NATIONAL OPEN UNIVERSITY OF NIGERIA

SCHOOL OF SCIENCE AND TECHNOLOGY

COURSE CODE: PHY 455

COURSE TITLE: LOWER ATMOSPHERIC PHYSICS

COURSE GUIDE

PHY 455
LOWER ATMOSPHERIC PHYSICS

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INTRODUCTION

You have studied the behaviour of gases under controlled conditions. The knowledge you gained in your study of gas laws shall now be extended to atmospheric gases. The atmosphere itself is a relatively thin gaseous envelop that surrounds the earth. It provides the air we breathe and protects us against some harmful radiations from the sun. You certainly know the statement of Snell's law. In this course, we shall use Snell's law to explain some of the observed characteristics of radio waves. At the end of this course, you should be able to answer each of the following questions: Why does most of our weather occur within the troposphere? What is the importance of the ionosphere in relation to radio-wave propagation? How does the temperature at any point in the atmosphere depend on height? What happens within the saturation region? How can we relate the internal energy, the enthalpy and the entropy to the temperature and specific volume of a pure substance? What are the effects of the troposphere on radio wave propagation? How does the flow of radiant energy change as one moves through an atmosphere that emits and absorbs radiation? What are the factors responsible for the broadening of spectral lines? The answer to each of these questions is of interest to us because the phenomena involved have significant effects on our life here on earth.

THE COURSE

PHY 435 Lower Atmospheric Physics comprises a total of 6 units arranged into 2 modules as follows:

Module 1 is composed of 3 units
Module 2 is composed of 3 units

Module 1 deals with (a) the structure and composition of the atmosphere of the earth (b) the basic thermodynamic concepts necessary for a good understanding of the terrestrial atmosphere and (c) the behaviour of atmospheric air and water vapour. In unit 1 of module1, you shall learn how the atmosphere is stratified. We shall then proceed to examine the factors that affect *space weather* and their impact on human activity.

Unit 2 of module 1 deals with the laws of thermodynamics. You shall be introduced to different processes involving expansion, compression, heating and cooling of gases. Enthalpy, entropy and internal energy shall be studied in this unit. In unit 3 of module 1, you will examine the behaviour of air and water vapour. In particular, you will find out how exciting the saturation regions can be. You will see how water can exist in three pure phases and four equilibrium mixture phases.

Module 2 is devoted to the physics of a neutral atmosphere. In Unit 1 of Module 2, you will see how the equation of sate and the equation of hydrostatic equilibrium may be used to study the pressure profile of the atmosphere. You will find out that the density and temperature profiles are also derivable from the expression for the variation of pressure with height.

In Unit 2 of Module 2, you shall be introduced to the concept of intensity, flux and luminosity. The equation of radiative transfer shall be derived in this unit. You will see the *local thermodynamic equilibrium (LTE) approximation* simplies the solution of the equation of transfer. In Unit 3 of Module 2, you will be introduced to the basic *transition* and *selection* rules that are useful in understanding the formation of spectral lines. The Lorentzian profile and the Gaussian profile shall be discussed in this unit. You will see the three mechanisms by which a spectral line may be broadened.

COURSE AIMS AND OBJECTIVES

This course aims at providing you with a good knowledge of the lower portion of the atmosphere of the earth. You will learn about the structure and composition of the atmosphere. You will also be introduced to the portion of the atmosphere that has a significant influence on the propagation of radio waves; that part of the atmosphere is called the ionosphere. You will find out how the terrestrial atmosphere provides protection against some of the harmful radiations from the sun, and how the atmosphere helps to maintain warm temperatures on the earth.

A sound knowledge of the basic principles of thermodynamics is necessary for you to understand the dynamics of the terrestrial atmosphere. This explains why a section of this course is devoted entirely to the study of elementary principles of thermodynamics. These thermodynamic principles shall be very useful to you in understanding the behaviour of the air and the water vapour present in the atmosphere of the earth.

Many radiative processes are constantly taking place within the atmosphere. At the end of this course, you should be able to discuss the concept of intensity, flux and luminosity of a pencil of radiation. You should also be able to derive and solve the equation of radiative transfer using the LTE approximation.

WORKING THROUGH THE COURSE

This course is presented in a simple and readable style. The coverage is comprehensive. You should strive to study the materials contained in this course carefully and repeatedly until a full mastery of the subject matter is gained. The examples provided in the text should be studied and mastered. In this way, you will be able to do the assignment at the end of each unit. The equation of transfer is sufficiently straightforward. However, you are advised to review some of the mathematical tools needed to solve elementary differential equations. This will provide you with the instrument necessary to solve, without difficulty, the equation of radiative transfer. You should also do your best to participate in all the tutorial sessions. In this way, you shall be able to learn from your facilitators, your course mates and your study units.

COURSE MATERIALS

You will be provided with adequate course materials including your course guide and study units. You will also be provided with a list of recommendable textbooks. All these study materials are extremely useful. You should therefore strive as much as possible to get all the needed materials. You must also study the materials diligently.

STUDY UNITS

This course contains 6 study units organised into 2 modules. Module 1 comprises 3 units and Module 2 comprises 3 units also. The 6 study units are arranged in the following order.

Module 1

Unit 1	Structure and Composition of the Atmosphere
Unit 2	Basic Thermodynamic Concepts
Unit 3	Air and Water Vapour

Module 2

Unit 1	The Neutral Atmosphere
Unit 2	Radiative Transfer
Unit 3	Line Profiles

TEXTBOOKS AND REFERENCES

A.De Vos, A. (1992). *Endoreversible Thermodynamics of Solar Energy Conversion*. New York: Oxford University Press, New York.

Emerson, D. (1996). *Interpreting Astrophysical Spectra*. Chichester, England: John Wiley & Sons, Chichester, England.

J.P. Holman, J.P. (1980). *Thermodynamics*. Tokyo: McGraw-Hill Kogakusha Ltd, Tokyo.

E.Tandberg-Hanssen, E. & A.G Emslie, A.G. (1988). *The Physics of Solar Flares*. Cambridge Astrophysics Series, Cambridge: Cambridge University Press, Cambridge.

Yeh, K.C & Liu, C.H.(1977). *Theory of Ionospheric Waves*. New York: Academic Press Inc.

ASSESSMENT

The National Open University of Nigeria assesses your performance by means of the following instruments:

(a) Tutor-Marked Assignment (TMA)
(b) End of Course Examination.

TUTOR-MARKED ASSIGNMENTS (TMAs)

Tutor-Marked Assignment (TMA) represents the traditional Continuous Assessment. It accounts for 30% of your total score. In this course, you are provided with six TMAs. You should try to do all the 6 TMAs. Please, note that you will not be allowed to sit for the end of course examination without completing at least 5 out of the 6 TMAs. Your course facilitator shall provide the TMAs. You are required to complete each TMA and return same to your course facilitator within the stipulated period.

END OF COURSE EXAMINATION

The end of course examination carries 70% of your total score. The date, time and venue of the examination shall be made known to you in advance. This may or may not coincide with the semester examination of the National Open University of Nigeria. You must sit for the end of course examination.

SUMMARY

This course is split into 2 modules. Each of the two modules has 3 units. Each unit is carefully designed to ensure that you understanding its *message* with relative ease. Abstract concepts are systemically translated into concrete applications following a simple *logical* sequence.

