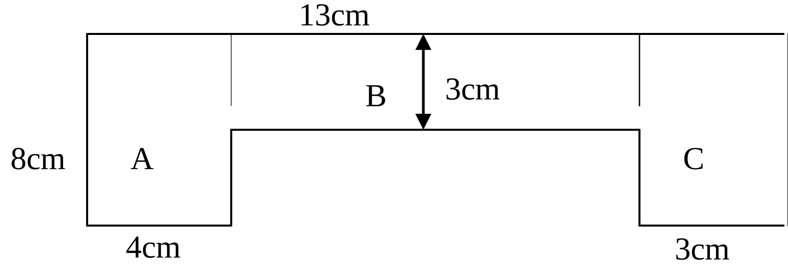
Shapes	**Area**
Rectangle sides	S^2
Rectangle length 1 Breadth	*1b*
Triangle Base b, length 1	$\frac{1}{2}bh$
Parallelogram Base b, length 1	*1b*
Trapezium Length 1 Parallel line a & b	$\frac{1}{2}(a+b)h$
Circle Radius r	π^2
Sector of circle radius r angle θ	$\frac{\theta}{360}\pi r^2$

3.2 Formula for Solid Shapes

Shapes	**Area**	**Volume**
Cube	$6S^2$	S^3
Cuboid Length 1 Breadth b, Height h	*2(ib + bh + 1h)*	*1bh*
Prison Height h Base area A	-	Ah
Cylinder Radius r Height 1	$2\pi rh + 2\pi r^2$ $2\pi r(h + r)$	$\pi r^2 h$
Cone Radius r Slant height 1 Height 1	$\pi rl + \pi r^2$ $\pi r(1 + r)$	$\frac{1}{3}\pi r^2 h$
Sphere Radius r	$4\pi r^2$	$\frac{4}{3}\pi r^3$

3.3 Area of Plane and Solid Shapes

1. What is the area of the figure below.



Solution

Area of rectangle = length x breadth

Area of section A = 8 x 4 = 32cm^2

Area of section B = 6 x 3 = 18cm^2

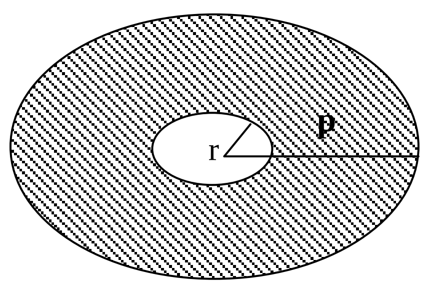
Area of section C = 8 x 3 = 24cm^2

Area of the figure = (32 + 18 + 24)cm^2

= 74cm^2

2. Calculate the area of the shaded portion in the figure below with diameter of the big circle as 30cm and the small circle as 10cm.

Solution



Let the big circle diameter be D and the radius be R.

The small circle diameter is d and the radius is r.

D = 30cm $\therefore$ R = 15cm

d = 10cm $\therefore$ r = 5cm

Area of big circle = $\pi R^2 = \frac{22}{7} \; x \; \frac{15}{1} \; x \; \frac{15}{1}$

= 707.14cm^2

Area of small circle = $\pi r^2 = \frac{22}{7} x \frac{5}{1} x \frac{5}{1}$

$= 78.57cm^2$

Area of the shaded portion is

$(707.14 – 78.57)cm^2 = 628.57cm^{2+}$

or

Area of shaded portion

$= \pi R^2 - \pi r^2$

$= \pi(R^2 - r^2)$

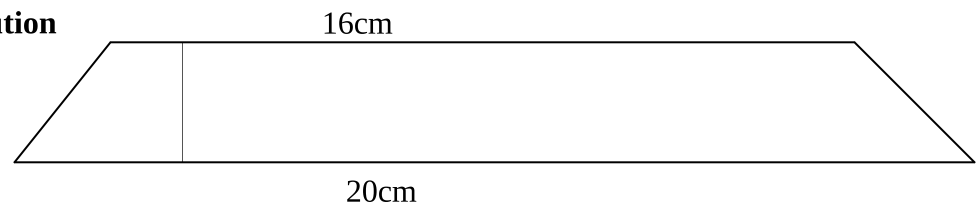
$= \frac{22}{7}(15^2 - 5^2)$

$= \frac{22}{7}(15-5)(15-5) = \frac{22}{7} x\ 20\ x\ 10$

$= \frac{4400}{7} = 628.57cm^2$

3. The parallel sides of a trapezium are 10cm and 16cm. If the area of the trapezium is $98cm^2$. Calculate the distance between the parallel sides.

Solution



Area of trapezium = ½ (sum ot //sides) x height

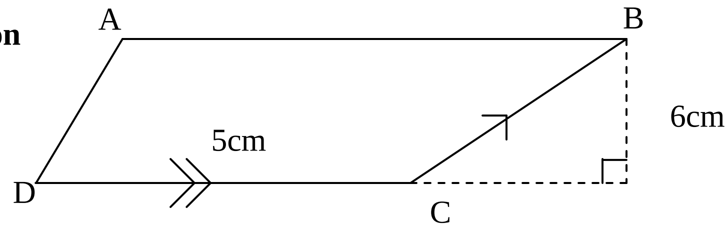
$98 = \frac{1}{2}(20+16)h$

$98\ x\ 2 = 36h$

$h = \frac{196}{36} = 5.44cm$

4. Calculate the area of the parallelogram in the figure below.

Solution



Solution

The height ofthe parallelogram is 6cm the base is 5cm.

Area = 6 x 5 = 30cm^2

5. Find the area of a sector of a circle with radius 14cm and 160^0 as angle at the centre.

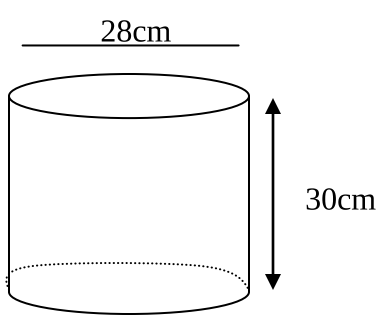
Solution

$$Area = \frac{\theta}{360}\pi r^2$$

$$= \frac{160}{360} x \frac{22}{7} x \frac{14}{1} x \frac{14}{1}$$

$$= 273.78 cm^2$$

6. Calculate the area of the curved surface of a cylinder shown below



Solution

Given: diameter = 28cm ⇒ radius = 14cm height = 30cm

Area of curved surface (A) = 2πr h

$$A = 2 x \frac{22}{7} x \frac{14}{1} x \frac{30}{1} = 2640 cm^2$$

7. Calculate the total surface area of a solid cylinder of radius 7cm and height 20cm. Total surface area = Area of the top + Area of base + Area of curved surface.

(i) Area of top = πr^2 (ii) Area of base = πr^2 (iii) Area of curved surface = πrh^2

Total surface area = $2\pi r^2 + \pi r^2 + \pi rh$

i. Area of top = $\pi r^2 = \frac{22}{7} x \frac{7}{1} x \frac{7}{1} = 154cm^2$

ii. Area of base = $\pi r^2 = \frac{22}{7} x \frac{7}{1} x \frac{7}{1} = 154cm^2$

iii. Area of curved surface = $2\pi rh = 2 x \frac{22}{7} x \frac{7}{1} x \frac{20}{1} = 880cm^2$

Total surface area = $(154 + 154 + 880)cm^2$

= $1180cm^2$

(a) What is the length of the cylinder with curved surface area $186cm^2$ and diameter 14cm.

Solution

$2\pi rh = 186$

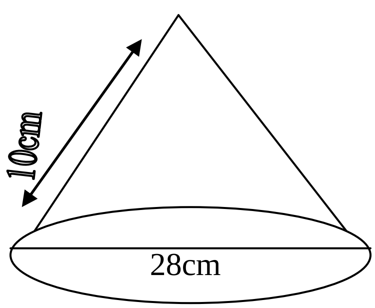
i.e $2 x \frac{22}{7} x \frac{7}{1} x h = 186$

Curved surface area of cylinder = $2\pi rh$

$44h = 186$

$h = \frac{186}{44} = 4.22cm$

10. Find the total surface area of the cone below.



Solution

Total surface of a cone = Area of base + Area of curved surface

Area of base = πr^2

Area of curved surface = πrl.

