



NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: PHY 402

COURSE TITLE: NUCLEAR PHYSICS

**COURSE
GUIDE**

PHY 402

NUCLEAR PHYSICS

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INTRODUCTION

Nuclear Physics is the field of [physics](#) that studies the constituents and interactions of [atomic nuclei](#). The most commonly known applications of nuclear physics are [nuclear power](#) generation and [nuclear weapons](#) technology, but the research has provided application in many fields, including those in [nuclear medicine](#) and [magnetic resonance imaging](#), [ion implantation](#) in [materials engineering](#), and [radiocarbon dating](#) in [geology](#) and [archaeology](#). The field of [particle physics](#) evolved out of nuclear physics and is typically taught in close association with nuclear physics.

THE COURSE

PHY 402: Nuclear Physics is 3-unit course. We shall start the course with a review of the atom and its constituents. Unit 1 addresses the nuclear structure. Unit 2 opens with the nuclear models. These proposed models are used to describe the nuclear forces in the nucleus of an atom. Unit 3 teaches you the spontaneous disintegration of the nuclei of an atom either naturally or artificially. Unit 4 discusses the characteristics of particles emitted during radioactive disintegration. Unit 5 teaches the disintegration of a nuclide through the bombardment of its nucleus with energetic particles (Nuclear fission) and also the coming together of light nuclei to form a larger nucleus (Nuclear fusion). Unit 6 discusses the interaction of the radiation emitted during radioactive disintegration with matter and most especially human body.

COURSE AIMS

The aim of this course is to teach you nuclear structures, disintegration of nuclides which is accompanied with the emission of particles in form of radiation, and the effect of these radiations on matter.

COURSE OBJECTIVES

At the end of this course, you should be able to

- explain the constituents of the nucleus of an atom
- describe the nuclear models
- describe radioactive processes
- describe the nuclear reactions
- explain the different kind of ways by which radiations interact with matter.

WORKING THROUGH THE COURSE

Nuclear Physics is the foundational material for a good understanding of nuclear structure and its properties. It is hoped that bearing this in mind, you should be able to find enough motivation to thoroughly work on this course.

THE COURSE MATERIAL

You will be provided with the following materials:

1. [Course Guide](#)
2. [Study material containing study units](#)

At the end of the course, you will find a list of recommended textbooks which are necessary as supplements to the course material. However, note that it is not compulsory for you to acquire or indeed read them.

STUDY UNITS

The following study units are contained in this course:

Unit 1	Nuclear Structure
Unit 2	Nuclear Models
Unit 3	Radioactivity
Unit 4	The Energetic of Particles
Unit 5	Nuclear Reactions
Unit 6	Interaction of Radiation with Matter

TEXTBOOKS

Some reference books which you may find useful are given below:

Gautreaux, R. & W. Savin. *Schaum's Outline of Theory and Problems of Modern Physics*. (2nd ed.).

Greiner, W. & J. A. Maruhn. *Nuclear Models*.

ASSESSMENT

There are two components of assessment for this course. The Tutor-Marked Assignment (TMA) and the end-of-course examination.

TUTOR-MARKED ASSIGNMENT

The TMA is the continuous assessment component of your course. It accounts for 30% of the total score. You will be given some TMAs to answer. Three of these must be answered before you are allowed to sit for the end-of-course examination. The TMAs would be given to you by your facilitator and returned after they have been graded.

END-OF-COURSE EXAMINATION

This examination concludes the assessment for the course. It constitutes 70% of the whole course. You will be informed of the time for the examination.

SUMMARY

This course is designed to lay a foundation for further studies in nuclear physics. At the end of this course, you should be able to answer the following types of questions:

- What is Nuclear Physics?
- What are referred to as nucleons?

- What are the different kinds of nuclear model?
- What are nuclear reactions?
- What are the various effects of the interaction of radiation with matter?

We wish you success.



**MAIN
COURSE**

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MODULE 1

Unit 1	Nuclear Structure
Unit 2	Nuclear Models
Unit 3	Radioactivity
Unit 4	The Energetic of Particle
Unit 5	Nuclear Reactions
Unit 6	Interaction of Radiation with Matter

UNIT 1 NUCLEAR STRUCTURE

CONTENTS

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3.2	The Nucleus
3.3	Nuclear Binding Energy and Separation Energy
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1.0 INTRODUCTION

Matter was once considered to be made simply of atoms. It was soon discovered that atoms are made of elementary particles.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- highlight the constituents of an atom
- explain nuclear size, nuclear masses and nuclear forces.

3.0 MAIN CONTENT

3.1 Nuclear Structure

Although there are numerous elementary particles, the only relevant particles in our earthly life and in nuclear reactors, which we are going to discuss are photons and the particles that constitute material, that is, protons, neutrons, and electrons. Among these, the proton and neutron have approximately the same mass. However, the mass of the electron is only 0.05% that of these two particles. The proton has a positive charge and its absolute value is the same as the electric charge of one electron (the elementary electric charge). The proton and neutron are called nucleons and they constitute a nucleus. An atom is constituted of a nucleus and electrons that circle the nucleus due to Coulomb attraction.

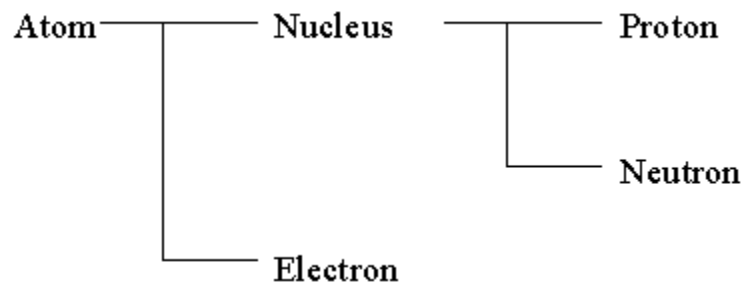


Fig 1.0: Constitution of an Atom (No. 1)

Species of atoms and nuclei are called elements and nuclides respectively. An element is determined by its proton number (the number of protons). The proton number is generally called the atomic number and is denoted by Z . A nuclide is determined by both the proton number and the neutron number (the number of neutrons denoted by N). The sum of the proton number and neutron number, namely, the nucleon number, is called the

mass number and is denoted by A ($A=Z+N$). Obviously, a nuclide can also be determined by the atomic number and mass number.

In order to identify a nuclide, A and Z are usually added on the left side of the atomic symbol as superscript and subscript respectively. For example, there are two representative nuclides for uranium, described as ${}^{235}_{92}\text{U}$ and ${}^{238}_{92}\text{U}$. If the atomic symbol is given, the atomic number can be uniquely determined; thus Z is often omitted like ${}^{235}\text{U}$ and ${}^{238}\text{U}$. The chemical properties of an atom are determined by the atomic number, so even if the mass numbers of nuclei are different, if the atomic numbers are the same, their chemical properties are the same. These nuclides are called isotopic elements or isotopes. If the mass numbers are the same and the atomic numbers are different, they are called isobars. If the neutron numbers are the same, they are called isotones.

The above examples for uranium are isotopes. Summarising these and rewriting the constitution of an atom we obtain Figure 1.1.

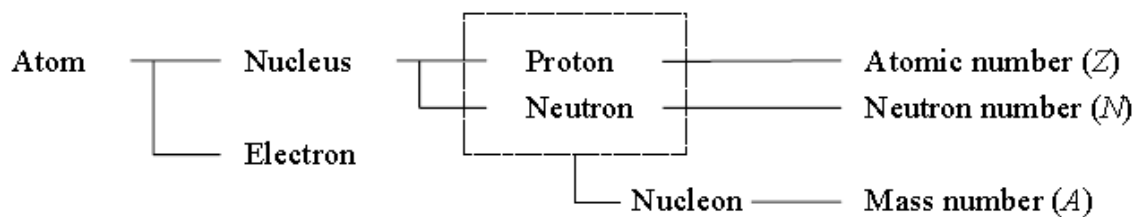


Figure 1.1: Constitution of an Atom (No. 2)

A nuclide is any nuclear specie with the combination of neutrons and protons i.e. ${}^A_Z\text{X}$, A = atomic mass, Z = atomic number = number of proton = number of electrons while number of neutron, $N = (A - Z)$.

- nuclides with the same Z = Isotopes
- nuclides with the same A = Isobars
- nuclides with the same N = Isotones
- nuclides with the same A and Z but different states of excitations = Isomers
- the charge distribution within the nucleus can be assumed uniform with charge density given by:

$$P(r) = \frac{\rho_0}{1 + e^{\frac{(r-R)}{b}}}$$

Where ρ_0 = density at the nuclear centre

R = radius at which ρ falls to $\rho_0/2$

b = measures how rapidly the density falls to zero at the nuclear surface.

R = radius of nucleus $R = r_0 A^{1/3}$,

R = radius of nucleus and $r_0 = 1.23 \times 10^{-15} \text{ m}$

This implies that:

- the volume of the nucleus is proportional to number of particle A
- charge density $\rho(r)$ decreases slowly with increasing A .

This implies that:

- the volume of nucleus is proportional to number of particle A
- charge density $P(r)$ decreases slowly with increasing A .
- 1 a.m.u. = $1.6660053 \times 10^{-27} \text{ kg}$.
= 931 MeV

To show that the electron is not a constituent of the nucleus

Uncertainty principle is applied here.

- Typical nuclear are less than 10^{-14} m in radius
- Therefore for an electron to be confined within such nucleus, the uncertainty in its position may not exceed 10^{-14} m .
- The corresponding uncertainty in the electrons momentum is:

$$\Delta p \approx \frac{\hbar}{\Delta x}$$

$$\hbar = \frac{h}{2\pi} = \frac{6.625 \times 10^{-34}}{2 \times 3.14}$$

$$\cong 1.055 \times 10^{-34} \text{ JS}^{-1}$$

$$\text{and } \Delta x = 2r = 2 \times 10^{-14} \text{ m}$$

$$\therefore \Delta p = \frac{\hbar}{\Delta x} = \frac{1.055 \times 10^{-34}}{2 \times 10^{-14}}$$

$$\approx 5.275 \times 10^{-21} \text{ kgms}^{-1}$$

- If this is uncertainty in the momentum of the electron, the momentum itself must be at least comparable in magnitude.
- The K.E. of the electron of mass, m may be put as follows:

$$T = \frac{\rho^2}{2m}$$

where $m = 9 \times 10^{-31} \text{ kg}$

$$= 9 \times 10^{-31} \text{ kg} \times 1.6 \times 10^{-19} \text{ eV}$$

$$= 1.44 \times 10^{-49} \text{ eV}$$

$$\approx \frac{(5.275 \times 10^{-21})^2}{2 \times 1.44 \times 10^{-49}}$$

$$\approx 9.7 \times 10^7 \text{ eV}$$

$$\approx 97 \text{ MeV}$$

From the above, it follows that if the electrons are present inside the nucleus, their K.E must be of the order 97MeV. But experimental data reveal that no electron in the atom has energy greater than 4MeV. This clearly reveals that e-s do not exist in the nucleus.

Excess mass and packing fraction

Excess mass is defined as the difference between the masses of the nucleons (M) and the atomic mass (A). This implies that excess mass = (M - A), which can be either positive or negative.

Packing fraction (f) can be defined as the ratio of the excess mass to the atomic mass. This implies that packing fraction is expressed as $(M - A)/A = f$.

It is only Carbon -12 that has its $M - A = 0$.

3.2 The Nucleus

The alpha (α) scattering experiment led to the discovery of a nucleus of an atom. The mass of the atom seems to be concentrated at the nucleus and it is surrounded by cloud of electrons which makes the entire atom electrically neutral.

One of the goals of Rutherford's α -scattering is the determination of the radius (R) of the nucleus that is as α -particle approaches the gold nucleus it slows down due to coulomb force but later speeds up on its way-out.

The coulomb repulsive force in the region close to the scattering gold nucleus is given by:

$$F = \frac{2eZe}{4\pi\epsilon_0 b^2}$$

The time of operation of force

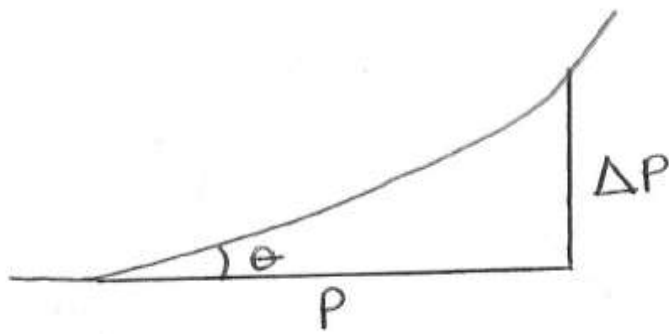
$$\Delta t = \frac{b}{v}$$

The force produces a momentum, Δp , which is perpendicular to the direction of α -particle.

$$F = \frac{\Delta p}{\Delta t}$$

$$\Delta p = f \cdot \Delta t$$

$$\Delta p = \frac{2eZe}{4\pi\epsilon_0 b^2} \cdot \frac{b}{v}$$



$$\theta = \frac{\Delta p}{p} = \frac{2Ze^2}{4\pi\epsilon_0 b v} \div mv$$

$$\theta = \frac{2Ze^2}{4\pi\epsilon_0 b m v^2}$$

Making b the subject of formula

$$b = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{m v^2} \cdot \frac{1}{\theta}$$

Make b = R and putting $\theta = 1$

$$R = \frac{1}{4\pi\epsilon_0} \cdot \frac{2Ze^2}{m v^2}$$

$$R \approx 10^{-14} m$$

Radius of nucleus, R is smaller than the radius of the atom.

SELF-ASSESSMENT EXERCISE 1

- i. What do you understand by the term Nuclear force?
- ii. Define the following terms:
 - (a) Excess mass
 - (b) Packing fraction
- iii. Show that the electron is not a constituent of the nucleus of an atom.

3.3 Nuclear Binding Energy and Separation Energy

Binding energy is the energy that must be supplied to dissociate the nucleus into separate nucleus or the energy released when the separated nucleons were assembled into a nucleus.

$$B(A, Z) = [ZM_H + NM_N - M(A, Z)] 931 \text{MeV} \text{ ----- (1)}$$

Also, the difference between the actual nuclear mass and the mass of the entire individual nucleus is called the mass defect (Md) which is equal to $W - M$.