Upon satisfactory completion of this course, you should be able to explain why most of our weather occurs within the troposphere. You should also be able to discuss the variations of *temperature, pressure* and *number density* with height in the atmosphere of the earth. Furthermore, you should be able to use the LTE approximation in solving the equation of radiative transfer (in relation to radio-wave propagation).

It is advisable for you to study each of the 6 units contained in this course carefully. It is also highly recommended that you read as many of the recommended textbooks as possible. In this way, you shall be able to derive the maximum benefit from this course.

I wish you the success in your effort to acquire knowledge.

MAIN COURSE

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MODULE 1

UNIT 1 STRUCTURE AND COMPOSITION OF THE ATMOSPHERE

CONTENTS

1.0 INTRODUCTION

The atmosphere of the earth is a relatively thin spherical envelop that surrounds the earth. It contains gases, clouds and other air-borne particles called aerosols. The terrestrial atmosphere provides protection against the harmful radiations from the sun. It also helps to maintain warm temperatures on the earth. In this unit, you shall be introduced to the structure and composition of the terrestrial atmosphere.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- explain why most of our weather occurs within the troposphere
- describe the different layers of the atmosphere
- describe the importance of the ionosphere in relation to radio wave propagation
- describe the D, E and F layers of the ionosphere.

3.0 MAIN CONTENT

3.1 Earth's Atmosphere

3.1.1 Layers of the Atmosphere

The atmosphere of the earth is composed of different layers. It starts from the surface of the earth and thins out with height until it merges with space. In terms of the variation of temperature with height, the layers of the atmosphere are:

a) the troposphere
b) the *stratosphere*
c) the mesosphere, and
d) the thermosphere.

These layers are listed in order of increasing height. Thus, the troposphere is the lowest layer followed by the stratosphere, the mesosphere and the thermosphere. The thermosphere is the highest layer of the earth's atmosphere (in terms of temperature).

In terms of composition, the atmosphere is classified into two:

a) the homosphere (up to 85km height), and
b) the heterosphere (above 85km height).

In the homosphere, the mean molecular mass is constant due to mixing. In the heterosphere, the mean molecular mass varies due to diffusive separation. *In terms of the escape properties of neutral particles, the terrestrial atmosphere is called the exosphere.*

3.1.2 Troposphere

The troposphere is the first layer above the surface of the earth. It stretches from the surface of the earth to an altitude of about 10 kilometres. The gases in the troposphere are predominantly molecular nitrogen (N_2) and molecular oxygen (O_2). The troposphere contains 90% of the mass of the earth's atmosphere. It also contains 99% of all the water vapour present in the atmosphere.

Within the troposphere, temperature deceases with height at the rate of about 6.5 ^{o}C per kilometer. This rapid decrease in temperature with height is almost linear. It continues until the temperature reaches a local minimum value which defines the upper boundary of the troposphere known as the tropopause. At the tropopause, the temperature remains constant. The tropopause may lie somewhere between 10km and 15km above the surface of the earth.

All our day-to-day activities occur within the troposphere: the top of Mount Everest lies within the troposphere; air planes fly within the troposphere; and 99% of all atmospheric weather is confined to the troposphere. It is the decrease in temperature with height that promotes vertical motion of the atmosphere. This explains why most of our weather occurs within the troposphere.

3.1.3 Stratosphere

The stratosphere lies above the tropopause. In the stratosphere, the temperature increases with altitude. The increase in temperature with height continues until a local maximum is reached at a height of about 50 kilometers. This local maximum defines the stratopause. The stratopause is the upper boundary region of the stratosphere.

The stratosphere is very stable. For that reason, many jet planes fly within the stratosphere. However, stratospheric air remains sufficiently dense to enable hot air balloons to ascend to altitudes of between 15km and 20km. Helium balloons are capable of rising to a height of about 35 kilometres. The ozone layer exists in the stratosphere.

The stratospheric region is relatively calm. The variation of stratospheric temperature with height is not as rapid as the variation of tropospheric temperature with height. The stratosphere is relatively warm because the sun's ultraviolet radiation is absorbed by oxygen and ozone.

3.1.4 Mesosphere

The mesosphere lies above the stratopause. In the mesosphere, the temperature decreases with altitude up to a local minimum. This local minimum, called the *mesopause,* defines the upper boundary of the mes osphere. At the mesopause, the temperature can be as low as -90^{o}C. This is the lowest temperature anywhere in the entire terrestrial atmosphere.

The mesopause is located at an altitude of about 80 kilometres. Meteors are found in the neighbourhood of the mesopause.

3.1.5 Thermosphere

The thermosphere lies above the menopause. It extends to an altitude of about 500 kilometres. The air in the thermosphere is extremely thin. Large amounts of nitrogen and oxygen atoms are produced through the process of photo-dissociation of the dominant N_2 and O_2 molecules. Above the mesopause, the temperature increases dramatically to an overall maximum value that may exceed 1000K. Beyond this point, the thermospheric temperature remains almost constant with altitude. The thermosphere includes the ionised portion of the atmosphere called the *ionosphere*.

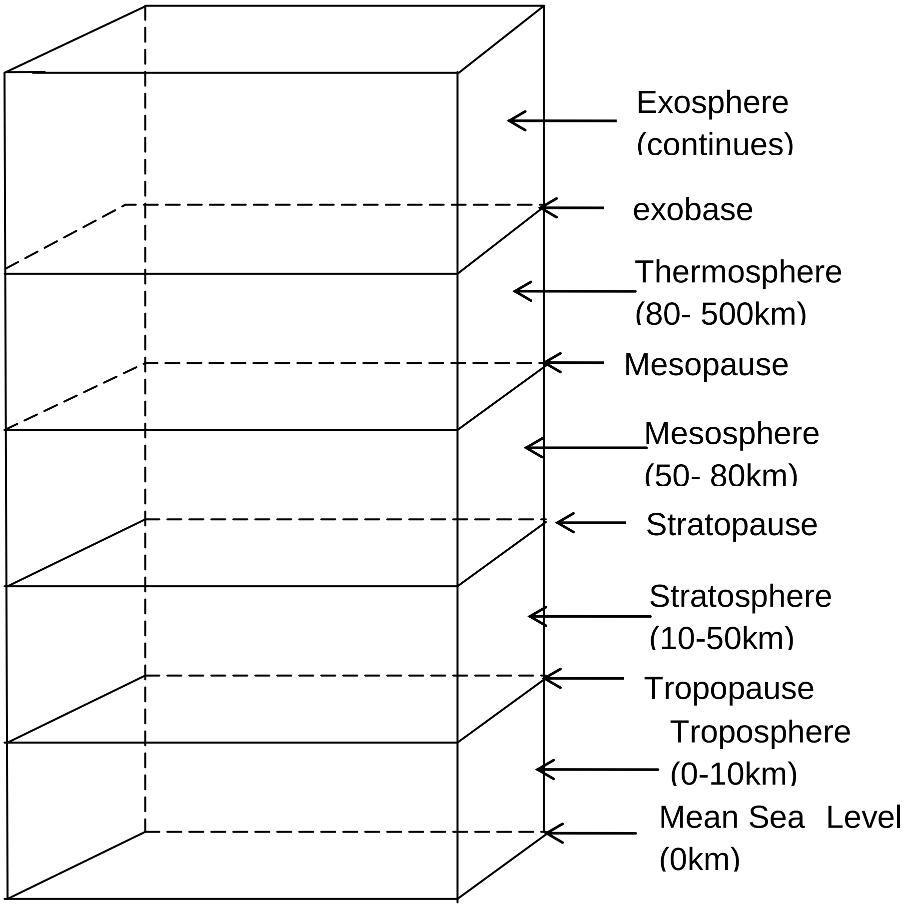

Fig. 1 (a): Possible Classification Scheme for the Layers of the Earth's Atmosphere (not drawn to scale)

3.1.6 Exosphere

The exosphere continues beyond the thermosphere. The transition region between the thermosphere and the exosphere is known as the *exobase*. The region above the exobase is still called the exosphere. The exobase is located at a height of about 600km. In this portion of the exosphere, the neutral densities are so low that collisions become unimportant. For this reason, the upper atmosphere can no longer be characterised as *fluid*. The space shuttle orbits around the earth within this region of the exosphere.