Total surface area = $\pi r l + \pi r^2$

= $\pi r (l + r)$

Area of base = $\pi rl = \frac{22}{7} x \frac{25}{2} x \frac{25}{2}$

$= \frac{22}{7} x \frac{14}{1} x \frac{14}{1} = 616cm^2$

Area of curved surface = $\pi rl = \frac{22}{7} x \frac{14}{1} x \frac{10}{1}$

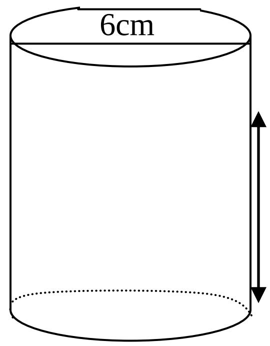
$= 440cm^2$

Total surface area = $(616 + 440)cm^2$

$= 1056cm^2$

3.3 Volume of Plane and Solid Shapes

1. Calculate the volume of a cylinder in the figure below.



Solution

D = 6cm, r = 3cm, h = 21cm

$Volume = \pi r^2 h = \frac{22}{7} x \frac{3}{1} x \frac{2}{1} x \frac{21}{1} = 594cm^3$

2. Which of the cylinder has the lower volume; a cylinder with height 15cm and diameter 8cm or a cylinder with height 8cm and diameter 15cm?

Solution

Volume of cylinder = $\pi r^2 h$

Volume of the first cylinder = $\pi r^2 h$

$= \frac{22}{7} x \ 4 \ x 4 \ x \frac{15}{1}$

$= 745.29cm^3$

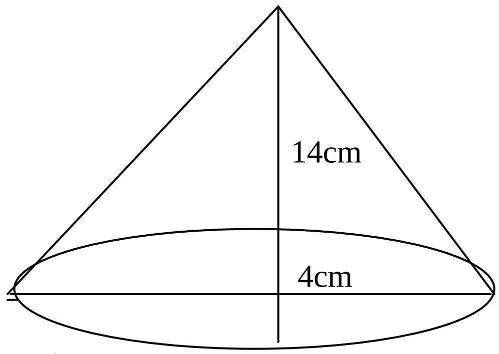
Volume of the second cylinder = $\pi r^2 h$

$= \frac{22}{7} x \frac{15}{2} x \frac{15}{2} x \frac{8}{1}$

$= 1,414.29cm^3$

The cylinder with diameter 8cm has the lower volume.

4. Calculate the volume of the cone below.

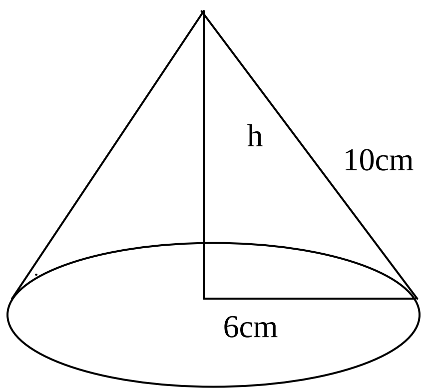


Volume of cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} x \frac{22}{7} x \frac{4}{1} x \frac{4}{1} x \frac{4}{1}$

$= 234.67 cm^3$

4. Calculate the volume of the figure below.



Solution

Height (h) is not known so we have to calculate for h first.

$10^2 = h^2 + 6^2$

$100 - 36 = h^2$

$h^2 = 64$

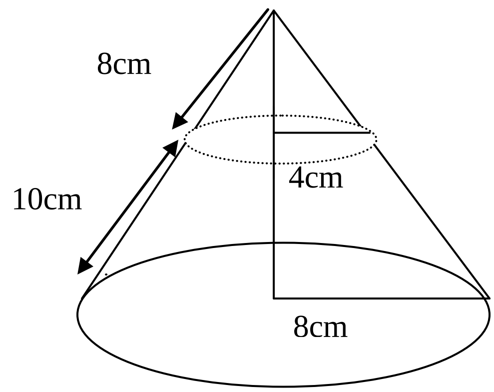
$h = \sqrt{64} = 8$

volume of cone $= \frac{1}{3}\pi r^2 h$

$= \frac{1}{3} x \frac{22}{7} x \frac{6}{1} x \frac{6}{1} x \frac{8}{1}$

$= 301.71 cm^3$

6. The upper part of the cone in the figure below is cut off from the cone. What is the volume of the cone left.

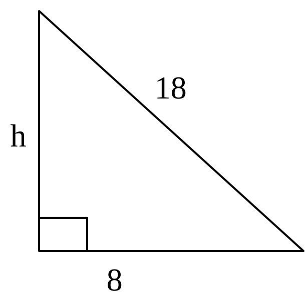


Solution

Volume of the whole cone = $\frac{1}{3}\pi r^2 h$

$$= \frac{1}{3} x \ \frac{22}{7} x \ 8 \ x 8 x \ h$$

Height of full cone



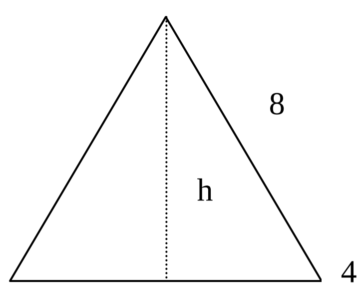
$h^2 + 8^2 = 18^3$

$h^2 = 324 – 64$

$h^2 = 260$

$h = \sqrt{260} = 16.12cm$

height of the upper cone



$h^2 + 4^2 = 8^2$

$h^2 = 64 - 16$

$h = 6.93cm$

Volume of full cone = $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} x \frac{22}{7} x \frac{8}{1} x \frac{8}{1} x \frac{16.12}{1}$

$= 1{,}080.81cm^3$

Volume of the upper cone $\frac{1}{3}\pi r^2 h$

$= \frac{1}{3} x \frac{22}{7} x \frac{4}{1} x \frac{4}{1} x \frac{6.93}{1}$

$= 116.161cm^3$

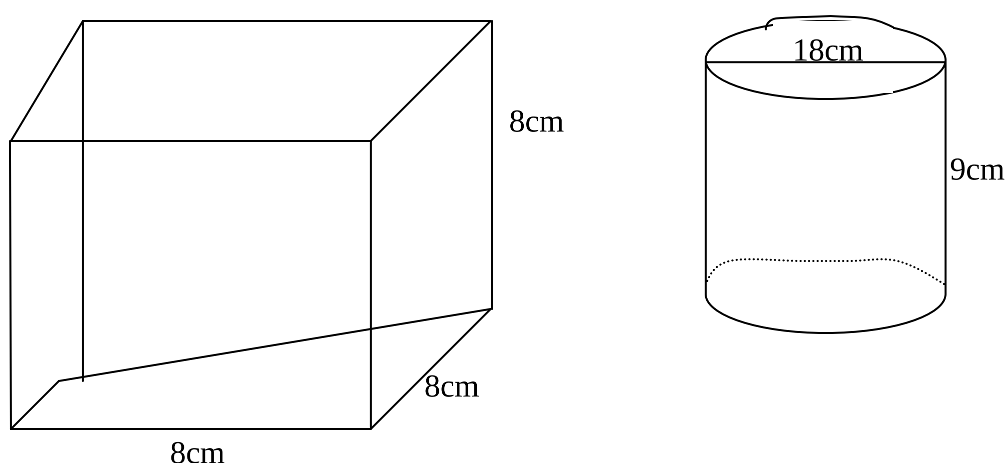
The volume of the cone left =

$(1080.81 - 116.16)cm^3$

$= 964.65cm^3$

Self Assessment Questions

1. What is the difference between the total surface area of the three-dimensional shapes in the figure below.



2. A cylinder point container is of height 55cm and radius 22cm. calculate the total surface area.

3. A cylinder cake of height 12cm and diameter 36cm, wishes to be wrap by the cake designer.

a. How many cm^2 paper wrapper would be need to the nearest cm^2.

b. If a centimeter square paper cost ₦1.50k, how much would be spend in buying the paper to the nearest naira.