Binding energy is a measure of cohesiveness of a nucleus that is between the proton and neutron. Also, a more useful measure of cohesiveness is the binding energy per nucleon.

$$\frac{B(A, Z)}{A} = \frac{[ZM_H + NM_N - M(A, Z)] 931 \text{MeV}}{A} \text{ ----- (2)}$$

From equation (1) the mass of a nucleon becomes.

$$\frac{B(A, Z)}{931} = [ZM_H + NM_N - M(A, Z)] \text{MeV}$$

$$M(A, Z) = \left[ZM_H + NM_N - \frac{B(A, Z)}{931} \right] \text{a.m.u}$$

The binding energy can also be written in terms of the mass number or the atomic mass number

$$B = AM_N - Z(M_N - M_H) - M(A, Z) \text{a.m.u}$$

Dividing through by A

$$B = M_N - \frac{Z}{A}(M_N - M_H) - \frac{M}{A}$$

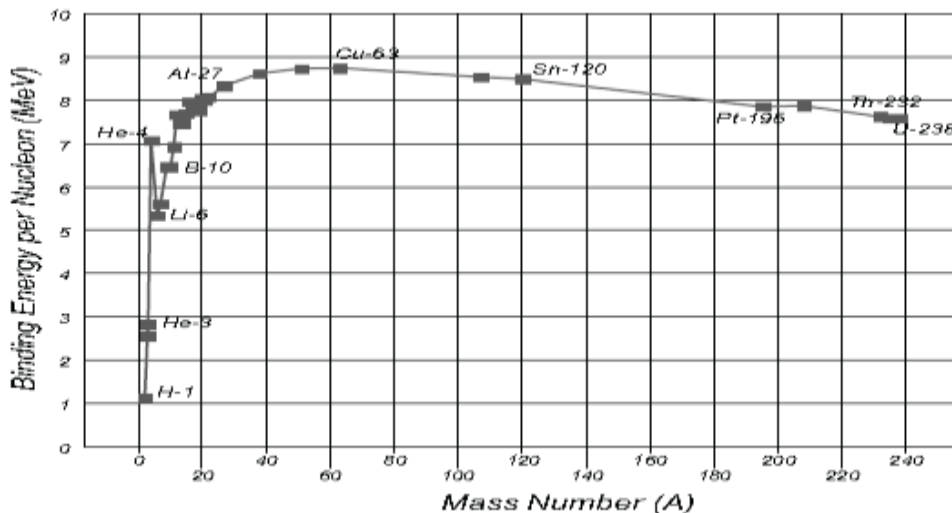


Figure 1.3: Dependence of Binding Energy per Nucleon on Mass Number

For $A < 28$ there is a prominent cyclic re-occurrence because of strong binding energy and we give them $A = 4n$ where n is integer.

Separation Energy

The work necessary to separate a proton, neutron, deuteron or α -particle from a nucleus is called Separation Energy.

$$S_n = M[A-1, Z] + M_N - M(A, Z) \text{ MeV}$$

$$S_p = M[A-1, Z-1] + M_H - M(A, Z) \text{ MeV}$$

Examples

For ${}^{16}_8\text{O}$

$$\begin{aligned} S_n &= [M(A-1, Z) + M_N - M(A, Z)] \text{ MeV} \\ &= (15.003070 + 1.008665 - 15.99491) \text{ MeV} \\ &= 0.01682 \text{ MeV} \\ &= 15.65942 \text{ MeV} \end{aligned}$$

$$\begin{aligned} S_p &= [M(A-1, Z-1) + M_H - M(A, Z)] \text{ MeV} \\ &= (15.000108 + 1.007825 - 15.99491) \text{ MeV} \\ &= 12.18679 \text{ MeV} \end{aligned}$$

for ${}^{17}_8\text{O}$

$$\begin{aligned} S_n &= [M(A-1, Z) + M_N - M(A, Z)] \text{ MeV} \\ &= (15.994915 + 1.008665 - 16.999133) \text{ MeV} \\ &= 4.140157 \text{ MeV} \end{aligned}$$

$$\begin{aligned} S_p &= [M(A-1, Z-1) + M_p - M(A, Z)] \text{ MeV} \\ &= M(A-1, Z-1) + 1.007825 - 16.999133) \text{ MeV} \end{aligned}$$

Generally, S_p or S_n is large for nuclei with even N or even Z , then odd N or odd Z . If S_n or S_p is plotted against A , there are some areas of discontinuities at $A=2, 8, 20, 50, 82, 126$ (magic numbers). The energy required to remove magic numbers are higher than ordinary numbers.

SELF-ASSESSMENT EXERCISE 2

- i. Calculate the binding energy and separation energy for the following atoms using the table in the appendix: (a) ${}^{16}_8\text{O}$ (b) ${}^{17}_8\text{O}$
- ii. Define the following terms:
 - (a) Nuclear binding energy - Binding energy is the energy that must be supplied to dissociate the nucleus into separate

nucleus or the energy released when the separated nucleons were assembled into a nucleus.

(b) Separation energy – This is the work necessary to separate a proton, neutron, deuteron or α -particle from a nucleus.

3.4 Nuclear Force

Previously, we considered the curve of binding energy per nucleon against mass number (B/A against A curve). The value of B/A is approximately constant and it is about 8 MeV/nucleon. This is about a million time higher than the binding energy of an electron in the hydrogen atom (which is 13.6 eV). In other words, the force that keeps the nucleus together is much stronger than the electrical force which keeps the atom together. Also, this nuclear force must be stronger than the electrical force between the protons since protons are bound in the nuclei. The nuclear force is often called the strong interaction, because it is the strongest of the four basic forces or interactions found in nature.

3.5 Magnetic Dipole and Electric Quadrupole Moments of the Deuteron

In our discussion so far, we have assumed that the deuteron is in an $l = 0$ state and so we do not expect any orbital angular momentum. In other words, one expects the total magnetic moment of the deuteron to be the sum of the magnetic moments due to the spin motions of the neutron and the proton. Thus, we expect a value:

$$\mu (\text{neutron}) + \mu (\text{proton}) = 0.8797 \frac{e\hbar}{2M_p}$$

for the deuteron magnetic moment.

However, the experimentally obtained value of the magnetic moment of the deuteron was:

$$\text{Deuteron magnetic moment, } \mu_d = 0.8574 \frac{e\hbar}{2M_p}$$

the difference is about 2.5 per cent, which is small but significant. The simplest explanation for this discrepancy is that our assumption, that the deuteron is in $l = 0$ state is not entirely correct. There must be some orbital momentum in the deuteron nuclear system. This implies that the wave functions describing the deuteron are not spherically symmetric. The spherically symmetric wave functions have only r dependence and no (θ, φ) dependence. This is what happens in an $l = 0$ state; the wave function is a function of r only and so, is spherically symmetric.

More experimental evidence for the lack of spherical symmetry is provided by the observation of an electric quadrupole moment for the deuteron:

Deuteron electric quadrupole moment = $0.282 F^2$

This value though small, is non-zero and significant. A non-zero quadrupole moment for the deuteron means that the electric charge in the neutron is not spherically distributed, but it is slightly deformed in a prolate, non-spherical shape. This clearly indicates that the wave function is not the simple spherically symmetrical ($l = 0$) one, a fact in agreement with the conclusion from the value of the magnetic dipole moment.

4.0 CONCLUSION

In conclusion, we have been able to examine the nuclear structure and its constituents. Also, we studied the nuclear size and the nuclear binding forces.

5.0 SUMMARY

In this unit, we have been able to understand that the nucleus of an atom consists of the proton and neutron which is surrounded by electrons. The nuclear binding force and separation energies were examined and calculated for some atoms.

6.0 TUTOR-MARKED ASSIGNMENT

1. A nucleus with $A = 235$ splits into two nuclei whose mass numbers are in the ratio 2:1, find the radii of the new nuclei.
2. Calculate the binding energy and separation energy of protons and neutrons of the following atoms: (i) ${}_{12}^{24}\text{Mg}$ (ii) ${}_{13}^{27}\text{Al}$.
3. Define the following terms: (i) isotopes (ii) isobars (iii) isotones.

7.0 REFERENCES/FURTHER READING

Greiner, W. & Maruhn, J. A. *Nuclear Models*.

UNIT 2 NUCLEAR MODELS

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- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Nuclear Models
 - 3.2 The Liquid Drop Model
 - 3.3 The Shell Model
 - 3.4 The Collective Model
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

These models are proposed models that are used to explain the nuclear forces in the nucleus of an atom.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- highlight the properties of nuclear forces
- explain all the proposed models used to describe the nuclei binding energy of forces.

3.0 MAIN CONTENT

3.1 Nuclear Models

These are just meant to explain the nuclear forces in the nucleus of an atom. All that is known about the nuclear force is that:

- i. Short range of operation of the order of $\approx 10\text{cm}$
- ii. Independent of charge i.e. exists equally between proton and neutron.
- iii. Strong force which can overcome the Coulomb force.
- iv. It is a repulsive force to certain extent in order to prevent the collapse of the nucleus.

The development of nuclear models is connected with two observations i.e. the stability of nuclides with the number of protons or neutrons equal to any one of the magic number and the relation between binding energy

and mass number have been used as the tests for the validity of the models. With this in mind there are three nuclear models that are of importance. These are:

- i. The liquid drop model
- ii. The shell model or independent particle model
- iii. The collective model.

3.2 The Liquid Drop Model

In the liquid drop model, nuclei are considered to behave like drops of incompressible liquid, i.e., like drops of very high density (density of the order of 10^{14}kg/m^3). With this point of view and using concepts from classical physics (i.e. physics of continua), concepts like surface tension and surface energy, volume, and energy predictions are made about the overall behaviour of nuclides. One of the predictions, as mentioned above, is about the relation between the binding energy and mass number for nuclides.

Using the liquid drop point of view, the binding energy of a nuclide would be the resultant of five energies, the volume energy (E_v), the surface energy (E_s), the energy due to asymmetry (i.e., deviation from a stable configuration), (E_a), the energy due to even-odd combination of nucleons in a nuclide, (E_δ) and coulomb energy (for protons), (E_c). With this, the total binding energy for a nuclide is

$$B.E = E_v + E_s + E_a + E_\delta + E_c \quad (2.1)$$

The relations for $E_v, E_s, E_a, E_\delta, E_c$ are

$$\begin{aligned} E_v &= C_v A \\ E_s &= -C_s A^{2/3} \\ E_a &= \frac{-C_a[(A - Z) - Z]^2}{A} = \frac{-C_a(A - 2Z)^2}{A} \\ E_\delta &= \begin{cases} \delta/2a, & \text{for even-even nuclides;} \\ 0, & \text{for even-odd or odd-even nuclides;} \\ -\delta/2a & \text{for odd-odd nuclides.} \end{cases} \\ E_c &= \frac{-4C_c Z(Z - 1)}{A^{1/3}} \end{aligned}$$

where

$$\begin{aligned} C_v &\approx 14 \text{ MeV} \\ C_s &\approx 13.1 \text{ MeV} \\ C_a &\approx 19.4 \text{ MeV} \\ \delta &\approx 270 \text{ MeV} \end{aligned}$$

$$C_c \approx 14 \text{ MeV}$$

The mass of a nuclide is given by

$$M_n = (A - Z)m_n + Zm_p - \frac{B.E.}{c^2} \quad (2.2)$$

where the B.E. is as given in Eq.2.1. This relation is referred to as the semi-empirical mass relation. These predictions by the liquid drop model and other predictions such as predictions about the fission of nuclides are in agreement with observation.

3.3 The Shell Model

In the shell model, nucleons are treated as individual particles existing within the potential created by the nucleons of a nuclide. Hence the shell model is also referred to as the independent particle model. This is similar to the treatment of the electrons of atoms in atomic physics. In the shell model of nuclear physics though, the potential is due to both the electromagnetic potential and the nuclear potential. Thus, the potential that a nucleon finds itself in a nuclide is

$$V(r) = V_n(r) = \frac{-V_0}{1 + e^{-(r-R)/a}} \quad \text{for a neutron}$$

$$V(r) = V_n(r) + V_e(r) = \frac{-V_0}{1 + e^{-(r-R)/a}} + V_e(r) \quad \text{for a proton}$$

$$V(r) = \begin{cases} \frac{Ze^2}{4\pi\epsilon_0 R_e} \left[1 + \frac{1}{2} \left(1 - \left(\frac{r}{R_e} \right)^2 \right) \right], & \text{for } r < R_e; \\ \frac{Ze^2}{4\pi\epsilon_0 r}, & \text{for } r \geq R_e \end{cases}$$

where

$$V_0 = 57 \pm \frac{27(A-2Z)}{A} \text{ MeV}$$

(+) for protons and (-) for neutrons

$$R = 1.25 A^{4/3} F, \text{ a constant for a nuclide}$$

$$a = 0.65F, \text{ a constant.}$$

With this relation for the potential and the assumption that for nucleons there is a strong spin orbit coupling, solving Schrödinger's equation for nucleons in nuclides predicts the fact that for values of Z or (A-

Z)=2,8,20,28,50,82,126, and 184 there would be closed shells, i.e., stable nuclides in agreement with what is observed experimentally.

3.4 The Collective Model

The term collective model is not a specific and clear term. In much of the literature on nuclear model, the term is used to imply any model that deals only with the collective behavior of nucleons. In view of this, even the liquid drop model can be looked upon as a collective model. In some parts of the literature, the term is used with any model that takes collective effects into account, and this is what we shall take as the meaning in this text. Here it is the latter usage that is being taken.

Nuclide can have rotational energy or vibrational energy. In both cases, the energies will be integer multiples of a phonon $h\nu_\lambda$. With this, in the overall modeling of the structure of nuclei, first making certain assumptions about the nature of nuclei, the Hamiltonian for a certain model is derived. Then the Hamiltonian is solved and the wave function for the nuclide or nucleon of interest determined. Then predictions with the wave function are compared with experimental observation. From this, the model is evaluated depending on the degree of agreement and disagreement. The experimental observation that played an important role in the development of the collective nuclear model is that of photon-nuclear reaction, the occurrence of giant resonances in photonuclear reactions. For the collective behaviour of nucleons, nuclides are considered to consist of two fluids: Proton fluids and a neutron fluids.

The proton and neutron liquids could undergo rotational and vibrational motion at their surfaces. In addition to this, in the presence of electromagnetic fields there could be density fluctuations of the density of proton $\rho_p(r, t)$, and the density of neutron $\rho_n(r, t)$ and resulting dipole, quadruple, etc., resonances. This is due to the fact that electromagnetic fields (and photons) react only with the protons.

In addition to these two collective behaviors, the structure of a nuclide can be affected by the individual motion of the individual nucleons comprising it.

Putting these three factors together, the Hamiltonian for a nucleus is:

$$\hat{H} = \hat{H}_{surface\ effects} + \hat{H}_{giant\ resonance} + \hat{H}_{interaction} \quad (2.3)$$

Where

- \hat{H}_s , the halmitonian due to surface effects
- \hat{H}_{gr} , the halmitonian due to giant resonance

\hat{H}_{int} , the hamiltonian due to the interaction between individual motion and coll

The Hamiltonian for the nucleus is

$$\hat{H}_s = \hat{H}_s + \hat{H}_{gr} + \hat{H}_{int} \quad \& \quad \hat{H}_s = \hat{H}_{vib} + \hat{H}_{rot}$$

Therefore $\hat{H} = \hat{H}_{vib} + \hat{H}_{rot} + \hat{H}_{gr} + \hat{H}_{int}$

For the individual particle (nucleon), the Hamiltonian would be the sum of the Hamiltonian of the collective motion, the Hamiltonian of the individual particle and the Hamiltonian that takes into account the interaction between the individual motion and the collective motion. Thus for a particle,

$$\hat{H} = \hat{H}_{part} + \hat{H}_{coll} + \hat{H}_{int}$$

(2.4)

In collective models, therefore individual particles move in a deformed shell potential and the nucleus as a whole behave like an incompressible fluid with its motions (vibratory and rotational) being influenced and affected by the motion of the individual particles inside the nuclei.

SELF-ASSESSMENT EXERCISE

- i. List the properties of nuclear force..
- ii. List the Nuclear models..
- iii. Briefly explain each of the listed models in question 2 above.

4.0 CONCLUSION

In conclusion, we have been able to examine the different models proposed to describe the nucleus of an atom.

5.0 SUMMARY

In this unit, we have been able to understand that the nucleus of an atom can be likened to different forms of matter. This likeness led to the development of equations for the binding energies of a nucleus.

6.0 TUTOR-MARKED ASSIGNMENT

1. What do you understand by the “nuclear binding energy”.
2. State the Weizsacher’s semi empirical equation and explain each term of the equation.

7.0 REFERENCES/FURTHER READING

Greiner, W. & Maruhn, J. A. *Nuclear Models*.