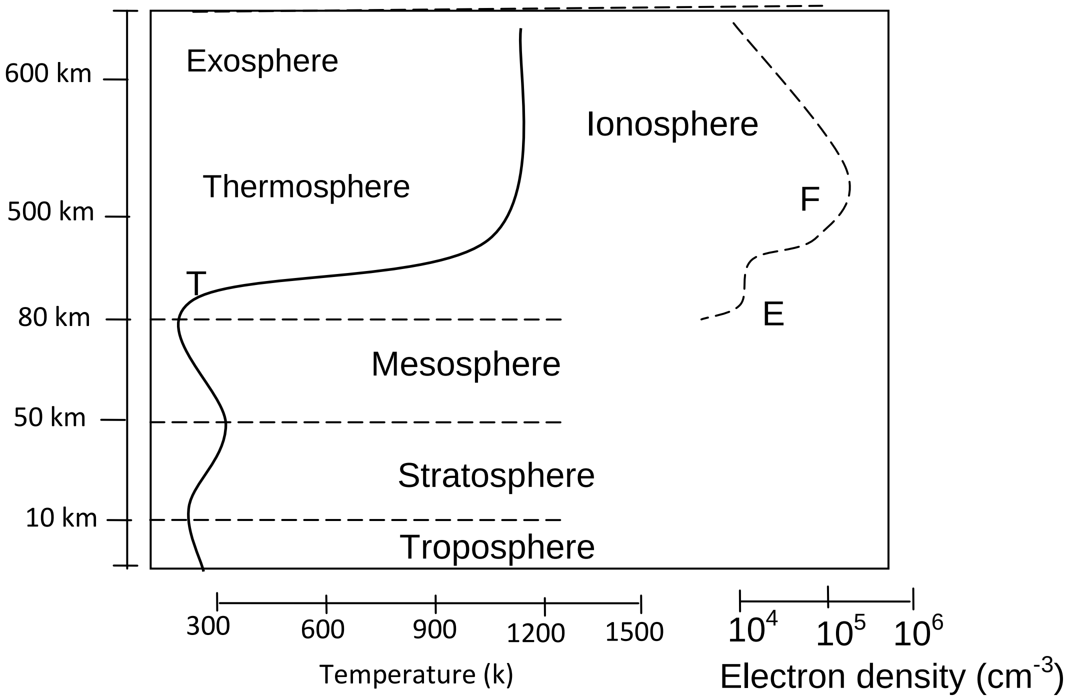

Fig. 1 (b): Temperature Profile of the Earth's Atmosphere

3.2 Ionosphere

3.2.1 Layers of the Ionosphere

The ionosphere is that part of a planetary atmosphere where ions and electrons are present in quantities sufficient to affect the propagation of radio waves. The ionosphere extends from an altitude of about 60 kilometres to an altitude of about 2000 kilometres.

The composition of the atmosphere changes with height. As a result, the ion production rate also changes with height. This leads to the formation of the following three *distinct ionisation peaks*:

a) the D layer
b) the E layer, and
c) the F layer.

The ionosphere is a very important region of the atmosphere because of its influence on radio waves.

3.2.2 D Layer

The D layer is mainly responsible for the absorption of high frequency (HF) radio waves. The absorption is strongest for radio frequencies below 10MHz. The absorption becomes progressively smaller as the frequency increases. The absorption is higher in the day and lower at night. The ionisation is due to Lyman series-alpha hydrogen radiation at a wavelength of 121.5 nanometre (nm) which ionises nitric oxide (NO). Thus, the dominant ions are NO^+ and $O_2^+ \cdot$ The D layer extends in height from about 40km to 90km. The electron density of the D layer is about $2.5\times10^9 m^{-3}$ in the day. The electron density diminishes to a negligible value at night.

3.2.3 E Layer

The E layer lies above the D layer. It is the region of the ionosphere between an altitude of about 90km and an altitude of about 160km. In relation to the *solar zenith angle* and *solar activity*, the electron density in the E layer behaves in a regular manner. In the daytime, the electron density may reach about $2\times10^{11} m^{-3}$. This is high enough to reflect radio waves with frequencies of several megahertz. At night, the electron density of the E region may decrease to about $10^{10} m^{-3}$. The E region can only reflect radio waves that have frequencies lower that 10MHz.

The ionisation is due to soft X-ray (1nm-10nm) and far ultraviolet (UV) solar radiations which ionise molecular oxygen (O_2). The dominant ions are O_2^+ and NO^+. At night, the primary source of ionisation is no longer there. For that reason, the E layer begins to disappear. An increase in the height of the E layer increases the range over which radio communication (through reflection) is possible (figure 2).

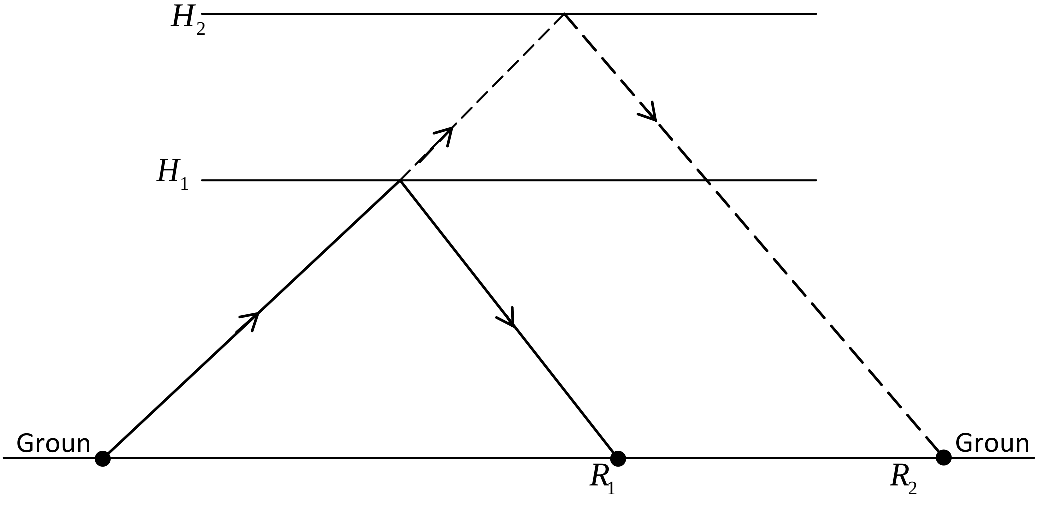

Fig. 1.2: Increasing the Height of the E Layer (from H_1 to H_2) Increases the Range (from R_1 to R_2).