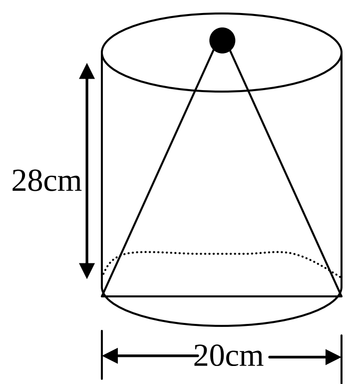
4. A cone has the radius of the base as 8cm and the slant height 21cm. Calculate.

a. The curved surface area.

b. The total surface area of the cone. To the nearest cm^2.

5. A cone of diameter 8cm and height 6cm and a cylinder of diameter 14cm and height 6cm. Find the difference in the total surface area.

6. A cone of base diameter 20cm and height 28cm is placed in a cylinder of the same dimensions as in the figure below. Find the volume of space tying between the cylinder and the cone to the nearest cm^3.



4.0 Conclusion

This unit considered the area of plane and solid shapes as well as the volume of plane and solid shapes.

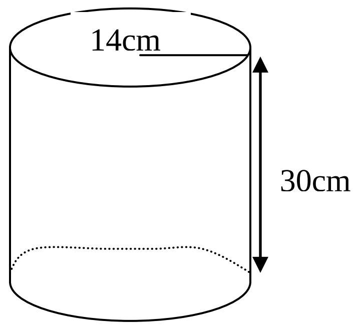
5.0 Summary

The highlight of the unit includes the following

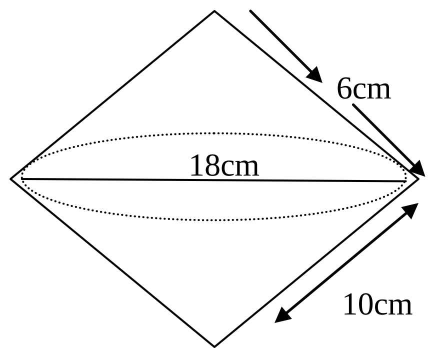
- Formula for area of plane shapes.
- Formula for area of solid shapes.
  (arc and volume)
- Area of plane and shapes
- Volume of plane and solid shapes.

6.0 Tutor Marked Assignment

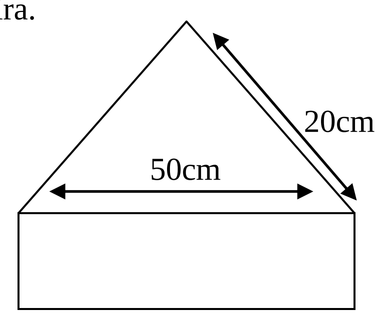
1. Find the total surface area of a cylinder whose radius is 14cm and whose height is 30cm.



2. A cylindrical water tank is closed at both ends. If the radius of the rank is 21cm and the height is 80cm. find the total surface area of the tank.
3. Calculate the curved surface area of the object below.



4. Material is needed to cover up the roof of a church in from of a cone as in the diagram below. If the material cost N215 per square metre, how much would be needed for the cover to the nearest naira.



7.0 References/Further Reading

Galadima, I (2004): Teaching Secondary School Algebraic World Problem: A Heuristic, Approach. Lagos: Biga educational serviced.

Channon, J.B. et. al (2002): New General Mathematics for Senior Secondary Schools. United Kingdom: Longman Group (FE) Ltd.

Mathematical Association of Nigeria (2008): MAN Mathematics for Junior Secondary Schools (3^{rd} ed). Ibadan: University Press Plc.

MODULE 4: MEASUREMENT

UNIT 1: Length and Perimeter

Content

1.0 Introduction

This unit is on length and perimeter. The length of chord and arc of circles are discussed. The perimeter of segment and sector of circles are also discussed. You are encouraged to study the unit carefully from section to section. Make sure you understand one section before you move to the next.

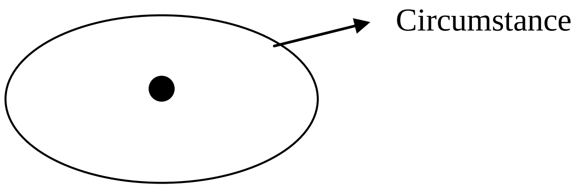
2.0 Objectives

At the end of this unit, you should be able to calculate; (i) the length of a chord of a circles; (ii) the length of an arc of a circle; (iii) the perimeter of a segment of a circle and (iv) the perimeter of a sector of a circle.

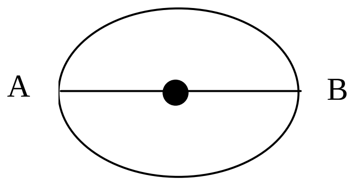
3.0 Main Body

3.1 Basic Definitions

1. **Circumference:** This is the line around a circle.

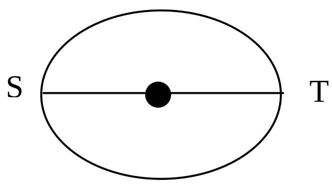


2. **Chord:** A line that joins any two points on the circumstance.



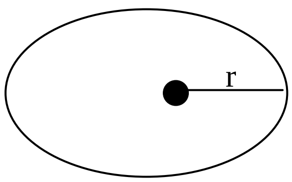
Line AB is a chord

3. Diameter: This is a chord that passes through the centre of the circle.



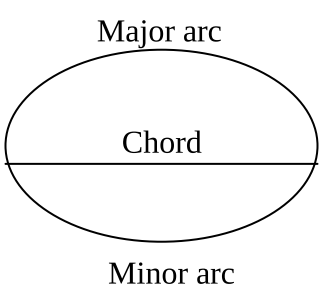
Line ST is a diameter

4. **Radius:** This is defined as half of a diameter. That is a line from any part of the circumstance to the centre of the circle.

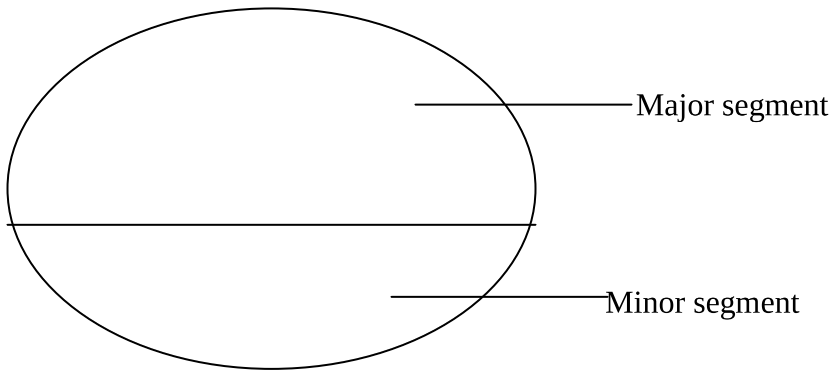


r is a radius

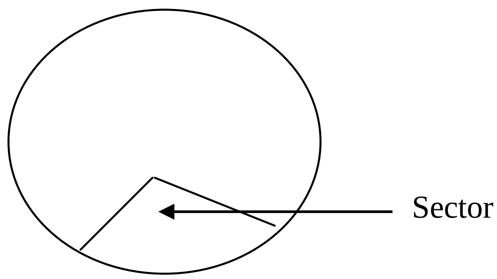
5. **Arc:** An arc is a part of a circumstance. A chord that is not a diameter divides that circumstance into two arcs of different lengths. The longer arc is called the major arc while the shorter arc is called the minor arc.