UNIT 3 RADIOACTIVITY

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 Stability of Nuclides
 - 3.2 Kinematics of Radioactivity
 - 3.3 Radioactive Series and Age Determination Using Radioisotopes
- 4.0 Conclusion
- 5.0 Summary
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1.0 INTRODUCTION

The decay of the nucleus of an atom can either be natural or artificial. This decay occurs in nuclide in order that they may attain stability. This disintegration or decay occurs with the emission of some particles or energy.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain radioactivity and its kinematics
- highlight and list the properties of particles released during radioactivity
- explain the radioactive series
- explain age determination using radioisotopes.

3.0 MAIN CONTENT

3.1 Stability of Nuclides

The stability of nuclides is mainly determined by the atomic mass (A) and the (N/Z) ratio. The condition for the stability of light elements is

$N/Z=1$, and for heavy elements, $N/Z \approx 1.5$ Nuclides that are not stable

due to this ratio seek stability by undergoing inter-nuclear spontaneous transformation which shifts the N/Z ratio to a more stable configuration.

During this transformation, the following could occur:

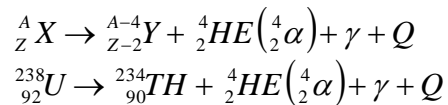
1. ${}_1^1p \rightarrow {}_0^1n + {}_1^0\beta + V.$
2. ${}_0^1n \rightarrow {}_1^1p + {}_1^0\beta + V.$
3. α particles may be emitted (that is 2 protons and 2 neutrons)
4. Splitting of nucleus into two nearly equal fragments through nuclear fission.

Radioactivity can therefore be defined as the tendency of unstable nuclides, seeking to become stable through the emission of particles and energy. The emitted particles during radioactivity are referred to as nuclear radiation.

Nuclear radiations can well be referred as ionizing radiation because they have sufficient energy to cause the production of ion pairs in any medium which they pass through. The most common particles usually emitted are β^+ , β^- , α and γ particle.

– α – particles

The nuclear transformation equation for an α - particle is given by:



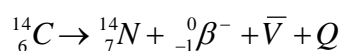
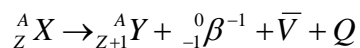
Properties

1. α - particles can be stopped by a thin sheet of paper
2. They cause intense ionization in air
3. a group of α - particle emitted from the same type of nuclides usually have definite velocity and energy
4. α - particles cover a definite distance in a given material practically without any loss of intensity and suddenly in a small distance are absorbed completely

The definite distance traveled with a given material is called the RANGE

- β - Particles

The nuclear transformation equation for an β -particle is given by:



Properties

1. They cause less ionization in air.
2. 100 times more penetrating than α particle and can penetrate a sheet of aluminum, a few millimeters thick.
3. A particular β - active element emits β - particles with energies varying between zero and certain maximum. This maximum energy is called the end point energy.

X-Rays and γ Rays

These are part of electromagnetic radiation.

Table 3.0

γ RAYS.	X-RAYS
Short wavelength compared with X-ray	Longer wavelength than γ - ray
More energetic than x-rays and more penetrating than β RAYS. (\approx 100 times).	Less energetic than γ - ray
They are emitted from the atomic nucleus	They are emitted by electrons

The ability of the γ ray to be able to dislodge electrons from both the outer and inner orbit can be done in the following ways:

1. Photoelectric effect
2. Compton effect
3. Pair production

SELF-ASSESSMENT EXERCISE 1

- i. Define the term radioactivity.
- ii. What are the nuclear radiations?
- iii. List and briefly explain the different forms of nuclear radiations.

3.2 Kinematics of Radioactivity

When a nucleus disintegrates by emitting particle α, β, γ or capture an electron from an atomic shell (k-capture), this process is called *Radioactive decay*. All nuclear decay follows a single law called a *Decay law*.

The number of nuclei of a given radioactive sample disintegrating per second is called the *Activity* of the sample i.e. $A = \frac{dN}{dt}$.

The activity ($\frac{dN}{dt}$) at any instant of time is proportional to the number N of the parents type present at that time.

$$\text{i.e. } \frac{dN}{dt} \propto N$$

$$A = \frac{dN}{dt} = -\lambda N \quad \text{----- (1)}$$

Where λ is the decay constant or disintegration constant which only depends on the nature or characteristics of the radioactive sample and not on the amount of substance. Also λ - gives the probability of decay per unit interval of time.

From (1) above

$$\int_{N_0}^N \frac{dN}{N} = \int_0^t -\lambda dt$$

$$[\ln N]_{N_0}^N = -\lambda [t]_0^t$$

$$\ln N - \ln N_0 = -\lambda t$$

$$\ln \left(\frac{N}{N_0} \right) = -\lambda t$$

$$\frac{N}{N_0} = e^{-\lambda t}$$

$$N = N_0 e^{-\lambda t} \quad \text{----- (2)}$$

Multiply both sides of equation (2) by λ

$$\lambda N = \lambda N_0 e^{-\lambda t}$$

$$A = A_0 e^{-\lambda t} \quad \text{----- (3)}$$

From equation (1)

Where A stands for the activity. Activity is measured in Becquerel (1 dps) and 1 curie (1 ci) = 3.7×10^{10} Bq.

A time interval during which half of given sample of radioactive substance decays is referred to as the *Half Life*.

$$\text{i.e. } A = \frac{A_0}{2} = A_0 e^{-\lambda T_{1/2}}$$

$$\frac{1}{2} = e^{-\lambda T_{1/2}}$$

Taking the log of both sides

$$-\ln 2 = -\lambda T_{1/2}$$

$$T_{1/2} = \frac{\ln 2}{\lambda} = \frac{0.693}{\lambda}$$

Since individual radioactive atoms may have life spans between 0 and ∞ , we can then talk of average life or mean life.

$$T_{\text{mean}} = \frac{\text{total life time of all nuclei in a given sample}}{\text{total number of nuclei in that sample}}$$

$$= \frac{t_1 dN_1 + t_2 dN_2 + \dots + t_N dN_N}{dN_1 + dN_2 + \dots + dN_N}$$

$$T_{\text{mean}} = \frac{\int_0^{N_0} t dN}{\int_0^{N_0} dN} = \frac{-1}{N_0} \int_0^{N_0} t dN \quad \text{--- (1)}$$

$$\text{but } N = N_0 e^{-\lambda t}$$

$$\lambda N = \lambda N_0 e^{-\lambda t}$$

$$\text{Since } \frac{dN}{dt} = -\lambda N$$

$$\frac{dN}{dt} = -\lambda N_0 e^{-\lambda t}$$

$$dN = -\lambda N_0 e^{-\lambda t} dt \quad \text{--- (2)}$$

Substitute (2) into (1)

$$= -\frac{1}{N_0} \int_0^{N_0} t (+\lambda) N_0 e^{-\lambda t} dt$$

$$T_{\text{mean}} = \lambda \int_0^{\infty} t e^{-\lambda t} dt \quad \text{--- (3)}$$

it is 0 \rightarrow ∞ because from

$$N = N_0 e^{-\lambda t}$$

$$\text{for } N = 0 = N_0 e^{-\lambda t}$$

$$0 = e^{-\lambda t}$$

$$O = \frac{1}{e^{\lambda t}} \Rightarrow t \rightarrow \infty$$

$$\text{and for } N = N_o = N_o e^{-\lambda t}$$

$$1 = e^{-\lambda t}$$

$$1 = \frac{e}{e^{\lambda t}} = t \rightarrow 0$$

For the relationships to hold, then integrate equation (3) by parts.
From equation (3).

$$T_{mean} = \lambda \int_0^{\infty} t e^{-\lambda t} dt.$$

$$\Rightarrow \int u dv = uv - \int v du$$

$$\text{where } u = t \text{ and } dv = e^{-\lambda t}$$

$$\Rightarrow du = 1 dt \text{ and } v = \int e^{-\lambda t} = -\frac{e^{-\lambda t}}{\lambda} + c$$

$$T_{mean} = \lambda \left[t \left(\frac{-e^{-\lambda t}}{\lambda} \right) - \int_0^{\infty} -\frac{e^{-\lambda t}}{\lambda} dt \right]$$

$$= \lambda \left[\frac{-te^{-\lambda t}}{\lambda} - \left(\frac{e^{-\lambda t}}{\lambda^2} \right) \right]_0^{\infty}$$

$$T_{mean} = \lambda \left[\frac{-te^{-\lambda t} (\lambda) - e^{-\lambda t}}{\lambda^2} \right]_0^{\infty}$$

$$= \frac{\lambda}{\lambda^2} \left[-te^{-\lambda t} (\lambda) - e^{-\lambda t} \right]_0^{\infty} + c$$

$$= \frac{1}{\lambda} [0 + 1] + c$$

$$= \frac{1}{\lambda}$$

Radioactive Equilibrium

Considering this decay process

$A \rightarrow B \rightarrow C$ (Stable). Since the number of nuclei entering B will be the decay of A.

$$\Rightarrow \frac{-dN_A}{dt} = \lambda_A N_A$$

The number of nuclei leaving B will be $\lambda_B N_B$

Therefore, the net change in the number of nuclei per second of B is

$$\frac{dN_B}{dt} = \lambda_A N_A - \lambda_B N_B \text{ ————— (1)}$$

But this is a first order (linear d.e.) and $N_A = N_o e^{-\lambda_A t}$

Rewriting equation (1)

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_A$$

$$\frac{dN_B}{dt} + \lambda_B N_B = \lambda_A N_o e^{-\lambda_A t} \quad \text{---(2)}$$

Comparing this with $\frac{dy}{dx} + py = Q$

Where our integrating factor = $e^{\int p dx}$

Therefore, our integrating factor = $e^{\int \lambda_B dt} = e^{\lambda_B t}$

Multiply both sides of equation by integrating factor = $e^{\lambda_B t}$

$$e^{\lambda_B t} \frac{dN_B}{dt} + \lambda_B N_B (e^{\lambda_B t}) = \lambda_A N_o e^{-\lambda_A t} (e^{\lambda_B t}).$$

$$\frac{d}{dt} (e^{\lambda_B t} N_B) = \lambda_A N_o e^{-\lambda_A t} (e^{\lambda_B t})$$

Now integrating both sides with t

$$e^{\lambda_B t} N_B = \int \lambda_A N_o e^{-\lambda_A t} e^{\lambda_B t} dt$$

$$= \lambda_A N_o \int e^{-\lambda_A t} \cdot e^{\lambda_B t} dt$$

Using integration by parts

$$\int e^{-\lambda_A t} \cdot e^{\lambda_B t} dt$$

$$u = e^{-\lambda_A t} \text{ and } dv = e^{\lambda_B t}$$

$$du = -\lambda_A e^{-\lambda_A t} \text{ and } v = \frac{e^{\lambda_B t}}{\lambda_B}$$

$$\int u dv = uv - \int v du$$

$$= e^{-\lambda_A t} \left(\frac{e^{\lambda_B t}}{\lambda_B} \right) - \int \frac{e^{\lambda_B t}}{\lambda_B} \cdot -\lambda_A e^{-\lambda_A t} dt$$

$$= \frac{e^{\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} + \frac{\lambda_A}{\lambda_B} \int e^{\lambda_B t} \cdot e^{-\lambda_A t} dt \quad \text{---(3)}$$

Since we are back to the initial integral, put

$$I = \int e^{\lambda_B t} \cdot e^{\lambda_A t}$$

Then equation (3) is

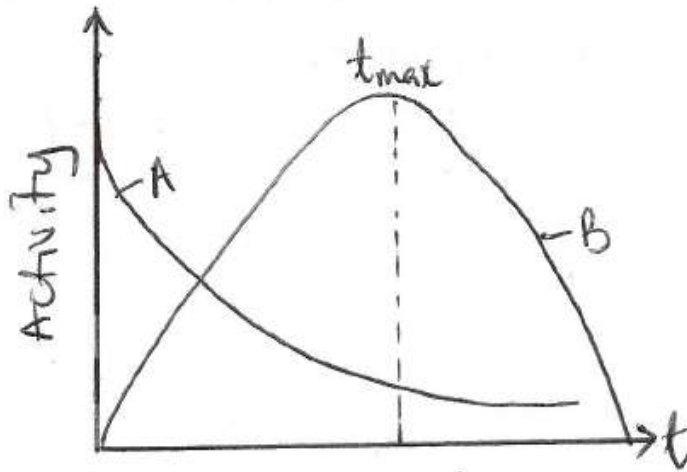
$$\begin{aligned}
 I &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} + \frac{\lambda_A}{\lambda_B} I \\
 I - \frac{\lambda_A}{\lambda_B} \cdot I &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \\
 I \left(1 - \frac{\lambda_A}{\lambda_B} \right) &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \\
 I &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \div \left(1 - \frac{\lambda_A}{\lambda_B} \right) + C \\
 &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \div \left(\frac{\lambda_B - \lambda_A}{\lambda_B} \right) + C \\
 &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B} \times \frac{\lambda_B}{\lambda_B - \lambda_A} + C \\
 &= \frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B - \lambda_A} + C
 \end{aligned}$$

Then the equation will now be

$$\begin{aligned}
 e^{-\lambda_B t} N_B &= \lambda_A N_0 \left[\frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B - \lambda_A} \right] + C \\
 N_B &= \frac{\lambda_A N_0}{e^{\lambda_B t}} \left[\frac{e^{-\lambda_A t} \cdot e^{\lambda_B t}}{\lambda_B - \lambda_A} \right] + C \\
 N_B &= \frac{\lambda_A N_0}{\lambda_B - \lambda_A} \left[e^{-\lambda_A t} \right] + C \\
 N_B &= \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} \left[e^{-\lambda_A t} - e^{-\lambda_B t} \right] \text{-----(4)}
 \end{aligned}$$

Cases

- At t maximum, $\frac{dN}{dt} = 0$. i.e. $\lambda_A N_A = \lambda_B N_B$ and the activity of the parent and daughter are said to be at equilibrium. This is called the *Ideal Equilibrium*



2. Considering a case whereby the daughter is short lived than the parent i.e. $T_A > T_B$ from the equation (1) above, the activity of B is

$$\lambda_B N_B = \frac{N_0 \lambda_A}{\lambda_B - \lambda_A} \cdot \lambda_B [e^{-\lambda_A t} - e^{-\lambda_B t}] \text{-----(5)}$$

but $\lambda_A N_A = \lambda_A N_0 e^{-\lambda_A t}$

$$N_0 \lambda_A = \frac{\lambda_A N_A}{e^{-\lambda_A t}} \text{-----(6)}$$

Introducing (6) into (5)

$$\lambda_B N_B = \frac{\lambda_A N_A}{e^{-\lambda_A t}} \cdot \frac{\lambda_B}{\lambda_B - \lambda_A} [e^{-\lambda_A t} - e^{-\lambda_B t}]$$

$$= \frac{\lambda_A N_A \cdot \lambda_B}{\lambda_B - \lambda_A} [1 - e^{-\lambda_B t} \cdot e^{\lambda_A t}]$$

$$\lambda_A N_A \cdot \frac{\lambda_B}{\lambda_B - \lambda_A} = [1 - e^{-\lambda_B t + \lambda_A t}]$$

$$\lambda_B N_A = \lambda_A N_A \cdot \frac{\lambda_B}{\lambda_B - \lambda_A} [1 - e^{-(\lambda_B t - \lambda_A t)}]$$

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{\lambda_B}{\lambda_B - \lambda_A} [1 - e^{-(\lambda_B t - \lambda_A t)}]$$