The E region peaks at an altitude of about 100km. However, sporadic E propagation (Es) is also possible. It is characterised by small clouds of intense ionisation. Such clouds can reflect radio waves ranging in frequency from about 25MHz to about 225MHz. Sporadic-E events occur most frequently during the hot season. Occasionally, the sporadic-E event may also occur during the cold season. As a result of the sporadic-E events, some propagation channels (which are generally unreachable) can open up. This may last for just a few minutes. It may also have an extended period of several hours.

3.2.4 F Layer

The F layer lies above the E layer. Its behaviour is fairly irregular. It is usually classified into a number of *anomalies* such as the *equatorial anomaly* and the *seasonal anomaly*. The day-time electron density is quite different from the night-time electron density. In the day, the average peak value of the electron density is about $2\times10^{12}m^{-3}$. At night, the average peak value of the electron density falls to about $2\times10^{11}m^{-3}$. The F region is responsible for *reflection* of radio waves.

According to the Institute of Electrical and Electronics Engineers (IEEE), the region extends to the magnetospheric shock boundary. This magnetospheric shock boundary is several earth radii away. However, some scientists believe that the F region stops at about 800km above the surface of the earth. *When classified according to its composition, the atmosphere above this point is called the protonosphere. When classified according to its dynamic property, the protonosphere is called the magnetosphere.*

In the F region, the dominant ion is O^+. In terms of high frequency (HF) communications, the F region is the most important part of the ionosphere. Ionisation of the atomic oxygen (O) is due to extreme ultraviolet solar radiation $(10-100nm)$. The F region is thickest and *most reflective* of radio waves on the side of the earth facing the sun. During the day, the F region splits into two layers. These two layers are called the F_1 and F_2 layers. Above the F region, there is a region in which the density decreases exponentially with height. This region is known as the *topside ionosphere.* The topside ionosphere extends to an altitude of a few thousand kilometres.

The ionosphere is an extremely interesting medium. In going from the D-region to the upper F region, the electron density varies over *four* orders of magnitude. In the D region, the ionosphere manifests a *collision-dominated* behaviour. Here, the electron collision frequency is as high as 8×10^7 per second. In the upper layer of the F region, the ionosphere manifests a *collisionless* behaviour. As a result, the concept of *frozen-in* magnetic field becomes applicable. The ionosphere exhibits non-linear properties due to the fact that the region is inhomogeneous and anisotropic. The Luxembourg effect (or cross-modulation) is a good example of the non-linear properties of the ionosphere. Idealised ionisation profiles of the ionosphere are presented in figure 3. The ionisation profiles show how the electron density changes with height at temperate latitudes near *sunspot maximum.*

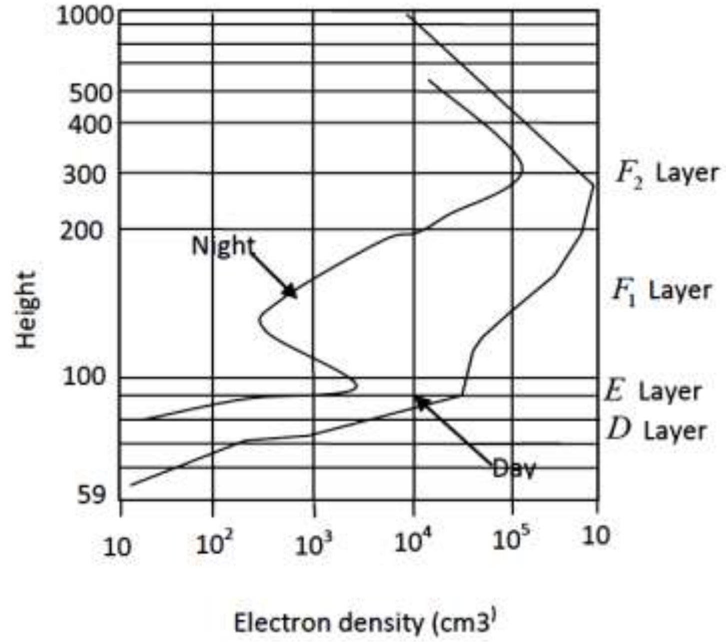

Fig.1. 3: **Day-Time and Night-Time Ionisation Profiles of the Ionosphere**

3.3 Aurora

The auroras are found in the thermosphere. Auroras are the light produced when protons and electrons from the sun travel along the earth’s magnetic field lines above the North Pole (and above the South Pole) and excite neutral atoms and molecules. The aurora associated with the North Pole is sometimes called the northern lights; the aurora associated with the South Pole is sometimes called the southern lights. The aurora is a spectacular display of lights requiring tens of thousands of volts to produce. This fascinating display of energy takes place within the ionosphere (60-200km above the surface of the earth). In particular, the *aurora borealis* over northern Canada occurs at a height of about 100 kilometres.

The aurora is beautiful, spectacular, and splendid. They appear in the form of majestic, colourful, irregular lights in the night sky. The aurora has various shapes, colours and structures. They appear almost every night in the sky of the north and south poles.

The aurora is a large-scale electrical discharge phenomenon in the high-altitude atmosphere, resulting from *quantum leaps* in oxygen and nitrogen atoms. Above an altitude of about one hundred kilometres, oxygen and nitrogen atoms (and molecules) are energised and/or ionised by energetic electrons. In the transition region between the earth's atmosphere and the near-earth space, electrons are in abundance. The electrons are accelerated to high energies by electric fields in the magnetosphere. As the energetic electrons stream along the magnetic field lines, they strike and excite atoms and molecules of oxygen and nitrogen. When these excited particles are de-excited, they emit the colourful radiation, known as aurora, over the polar sky. The colour, shape and intensity of aurora depend on the nature of the electromagnetic forces that shoot the electrons downward into the upper atmosphere.

3.4 Solar Wind

The surface of the sun is like a pot of boiling water. Bubbles of hot, electrified gas circulate from the interior to the surface of the sun. This gas is made of electrons and protons in the fourth state of matter known as *plasma.* As the circulation continues, the plasma continuously bursts out into space. This steady stream of ionised plasma, which escapes from the solar corona and pervades the whole interplanetary space, is known as the solar wind.

Solar wind is mainly composed of electrons and protons. However, about 10% of the solar wind are helium ions. The solar wind induces geomagnetic activity by variation of its pressure and magnetic field. At the orbit of the earth, the solar wind has a density of about 10 particles per cubic centimetres, a temperature of about $10{,}000{,}000\,\mathrm{K}$ (equivalent to about 1 keV) and an *average speed* of about 500 km/sec. In general, though, the solar wind blows with speeds ranging from 400km/sec to 2,500km/sec. It carries a million tones of matter into space every second. In particular, the solar wind carries mass away from the sun at the rate of about $1.8\times10^{27}\,erg/\mathrm{sec}$. Yet, the effect of the solar wind remains negligible in the overall mass-energy balance of the sun.

The magnetic fields from the sun are embedded in the solar wind. These magnetic fields have strength of only about $10nT$ at the orbit of the earth. They are about 3,000 times weaker than the magnetic field of the earth (at the surface of the earth).

It is neither the mass nor the speed of the solar wind that makes it potent. It is the energy stored in the plasma and the magnetic fields associated with the plasma that enables the solar wind to shape the protective magnetic shield of the earth known as the magnetosphere.

Less than 1% of the solar wind actually penetrates the magnetosphere. Yet, it is enough to generate millions of *amps* of electric current in our atmosphere. It is also enough to cause occasional *storms* in the space around the earth.