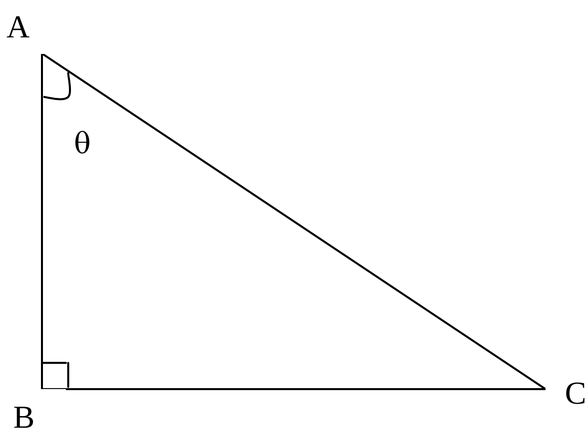
6. **Segment:** This is the space bounded by an arc and a chord. A chord also divides a circle into a major segment and a minor segment.



7. **Sector:** A sector of a circle is the space bounded by two radial and an arc.



8. Trigonometric Ratio: In a right angled triangle, the following ratios are defined.



$$Sin\,\theta = \frac{Opposite}{Hypothenus} = \frac{BC}{AC}$$

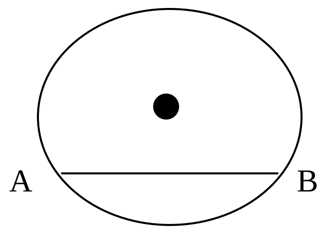
$$Cos\,\theta = \frac{Adjacent}{Hypothenus} = \frac{AB}{AC}$$

$$Tan\,\theta = \frac{Opposite}{Adjacent} = \frac{BC}{AB}$$

3.2 Length of a Chord of a Circle

A chord of a circle is a line that join any two points on the circumstance of the circle.

For example

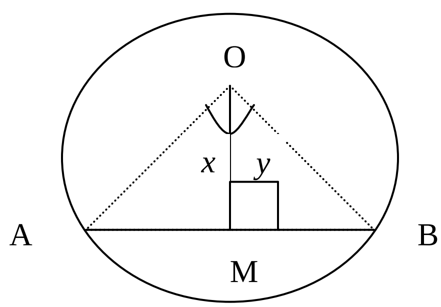


AB in the figure above is a chord.

From chord properties of a circle, you will recall that:

1. A line joining the centre of a circle to the mid-point of a chord is perpendicular to the chord, that is, the line meet the chord at angle 90^0. Conversely, the perpendicular drawn from the centre of a circle to a chord, bisects the chord.
2. A perpendicular from the centre of chord to the centre of the circle bisect the angle subtended by the chord at the centre of the circle.

These are illustrated in the diagram below.



AM = MB

<AOM = <BMO

and

<x = <y

Consider the following problems

1. The distance of a chord of a circle of radius 5cm, from the centre of the circle is 4cm.

Calculate the length of the chord.

Solution

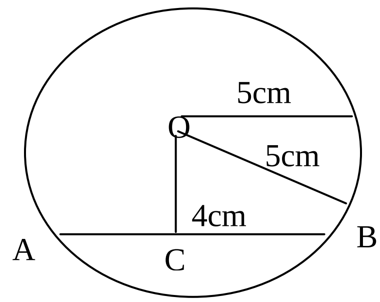
State the information given in the question

Radius = 5cm

Distance from chord to centre = 4cm

Length of chord = ?

Represent the information in a diagram



The required calculation is the length of AB which is the chord, applying Pythagoras theorem.

$5^2 = 4^2 + CB^2$

$25 = 16 + CB^2$

$CB^2 = 25 – 16$

$CB^2 = 9$

$CB = \sqrt{9} = 3cm$

Recall that AC = CB

$\therefore$ AC = CB

i.e., AB = 3 + 3 = 6cm

The length of the chord is 6cm.

2. The radius of a circle is 12cm. A chord of the circle is 16cm long. Calculate the distance of the chord from the centre of the circle. Give the answer to 1 decimal place.

Solution

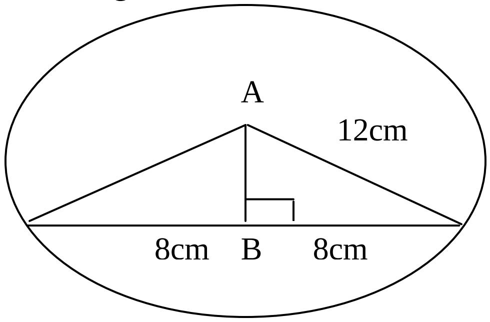
State the information given in the question,

Radius = 12cm

Chord = 16cm

Distance of chord from centre = ?

Represent this information in a diagram



The required calculation is AB which is the distance of the chord from of the centre the circle.

Recall that

$12^2 = AB^2 + 8^2$

$144 = AB^2 + 64$

$AB^2 = 144 – 64$

$AB^2 = 80$

$AB = \sqrt{80} = 8.94427cm$

- *8.9cm* to 1dp

3. A chord of length 24cm is 13cm from the centre of the circle. Calculate the radius of the circle. Giving the answer to 2 decimal places.

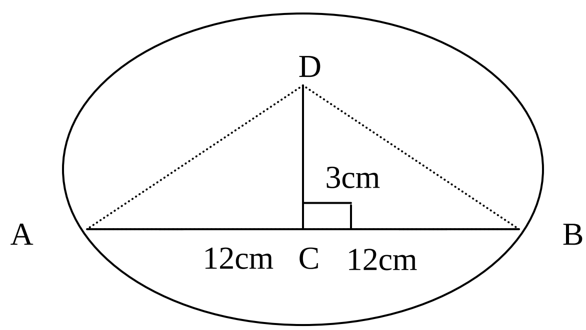
Solution

State the information given length of chord = 24cm.

Distance between chord and centre = 13cm

Radius = ??

Represent the information in a diagram.



The required calculation is BD which is the radius of the circle.

Recall that;

$BD^2 = 13^2 + 12$

$BD^2 = 169 + 144$

$BD^2 = 313$

$BD = \sqrt{313} = 17.6918$

$BD = 17.69$ to 2dp

(4) A chord subtends an angle 94^0 at the centre of a circle of radius 4cm find the length of the chord. Give the answer to 2 decimal places.

Solution

State the information given in the question

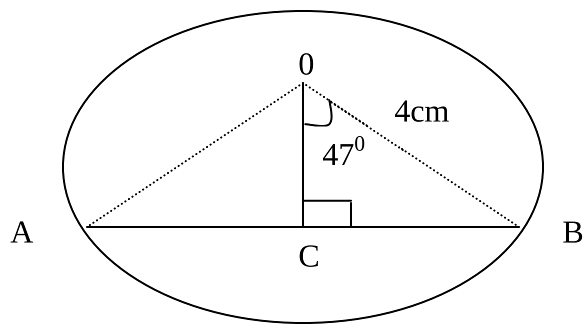
Angle subtend by chord at the centre = 94^0

radius = 4cm

length of chord = ?

represent the information is a diagram

$\frac{94}{2} = 47^0$



Since the perpendicular bisects the angle subtend, the required calculation is AB which is the length of the chord.

Recall that AB = 2CB

$\text{Sin } 47^0 = \frac{CB}{4}$

$CB = 4\sin 47^0$

$CB = 4(0.73134)$

$= 2.93CM$

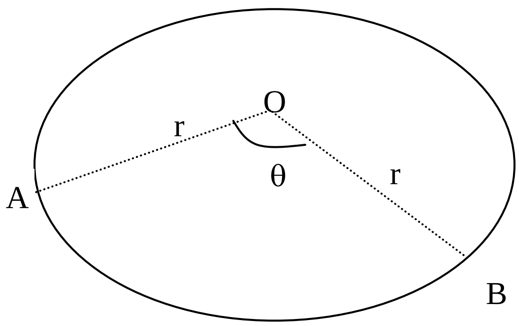
$\therefore 2CB = 2 \times 2.93.$

$= 5.86$ *to 2* decimal place

Self Assessment Exercise 1

The radius of a circle is 8cm. The length of a chord of the circle is 14cm. Calculate the distance of the chord from the centre of circle.