Since $T = 1/\lambda$ therefore $\lambda = 1/T$

$$\Rightarrow \lambda_B = \frac{1}{T_B}, \lambda_A = \frac{1}{T_A}$$

$$\frac{\lambda_B}{\lambda_B - \lambda_A} = \frac{\frac{1}{T_B}}{\frac{1}{T_B} - \frac{1}{T_A}} = \frac{\frac{1}{T_B}}{\frac{T_A - T_B}{T_B T_A}} = \frac{1(T_B T_A)}{T_B(T_A - T_B)} = \frac{T_A}{T_A - T_B}$$

Then the equation becomes

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} [1 - e^{-(\lambda_B - \lambda_A)t}]$$

Since $(\lambda_B - \lambda_A)_t = \left(\frac{T_A - T_B}{T_B T_A}\right) \cdot t$

$$= \left(\frac{T_A - T_B}{T_A}\right) \frac{1}{T_B} \cdot t$$

$$= \left(\frac{T_A - T_B}{T_A}\right) \lambda_B \cdot t$$

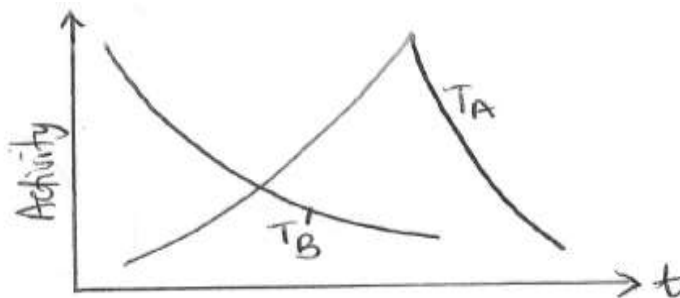
$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} [1 - e^{-(T_A - T_B)\lambda_B t}]$$

At large time, t

$$\frac{\lambda_B N_B}{\lambda_A N_A} = \frac{T_A}{T_A - T_B} \text{----- (7)}$$

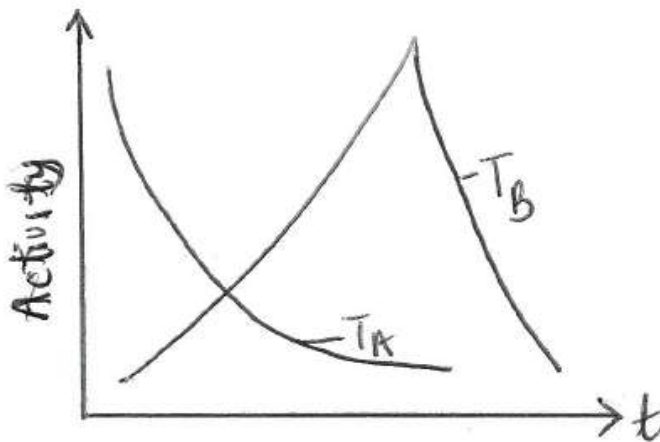
When this equation (7) holds, this implies that transient equilibrium exists between the parent and daughter, and its correlation is that

$$\frac{T_A}{T_A - T_B} > 1. \text{ For } (T_A > T_B)$$



3. For the case in which the daughter is long-lived, then the parent i.e. $(T_A < T_B)$. It follows from equation (7) that the ratio $\frac{\lambda_B N_B}{\lambda_A N_A}$

increases as t increases. Therefore, after sufficient time, the activity of the daughter becomes independent of that of the residual activity of the parents.



4. If $T_A \gg T_B, \lambda_A \ll \lambda_B$
 then $\lambda_B N_B = \lambda_A N_A [1 - e^{-\lambda_B t}]$
 for $t = T_B$
 $e^{-\lambda_B t} = 0$

$\lambda_B N_B = \lambda_A N_A$ (secular equilibrium)

i.e. $\frac{\lambda_B}{\lambda_A} = \frac{T_A}{T_B} = \frac{N_A}{N_B}$

All these can as well be called serial transformation.

SELF-ASSESSMENT EXERCISE 2

- i. Define the following terms:
 - (a) Activity
 - (b) Half life
 - (c) Decay
- ii. How many kinds of radioactivity equilibrium exist?

3.3 Radioactive Series

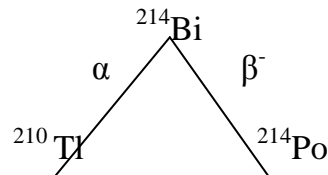
Radio nuclides that are related constitute a decay chain or series. The successive daughter products are formed through the emission of β and α particles leading to stable end-product. There are four known series in nature but that of neptinium is artificial. They are:

1. Thorium $4n$
2. Neptunium $4n+1$
3. Uranium $4n+2$
4. Actinium $4n+3$

There are also some that are not of large atomic number e.g. ^{40}K and all these constitute a source of radioactivity in the earth crust.

Branching

Normally, a particular radio nuclide is supposed to decay through either α and β decay. In some cases, some will decay through α and β or both. This phenomenon is known as branching e.g.

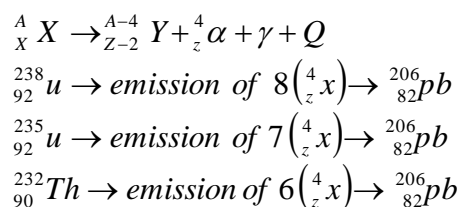


Age Determination Using Radioisotopes

Radioactivity is the best clock in determining or usually applied to estimate the absolute age of geological materials because it is totally not affected by environmental changes of natural processes like earthquakes, storms etc. Some of the radioisotopes that are useful for geological age dating include $^{87}\text{Sr} / ^{87}\text{Rb}$, $^{14}\text{C} / ^{12}\text{C}$, Pb / U , Pb / Th etc.

The half life of these radioisotopes is usually used for determining the age ranges of interest.

The determination of geological ages is done very often by the *lead method* which involves



These are natural decay series in which after sufficient time e.g. billion years only uranium and lead are the elements left in appreciable amounts. This is because all elements in the uranium series are in secular equilibrium with the parents except ${}^{206}_{82}\text{Pb}$ which is not in secular equilibrium.

From

$$N_B = \frac{N_o \lambda_A}{\lambda_B - \lambda_A} [e^{-\lambda_A t} - e^{-\lambda_B t}]$$

where $N_B = N_{Pb}$

$$\lambda_B = \lambda_{Pb} - 0 \text{ (stable)}$$

$$\lambda_A = \lambda_u$$

$$N_o = N_v$$

Then substitute

$$\begin{aligned} \Rightarrow N_{Pb} &= \frac{N_v \lambda_A}{\lambda_B - \lambda_A} [e^{-\lambda_u t} - e^{-0}] \\ &= -N_v [e^{-\lambda_u t} - 1] \\ N_{Pb} &= N_v [1 - e^{-\lambda_u t}] \text{-----(1)} \end{aligned}$$

Therefore, number of uranium atoms originally present = present number of lead atoms + present number of uranium i.e.

$$N_v = N_{Pb} + N_u \text{-----(2)}$$

Solving equation (1) and (2) simultaneously

$$\begin{aligned} N_{Pb} &= N_v [1 - e^{-\lambda_u t}] \\ N_{Pb} &= N_v - N_v e^{-\lambda_u t} \text{-----(3)} \\ N_{Pb} &= N_v - N_u \text{-----(4)} \end{aligned}$$

Then we have

$$\begin{aligned} 0 &= -N_v e^{-\lambda_u t} + N_u \\ N_v e^{-\lambda_u t} &= N_u \\ e^{-\lambda_u t} &= \frac{N_u}{N_v} \\ -\lambda_u t &= \log \frac{N_u}{N_v} \\ t &= -\frac{1}{\lambda_u} \log \frac{N_u}{N_v} \\ t &= -\frac{1}{\lambda_u} \log \left[\frac{N_u}{N_v + N_{Pb}} \right] \\ t &= -\frac{1}{\lambda_u} [\log N_u - \log (N_u + N_{Pb})] \\ &= \frac{1}{\lambda_u} [\log (N_u + N_{Pb}) - \log N_u] \end{aligned}$$

$$t = \frac{1}{\lambda_u} \log \left[\frac{N_u + N_{Pb}}{N_u} \right]$$

SELF-ASSESSMENT EXERCISE 3

- i. What are radioactive series?
- ii. List the four known radioactive series.
- iii. Give the reasons for using radioactivity to determine the age of matter.

4.0 CONCLUSION

In conclusion, we have been able to examine radioactivity and its kinematics. Also, we examined the use of radioactivity such as in age determination.

5.0 SUMMARY

In this unit, we have been able to explain that nuclides decay to attain stability. Also, this decay is accompanied by the release of particles. This phenomenon can be used to determine the age of matter.

6.0 TUTOR-MARKED ASSIGNMENT

1. Differentiate between the following radioactive particles (α , β and γ).
2. Briefly explain the different kinds of radioactive equilibrium.
3. Briefly explain Branching in radioactive decay.
4. Explain age determination using radioisotopes.

7.0 REFERENCES/FURTHER READING

Greiner, W. & Maruhn, J. A. *Nuclear Models*.

UNIT 4 THE ENERGETICS OF PARTICLE

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Content
 - 3.1 α -decay
 - 3.2 β -decay
 - 3.3 γ -decay
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Different particles are emitted during the disintegration of nuclides. These particles exhibit different characteristics when they are emitted. Therefore, it is important we study these particles in detail to enable us handle them properly when they are released.

2.0 OBJECTIVES

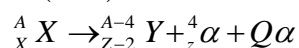
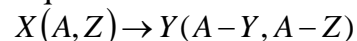
At the end of this unit, you should be able to:

- explain the conditions necessary for α , β and γ decay to be possible.
- list the properties of these particles when they are released.

3.0 MAIN CONTENT

3.1 α -Decay

α -particles are stable and exhibit a definite range when they transverse a medium. For an α decay to be possible, there is a minimum energy requirement.



Where $Q_\alpha = \Delta mc^2$

Since is due to mass defect between m_i and m_f (initial and final masses).

$$m_i = [M(A, Z)] = ZM_p + NM_n - E_{Bi}$$

$$m_f = [M(A - 4, Z - 2)] = (Z - 2)M_p + (N - 2)M_n - E_{Bf}$$

$$M\alpha = [M(4, 2)] = 2M_p + 2M_n - E_{B\alpha}$$

$$\begin{aligned}
 Q_\alpha &= m_i - m_f - m_\alpha \\
 &= -E_{Bi} + E_{Bf} + E_{B\alpha} \\
 &= E_{B\alpha} + E_{Bf} - E_{Bi}
 \end{aligned}$$

And $E_{B\alpha} = 28.3 \text{ MeV}$

$$Q_\alpha = (28.3 + \Delta E_B) \text{ MeV}$$

From semi-empirical formula

Binding energy = $E(Z, A)$

Therefore, the disintegration energy of nuclei Q_α or total energy released

$$\text{in } \alpha\text{-decay is given as } Q_\alpha = 28.3 + \left(\frac{2E}{2A}\right)_2 \Delta A + \left(\frac{2E}{2Z}\right)_A \Delta Z$$

α emission is not possible if $Q_\alpha < 0$, that is Q_α must be > 0 .

From studies, it has been found that $Q_\alpha > 0$ for nuclide for which $Z > 82$

Suppose, the mass of parents = M_P .

Mass of daughter = M_d

Mass of α particle = M_α

Velocity of α particle when emitted = V_α

Velocity of record of daughter = V_d

From the conservation of momentum

$$M_\alpha V_\alpha = M_d V_d \text{ ----- (1)}$$

Total energy = Q_α = final kinetic energy – initial kinetic energy

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha^2 + \frac{1}{2} M_d V_d^2 \text{ ----- (2)}$$

From (1)

$$V_d = \frac{M_\alpha V_\alpha}{M_d} \text{ ----- (3)}$$

Substitute (3) into (2)

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha^2 + \frac{1}{2} M_d \left[\frac{M_\alpha V_\alpha}{M_d} \right]^2$$

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha^2 + \frac{1}{2} M_\alpha^2 V_\alpha^2 / M_d$$

$$= \frac{1}{2} M_\alpha V_\alpha^2 \left[1 + \frac{M_\alpha}{M_d} \right] \text{ ----- (4)}$$

For small approximation

$$\frac{M_\alpha}{M_d} \approx \frac{4}{A-4} \text{ ----- (5)}$$

Substitute (5) into (4)

$$Q_\alpha = \frac{1}{2} M_\alpha V_\alpha^2 \left[1 + \frac{4}{A-4} \right]$$

$$Q_\alpha = E_\alpha \left[\frac{4}{A-4} + 1 \right]$$

$$= E_\alpha \left[\frac{4 + A - 4}{A - 4} \right]$$

$$Q_\alpha = E_\alpha \left[\frac{A}{A-4} \right]$$

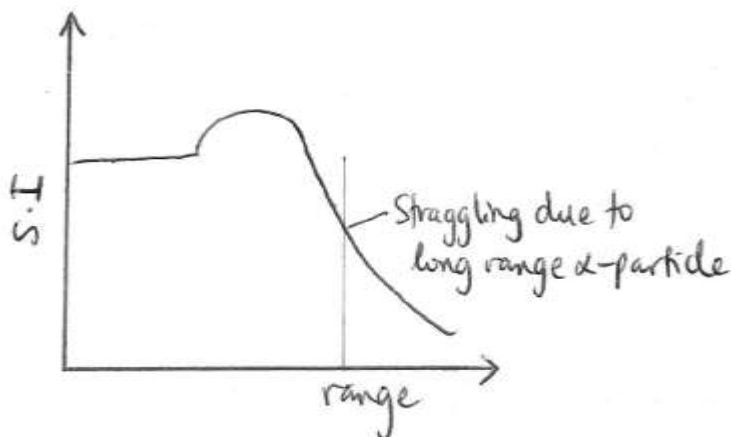
Since A is large

$$Q_\alpha \approx E_\alpha$$

This means most of the energy released is carried away by the α -particle.

Range of α -Particle

α - particles are densely ionizing and lose their energies in quick succession in air or any medium. The number of ions pairs produced per unit length is called *specific ionization* (S.I)



The mean distance travelled by α -particle before absorption is called *range*.

The intersection of α -particle with atoms or molecules of a medium are purely statistical and as a result they do not have the same range as in air.

$$\text{Range, } R = 318E^{3/2}$$

An empirical relation between the range of α -particle and disintegration constant is given by Geiger Nuttal Law

$$\log \lambda = A \log R + B$$

α -decay paradox

Because α -particle is a tightly bound entity we can assure it pre-exists in the nucleus before its emission. For a α -particle to come out or go into the nucleus it implies it must have an energy in the neighborhood of the potential well of the nucleus.

The energy of α -particle usually ranges between 4-8MeV which is far less than what is required to surmount the potential barrier. Classically it is impossible to understand this because it has no chance of leaving the nucleus.

In 1928, George Gamow and independently with others applied wave mechanics to the problem of α -decay paradox and they were able to resolve it. They considered an α -particles as a matter wave. This implies that α -particle as a finite probability of penetrating the wall of thickness where it undergoes series of collisions per second.

It was also discovered that the probability of finding the α -particle outside the nucleus is small but not zero.

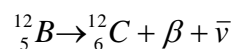
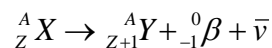
SELF-ASSESSMENT EXERCISE 1

- i. Show that ${}_{94}^{236}\text{Pu}$ is unstable against α -decay.
- ii. List the condition necessary for an α -decay to occur.
- iii. What is meant by the range of an α -particle?