Every eleven years, the sun reaches a peak period of activity called the *solar maximum.* A few years later, a quiet period called the *solar minimum* follows suit. The sun-earth system is driven by this 11-year solar cycle. During the solar maximum, there are many *sunspots*, *eruptive prominences*, *solar flares*, and *coronal mass ejections* (*CMEs*). All these events have strong influence on communications and other technologies here on earth.

3.5 Electromagnetic Coupling

Electromagnetic coupling arises as a result of the interaction of the magnetised solar wind with the geomagnetic field of the earth. When a supersonic solar wind first encounters the geomagnetic field, a free-standing bow shock is formed. As a result, the solar wind is deflected around the earth in a region known as the *magnetosheath* (figure 4). The subsequent interaction of the *magnetosheath flow* with the geomagnetic field leads to the formation of the magnetopause.

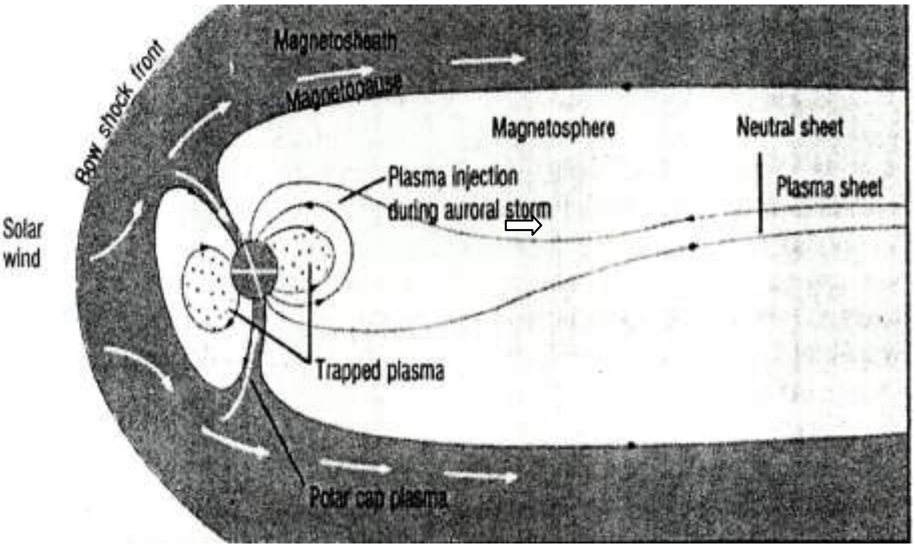

Fig. 1.4: The Interaction of Plasma with the Magnetosphere (after Moore, 1987)

The magnetopause is a relatively thin boundary layer that serves to separate the magnetic field of the solar wind (also known as interplanetary magnetic field, IMF) from the geomagnetic field. The separation is accomplished by means of the *current system* of the magnetopause. However, the magnetopause current layer does not provide a perfect shield. This explains why a portion of the magnetic field of the solar wind penetrates the magnetopause and *connects* with the geomagnetic field (figure 4).

The connection of the IMF and the geomagnetic field occurs in a region known as the *polar cap*. The connected field lines are referred to as *open lines*. The intermediate region that lies between the polar region (i.e. open field line region) and the low-latitude region is called the *auroral oval*.

3.6 Geomagnetic Storms and Substorms

The magnetosphere is the near-earth region of space where the dynamics are governed by the internal geomagnetic field. The solar wind is a highly conducting, collisionless, magnetised plasma. Its behaviour is governed by the ideal *magneto hydrodynamic* (*MHD*) equations. The solar wind magnetic field (or interplanetary magnetic field IMF) penetrates the magnetopause and connects with the geomagnetic field.

The aurora oval contains dipolar magnetic field lines. The field lines are closed. However, as the solar wind blows across the earth, the dipolar magnetic field lines are distorted (Figure 4). On the *dayside*, the field lines are compressed. On the *nightside*, the field lines are elongated to form a very long geomagnetic tail. The geomagnetic tail plays a dominant role in the dynamic processes associated with the magnetosphere. For instance, the geomagnetic tail acts as an energy reservoir that powers *magnetospheric* dynamic processes.

The magnetosphere has several distinct regions. Each of the regions has its characteristic plasma properties. The *tail lobes* at high latitudes are regions of low plasma density and low energy. The *plasma sheets* are hot regions of high plasma density and high energy. In the inner magnetosphere, large populations of high energy particles are trapped. These trapped particles are electrons and ions. They reside in the ring current and in the *Van Allen* radiation belts. For this reason, the inner magnetosphere is the most hazardous region for technological systems.

The solar wind changes constantly and the magnetosphere respond to such changes. The dynamic response of the magnetosphere to varying conditions of the solar wind (and the interplanetary magnetic field) is called *magnetospheric substorm*. During the periods of south wind IMF, the energy input is enhanced. The energy extracted from the solar wind is stored in the form of magnetic energy in the *magnetotail*. This defines the growth phase of the substrom. About an hour later, the magnetotail becomes unstable, and the energy stored is dissipated in driving some *highly dynamic* magnetospheric processes. After about four hours, the energy dissipation stops. This marks the end of the substorm process.

Sometimes, large disturbances in the near-earth space environment are caused by coherent solar wind and interplanetary magnetic field structures that originate from violent events (like coranal mass ejections or solar flares) from the sun. Such large disturbances in the near-earth environment are called geomagnetic storms. Geomagnetic storms are generally associated with major disturbances in the geomagnetic field and strong enhancement of the injection of energetic particles into the outer part of the *Van Allen* radiation belt. Typically, the injected flux of ions has energies of the order of 100keV, while the injected flux of electrons has energies of the order of 100MeV.

Thus, a geomagnetic storm may be described as an intense disturbance of the earth's magnetosphere resulting from a sudden change in the dynamic pressure of the solar wind at the magnetopause. The disturbance may propagate toward the lower altitudes, into the ionosphere, where a traveling ionospheric wave is set up.

3.7 Impact of Space Weather on Human Activity

Space weather is defined in terms of the response of the near-earth space environment to the violent and stormy events emanating from the sun. But, why do scientists study space weather? It is an established fact that modern technological systems are vulnerable to the effects of some dynamic processes taking place in the sun. For instance, the geomagnetic storm of March 24, 1940 caused a temporary disruption of electrical services in New England, New York, Pennsylvania, Minnesota, Quebec and Ontario. On the 9th and 10th days of February 1958, a geomagnetic storm caused a power transformer failure in the British Columbia Hydro and Power Authority. On the 2nd day of August 1972, the following technical problems were reported:

a) A 230,000-volt transformer at the British Columbia Hydro and Power Authority exploded.
b) In Canada, the Minatoba Hydro recorded power drops from 164 to 44 megawatts (in a matter of minutes) in the power being supplied to Minnesota.
c) The Bureau of Reclamation Power Station in Water Town (South Dakota) was subjected to large swings in power line voltages up to 25,000 volts.
d) Wisconsin Power and Light reported voltage swings up to 25,000 volts.
e) Madison Gas and Electric reported voltage swings up to 25,000 volts.
f) Wisconsin Public Service Corporation also reported voltage swings up to 25,000 volts.

Obviously, these technical problems are traceable to *hostile* space weather.

Scientists study space weather in order to help protect both space-based and ground-based technical and biological systems. The technical systems are all the bye-products of technology which are vulnerable to the *harsh* space environment. The biological systems include humans as well as other living things that may be adversely affected by space weather.