3.3 Length of an arc of a Circle



Consider the diagram above. The length of the arc AB is defined by the formula.

$$AB = \frac{\theta}{360^0} \; x \, 2\pi$$

Where r is the radius of the circle and θ is the angle subtend by the arc at the centre of the circle. The following examples will clarify the idea.

Worked Examples

(1) An arc of a circle of radius 4cm subtends an angle of 85^0 at the centre of the circle. Calculate the length of the arc $\left(take \;\; \pi \; = \frac{22}{7} \right)$, give the answer to 2dp.

Solution

radius = 4cm

$\theta = 85^0$

Length of Arc = ??

Let Length of arc be AB

$$AB = \frac{\theta}{360^0} \; x \, 2\pi$$

$$= \frac{85}{360} \; x \, 2 \; x \; \frac{22}{7} \, x \, 4.$$

= 5.94cm

2. An arc of a circle of radius 7cm is 14cm long. What angle does the arc subtend at the centre of the circle? $\left(take\ \pi = \frac{22}{7}\right)$.

Solution

Radius = 7cm

$\pi = \frac{22}{7}$

Length of arc = AB = 14cm.

θ = ?

$AB = \frac{\theta}{360}\ x\ 2\lambda r$

Substituting into the formular.

$14 = \frac{\theta}{360}\ x\,2\,x\ \frac{22}{7}\ x\ 7$

$14 = \frac{\theta\, x\ 2\ x\ 22\ x7}{360\, x\, 7}$

Make θ the subject of the formula

$\theta = \frac{14\,x\,360\ x7}{2\,x\,2\,2\ x\,7} = \frac{35280}{308} = 114.5454$

$\theta = 114.55^0$

Self Assessment Exercise 2

An arc of radius subtends an angle of 105^0 at the centre of the circle. Calculate the length of the arc. Take $\pi = \frac{22}{7}$ and give the answer to 2dp.

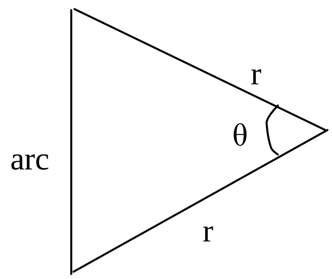
3.4 Perimeter of Segment and Sector of a Circle

3.4.1 Perimeter of a Circle

(a) Perimeter of a circle is the circumference of the circle = 2πr.

(b) A segment is that part of a circle which lies on either side of a chord. The perimeter of a segment = length of arc + length of chord $= \frac{\theta}{360}\,x\,2\pi r + 2r\ Sin\left(\frac{\theta}{2}\right)$

(c) Perimeter of a sector



Examples

(1) Find the perimeter of a circle of a radius 7.cm $\left(take\,\pi = \frac{22}{7}\right)$

Solution

Perimeter of a circle = $2\pi r$

$$= 2\,x\,\frac{22}{7}\,x\,7.0$$

$$= 44.0cm$$

Solution

Length of arc $= \frac{\theta}{360}\,x\,2\pi r$

Perimeter of a sector = length of arc + 2r

Length of arc $= \frac{60}{360}\,x\,2\;x\,\frac{22}{7}\,x\,\frac{8.5}{1}$

$$= \frac{1}{6}\,x\,\frac{44}{7}\,x\,\frac{8.5}{1} = 8.90cm$$

Perimeter of the sector = 8.90 + 2 (8,5)

= 8.9 + 17 = 25.9cm

Self Assessment Question

(1) Calculate the length of a chord that is 6cm from the centre of the circle and circle is of radius 4cm.

(2) Calculate the length of the arc of a circle of radius 6.5cm and subtend the angle of 60^0 at the centre of the circle. Also calculate the perimeter of the arc.

4.0 Conclusion

This unit discussed basic definitions in respect to circle, length of chord and arc of a circle and the perimeter of segment and sector of a circle and the perimeter of segment and sector of a circle.

5.0 Summary

In this unit, you have learnt the following:

- The length of chord and an arc of a circle.
- The perimeter of segment and sector of a circle.

6.0 Tutor Marked Assignment

$$\left(take\, \pi = \frac{22}{7} \right)$$

(1) A chord subtends an angle of 96^0 at the centre of a circle of radius 6cm. Find the length of the chord.

(2) An arc of a circle of radius 9cm is 18cm long. What angle does the arc subtend at the centre of the circle.

(3) The angle of a sector of a circle is 100^0, if the radius of the circle is 3.5cm, find the perimeter of the sector.

(4) A rope of length 20m is used to form a sector of a circle of radius 7.5cm on a school playing field. Find the perimeter.

7.0 References/Further Reading

Galadima, I. (2004). Teaching Secondary School Algebraic World Problem: A Heuristic Approach. Lagos: Biga Educational Services.

Channon, J.B. et al (2002). New General Mathematics for Senior Secondary Schools. United Kingdom: London Group (FE) Ltd.

Mathematical Association of Nigeria (2008). MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 4: UNIT 2

Unit 2: Areas of Circles, Sector and Segments of a Circle

Content

1.0 Introduction

A circle can be defined as a plane shape bounded by a closed curve called the circumstance. In module 3, unit 1, the definition of sector and segment of a circle were given. In this unit, you will be exposed to how to calculate area of a circle as well as areas of sectors and segments of a circle. It is important that you study the unit carefully.

2.0 Objectives

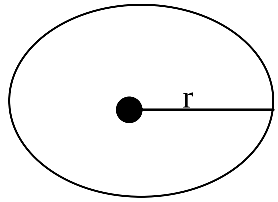
At the end of this unit, you should be able to calculate.

i. area of a circle
ii. area of a sector
iii. area of a segment

3.0 Main Body

3.1 Area of a Circle

The area of a circle is given by the formula πr^2, where r is the radius of the circle and π is a constant.



Area = πr^2

Consider the following examples.

1. Use the value $\frac{22}{7}$ for π to find the area of a circle of radius.

(a) 4.5cm (b) 70cm

Solution

(a) Area of a Circle = πr^2

$\pi = \frac{22}{7}$, r = 4.5

Area of circle = $\frac{22}{7} \times (4.5)^2$

$= \frac{22}{7} \times 20.25$

$= 63.64cm^2$

(b) Area of Circle = π^2

$\pi = \frac{22}{7}$, $r = 70$

Area of Circle = $\frac{22}{7} \times (70)^2$

$= \frac{22}{7} \times 4900$

$= 15.400m^2$

2. The area of a circle is given as $3850m^2$ if π is a given as $\frac{22}{7}$, calculate the radius of the circle.

Solution

Area of circle = πr^2

Area = 3850

$\pi = \frac{22}{7}$

$r = ?$

$3850 = \frac{22}{7} \times r^2$

$3850 = \frac{22 x r^2}{7}$

$7 x 3850 = 22 x r^2$

$26,950 = 22r^2$

$r^2 = \frac{26950}{22}$

$r^2 = 1,225$

$r = \sqrt{1,225}$

$r = 35M$

(3) Use the value 3.14 for π to find the area of a circle of radius

(a) *6cm* (b) *14cm*

Solution

(a) Area of circle = πr^2

$\pi = 3.14$

$r = 6$

$\therefore$ Area = $3.14 x 6^2$

$= 3.14 x 6 x 6$

$= 113.04cm^2$

(b) Area of circle = πr^2

$\pi = 3.14$

r = 14m

$\therefore$ Area = $3.14 \text{ x } 14^2$

= 3.14 x 14 x 14

= $615.44m^2$

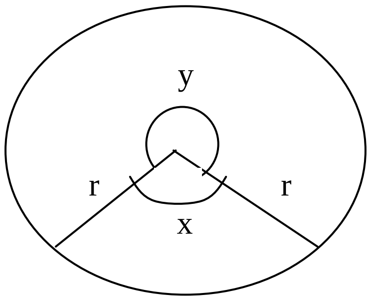
Self Assessment Exercise 1

The radius of a circle is 6cm. Calculating the area of the circle if (a) π is $\frac{22}{7}$ (b) π is 3.14.