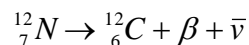
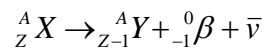
3.2 β Decay

A decay process in which the charge of the nucleus changes without a change in the number of nucleons. There are three types of β decay:

- i. β^- decay e.g

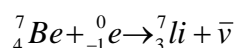
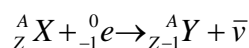


- ii. β^+ decay e.g.

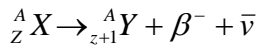


- iii. Electron capture or k-capture

A process through which the nucleus captures an orbital electron, most often from the closest shell to convert a proton to neutron.



Energetic of β^- decay



In terms of nuclear masses

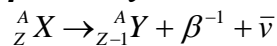
$$\frac{Q}{c^2} = Mn({}^A_Z X) - Mn({}^A_{z+1} Y) - Me$$

And in terms of atomic masses

$$\frac{Q}{c^2} = Ma({}^A_Z X) - Ma({}^A_{z+1} Y)$$

For β^- to be possible, $Q > 0$

β^+ Decay



Nuclear masses

$$\frac{Q}{c^2} = Mn({}^A_Z X) - Mn({}^A_{z-1} Y) - Me$$

Atomic masses

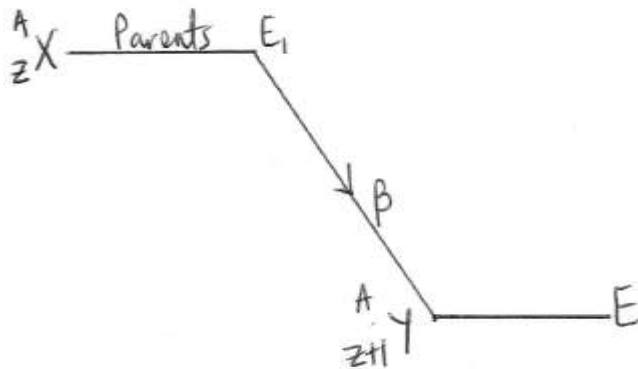
$$\frac{Q}{c^2} = Ma({}^A_Z X) - Ma({}^A_{z-1} Y) - 2Me$$

Electron capture

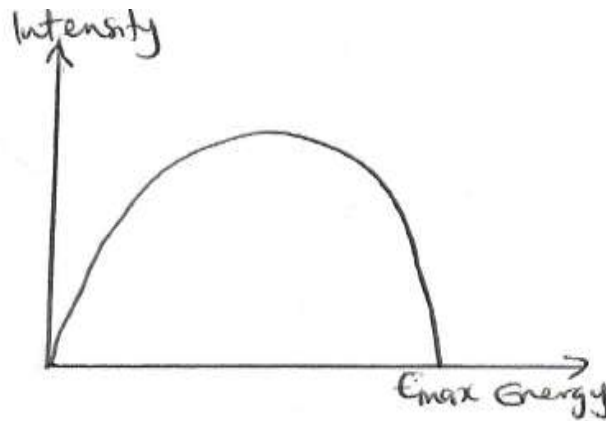
$$\frac{Q}{c^2} = Ma({}^A_Z X) - Ma({}^A_{z-1} Y)$$

β – Spectrum

- 1) Unlike α -rays, the spectrum of β -rays happens to be continuous that is the electrons emitted have different kinetic energies.
- 2) It is also an energy transition between two definite energy states.



- 3) Mono-energetic β -rays forming a line spectrum are expected.



From the figure above, most of the electrons are emitted with only $\frac{1}{3}$ of the energy.

Therefore, this makes one to imagine where the remaining of the $\frac{2}{3}$ of the maximum energy would have gone to. Since measurements like momentum and angular momentum are not conserved. These suggest that a third particle must exist that always accompany the β -decay. It was detected to be neutrino(μ).

Neutrino (μ)

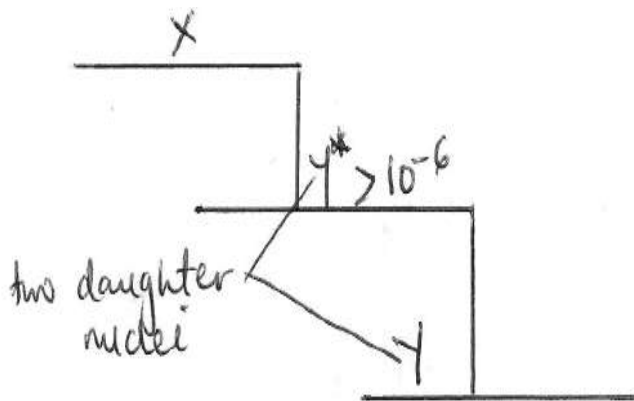
1. Carries away energy equal to the energy different between the observed energy for a β -decay and the maximum energy of the continuous spectrum.
2. To maintain the principle of conservation of energy, neutrino was given the following properties
 - a). 0 charge b).0 mass c). Moves with speed of light d). Spin of $\frac{1}{2}\hbar$.
3. The antiparticle of neutrino, (antineutrino) has the following properties
 - a). 0 charge b). 0 mass c). spin of $\frac{1}{2}\hbar$

3.3 γ - Decay

When a nucleus is in an excited gamma rays are emitted and it is brought to the ground state.

A nucleus is usually left in an excited state after emitting either α or β rays then it is de-excited by emitting gamma rays. Gamma rays are emitted with discrete and definite energies which is an indication of the nuclear structure. The energy carried away is $\Delta E = hf$.

When the mean life time of the excited nucleus is $>10^{-6}$ secs. , the daughter nucleus is said to exhibit nuclear isomerism.



γ^k and γ are nuclear isomers and are chemically and physically the same. The difference is that γ^k is more energetic than γ and it eventually emits the energy as γ ray and returns to ground state.

Sometimes, instead of γ ray being emitted, this excess energy of the excited nucleus may be transferred to an extra nuclear electron to get it from its shell (usually K or L shell). This process is called *internal conversion*.

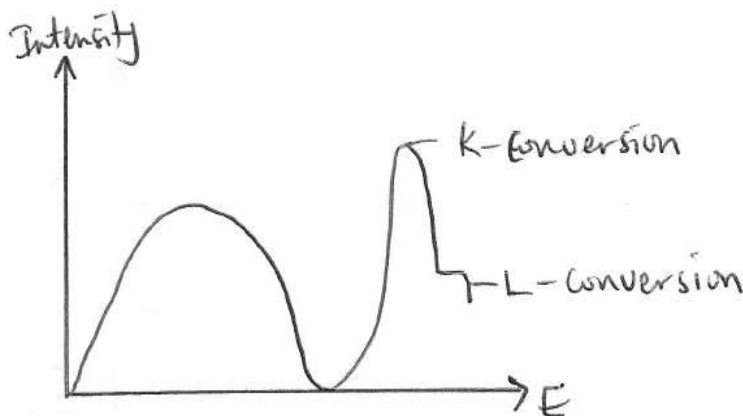
Kinetic energy of converted electron is

$$Ke = \Delta E - Be$$

Be – binding energy of electron

$$\Delta E = E_i - E_f$$

Generally, because of internal conversion, the yield of γ rays in a particular decay $<100\%$. Some of the spikes observed in continuous β – spectrum is usually due to internal conversion process.



SELF-ASSESSMENT EXERCISE 2

- i. Explain the processes involved in a γ decay scheme of a nuclide.
- ii. What is nuclear isomerism?

4.0 CONCLUSION

In conclusion, we have been able to examine the particles emitted during the disintegration of nuclides as well as their energetic.

5.0 SUMMARY

In this unit, we have been able to understand that different particles and antiparticles are emitted during disintegrations. Also, we have understood that conditions surrounding disintegration determines which of the particles are released during any disintegration.

6.0 TUTOR-MARKED ASSIGNMENT

- i. What is meant by the term specific ionization?
- ii. Write a formula relating the range of α particle and the disintegration constant.
- iii. Briefly explain the α decay paradox.
- iv. Show that ${}^{12}_7\text{N} \rightarrow {}^{12}_6\text{C} + \beta^+ + Q$ is energetically possible for β^+ decay.
- v. What happens to the remaining energy in β decay after the emission of a β particle?
- vi. What is internal conversion?

7.0 REFERENCES/FURTHER READING

Gautreau, R. & Savin, W. (1999). *Schaum's Outline of Theory and Problems of Modern Physics*.

UNIT 5 NUCLEAR REACTIONS

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Contents
 - 3.1 Nuclear Reaction
 - 3.2 Q – Value Equation
 - 3.3 Nuclear Fission Reaction
 - 3.4 Nuclear Fusion Reaction
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

In this unit, nuclear reactions are briefly explained. Here nuclei are bombarded with known projectiles and the final products are observed.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain process involved in nuclear reactions
- explain nuclear fission and fusion reactions
- explain the types of nuclear reactions which exist.

3.0 MAIN CONTENT

3.1 Nuclear Reaction

A nuclear reaction is a process whereby the mass number or the atomic number of target nuclei changes as a result of bombardment with projectile particles resulting in the release of energy. The necessary things for a nuclear reaction are:

1. A target nucleus
2. A projectile example ${}_0^1n$, ${}_1^1H$, ${}_2^4He$, ${}_1^2H$.

$X + x \rightarrow Y + y + Q$ or $X(x, y)Y$

e.g. ${}_{7}^{14}N(\alpha, p){}_{7}^{17}O$

$\Rightarrow {}_{7}^{14}N + {}_{2}^{4}He \rightarrow {}_{8}^{17}O + {}_{1}^{1}H + Q.$

Types of Nuclear Reaction

- 1) F - Fission - $X + x \rightarrow y_1 + y_2$
- 2) I - Inelastic nuclear reaction - $X + x \rightarrow X^k + x$
- 3) T - Transmutation - $X + x \rightarrow Y + y$
- 4) E - Elastic nuclear reaction - $X + x \rightarrow X + x$
- 5) C - Capture - $X + x \rightarrow Y^k$

The physical qualities which are conserved in any nuclear reaction include:

- 1) Total electric charge $EZ = K$
- 2) Total number of nuclei $EA = K$
- 3) Linear momentum $EP = K$
- 4) Sum of mass and energy $E(\text{mass} + Ke)$
- 5) Parity K

3.2 Q - Value Equation

This is the nuclear change or the amount of energy released in a nuclear reaction. For a nuclear reaction, the total rest mass and kinetic energy are conserved.

Example $X + x \rightarrow Y + y + Q$

$$\text{i.e. } [E_x + m_x C^2] + [E_x + m_x C^2]$$

$$= [E_Y + m_Y C^2] + [E_y + m_y C^2] + Q$$

Since the target nucleus X is at rest, then the equation turns to

$$[M_x C^2] + [E_x + m_x C^2] = [E_Y + M_Y C^2] + [E_y + M_y C^2] + Q$$

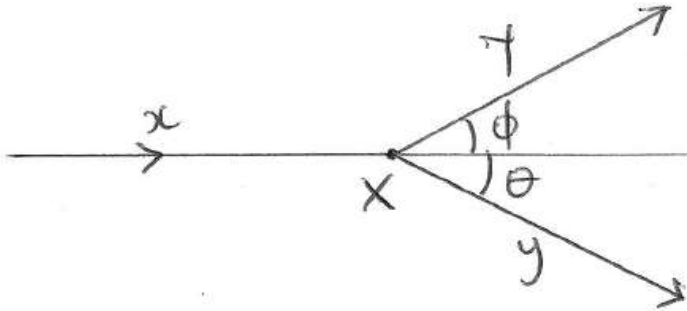
But $Q = \text{change in energy}$

Therefore $Q = E_x - (E_Y + E_y)$

$$Q = [(M_x + m_x) - (M_Y + M_y)] C^2$$

$$Q = \Delta mc^2$$

Conventional Q-value Equation



Applying the principle of the conservation of momentum

$$M_x V_x = M_Y V_Y \cos \phi + M_y V_y \cos \theta \text{ (x-direction)} \text{-----(1)}$$

$$0 = M_Y V_Y \sin \phi + M_y V_y \sin \theta \text{ (y-direction)} \text{-----(2)}$$

from $E = \frac{p^2}{2m}$, $p^2 = 2mE$

Therefore, momentum $mv = \sqrt{2ME}$

Equation (1) and (2) turns into

$$(M_x E_x)^{1/2} = (M_Y E_Y)^{1/2} \cos \phi + (M_y E_y)^{1/2} \cos \theta \text{-----(3)}$$

$$0 = (M_Y E_Y)^{1/2} \sin \phi + (M_y E_y)^{1/2} \sin \theta \text{-----(4)}$$

Square equation (3) and (4) and add

$$M_x E_x = M_Y E_Y \cos^2 \phi + M_y E_y \cos^2 \theta$$

$$0 = M_Y E_Y \sin^2 \phi + M_y E_y \sin^2 \theta$$

$$M_x E_x = M_Y E_Y (\cos^2 \phi + \sin^2 \phi) + M_y E_y (\cos^2 \theta + \sin^2 \theta)$$

$$M_x E_x = M_Y E_Y + M_y E_y .$$

The minimum energy a projectile must have before it can induce a nuclear reaction is called the *threshold energy*

Conservation of linear momentum

$$M_x V_x = M_c V_c$$

$$V_c = \frac{M_x V_x}{M_c}$$

$$\text{But } -Q = \frac{1}{2} M_x V_x^2 - \frac{1}{2} M_c V_c^2$$

$$-Q = \frac{1}{2} M_x V_x^2 - \frac{1}{2} M_c \frac{M_x^2 V_x^2}{M_c^2}$$

$$-Q = \frac{1}{2} M_x V_x^2 \left[1 - \frac{M_x}{M_c} \right]$$

$$\text{But } M_c = M_X + M_x$$

$$-Q = \frac{1}{2} M_x V_x^2 \left[1 - \frac{M_x}{M_X + M_x} \right]$$

$$-Q = \frac{1}{2} M_x V_x^2 \left[\frac{M_X + M_x - M_x}{M_X + M_x} \right]$$

$$-Q = \frac{1}{2} M_x V_x^2 \left[\frac{M_X}{M_X + M_x} \right]$$

$$\text{Take } \frac{1}{2} M_x V_x^2 = E_{thr}$$

$$-Q = E_{thr} \left[\frac{M_X}{M_X + M_x} \right]$$

$$E_{thr} = -Q \times \left[\frac{M_X + M_x}{M_X} \right]$$

$$E_{thr} = -Q \times \left[1 + \frac{M_x}{M_X} \right]$$

SELF-ASSESSMENT EXERCISE 1

- i. Define what is meant by
 - (i) Nuclear
 - (ii) Q-value energy
- ii. What is meant by threshold energy?
- iii. Calculate Q-Value of the following nuclear reaction:

3.3 Nuclear Fission Reaction

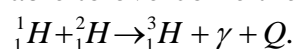
This is a reaction which involves the splitting of heavy nuclei into two or more lighter nuclei by bombarding the heavy nuclei with thermal neutrons and it is usually accompanied with a high energy released. Nuclides that can be fissioned by thermal neutrons as follow: ^{235}U , ^{237}U and ^{239}Pu .

Only ^{235}U occurs naturally while others are gotten from fertile materials. The process of conversion of fertile materials to fissionable material is referred to as *breeder reaction*.

Energy released in a fission reaction is $Q = \Delta mc^2 = (\sum m_i - \sum m_f)c^2$

3.4 Nuclear Fusion Reaction

This is the combination of two or more light nuclei to form a heavier one and this involves the supply of high energy. In which this energy will be able to overcome the coulombs force between them.