Many scientists believe that solar activity affects more than just satellites and power grids. For instance, the magnetic activity that accompanies sunspots and active regions can produce dramatic changes in the level of ultraviolent and soft X-ray emissions. These changes are capable of altering the dynamics, temperature and chemistry in the affected layers. This situation may affect the weather conditions of the earth negatively. As a result, humans may be directly affected. It is therefore, important that we understand the sun and the dynamic processes that give rise to *space weather*. In this way, we can be better prepared to deal with all the adverse effects which space weather has on the earth.

4.0 CONCLUSION

In this unit, you have been introduced to the structure and composition of the atmosphere. You have seen that the ionosphere is very important because of the fact that it is capable of reflecting radio waves. You have noted how the different layers of the ionosphere behave in the day and at night. You have also seen the adverse effects which space weather has on space-based and ground-based technical and biological systems.

5.0 SUMMARY

Having gone through this unit, you learned that:

- the atmosphere is a thin spherical envelop that surrounds the earth
- the atmosphere of the earth contains gases, clouds and other airborne particles called *aerosols*
- the atmosphere protects the earth against harmful radiations from the sun
- the atmosphere helps maintain warm temperatures on the earth
- the atmosphere of the earth may be classified (in terms of temperature) into four layers: the troposphere, the stratosphere, the mesosphere, and the thermosphere

- the atmosphere may be classified (in terms of composition) into two: the homosphere and the heterosphere
- the mean molecular mass is constant in the homosphere due to mixing
- the mean molecular mass changes in the heterosphere due to diffusive separation
- the atmosphere may be classified (in terms of the escape properties) into only one layer: the exosphere
- the gases in the troposphere are predominantly molecular nitrogen (N_2) and molecular oxygen (O_2)
- the temperature decreases with height in the troposphere
- the temperature increases with height in the stratosphere
- the temperature decreases with height in the mesosphere
- the temperature increases dramatically with altitude in the thermosphere
- the thermosphere includes the ionised portion of the atmosphere called the ionosphere
- the auroras are the light produced when electrons and protons from the sun travel along the magnetic field lines of the earth and excite neutral atoms and molecules above the north pole and above the south pole
- the ionosphere is that part of the earth's atmosphere where ions and electrons are present in quantities sufficient to affect the propagation of radio waves
- the ionosphere has three distinct ionisation peaks known as the D layer, the E layer and the F layer
- the ionosphere is capable of reflecting radio waves
- the steady stream of ionised gas (plasma) which escapes from the solar corona, and pervades the whole interplanetary space, is called *solar wind*
- the energy stored in the plasma and the magnetic fields (associated with the plasma) enable the solar wind to shape the magnetic shield of the earth known as the magnetosphere
- electromagnetic, coupling arises as a result of the interaction of the magnetised solar wind with the geomagnetic field of the earth.

6.0 TUTOR-MARKED ASSIGNMENT

1. The thin spherical envelop that surrounds the earth is called the_____
   a) lithosphere
   b) biosphere
   c) atmosphere
   d) hydrosphere

2. The atmosphere of the earth contains the following objects except____
   a) gases
   b) clouds
   c) aerosols
   d) sunspots
3. Which of the following protects the earth against harmful radiations from the sun?
   a) biosphere
   b) cryosphere
   c) hydrosphere
   d) atmosphere
4. In terms of temperature, the terrestrial atmosphere may be classified into____
   a) 2
   b) 3
   c) 4
   d) 6
5. In terms of composition, the atmosphere of the earth may be classified into_________
   a) 2
   b) 3
   c) 4
   d) 6
6. In terms of the escape properties, the atmosphere of the earth is called the___________
   a) troposphere
   b) exosphere
   c) exobase
   d) tropospause
7. In terms of composition, the region below an altitude of about 85km is called the_____________
   a) heterosphere
   b) thermosphere
   c) mesosphere
   d) homosphere
8. In the homosphere, the mean molecular mass___________
   a) remains constant
   b) increases with height
   c) decreases with height
   d) is nearly zero
9. In the troposphere, the gases are predominantly__________
   a) atomic oxygen and atomic nitrogen
   b) oxygen ions and nitrogen ions
   c) molecular nitrogen and molecular oxygen
   d) molecular nitrogen and atomic oxygen

10. In the troposphere, the temperature____________
    a) remains constant
    b) decreases with height
    c) increases with height
    d) is always zero Celsius
11. In the stratosphere, the temperature___________
    a) remains constant
    b) decreases with height
    c) increases with height
    d) is always 6.5^oC
12. In the mesosphere, the temperature________
    a) remains constant
    b) decreases with height
    c) increases with height
    d) is always -90^oC
13. The part of the terrestrial atmosphere where ions and electrons exist in quantities sufficient to affect the propagation of radio waves is called_____
    a) the mesopause
    b) the ionosphere
    c) the magnetosphere
    d) the stratopause
14. The D, E and F layers are distinct ionisation peaks of _____________
    a) the troposphere
    b) the tropopause
    c) the ionosphere
    d) the magnetosphere
15. Which of the following terms best describes the portion of the earth's atmosphere that is most reflective of radio wave?
    a) Tropopause
    b) Stratopause
    c) Mesopause
    d) Ionosphere
16. The steady stream of ionised gas which escapes from the solar corona and pervades the whole interplanetary medium is called________
    a) solar flare
    b) coronal mass ejection (CME)
    c) solar radiation
    d) solar wind
17. Electromagnetic coupling arises as a result of the interaction of the magnetised solar wind with__________________
    a) solar radiation
    b) geomagnetic field
    c) molecular nitrogen

d) molecular oxygen

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UNIT 2 BASIC THERMODYNAMIC CONCEPTS

CONTENTS

1.0 INTRODUCTION

The terrestrial atmosphere contains gases, clouds and other air-bone particles called aerosols. A gas may expand (or be compressed) in different ways. A gas may also be heated (or be cooled) in different ways. In this unit, you shall be introduced to two different processes involving expansion, compression, heating and cooling of a gas. You shall also be introduced to the basic principles of thermodynamics.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- discuss the first law of thermodynamics
- derive an expression for the work done by an ideal gas undergoing isothermal expansion
- derive the equation of a reversible adiabatic process
- discuss how temperature changes in an adiabatic process
- describe the concept of *enthalpy* and *internal energy* of a system.

3.0 MAIN CONTENT

3.1 Basic Concepts: First Law of Thermodynamics

For a closed system that undergoes no changes in kinetic or potential energy, the differential form of the first law of thermodynamics states that

$$\delta Q = \delta U + \delta W,$$

where δQ is the change in the heat content of the system, δU is the change in the internal energy of the system, and δW is the element of work done by the system. The internal energy of the system δU is given by

$$\delta U = c_v \delta T,$$

where c_v is the specific heat capacity of the gas at constant volume, and δT is the change in the temperature of the gas. In general, the specific heat capacity at constant volume c_v is given by

$$c_v = \left(\frac{\partial U}{\partial T} \right)_v ,$$

where T is the temperature of the gas.
The element of work δW done by the system is given by

$$\delta W = p\delta V,$$

where p is the pressure of the gas, and δV is the change in the volume of the same gas .
For an ideal gas, the equation of state is

$$pV = nRT,$$

where n is the number of moles, R is the universal gas constant and T is the absolute temperature of the gas. The parameter R is numerically equal to 8.314 $J\ mol^{-1} K^{-1}$.

In practice, a gas may expand (or be compressed) in different ways. The gas may also be heated (or be cooled) in different ways. In what follows, you shall be introduced to two different processes involving expansion, compression, heating and cooling of a gas. The two processes are the isothermal process and the adiabatic process.