3.2 Area of a Sector of a Circle

As defined earlier, a sector is the space bounded by an arc and two radia. Area of a sector is given by the formula.

$\frac{\theta}{360} x \pi r^2$ where r is the radius of the circle and θ is the angle between the two radius.



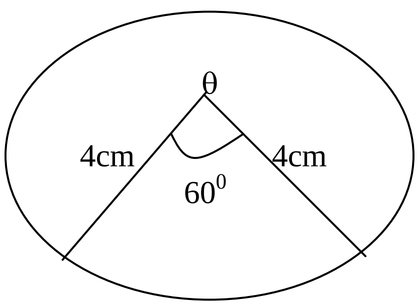
Area of the minor sector $\frac{x}{360} x \pi r^2$

Area of the major sector $\frac{y}{360} x \pi r^2$ *or* $\frac{360 - x}{360} x \pi r^2$

Consider the following examples.

1. An arc AB subtends an angle of 60^0 at the cenre O of a circle of radius 4cm. calculate (a) the area of the minor sector (b) the area of the major sector. $\left(take \ \pi = \frac{22}{7} \right)$

Solution



(a) Area of minor sector $\frac{\theta}{360} x \ \pi r^2$

$$\frac{60}{360} x \frac{22}{7} x \ 4 x 4$$

$$\frac{60 x 22 \ x 4 x 4}{360 x 7} = \frac{21{,}120}{2{,}520}$$

$= 8.38cm^2$

Area of minor sector is equal to 8.38cm^2

$<y = 360^0 - 60^0 = 300^0$

$\pi = \frac{22}{7}$

b. Area of the major sector $\frac{360 - \theta}{360} x \ \pi r^2$

Area = $\frac{800}{360} x \frac{22}{7} \ x 4 \ x 4$

$\frac{300 \, x \, 22 \, x \, 4 \, x \, 4}{360 \, x \ 7} = \frac{105600}{2520} = 41.90 cm^2$

Area of major sector is equal to *41.90cm^2*

2. The angle of a sector of a circle is 295^0. Calculate the radius if the area of the sector is 35.2cm^2.

Solution

$\theta = 295^0$

Area = 35.2cm^2

$r = ?$

Area of sector $= \frac{\theta}{360} x \ \pi r^2$

i.e, $35.2 = \frac{295}{360} x \frac{22}{7} x \, r^2$

$35.2 = \frac{6490 r^2}{2.520} = 2.575 P^2$

$r^2 = \frac{35.2}{2.575}$

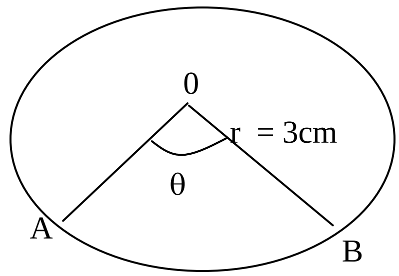
$r^2 = 13.67$

$r = \sqrt{13.67}$

$r = 3.7cm$

3. The length of arc bounding a sector of a circle of radius 3cm, is 7cm, calculate the area of the sector.

Solution



Arc AB = 7cm, radius = 3cm

First find angle θ.

$$Arc\ AB = \frac{\theta}{360} x\, 2\pi r$$

$$7 = \frac{\theta}{360} x\, 2\ x\ \frac{22}{7} x \frac{3}{1}$$

$$7 = \frac{132\theta}{7\, x\, 360} = \frac{132\theta}{2520}$$

$$\theta = \frac{7\, x\, 2520}{132}$$

θ = 133.64^0

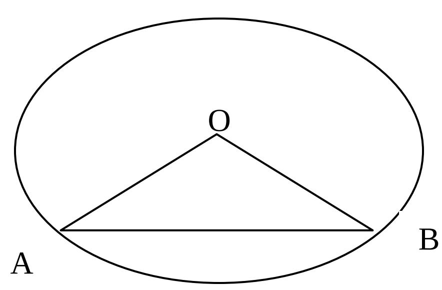
Area of sector = $\frac{\theta}{360}\pi r^2$

Area = $\frac{133.64}{360} x\ \frac{22}{7} x\ 3^2$

= $10.5cm^2$

3.3 Area of a Segment of a Circle

The segment of a circle is the space bounded by an arc and a chord.



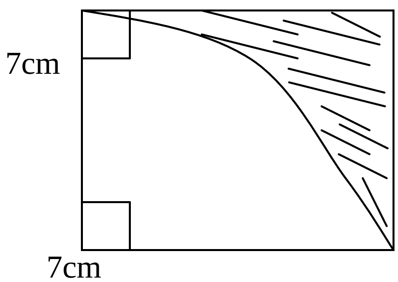
Area if segment

ABC is given by: (Area of sector OACB) – (Area of Triangle OAB)

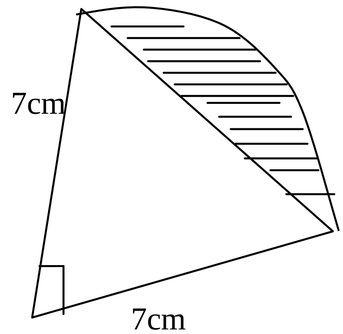
Examples

(1) Find the are of the shaded segment of the circles below

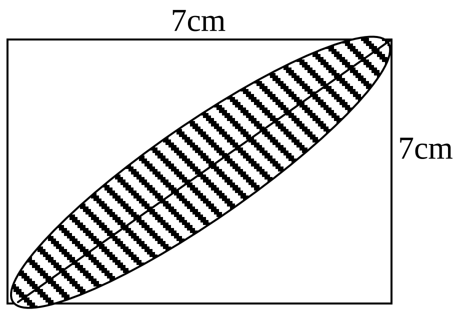
(a)



(b)



(c)



(a)

Solution

Area of Sector $= \dfrac{\theta}{360} \; x \; \pi r^2$

$= \dfrac{\theta}{360} \, x \, \dfrac{22}{7} \; x \; \dfrac{7}{1} \, x \, \dfrac{7}{1}$

$= \dfrac{1386}{360}$

$= 38.5m^2$

Area of square = Length x breath

= 7 x 7

= $49cm^2$

Area of shaded portion = $(49 – 38.5cm^2)$

= $10.5cm^2$

(b)

Area of sector = $\frac{\theta}{360} x \ \pi r^2$

$= \frac{90}{360} \ x \ \frac{22}{7} \ x \ \frac{7}{1} x \frac{7}{1}$

$= \frac{97020}{2520}$

Area of triangle = ½ base x Length

$= \frac{1}{2} x \frac{7}{1} \ x \ \frac{7}{1}$

= $24.5cm^2$

Area of shaded portion = (38.5 – 24.5)

= $14cm^2$

(c)

Area of sector = $\frac{\theta}{360} x \ \pi r^2$

$= \frac{90}{360} \ x \ \frac{22}{7} \ x \ \frac{7}{1} x \frac{7}{1}$

$= 38.5cm^2$

Area of triangle = ½ bh

$= \frac{1}{2} \ x \, 7 \, x \ \ 7 = 24.5cm^2$

Area of shaded portion = (38.5 – 24.5)cm^2

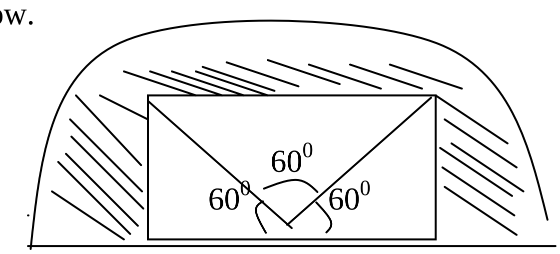
= $14cm^2$

For the two sides mean

$14cm^2$ x 2 = $28cm^2$

Self Assessment Question

(1) Find the area in terms of π and v, of the shaded portion of the semi-circular shape in the diagram below.