And in doing so they must overcome the potential barrier which is equal to

$$V = \frac{Z_1 Z_2 e^2}{4\pi\epsilon_0 r}$$

$$V = 0.15 Z_1 Z_2 \text{MeV}, \text{ for } r = 10^{-14} \text{ m}$$

This sort of energy (10^9 k) can only be acquired during nuclear explosions which is not practicable. Usually at this temperature, atoms exist as ions and are collected as plasma.

Because of the high temperature, it is also referred to as thermonuclear reaction.

SELF-ASSESSMENT EXERCISE 2

- i. Distinguish between nuclear fusion and nuclear fission.
- ii. What is the kinetic energy of a 300K thermal neutron?
- iii. What will be the energy released if two deuterium nuclei fuse into an α -particle?

4.0 CONCLUSION

In conclusion, we have been able to examine nuclear reactions in its different forms. Also nuclear fusion and nuclear fission were examined.

5.0 SUMMARY

In this unit, we have been able to understand the conditions necessary for nuclear reactions to take place. Also, the importance of the Q-value was examined. Also nuclear fusion and nuclear fission were examined with their kinematics.

6.0 TUTOR-MARKED ASSIGNMENT

1. Determine the unknown particle in the nuclear reactions: (a) ${}_{74}^{182}\text{W}({}_2^3\text{He}, n)X$, (b) ${}_{20}^{42}\text{Ca}({}_3^6\text{Li}, X){}_{21}^{45}\text{Sc}$.
2. Calculate the Q-value for the reaction ${}_{20}^{42}\text{Ca}(p, d){}_{20}^{41}\text{Ca}$.
3. Calculate the Q-value for the D-T fusion reaction ${}_1^3\text{H}(d, n){}_2^4\text{He}$.
4. Find the Q-value for the D-D reactions (a) ${}_1^2\text{H}(d, n){}_2^3\text{He}$, (b) ${}_1^2\text{H}(d, p){}_1^3\text{H}$.
5. Calculate the energy released in the fusion process ${}_2^4\text{He} + {}_2^4\text{He} + {}_2^4\text{He} \rightarrow {}_{12}^{12}\text{C}$.

7.0 REFERENCES/FURTHER READING

Gautreau, R. & Savin, W. (1999). *Schaum's Outline of Theory and Problems of Modern Physics*.

UNIT 6 INTERACTION OF RADIATION WITH MATTER

CONTENTS

- 1.0 Introduction
- 2.0 Objectives
- 3.0 Main Contents
 - 3.1 Heavy Charged Particle Interaction
 - 3.2 Beta Rays (fast electrons)
 - 3.3 Motions
 - 3.4 Neutrons
- 4.0 Conclusion
- 5.0 Summary
- 6.0 Tutor-Marked Assignment
- 7.0 References/Further Reading

1.0 INTRODUCTION

Radiation measurements are possible through interaction with matter either living or dead.

This interaction has made of possible to be used in diagnosis, researchers industries X-ray and radiotherapy. Because of the fundamental difference in energy transfer, radiation can be categorised into four:

1. Heavy charged particles
2. Fast electrons particles
3. Neutrons electrons particle
4. Protons electrons particles

Cross section and interaction co-efficient

The probability that an interaction will take place is expressed in cross sections. Cross section actually describes the effective area which the interaction center or entities presents to the radiation which if traversed by the radiation, it ensures that an interaction occur.

$$\delta = \frac{\text{prob of int eraction } p}{\text{no. of conc. center area}}$$

$$\delta = \frac{\text{prob of int eraction } p}{\text{particle fluence, } Q}$$

δ is expressed in m^2 or in barns

1 barns = $10^{-28} m^2$. Linear attenuation co-efficient is given as

$$\frac{d\phi}{\phi} = -\mu dl$$

$$\phi = \phi_0 e^{-\mu dl}$$

Where $\mu = p\delta$ that is probability of interaction per unit length.

Also number of atoms per unit volume of substances

$$n = \frac{N_A \rho}{M} = \frac{N}{V}$$

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain parameters used in quantifying the amount of radiation interacting with matter
- explain how the different categories of radiation interact with matter.

3.0 MAIN CONTENT

3.1 Heavy Charged Particle Interaction

This interaction can be divided into 3 broad groups which are:

- a. Interaction with individual electron of the atoms which leads to excitation or ionization of atoms and this collision can be:
 - i. *Inelastic* - sufficient energy received for excitation.
 - ii. *Elastic* - energy received is less than the smallest energy difference of the atomic level. It may be hard or soft collision. It may also be fast or slow depending on projectile velocity and orbital velocity
- b. Interaction with nuclei if incoming particle is heavy compared with electrons.
- c. Interaction with the whole coulomb field surrounding an atom. This occurs when the incident particle is of low energy or heavy particle with low velocities.

The outcome of a collision or interaction is determined by:

- i. Velocity of collision, V
- ii. Distance of closest approach of the participants.
- iii. The range of the potential which governs the interaction between the incident particle and the target.

For heavy particles like α particles interacts with the coulomb forces between positive and negative charges in matter. It then dissipates its

energy in succession to the electron through inelastic collision which then results in either excitation or ionization. The number of ion produced per unit distance is called *specific ionization*.

The linear rate of energy loss or linear stopping power S for heavy charged particle in a given absorber is

$$S = -\frac{dE}{dx}$$

S increase as the particle velocity decreases that is $S \propto \frac{1}{v}$

For a non-relativistic particle,

$$S = \frac{4\pi e^4 Z^2 N B(Z_1 V)}{M_o V^2} \text{ (Beth's formula)}$$

Where M_o - rest mass of electron

V - velocity of heavy particle

e - Electronic charge

Z - Atomic number of the absorber atom

$B(Z_1 V)$ - Beth's formula

$$B(Z_1 V) = Z \left[\frac{\ln 2M_o V^2}{I} - \ln \left(1 - \frac{V^2}{C^2} - \frac{V_o^2}{C^2} \right) \right]$$

I = average ionization potential of absorption

While for a non-relativistic charged particle

$$B(Z_1 V) = Z \left(\frac{\ln M_o V^2}{I} \right), S \propto \frac{1}{V^2} \propto \frac{1}{E}$$

3.2 Beta Rays (Fast Electrons)

The energy lost by fast electrons is due to excitation and ionization as well. Majorly, energy is usually lost due to:

a). Scattering of the fast electrons because they are colliding with another electron in target. The energy loss per unit length D

$$-\frac{dE}{dx} / C = \text{collision}$$

b). Through radiation process - $-\frac{dE}{dx} / r = \text{radiation}$

Which take place in the form of Bremsstrahlung (e-m radiation). (Usually for electrons with energy greater than rest mass energy).

Therefore, the total linear specific energy loss is;

$$\frac{dE}{dX} = \frac{dE}{dX} / c + \frac{dE}{dX} / r$$

$$\text{and } \frac{\frac{dE}{dX} / c}{\frac{dE}{dX} / r} = \frac{E_8}{7w}$$

c). Cerenkov radiation which is negligible at lower energy.

SELF-ASSESSMENT EXERCISE 1

- i. What do you understand by the term “specific ionization”?
- ii. Write the equation relating the specific ionization and the velocity of heavy particles.

$$\text{A: } B(Z_1V) = Z \left[\frac{\ln 2M_oV^2}{I} - \ln \left(1 - \frac{V^2}{C^2} - \frac{V_o^2}{C^2} \right) \right]$$

- iii. Describe one of the ways by which energy is lost when an electron interact with matter.

3.3 Photons

Interaction of E-M radiation (i.e. X and γ rays photons). Under an energy region of 0.01 - 10MeV, most interactions γ and X-rays can be explained under three different modes

- a. Photoelectric effect
- b. Compton effect
- c. Pair production

a). Photoelectric Effect

This is a kind of interaction whereby an incident photon transfers all its energy to the electron in a target and therefore these electrons are emitted as photoelectrons with a kinetic energy given as below;

$$E_e = E_\gamma - E_B$$

The incident photon must have an energy greater than or equal to E_B of the electron to the nucleus of the target.

Therefore, vacancies are created at the K-shells and filled by electrons from higher shells which results into X-rays. The cross section of the photoelectric effect is given as;

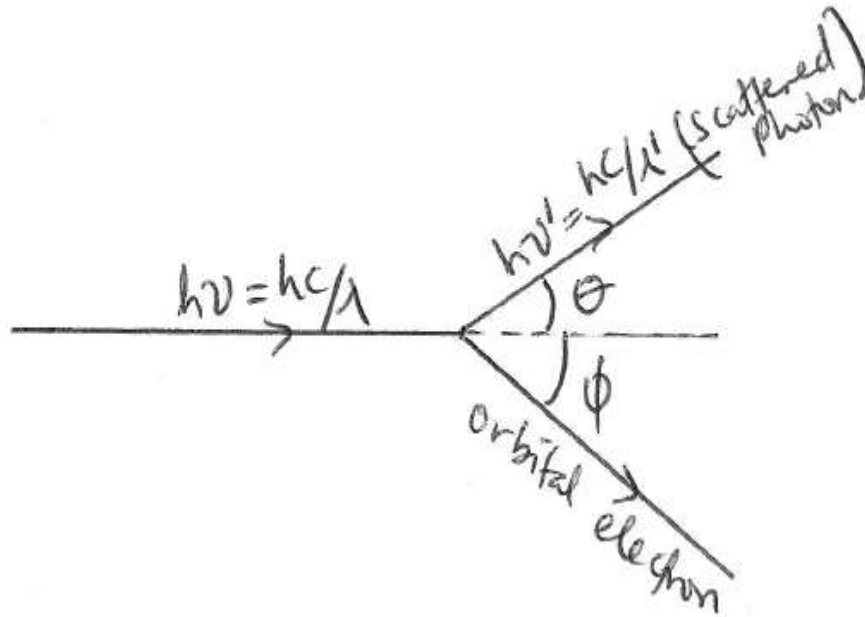
$$\sigma_{P.E} = \delta E_\gamma^{-7/2} \rho Z^5$$

ρ and Z^5 are density and atomic number of the absorbing material or target.

δ is constant

b). Compton Effect

This is a situation whereby the incident photon collides with the orbital electron of the target and the incident photon and orbital electrons are scattered at different angles.



Observations

- i. Reduction in photon energy from $h\nu \rightarrow h\nu'$.
- ii. Frequency is changed from $\nu \rightarrow \nu'$ (reduced).
- iii. The wavelength of photon increases from $\lambda \rightarrow \lambda'$.
- iv. Energy of scattered electron is $(h\nu - h\nu')$.
- v. The increase in wavelength is given as

$$\Delta\lambda = \lambda' - \lambda = \frac{h}{M_o C} [1 - \cos\theta]$$

Where M_o is the rest mass of the atom of which the electron is used.

- vi. The energy of the scattered photon is;

$$E_{\gamma'} = \frac{E_{\gamma}}{1 + \left[\frac{E_{\gamma}}{M_o C^2} \right] [1 - \cos\theta]}$$

- vi. The kinetic energy of the photo electron ($E_{K.E.}$) or ejected electron is

$$E_{K.E.} = \frac{\left[\frac{E_\gamma}{M_o C^2} \right] [1 - \cos\theta]}{1 + \left[\frac{E_\gamma}{M_o C^2} \right] [1 - \cos\theta]}$$

Where $M_o C^2$ is the rest energy of the electron

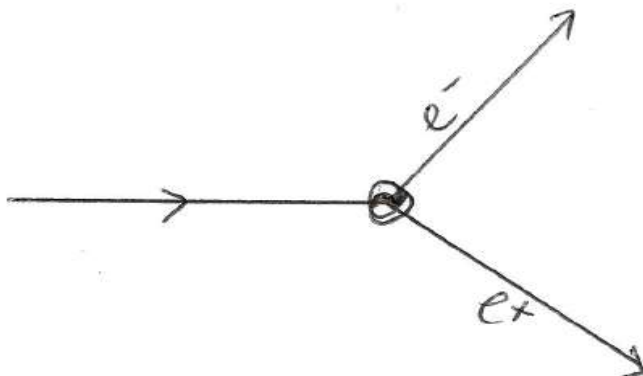
$E_{K.E.}$ is at minimum when $\theta=0$ and $E_{K.E.}$ is maximum when $\theta=180^\circ$ and this is called the *Compton edge energy*.

$$E_c = E_\gamma \left[\frac{2E_\gamma}{-M_o C^2 + 2E_\gamma} \right]$$

That is when photon is scattered backwards at $\theta=180^\circ$ which explain Compton plateau in γ spectroscopy.

c). Pair Production

This occurs when γ rays with sufficient energy interacts with the atom of the target in the coulomb field of the nucleus and disappears with electron positron pair in place of it.



The energy equation of the process is given as $h\nu = E_{e^-} + E_{e^+} + 2M_o C^2$
 Pair production can only take place if greater than or equal to $2M_o C^2 = 1.02\text{MeV}$.

The positron is an unstable particle once its kinetic energy = 0, it annihilates with the electron to form a photon which either escapes from the medium or interacts with the medium either photo electrically or photocompton.

$$\sigma_{p.p} = CZ^2 \delta \ln E_\gamma$$

The net effect of the three in γ -ray passing through an absorbing material or the linear cross-section is an exponential attenuation given by

$$I = I_o e^{-\sigma x}$$

3.4 Neutrons

The kind of reaction a neutron undergoes depends on its energy. Neutrons are classified according to their energy:

1. High energy neutron greater than 10MeV
2. Thermal neutrons is the same as average kinetic energy of gas molecules = 0.025eV.

All neutrons at the time of their birth are fast but are slowed down (thermalising) by colliding them elastically with atoms in their environment. After the slowing down, they are now been absorbed by nuclei of the absorbing material and interaction takes place. The interaction of neutron is different from past discussion because, it does not show a variation between the atomic mass and energy. The interactions instead produce:

1. Recurring nuclei
2. Subatomic particles
3. Photons which undergo previous processes.

Generally neutrons may collide with nuclei and undergo;

1. Elastic collisions

Fast neutrons react with low atomic number absorbers. The neutron is scattered with a reduced energy because part of the energy have been transferred to the recurring nuclei
e.g. ${}^1\text{H} (n, n) {}^1\text{H}$. (Here the nucleus moves).

2. Inelastic collisions

If the neutron is of low energy, the neutron may be momentarily captured by the nucleus and then emitted with diminished energy leaving the nucleus in an excited state and may return to its ground state with the emission of photon e.g. ${}^{16}\text{O}(n, n) {}^{16}\text{O}$. (Here the nucleus does not moved but brushed). If another particle is produced after interaction, it is called non-elastic collision e.g. ${}^{16}\text{O} (n, \alpha) {}^{13}\text{C}$. Thermal neutrons are captured by nuclei with reaction cross section as

$$\sigma \propto \frac{1}{v} \propto \frac{1}{\sqrt{E}} \Rightarrow \frac{\sigma}{\sigma_0} = \sqrt{\frac{E_0}{E}}$$

Passage of neutrons through a moderating material

The process of slowing down fast neutrons is known as moderation or thermalisation and this is done by making use of moderators (element with low atomic mass) e.g. graphite and heavy water, in such a way that no reaction is lost by absorption but merely have their kinetic energy being reduced by elastic collision with nuclei of the moderator.