3.2 Isothermal Process

An isothermal process is one which takes place at constant temperature. For one mole of an ideal gas, the equation of state may be written as

$$pV = RT,$$

where symbols have their usual meanings. In an isothermal process, the temperature T is constant. Thus, the equation of state of an ideal gas undergoing isothermal process is

$$pV = k,$$

where k is a constant. Hence, you may write

$$p = \frac{k}{V}.$$

This implies that the $p-V$ curve must be a rectangular hyperbola (figure 1).

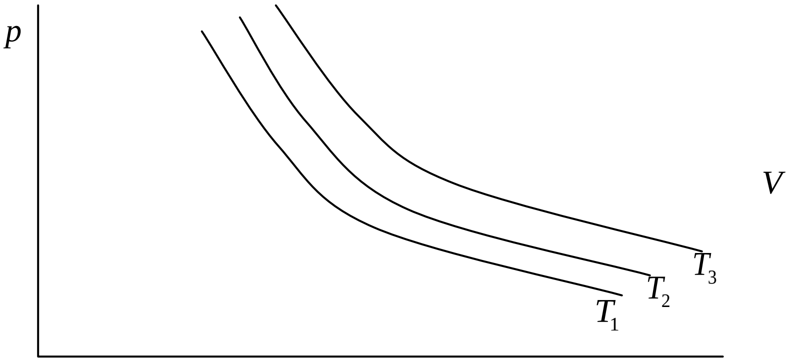

Fig. 1: The $p-V$ Curve of a Gas Undergoing Isothermals Process ($T_1 < T_2 < T_3$)

In an isothermal expansion, work is done by the system on the surroundings.

The element of work δW done by an ideal gas undergoing isothermal expansion is

$$\delta W = p\delta V.$$

Therefore, the work done in expanding from V_1 to V_2 is

$$W = \int dW$$

$$W = \int_{V_1}^{V_2} p\, dV$$

$$W = \int_{V_1}^{V_2} \left(\frac{RT}{V}\right) dV$$

$$W = RT\int_{V_1}^{V_2} \frac{dV}{V}$$

$$W = RT\left[\log_e V\right]_{V_1}^{V_2}$$

$$W = RT\left(\log_e V_2 - \log_e V_1\right)$$

$$W = RT\log_e\left(\frac{V_2}{V_1}\right).$$

The work W done by the gas is equal to the quantity of heat Q required to do the work. Thus, the heat Q required is also given by

$$Q = RT\log_e\left(\frac{V_2}{V_1}\right).$$

Now, for an isothermal process,

$$pV = k,$$

where k is a constant.

$$\therefore p_1V_1 = p_2V_2$$

$$\therefore \frac{V_2}{V_1} = \frac{p_1}{p_2}$$

Hence, the work done, in an isothermal process, may also be written as

$$W = Q = RT\log_e\left(\frac{p_1}{p_2}\right).$$

This quantity of heat Q must be supplied to the system that is doing work on the surroundings in order to keep the temperature constant. Otherwise, the temperature of the system falls, and the process ceases to be isothermal.

In the case of an isothermal compression, work is done on the system by the surroundings. In this case, the heat Q must be removed from the system in order to keep its temperature constant. Otherwise, the temperature rises, and the process ceases to be isothermal.

Perfect isothermality is a highly idealised concept. It is impossible to realise it in practice. For instance, the heat Q cannot flow through the walls of the vessel unless there is a temperature difference across the walls. Therefore, the temperature of the gas is bound to *fall a little* in an isothermal expansion. Also, the temperature of the gas is bound to *rise a little* in an isothermal compression. However, if the process is slow, and the gas is held in a thin, well-conducting vessel surrounded by a constant-temperature bath, the process becomes approximately isothermal.

3.3 Adiabatic Process

3.3.1 Equation of Reversible Adiabatic Change

An adiabatic process is one which takes place at constant heat. In an adiabatic process, the temperature may change but no heat enters or leaves the system.

Consider an adiabatic process in which a unit mass of an ideal gas expands from an initial volume V_1 to a final volume V_2. The first law of thermodynamics implies that the heat dQ supplied to the system is given by

$$dQ = dU + dW, \tag{1}$$

where dU is the increase in the internal energy of the system and dW is the external work done by the system.

You will recall that $dU = c_v dT$ and $dW = pdV$, so that

$$dQ = c_v dT + pdV, \tag{2}$$

where c_v is the specific heat capacity at constant volume, dT is the rise in temperature, dV is the increase in volume, and p is the pressure of the gas.

For a reversible adiabatic change $dQ = 0$.

Thus, Equation (2) reduces to

$$c_v dT + pdV = 0.$$

$$\therefore\ dT = \frac{-pdV}{c_v}. \tag{3}$$

Recall that the equation of state for a unit mass of an ideal gas is

$$pV = RT. \tag{4}$$

Differentiating Equation (4), you obtain

$$pdV + Vdp = RdT$$

$$\therefore\ dT = \frac{pdV + Vdp}{R} \tag{5}$$

Equating the RHS of (5) to the RHS of (3),

$$\frac{pdV + Vdp}{R} = \frac{-pdV}{c_v}$$

Cross-multiplying, you obtain

$$c_v\left(pdV + Vdp\right) = -RpdV,$$

so that

$$c_v\left(pdV + Vdp\right) + RpdV = 0.$$

Hence,

$$c_v pdV + c_v Vdp + RpdV = 0. \tag{6}$$

But c_p and c_v are related to the gas constant R in the following way:

$$R = c_p - c_v. \tag{7}$$

Using (7) in (6), you get

$$c_p pdV + c_v Vdp + \left(c_p - c_v\right)pdV = 0$$

$$c_v pdV + c_v Vdp + c_p pdV - c_v pdV = 0.$$

$$c_v Vdp + c_p pdV = 0.$$

$$Vdp + \frac{c_p}{c_v} pdV = 0.$$

If you set $\gamma = c_p/c_v$, you obtain

$$Vdp + \gamma pdV = 0.$$

Dividing each term by pV, you get

$$\frac{dp}{p} + \gamma \frac{dV}{V} = 0.$$

Integrating, you find that

$$\int \frac{dp}{p} + \gamma \int \frac{dV}{V} = \text{constant.}$$

or

$$\log_e p + \gamma \log_e V = a,$$

where a is a constant.

Applying the elementary laws of logarithm, you obtain

$$\log_e p + \log_e V^{\gamma} = a$$

or

$$\log_e \left(pV^{\gamma}\right) = a$$

That is,

$$pV^{\gamma} = e^{a}$$

where a is a constant.

Therefore, you may write:

$$pV^{\gamma} = \text{constant,} \qquad (8)$$

since a is a constant.

Equation (8) is the equation of a reversible adiabatic process.

3.3.2 Temperature Change in an Adiabatic Process

Recall that the general gas equation is

$$pV = RT \qquad (1)$$

so that

$$p = \frac{RT}{V}$$

and

$$pV^{\gamma} = \left(\frac{RT}{V}\right)V^{\gamma}$$

i.e.

$$pV^{\gamma} = RTV^{\gamma}V^{-1}$$

or

$$pV^{\gamma} = RTV^{\gamma-1}. \qquad (2)$$

But the equation of an adiabatic process is

$$pV^{\gamma} = \text{constant,}$$

$$\therefore RTV^{\gamma-1} = \text{constant.} \qquad (3)$$

Equation (3) is the equation for an adiabatic temperature change.