4.0 Conclusion

This unit discussed the area of circle. The area of sector of a circle and the segment of a circle.

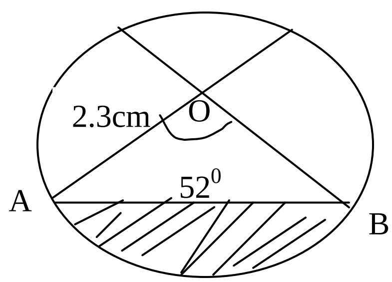
5.0 Summary

The following are the main points in this unit.

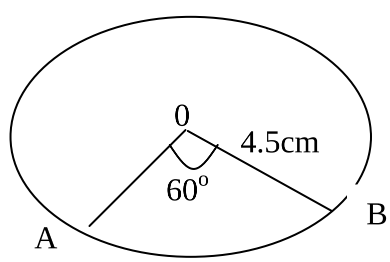
- Area of a circle = πr^2
- Area of a sector of a circle = $\frac{\theta}{360} x\ \pi r^2$ were θ is the angle of the sector & r is radius.
- Area of a segment of a circle. = (Area of sector) – (area of triangle).

6.0 Tutor Marked Assignment

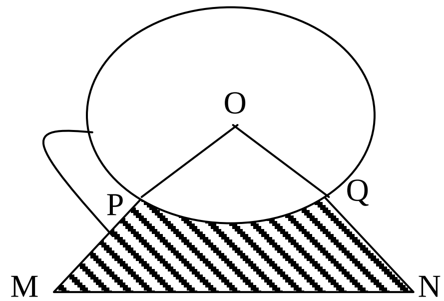
(1) In the figure below, O is the centre of the circle of radius 2.3cm. If <BAC = 32^0 calculate to 2d.p the area of the (i) minor sector (ii) shaded part.



(2) In the figure below, O is the centre of the circle of radius 4.5cm, <AOB = 60^0 calculate the area of the (i) minor sector (ii) major sector $\left(Take\ \pi = \frac{22}{7}\right)$



(3) In the diagram below POQ is a sector of the circle centre O. MNO is an equilateral triangle of area $8cm^2$. If the radius of the circle is 3cm, calculate the shaded portion of the triangle $\left(Take\ \pi = \frac{22}{7} \right)$



7.0 References/Further Reading

Galadima, I. (2004). Teaching Secondary School Algebraic World Problem: A Heuristic Approach. Lagos: Biga Educational Services.

Channon, J.B. et al (2002). New General Mathematics for Senior Secondary Schools. United Kingdom: London Group (FE) Ltd.

Mathematical Association of Nigeria (2008). MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.

MODULE 4: UNIT 3

Unit 3: Surface Area of Cube, Cubiod, Cylinder and Cone

Content

1.0 Introduction

From previous works, you have learnt how to find the area of some solid shapes. In this unit, you are going to learn how to find the surface area of cube, cuboid, cylinder and cone. Surface area is the total area of the solid. The formula for the various solid shapes are as follows.

(1) Surface area of a cube is $6l^2 cm^2$ where 1 is length of one side

(2) Surface area of cuboids is $2(1h+1h+bh)$ where 1 is length, b is breadth and h is height of the cuboid .

(3) Surface area of cylinder is $2\pi rh$

Where r is radius, he is height

(4) Surface Area of cone is (curved surfaced area of closed end) i.e πr (1+r).

2.0 Objectives

At the end of this unit you should be able to:

(1) State the expression for the surface area of (i) cube (2) cuboid (3) cylinder (4) cone

(2) Use the expression to solve problems.

3.0 Main Body

3.1 Surface Area of Cube

If a side of the cube is lcm, then area of each surface area of the cube (S) = $6 \times 1^2 cm^2$

$S = 61^2 cm^2$

(since a cube has six faces)

Examples

$l = 8cm$

$S = 6 \times 1^2 = 6 \times 8^2 = 6 \times 64 = 384cm^2$

(2) What is the total surface area of a cube of side 5cm?

Solution

$1 = 5cm$

Surface area $= 6 \times 1^2$

$= 6 \times 5^2$

$= 6 \times 25 = 150cm^2$

3.2 Surface Area of Cuboid

For a cuboid of length 1, width b, height h, the surface area is given by

$S = 21b + 21h + 2bh$

There are six faces with two equal faces. That is:

i.e $S = (1b + 1b) + (1h + 1h) + (bh + bh)$

$21b + 21h + 2bh$

$= 2(1b + 1h + bh)$ *sq. unit*

Examples

1. Calculate the surface area of a cuboid of length 5cm, width 5cm and height 8cm.

Solution

1 = 5cm, b = 6cm, h = 8cm

$S = 2 (1b + 1h + bh)$

$=2 (5 \times 6 + 5 \times 8 + 6 \times 8)$

$= 2(30 + 40 + 48)$

$= 2(118)$

$= 236cm^2$

(2) What is the total surface area of a cuboid of length 5cm, width 6cm and height 12cm.

Solution

1 = 5cm, b = 6cm, h = 12cm

$S = 2(1b + 1h + bh)$

$=2(5 \times 6 + 5 \times 12 + 6 \times 12)$

$=2(30 + 60 + 72)$

$=2(162)$

$=324cm^2$

3.3 Surface Area of a Cylinder

Given a cylinder of radius rcm and height hcm.

The curved surface area is given by the formula:

$S = 2\pi rh$

Area of a closed end = πr^2

$\therefore$ for a cylinder with one end close total surface area is:

$S = 2\pi rh + \pi r^2$

$= \pi r (2h + r)$

But for a cylinder with two ends closed total surface area

$S = 2\pi rh + 2\pi r^2$

$=2\pi r(h+r)$

Examples

(1) What is the total surface area of a cylinder of radius 3cm and height 7cm if

(i) both ends are closed

(ii) one end is closed

Solution

(i) Both ends are closed

Curved surface are $a = 2\pi r$

$$= 2 x \frac{22}{7} x\ 3\ x 7$$

$$= 132 cm^2$$

Area of two closed end $= 2\pi r^2$

$$= 2 x \frac{22}{7} x\ 3\ x 3$$

$$= \frac{396}{7} cm^2 = 56.57 cm^2$$

Total surface area = $(132 + 56.57)cm^2$

$= 188.57cm^2$

(ii) When one end is closed.

Curved surface area = $2\pi rh$

$$= 2 x \frac{22}{7} x\ 3\ x 7$$

$$= 132 cm^2$$

Area of one closed end = πr^2

$$= \frac{22}{7} x\ 3\ x 3$$

$$= 28.29 cm^2$$

Total surface area = $(132 + 28.29)cm^2 = 160.29cm^2$

2. A cylinder has a radius of 6cm, and a height of 4cm. What is the area of the curved surface?

Solution

Area of curved surface $= 2\pi rh = s$

$r = 6cm, h = 4cm$

$$S = 2 x \frac{22}{7} x\ 6\ x 4$$

$$\frac{1056}{7} cm^2$$

$$= 150.86 cm^2$$

(3) Calculate the surface area of a yellow cylinder which is closed at one end, if the base radius is 3.5cm and height 8cm.

Solution

$r = 3.5cm, h = 8cm$

Surface area = $2\pi rh + \pi r^2$

$$= \pi r(2h + r)$$

$$= \frac{22}{7} x \frac{3.5}{1} (2 x 8 + 3.54)$$

$$= \frac{77}{7} (19.5)$$

$$= 11(19.5) cm^2$$

$$= 214.5 cm^2$$

(4) Calculate the radius of the circular base of a cylinder whose height is 7cm and curved surface area is $88cm^2$.

Solution

Curved surface area = $88cm^2$

Height = 7cm, radius = ?