Velocity V_1 of a neutron after collision is

$$V_1^2 = V^2(1 + A^2 + 2A\cos\phi) \text{-----(1)}$$

and V is the velocity of neutron before collision also

$$V_0 = V(1 + A) \text{-----(2)}$$

And V_0 is the velocity of neutron in real frame.

$$E_0 = \frac{1}{2}mv_0^2 = \text{incident energy of neutron before collision}$$

$$E_1 = \frac{1}{2}mv_1^2 = \text{energy of neutron after collision.}$$

Therefore the fractional energy e_o is

$$\frac{E_1}{E_0} = \frac{\frac{1}{2}mv_1^2}{\frac{1}{2}mv_0^2} = \frac{v^2(1 + A^2 + 2A\cos\phi)}{v^2(1 + A)^2}$$

$$\frac{E_1}{E_0} = \frac{1 + A^2 + 2A\cos\phi}{(1 + A)^2} \text{-----(3)}$$

Cases

- 1. Glancing angle i.e. where $\phi = 0$

From $\frac{E_1}{E_0} = \frac{1 + A^2 + 2A\cos\phi}{(1 + A)^2}$

$$\frac{E_1}{E_0} = 1 \text{-----(4)}$$

- 2. Head-on-collision where $\phi = \pi = 180^0$

$$\frac{E_1}{E_0} = \frac{(A-1)^2}{(A+1)^2} \text{-----(5) (Neutron energy loss here is maximum)}$$

let $\alpha = \left(\frac{A-1}{A+1}\right)^2$

$$\frac{E_1}{E_0} = \alpha$$

From $\Delta E_{\text{max}} = (E_0 - E_1)_{\text{max}}$

$$= E_0 \left(1 - \frac{E_1}{E_0}\right)_{\text{max}}$$

$$\Delta E_{\text{max}} = E_0(1 - \alpha)_{\text{max}}$$

The maximum fractional energy loss can be deformed as

$$\frac{\Delta E_{\max}}{E_0} = 1 - \alpha = 1 - \frac{(A-1)^2}{(A+1)^2} \text{----- (6)}$$

For a good moderator, ΔE_{\max} must be large and therefore A must be small. From past discussion we have been dealing with flux of neutron, but now we want to discuss about one which will now be a statistical problem.

Let's assume the neutron is scattered between angle ϕ and $\phi + d\phi$ and the energy (that is E and E + dE) between E and E+dE. It will be observed that the entire range of energy through which the neutron can be scattered is between 1. $E_1 = E_0$ (from glancing) and (2) $E_1 = \alpha E_0$ (from head on collision). This implies that $E_0 - \alpha E_0 = E_0(1 - \alpha)$.

The probability $P(E)dE$ that a neutron will have an energy E between E_0 and αE_0 is 1.

The probability that it will lie between E and E+dE

$$= P(E) = \frac{1}{E_0(1-\alpha)}$$

$$\int_{\alpha E_0}^{E_0} P(E)dE = \int_{\alpha E_0}^{E_0} \frac{dE}{E_0(1-\alpha)} = 1$$

Therefore, the average energy (E) of a neutron after a series of scattering or the probability that a single collision will make a neutron have energy E is:

$$\begin{aligned} \langle E \rangle &= \frac{\int_{\alpha E_0}^E EP(E)dE}{\int_{\alpha E_0}^E P(E)dE} = \int_{\alpha E_0}^E \frac{EdE}{E_0(1-\alpha)} \\ &= \frac{1}{E_0(1-\alpha)} \int_{\alpha E_0}^{E_0} EdE \\ &= \frac{1}{E_0(1-\alpha)} \left[\frac{E^2}{2} \right]_{\alpha E_0}^{E_0} \\ &= \frac{1}{2E_0(1-\alpha)} [E_0^2 - \alpha^2 E_0^2] \\ &= \frac{1}{2E_0(1-\alpha)} E_0^2 [1 - \alpha^2] \\ &= \frac{1}{2(1-\alpha)} E_0 [(1-\alpha)(1+\alpha)] \\ &= \frac{1}{2} E_0 (1 + \alpha) \end{aligned}$$

Average log energy decrement

This is used to obtain the average number of collisions which a fast neutron, will make before its energy E_0 is reduced to thermal energy E_t .

Take $E_t = E$

Log energy decrement is $\log_e E_o - \log_e E = \log_e (E_o/E)$

Therefore average log = $[\log_e (E_o/E)]$

$$\xi = \int_{\alpha E_o}^{E_o} \log\left(\frac{E_o}{E}\right) P(E) dE$$

$$= \int_{\alpha E_o}^{E_o} \log\left(\frac{E_o}{E}\right) \frac{dE}{E_o(1-\alpha)} \text{----- (1)}$$

Since $\int_{\alpha E_o}^{E_o} \frac{dE}{E_o(1-\alpha)} = 1$

Then put $x = E/E_o$

For limits to change

For $E = E_o$; $x = 1$

And $E = \alpha E_o = x = \alpha$

From $x = E/E_o$, $dE = E_o dx$

And $\log E_o/E = -\log x$.

Then integral (1) turns into

$$= \int_{\alpha}^1 (-\log x) \frac{E_o dx}{E_o(1-\alpha)}$$

$$= -\frac{1}{1-\alpha} \int \log x dx$$

$$= 1 + \frac{\alpha}{1-\alpha} \log \alpha$$

Then substituting for $\alpha = \left(\frac{A-1}{A+1}\right)^2$

$$\xi = 1 + \frac{\frac{(A-1)^2}{(A+1)^2} \log \frac{(A-1)^2}{(A+1)^2}}{1 - \frac{(A-1)^2}{(A+1)^2}}$$

$$\xi = 1 - \frac{(A-1)^2}{2A} \log \frac{A-1}{A+1}$$

For $A > 1$

$$\xi = \frac{2}{A + 2/3}$$

Generally the number of collision required to reduce $E_o + E_t$ is given by

$$n = \frac{1}{\xi} \log E_o/E_t$$

The distance travelled by a fast neutron between its introduction into a slowing down medium and its thermalisation is called *fast diffusion*

length or slowing down length and square of the fast diffusion length is the *Fermi-age*.

Also, the distance travelled by the thermalised diffusion length and it is defined as the thickness of the slowing down medium.

$$n = n_0 e^{-t/l}$$

n and n_0 are number of neutrons before and after collision and L is thermal diffusion length. But for large absorption cross section

$$I = I_0 e^{-\sigma N t}.$$

SELF-ASSESSMENT EXERCISE 2

- i. Name the electromagnetic radiations (photons) which can interact with matter.
- ii. What is photoelectric effect?
- iii. Explain the processes of pair production.

4.0 CONCLUSION

In conclusion, we have been able to examine how the different radiations interact with matter as well as their resulting effect.

5.0 SUMMARY

In this unit, we have been able to understand that radiations which interact with matter can be categorised into four. The effect of each of these radiations were examined and quantified.

6.0 TUTOR-MARKED ASSIGNMENT

1. Mention five important applications of the interaction of radiation with matter.
2. Define the following terms:
 - Cross section
 - Cerenkov radiations
 - Moderation
 - Bremstrahlung
3. Briefly explain Compton Effect.
4. Distinguish between Compton Effect and Pair Production.

7.0 REFERENCES/FURTHER READING

Greiner, W. & Maruhn, J. A. *Nuclear Models by Springer*.

APPENDIX

11/10/12 Atomic Weights and Isotopic Compositions for All Elements

Atomic Weights and Isotopic Compositions for All Elements

<u>Isotope</u>		<u>Relative Atomic Mass</u>	<u>Isotopic Composition</u>	<u>Standard Atomic Weight</u>	<u>Notes</u>
1 H	1	1.007 825 032 07(10)	0.999 885(70)	1.007 94(7)	g,m,r,b,w
D	2	2.014 101 777 8(4)	0.000 115(70)		
T	3	3.016 049 2777(25)			
2 He	3	3.016 029 3191(26)	0.000 001 34(3)	4.002 602(2)	g,r,a
	4	4.002 603 254 15(6)	0.999 998 66(3)		
3 Li	6	6.015 122 795(16)	0.0759(4)	6.941(2)	g,m,r,c,i
	7	7.016 004 55(8)	0.9241(4)		
4 Be	9	9.012 182 2(4)	1.0000	9.012 182(3)	
5 B	10	10.012 937 0(4)	0.199(7)	10.811(7)	g,m,r
	11	11.009 305 4(4)	0.801(7)		
6 C	12	12.000 000 0(0)	0.9893(8)	12.0107(8)	g,r
	13	13.003 354 8378(10)	0.0107(8)		
	14	14.003 241 989(4)			
7 N	14	14.003 074 004 8(6)	0.996 36(20)	14.0067(2)	g,r,a,d
	15	15.000 108 898 2(7)	0.003 64(20)		
8 O	16	15.994 914 619 56(16)	0.997 57(16)	15.9994(3)	g,r,e,w
	17	16.999 131 70(12)	0.000 38(1)		
	18	17.999 161 0(7)	0.002 05(14)		
9 F	19	18.998 403 22(7)	1.0000	18.998 403 2(5)	
10 Ne	20	19.992 440 1754(19)	0.9048(3)	20.1797(6)	g,m,a
	21	20.993 846 68(4)	0.0027(1)		
	22	21.991 385 114(19)	0.0925(3)		
11 Na	23	22.989 769 2809(29)	1.0000	22.989 769 28(2)	
12 Mg	24	23.985 041 700(14)	0.7899(4)	24.3050(6)	

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Atomic Weights and Isotopic Compositions for All Elements

		25	24.985 836 92(3)	0.1000(1)		
		26	25.982 592 929(30)	0.1101(3)		
13	Al	27	26.981 538 63(12)	1.0000	26.981 538 6(8)	
14	Si	28	27.976 926 5325(19)	0.922 23(19)	28.0855(3)	r
		29	28.976 494 700(22)	0.046 85(8)		
		30	29.973 770 17(3)	0.030 92(11)		
15	P	31	30.973 761 63(20)	1.0000	30.973 762(2)	
16	S	32	31.972 071 00(15)	0.9499(26)	32.065(5)	g,r
		33	32.971 458 76(15)	0.0075(2)		
		34	33.967 866 90(12)	0.0425(24)		
		36	35.967 080 76(20)	0.0001(1)		
17	Cl	35	34.968 852 68(4)	0.7576(10)	35.453(2)	m
		37	36.965 902 59(5)	0.2424(10)		
18	Ar	36	35.967 545 106(29)	0.003 365(30)	39.948(1)	g,r,a
		38	37.962 732 4(4)	0.000 632(5)		
		40	39.962 383 1225(29)	0.996 003(30)		
19	K	39	38.963 706 68(20)	0.932 581(44)	39.0983(1)	g
		40	39.963 998 48(21)	0.000 117(1)		
		41	40.961 825 76(21)	0.067 302(44)		
20	Ca	40	39.962 590 98(22)	0.969 41(156)	40.078(4)	g,f
		42	41.958 618 01(27)	0.006 47(23)		
		43	42.958 766 6(3)	0.001 35(10)		
		44	43.955 481 8(4)	0.020 86(110)		
		46	45.953 6926(24)	0.000 04(3)		
		48	47.952 534(4)	0.001 87(21)		
21	Sc	45	44.955 911 9(9)	1.0000	44.955 912(6)	
22	Ti	46	45.952 631 6(9)	0.0825(3)	47.867(1)	
		47	46.951 763 1(9)	0.0744(2)		
		48	47.947 946 3(9)	0.7372(3)		

		49	48.947 870 0(9)	0.0541(2)		
		50	49.944 791 2(9)	0.0518(2)		
<hr/>						
23	V	50	49.947 1585(11)	0.002 50(4)	50.9415(1)	
		51	50.943 9595(11)	0.997 50(4)		
<hr/>						
24	Cr	50	49.946 0442(11)	0.043 45(13)	51.9961(6)	
		52	51.940 507 5(8)	0.837 89(18)		
		53	52.940 649 4(8)	0.095 01(17)		
		54	53.938 880 4(8)	0.023 65(7)		
<hr/>						
25	Mn	55	54.938 045 1(7)	1.0000	54.938 045(5)	
<hr/>						
26	Fe	54	53.939 610 5(7)	0.058 45(35)	55.845(2)	
		56	55.934 937 5(7)	0.917 54(36)		
		57	56.935 394 0(7)	0.021 19(10)		
		58	57.933 275 6(8)	0.002 82(4)		
<hr/>						
27	Co	59	58.933 195 0(7)	1.0000	58.933 195(5)	
<hr/>						
28	Ni	58	57.935 342 9(7)	0.680 769(89)	58.6934(4)	
		60	59.930 786 4(7)	0.262 231(77)		
		61	60.931 056 0(7)	0.011 399(6)		
		62	61.928 345 1(6)	0.036 345(17)		
		64	63.927 966 0(7)	0.009 256(9)		
<hr/>						
29	Cu	63	62.929 597 5(6)	0.6915(15)	63.546(3)	r
		65	64.927 789 5(7)	0.3085(15)		
<hr/>						
30	Zn	64	63.929 142 2(7)	0.482 68(321)	65.38(2)	
		66	65.926 0334(10)	0.279 75(77)		
		67	66.927 1273(10)	0.041 02(21)		
		68	67.924 8442(10)	0.190 24(123)		
		70	69.925 3193(21)	0.006 31(9)		
<hr/>						
31	Ga	69	68.925 5736(13)	0.601 08(9)	69.723(1)	
		71	70.924 7013(11)	0.398 92(9)		
<hr/>						
32	Ge	70	69.924 2474(11)	0.2038(18)	72.64(1)	

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		72	71.922 0758(18)	0.2731(26)		
		73	72.923 4589(18)	0.0776(8)		
		74	73.921 1778(18)	0.3672(15)		
		76	75.921 4026(18)	0.0783(7)		
33	As	75	74.921 5965(20)	1.0000	74.921 60(2)	
34	Se	74	73.922 4764(18)	0.0089(4)	78.96(3)	
		76	75.919 2136(18)	0.0937(29)		
		77	76.919 9140(18)	0.0763(16)		
		78	77.917 3091(18)	0.2377(28)		
		80	79.916 5213(21)	0.4961(41)		
		82	81.916 6994(22)	0.0873(22)		
35	Br	79	78.918 3371(22)	0.5069(7)	79.904(1)	
		81	80.916 2906(21)	0.4931(7)		
36	Kr	78	77.920 3648(12)	0.003 55(3)	83.798(2)	g,m,a
		80	79.916 3790(16)	0.022 86(10)		
		82	81.913 4836(19)	0.115 93(31)		
		83	82.914 136(3)	0.115 00(19)		
		84	83.911 507(3)	0.569 87(15)		
		86	85.910 610 73(11)	0.172 79(41)		
37	Rb	85	84.911 789 738(12)	0.7217(2)	85.4678(3)	g
		87	86.909 180 527(13)	0.2783(2)		
38	Sr	84	83.913 425(3)	0.0056(1)	87.62(1)	g,r,f
		86	85.909 2602(12)	0.0986(1)		
		87	86.908 8771(12)	0.0700(1)		
		88	87.905 6121(12)	0.8258(1)		
39	Y	89	88.905 8483(27)	1.0000	88.905 85(2)	
40	Zr	90	89.904 7044(25)	0.5145(40)	91.224(2)	g
		91	90.905 6458(25)	0.1122(5)		
		92	91.905 0408(25)	0.1715(8)		
		94	93.906 3152(26)	0.1738(28)		

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	72	71.922 0758(18)	0.2731(26)		
	73	72.923 4589(18)	0.0776(8)		
	74	73.921 1778(18)	0.3672(15)		
	76	75.921 4026(18)	0.0783(7)		
33 As	75	74.921 5965(20)	1.0000	74.921 60(2)	
34 Se	74	73.922 4764(18)	0.0089(4)	78.96(3)	
	76	75.919 2136(18)	0.0937(29)		
	77	76.919 9140(18)	0.0763(16)		
	78	77.917 3091(18)	0.2377(28)		
	80	79.916 5213(21)	0.4961(41)		
	82	81.916 6994(22)	0.0873(22)		
35 Br	79	78.918 3371(22)	0.5069(7)	79.904(1)	
	81	80.916 2906(21)	0.4931(7)		
36 Kr	78	77.920 3648(12)	0.003 55(3)	83.798(2)	g,m,a
	80	79.916 3790(16)	0.022 86(10)		
	82	81.913 4836(19)	0.115 93(31)		
	83	82.914 136(3)	0.115 00(19)		
	84	83.911 507(3)	0.569 87(15)		
	86	85.910 610 73(11)	0.172 79(41)		
37 Rb	85	84.911 789 738(12)	0.7217(2)	85.4678(3)	g
	87	86.909 180 527(13)	0.2783(2)		
38 Sr	84	83.913 425(3)	0.0056(1)	87.62(1)	g,r,f
	86	85.909 2602(12)	0.0986(1)		
	87	86.908 8771(12)	0.0700(1)		
	88	87.905 6121(12)	0.8258(1)		
39 Y	89	88.905 8483(27)	1.0000	88.905 85(2)	
40 Zr	90	89.904 7044(25)	0.5145(40)	91.224(2)	g
	91	90.905 6458(25)	0.1122(5)		
	92	91.905 0408(25)	0.1715(8)		
	94	93.906 3152(26)	0.1738(28)		

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	96	95.908 2734(30)	0.0280(9)		
41 Nb	93	92.906 3781(26)	1.0000	92.906 38(2)	
42 Mo	92	91.906 811(4)	0.1477(31)	95.96(2)	g
	94	93.905 0883(21)	0.0923(10)		
	95	94.905 8421(21)	0.1590(9)		
	96	95.904 6795(21)	0.1668(1)		
	97	96.906 0215(21)	0.0956(5)		
	98	97.905 4082(21)	0.2419(26)		
	100	99.907 477(6)	0.0967(20)		
43 Tc	97	96.906 365(5)		[98]	
	98	97.907 216(4)			
	99	98.906 2547(21)			
44 Ru	96	95.907 598(8)	0.0554(14)	101.07(2)	g
	98	97.905 287(7)	0.0187(3)		
	99	98.905 9393(22)	0.1276(14)		
	100	99.904 2195(22)	0.1260(7)		
	101	100.905 5821(22)	0.1706(2)		
	102	101.904 3493(22)	0.3155(14)		
	104	103.905 433(3)	0.1862(27)		
45 Rh	103	102.905 504(3)	1.0000	102.905 50(2)	
46 Pd	102	101.905 609(3)	0.0102(1)	106.42(1)	g
	104	103.904 036(4)	0.1114(8)		
	105	104.905 085(4)	0.2233(8)		
	106	105.903 486(4)	0.2733(3)		
	108	107.903 892(4)	0.2646(9)		
	110	109.905 153(12)	0.1172(9)		
47 Ag	107	106.905 097(5)	0.518 39(8)	107.8682(2)	g
	109	108.904 752(3)	0.481 61(8)		
48 Cd	106	105.906 459(6)	0.0125(6)	112.411(8)	g
	108	107.904 184(6)	0.0089(3)		
	110	109.903 0021(29)	0.1249(18)		

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		111	110.904 1781(29)	0.1280(12)		
		112	111.902 7578(29)	0.2413(21)		
		113	112.904 4017(29)	0.1222(12)		
		114	113.903 3585(29)	0.2873(42)		
		116	115.904 756(3)	0.0749(18)		
49	In	113	112.904 058(3)	0.0429(5)	114.818(3)	
		115	114.903 878(5)	0.9571(5)		
50	Sn	112	111.904 818(5)	0.0097(1)	118.710(7)	g _e
		114	113.902 779(3)	0.0066(1)		
		115	114.903 342(3)	0.0034(1)		
		116	115.901 741(3)	0.1454(9)		
		117	116.902 952(3)	0.0768(7)		
		118	117.901 603(3)	0.2422(9)		
		119	118.903 308(3)	0.0859(4)		
		120	119.902 1947(27)	0.3258(9)		
		122	121.903 4390(29)	0.0463(3)		
		124	123.905 2739(15)	0.0579(5)		
51	Sb	121	120.903 8157(24)	0.5721(5)	121.760(1)	g
		123	122.904 2140(22)	0.4279(5)		
52	Te	120	119.904 020(10)	0.0009(1)	127.60(3)	g _h
		122	121.903 0439(16)	0.0255(12)		
		123	122.904 2700(16)	0.0089(3)		
		124	123.902 8179(16)	0.0474(14)		
		125	124.904 4307(16)	0.0707(15)		
		126	125.903 3117(16)	0.1884(25)		
		128	127.904 4631(19)	0.3174(8)		
		130	129.906 2244(21)	0.3408(62)		
53	I	127	126.904 473(4)	1.0000	126.904 47(3)	
54	Xe	124	123.905 8930(20)	0.000 952(3)	131.293(6)	g _{m,a}
		126	125.904 274(7)	0.000 890(2)		
		128	127.903 5313(15)	0.019 102(8)		
		129	128.904 779 4(8)	0.264 006(82)		
		130	129.903 508 0(8)	0.040 710(13)		

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		131	130.905 0824(10)	0.212 324(30)	
		132	131.904 1535(10)	0.269 086(33)	
		134	133.905 394 5(9)	0.104 357(21)	
		136	135.907 219(8)	0.088 573(44)	
<hr/>					
55	Cs	133	132.905 451 933(24)	1.0000	132.905 451 9(2)
<hr/>					
56	Ba	130	129.906 3208(30)	0.001 06(1)	137.327(7)
		132	131.905 0613(11)	0.001 01(1)	
		134	133.904 508 4(4)	0.024 17(18)	
		135	134.905 688 6(4)	0.065 92(12)	
		136	135.904 575 9(4)	0.078 54(24)	
		137	136.905 827 4(5)	0.112 32(24)	
		138	137.905 247 2(5)	0.716 98(42)	
<hr/>					
57	La	138	137.907 112(4)	0.000 90(1)	138.905 47(7) g
		139	138.906 3533(26)	0.999 10(1)	
<hr/>					
58	Ce	136	135.907 172(14)	0.001 85(2)	140.116(1) g,f
		138	137.905 991(11)	0.002 51(2)	
		140	139.905 4387(26)	0.884 50(51)	
		142	141.909 244(3)	0.111 14(51)	
<hr/>					
59	Pr	141	140.907 6528(26)	1.0000	140.907 65(2)
<hr/>					
60	Nd	142	141.907 7233(25)	0.272(5)	144.242(3) g,f
		143	142.909 8143(25)	0.122(2)	
		144	143.910 0873(25)	0.238(3)	
		145	144.912 5736(25)	0.083(1)	
		146	145.913 1169(25)	0.172(3)	
		148	147.916 893(3)	0.057(1)	
		150	149.920 891(3)	0.056(2)	
<hr/>					
61	Pm	145	144.912 749(3)		[145]
		147	146.915 1385(26)		
<hr/>					
62	Sm	144	143.911 999(3)	0.0307(7)	150.36(2) g
		147	146.914 8979(26)	0.1499(18)	
		148	147.914 8227(26)	0.1124(10)	

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		149	148.917	1847(26)	0.1382(7)		
		150	149.917	2755(26)	0.0738(1)		
		152	151.919	7324(27)	0.2675(16)		
		154	153.922	2093(27)	0.2275(29)		
<hr/>							
63	Eu	151	150.919	8502(26)	0.4781(6)	151.964(1)	g
		153	152.921	2303(26)	0.5219(6)		
<hr/>							
64	Gd	152	151.919	7910(27)	0.0020(1)	157.25(3)	g
		154	153.920	8656(27)	0.0218(3)		
		155	154.922	6220(27)	0.1480(12)		
		156	155.922	1227(27)	0.2047(9)		
		157	156.923	9601(27)	0.1565(2)		
		158	157.924	1039(27)	0.2484(7)		
		160	159.927	0541(27)	0.2186(19)		
<hr/>							
65	Tb	159	158.925	3468(27)	1.0000	158.925 35(2)	
<hr/>							
66	Dy	156	155.924	283(7)	0.000 56(3)	162.500(1)	g
		158	157.924	409(4)	0.000 95(3)		
		160	159.925	1975(27)	0.023 29(18)		
		161	160.926	9334(27)	0.188 89(42)		
		162	161.926	7984(27)	0.254 75(36)		
		163	162.928	7312(27)	0.248 96(42)		
		164	163.929	1748(27)	0.282 60(54)		
<hr/>							
67	Ho	165	164.930	3221(27)	1.0000	164.930 32(2)	
<hr/>							
68	Er	162	161.928	778(4)	0.001 39(5)	167.259(3)	g
		164	163.929	200(3)	0.016 01(3)		
		166	165.930	2931(27)	0.335 03(36)		
		167	166.932	0482(27)	0.228 69(9)		
		168	167.932	3702(27)	0.269 78(18)		
		170	169.935	4643(30)	0.149 10(36)		
<hr/>							
69	Tm	169	168.934	2133(27)	1.0000	168.934 21(2)	
<hr/>							
70	Yb	168	167.933	897(5)	0.0013(1)	173.054(5)	g
		170	169.934	7618(26)	0.0304(15)		

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		171	170.936 3258(26)	0.1428(57)		
		172	171.936 3815(26)	0.2183(67)		
		173	172.938 2108(26)	0.1613(27)		
		174	173.938 8621(26)	0.3183(92)		
		176	175.942 5717(28)	0.1276(41)		
71	Lu	175	174.940 7718(23)	0.9741(2)	174.9668(1)	g
		176	175.942 6863(23)	0.0259(2)		
72	Hf	174	173.940 046(3)	0.0016(1)	178.49(2)	f
		176	175.941 4086(24)	0.0526(7)		
		177	176.943 2207(23)	0.1860(9)		
		178	177.943 6988(23)	0.2728(7)		
		179	178.945 8161(23)	0.1362(2)		
		180	179.946 5500(23)	0.3508(16)		
73	Ta	180	179.947 4648(24)	0.000 12(2)	180.947 88(2)	
		181	180.947 9958(19)	0.999 88(2)		
74	W	180	179.946 704(4)	0.0012(1)	183.84(1)	
		182	181.948 204 2(9)	0.2650(16)		
		183	182.950 223 0(9)	0.1431(4)		
		184	183.950 931 2(9)	0.3064(2)		
		186	185.954 3641(19)	0.2843(19)		
75	Re	185	184.952 9550(13)	0.3740(2)	186.207(1)	
		187	186.955 7531(15)	0.6260(2)		
76	Os	184	183.952 4891(14)	0.0002(1)	190.23(3)	gf
		186	185.953 8382(15)	0.0159(3)		
		187	186.955 7505(15)	0.0196(2)		
		188	187.955 8382(15)	0.1324(8)		
		189	188.958 1475(16)	0.1615(5)		
		190	189.958 4470(16)	0.2626(2)		
		192	191.961 4807(27)	0.4078(19)		
77	Ir	191	190.960 5940(18)	0.373(2)	192.217(3)	
		193	192.962 9264(18)	0.627(2)		

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78 Pt	190	189.959 932(6)	0.000 14(1)	195.084(9)	
	192	191.961 0380(27)	0.007 82(7)		
	194	193.962 680 3(9)	0.329 67(99)		
	195	194.964 791 1(9)	0.338 32(10)		
	196	195.964 951 5(9)	0.252 42(41)		
	198	197.967 893(3)	0.071 63(55)		
79 Au	197	196.966 568 7(6)	1.0000	196.966 569(4)	
80 Hg	196	195.965 833(3)	0.0015(1)	200.59(2)	
	198	197.966 769 0(4)	0.0997(20)		
	199	198.968 279 9(4)	0.1687(22)		
	200	199.968 326 0(4)	0.2310(19)		
	201	200.970 302 3(6)	0.1318(9)		
	202	201.970 643 0(6)	0.2986(26)		
	204	203.973 493 9(4)	0.0687(15)		
	81 Tl	203	202.972 3442(14)		0.2952(1)
205		204.974 4275(14)	0.7048(1)		
82 Pb	204	203.973 0436(13)	0.014(1)	207.2(1)	g,r,f
	206	205.974 4653(13)	0.241(1)		
	207	206.975 8969(13)	0.221(1)		
	208	207.976 6521(13)	0.524(1)		
83 Bi	209	208.980 3987(16)	1.0000	208.980 40(1)	
84 Po	209	208.982 4304(20)		[209]	
	210	209.982 8737(13)			
85 At	210	209.987 148(8)		[210]	
	211	210.987 4963(30)			
86 Rn	211	210.990 601(7)		[222]	
	220	220.011 3940(24)			
	222	222.017 5777(25)			
87 Fr	223	223.019 7359(26)		[223]	

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88	Ra	223	223.018 5022(27)			[226]
		224	224.020 2118(24)			
		226	226.025 4098(25)			
		228	228.031 0703(26)			
89	Ac	227	227.027 7521(26)			[227]
90	Th	230	230.033 1338(19)			
		232	232.038 0553(21)	1.0000	232.038 06(2)	g
91	Pa	231	231.035 8840(24)	1.0000	231.035 88(2)	
92	U	233	233.039 6352(29)			
		234	234.040 9521(20)	0.000 054(5)	238.028 91(3)	g,m,c
		235	235.043 9299(20)	0.007 204(6)		
		236	236.045 5680(20)			
		238	238.050 7882(20)	0.992 742(10)		
93	Np	236	236.046 570(50)			[237]
		237	237.048 1734(20)			
94	Pu	238	238.049 5599(20)			[244]
		239	239.052 1634(20)			
		240	240.053 8135(20)			
		241	241.056 8515(20)			
		242	242.058 7426(20)			
		244	244.064 204(5)			
95	Am	241	241.056 8291(20)			[243]
		243	243.061 3811(25)			
96	Cm	243	243.061 3891(22)			[247]
		244	244.062 7526(20)			
		245	245.065 4912(22)			
		246	246.067 2237(22)			
		247	247.070 354(5)			
		248	248.072 349(5)			

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97	Bk	247	247.070	307(6)	[247]
		249	249.074	9867(28)	
98	Cf	249	249.074	8535(24)	[251]
		250	250.076	4061(22)	
		251	251.079	587(5)	
		252	252.081	626(5)	
99	Es	252	252.082	980(50)	[252]
100	Fm	257	257.095	105(7)	[257]
101	Md	258	258.098	431(5)	[258]
		260	260.103	65(34)#	
102	No	259	259.101	03(11)#	[259]
103	Lr	262	262.109	63(22)#	[262]
104	Rf	265	265.116	70(46)#	[265]
105	Db	268	268.125	45(57)#	[268]
106	Sg	271	271.133	47(70)#	[271]
107	Bh	272	272.138	03(65)#	[272]
108	Hs	270	270.134	65(31)#	[270]
109	Mt	276	276.151	16(73)#	[276]
110	Ds	281	281.162	06(78)#	[281]
111	Rg	280	280.164	47(80)#	[280]
112	Cn	285	285.174	11(78)#	[285]
113	Uut	284	284.178	08(86)#	[284]

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114 Uuq 289	289.187 28(79)#	[289]
115 Uup 288	288.192 49(92)#	[288]
116 Uuh 293		[293]
117 Uus 292	292.207 55(101)#	[292]
118 Uuo 294		[294]