Curved surface area = $2\pi rh$

$$88 = 2 x \frac{22}{7} x \frac{7}{1} x r$$

$$88 = 44r$$

$$\frac{88}{44} = r$$

$$r = 2cm$$

(5) What is the height of a cylinder, whose radius is 4cm and the area of the curved surface is $352cm^2$?

Solution

Curved surface area = $352cm^2$

Radius = 4cm, height = ?

Curved surface area = $2\pi rh$

$$352 = 2\,x\,\frac{22}{7}\;x\,\frac{4}{1}\,x\,h$$

$$352 = \frac{176}{7}h$$

$$\frac{352\,x7}{176} = h$$

$$h = 14cm$$

Self Assessment Exercise

(1) Find the total surface area of a cylinder whose radius is 14cm and height 20cm.

(2) Calculate the curved surface area of a cylinder whose base diameter is 14cm and height 5cm.

(3) A cylindrical water tank is closed at both ends. If the radius of the tank is 0.5cm and the height is 0.75cm. Find the total surface area of the tank to the nearest square metres.

3.4 Surface Area of Cone

If a cone has base radius r and slant height 1,

Curved surface area = $\pi r1$

Total surface area = $\pi r1 + \pi r^2$

$= \pi r\,(1 + r)$

$S = \pi r\,(1 + r)$

Examples

(1) Calculate the total surface area of a cone of slant height 5m and radius 2m.

Solution

Slant height (h) = 5m

Radius (r) = 2m

Curved surface area = $\pi r1$

$$= \frac{22}{7}\,x\,2\,x\,5 = \frac{220}{7}m^2$$

Area of closed end = πr^2

$= \frac{22}{7} x 2 x 2 = \frac{88}{7}$

Total surface area = $\left(\frac{220}{7} + \frac{88}{7}\right) m^2$

$= \frac{308}{7} m^2$

$= 44 m^2$

(2) Calculate the curved surface area of a cone of base radius 5cm and slant height 6cm.

Solution

r = 5cm, h = 6cm

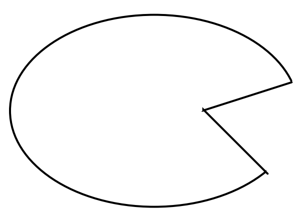
curved surface area *(S) = $\pi r1$*

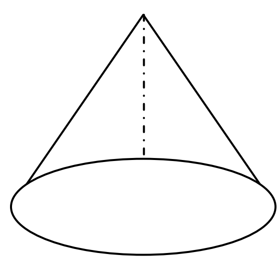
$S = \frac{22}{7} x \frac{5}{1} x \frac{6}{1}$

$\frac{660}{7} cm^2 = 94.29 cm^2$

(3) A cone is made from a sector of a circle of radius 14cm and angle $90^{0.}$ What is the area of the curved surface of the cone to the nearest whole number.

Solution





$\frac{\theta}{360} = \frac{r}{1}$

$\theta = 90^0$, r = ?, *1 = 14cm*

$\frac{90}{360} = \frac{r}{14}$

$\frac{90 x 14}{360} = r$

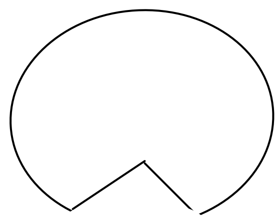
$r = \frac{7}{2}$

Area of curved surface = $\pi r1$

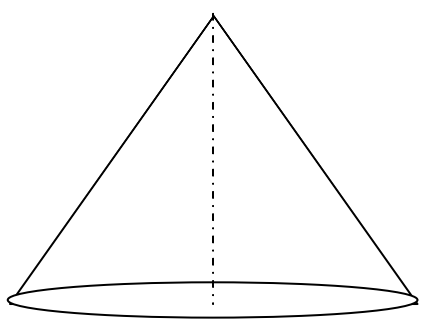
$= \frac{22}{7} x \frac{7}{2} zx \frac{14}{1}$

$= 154cm^2$

(4) Find the radius of the base of the cone in the figure below. If the figure is the net of the curved surface of the cone.



Solution



Then the curved surface area.

$\frac{\theta}{360} = \frac{r}{1}$

$\frac{240}{360} = \frac{r}{7}$

$\frac{240 x 7}{360} = r$

$r = \frac{14}{3} = 4.6$

Curved surface area = πr1

$= \frac{22}{7} \ x \ \frac{14}{3} x \frac{7}{1}$

$\frac{22 x 14}{3}$

$= 102.67cm^2$

(5) Find the total surface area of a solid cone of slant height 10cm and the base diameter 14cm.

Solution

1 = 10cm, d = 14cm i.e r = 7cm

total surface area = (curved surface + area of closed end)

Total surface area = = πr1 + = πr^2

$= \pi r1\ (1 + r)$

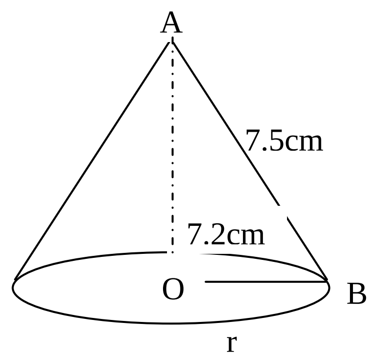
$= \frac{22}{7}\ x\ \ 7(10+7)$

$= 22(17)$

$= 374cm^2$

(6) Find the surface area of a right circular cone whose height is 7.2cm and slant height is 7.5cm.

Solution



h = 7.2cm, 1 = 7.5cm, r = ?

$AB^2 = AO^2 + BO^2$

$(7.5)^2 = (7.2)^2 + r^2$

$56.25 = 51.84 = r^2$

$12 = 56.25 – 51.84$

$r^2 = 4.41$

$r = 2.1$

Total surface area = $\pi r\ (l + r)$

$= \frac{22}{7}\ x\ 2.1(7.5+2.1)$

$= 63.36 =\ 63.4cm^2$

Self Assessment Exercise

(1) Calculate the area of the curved surface of a cone with radius 7cm and length of slanting side 15cm.

(2) What is the radius of a cone, with slant side of the length 10cm and curved surface area of $220cm^2$.

(3) Calculate the slant height and the curved surface area of a cone of base radius 5cm and height 11cm.

4.0 Conclusion

This unit discussed the surface area of cube, cuboid, cylinder and cone. The relationship between the real height and slant height of cylinder and cone. How to form a cone from the sector of a circle.

5.0 Summary

In this unit, we have learnt that the total surface area of

(1) a cube of side *1* is 61^2

(2) a cuboid of length 1, breadth b and height h is

$2(1b + 1h + bh)$

(3) a cone of radius r and slant height *1* is

$\pi r\ (1 + r)$

(4) a cylinder of radius r and height h is

$2\pi r\ (h + r)$

When one side is opened, and

$2\pi r\ (h + r)$

When both side are closed

6.0 Tutor Marked Assignment

(1) A cone has a base radius of 5cm and a height of 12cm calculate.

(a) Its slant height (b) its total surface area r leave your in term of π.

(2) The sector of a circle of radius 156cm and angle 192^2 is bent to form a cone. Find the following.

(a) The radius of the basic of the cone.

(b) The curved surface area of the open cone.

(3) Calculate the total surface area of a solid cone of slant height 25cm and base diameter 14cm.

(4) Calculate the total surface area of a solid cone of vertical height 24cm and base diameter 14cm.

(5) A cylindrical cup has a circular base of diameter 14cm and height 20cm. Calculate (a) its curved surface area (b) the area of its circular base (c) total surface area.

(6) A cylindrical water tank is closed to both ends. If the radius of the tanks is 21 x m and the height at 35cm. Find the total surface area of the tank.

7.0 References/Further Reading

Galadima, I. (2004). Teaching Secondary School Algebraic World Problem: A Heuristic Approach. Lagos: Biga Educational Services.

Channon, J.B. et al (2002). New General Mathematics for Senior Secondary Schools. United Kingdom: London Group (FE) Ltd.

Mathematical Association of Nigeria (2008). MAN Mathematics for Junior Secondary Schools (3rd ed). Ibadan: University Press Plc.