3.3.3 Work Done in an Adiabatic Process

The work W done by a unit mass of gas undergoing adiabatic expansion is given by

$$W = \int_{V_1}^{V_2} p\,dV \tag{1}$$

Since the process is adiabatic, it follows that

$$pV^{\gamma} = \beta \tag{2}$$

where β is a constant given by $\beta = e^{\alpha}$. Thus,

$$p = \beta V^{-\gamma} \tag{3}$$

Hence, the work W becomes

$$W = \int_{V_1}^{V_2} \beta V^{-\gamma} dV$$

$$W = \beta \int_{V_1}^{V_2} V^{-\gamma} dV$$

$$W = \beta \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$W = \frac{\beta}{1-\gamma}\left(V_2^{1-\gamma} - V_1^{1-\gamma}\right) \tag{4}$$

Now, Equation (4) states that

$$p = \beta V^{-\gamma},$$

so that

$$pV = \beta V^{-\gamma} \cdot V$$

or

$$pV = \beta V^{-\gamma+1}. \tag{5}$$

Expanding (4), you obtain

$$W = \frac{1}{1-\gamma}\left(\beta V_2^{1-\gamma} - \beta V_1^{1-\gamma}\right),$$

so that

$$W = \frac{1}{1-\gamma}\left(p_2 V_2 - p_1 V_1\right) \tag{6}$$

since, $pV = \beta V^{-\gamma+1}$.

Equation (6) represents the work W done by the system when a unit mass of an ideal gas expands adiabatically from some initial volume V_1 to a final volume V_2.

3.5 Enthalpy and Internal Energy

Recall that the first law of thermodynamics may be stated as follows:

$$dQ = dU + pdV. \qquad (1)$$

If the pressure p is constant then there is really no difference between pdV and $d(pV)$.

For instance,

$$d(pV) = pdV + Vdp,$$

and if the pressure p is constant, then $dp = o$

so that

$$d(pV) = pdV + 0.$$

Thus, for a constant-pressure process,

$$d(pV) = pdV. \qquad (2)$$

In this case, the first law of thermodynamics becomes

$$dQ = dU + d(pV).$$

$$dQ = d\left[U + (pV)\right]$$

This implies that

$$dQ = d(U + pV).$$

By defining a new thermodynamic property H such that.

$$H = U + pV, \qquad (3)$$

you find that, at constant pressure,

$$dQ = dH.$$

This new thermodynamic property H is called the *enthalpy* of the system. It follows, therefore, that

$$dH = c_p dT. \qquad (4)$$

You may wish to compare Equation (4) with the expression for the change in the internal energy dU of the system, namely,

$$dU = c_v dT. \qquad (5)$$

Equation (5) shows that the specific heat capacity at constant volume c_v may be written as:

$$c_v = \left(\frac{\partial U}{\partial T}\right)_V . \qquad (6)$$

Equation (4) shows that the specific heat capacity at constant pressure c_p may be written as

$$c_p = \left(\frac{\partial H}{\partial T}\right)_p . \tag{7}$$

These principal specific heat capacities play a significant role in the treatment of air and water vapour.

3.6 Specific Heats for Ideal Gases

A useful relation exists between the principal specific heats and the universal gas constant R. This relation may be expressed as follows:

$$R = c_p - c_v \tag{1}$$

You had earlier met this relation in your study of the equation of an adiabatic process. In what follows, you shall be introduced to a method of deriving the above relation. You will recall that the change in the internal energy of a system is given by

$$dU = c_v dT. \tag{2}$$

Also, the change in the enthalpy of the system is

$$dH = c_p dT. \tag{3}$$

If Eq. (2) is subtracted from Eq. (3), you obtain

$$dH - dU = c_p dT - c_v dT.$$

or

$$dH - dU = \left(c_p - c_v\right) dT. \tag{4}$$

But,

$$dH = dU + d(pV).$$

i.e.

$$dH = dU + d(RT),$$

since

$$pV = RT.$$

$$\therefore\ dH - dU = RdT \tag{5}$$

Comparing (4) and (5), you find that

$$RdT = \left(c_p - c_v\right) dT$$

or

$$R = c_p - c_v.$$

The equation given above shows that the difference of the principal specific heat capacities is numerically equal to the gas constant.

For air at room temperature, the molecular weight
$M = 28.97, \quad c_p = 1.005\,KJ/kg.{}^{o}C, \;\; c_v = 0.718\,KJ/kg.{}^{o}C, \;\; R = 287.1\,J/kg.K \text{ and } g = 1.4.$

For water vapour at room temperature, the molecular weight
$M = 18.02, \;\; c_p = 1867\,J/kg\,{}^{o}C, \;\; c_v = 1407\,J/kg.{}^{o}C, \;\; R = 460.6\,J/kg.K \text{ and } g = 1.33.$

3.7 Entropy and the Second Law of Thermodynamics

The second law of thermodynamics may be stated in any of the following ways:

a) *It is impossible to construct a device which operates in cycle and whose sole effect is to transfer heat from a cooler body to a hotter body. (This is the Clausius statement of the second law of thermodynamics).*
b) *It is impossible to construct a device which operates in a cycle and produces no other effect than the production of work and exchange of heat with a single reservoir. (This is the Kelvin-Plank statement of the second law of thermodynamics).*

Clausius statement of the second law of thermodynamics shows heat flows from a high-temperature region to a low temperature region in the absence of other effects. Thus, when a hot body is brought into contact with a cold body, the hot body becomes less hot and cold body becomes less cold. It is not possible for the cold body (in contact with a hot body) to be become more cold unless additional effects are brought to bear upon the system.

Kelvin-Plank statement shows that it is not possible to construct a device which will operate continuously while receiving heat from a single reservoir and producing an equivalent amount of work. Thus, a *perpetual-motion* machine cannot be built.

In general, any quantity which may be represented as an *exact differential* is a property (or point) function. This means that the quantity is a function of the state of the system. If the cyclic integral of a differential vanishes, then the differential is an exact differential.

It turns out that, for a reversible process, the cyclic integral of $\frac{dQ}{T}$ is zero. That is,

$$\oint \left(\frac{dQ}{T}\right)_{reversible} = 0.$$

Thus, the quantity $\frac{dQ}{T}$ is an exact differential which may be denoted by ds. In this case,

$$ds = \frac{dQ}{T},$$

where s is another new property of the system called *entropy*. It is important to note that entropy is so defined only for reversible processes. For such a reversible process a change in entropy may be with written as

$$\Delta s = s_2 - s_1 = \int_{a_1}^{a_2} \left(\frac{dQ}{T} \right)_{reversible},$$

where a_1 and a_2 are, respectively, the appropriate lower and upper limits of the integration. Here, a_1 and a_2 are two state points associated with the process.

In general,

$$s_2 - s_1 \geq \int_{a_1}^{a_2} \frac{dQ}{T},$$

where the *equality* holds for a reversible process and the *inequality* holds for an irreversible process.

A reversible process is an idealised state path. A process is said to be reversible if the initial state of the system can be restored following exactly the same state path in reverse direction with no observable effects on the system and its surroundings. In practice, it is impossible to find a perfectly reversible process. Since all real life processes are irreversible, we conclude that the entropy is always greater than zero.

4.0 CONCLUSION

In this unit, you have been introduced to the basic concepts of thermodynamics. You have seen the difference between an isothermal process and an adiabatic process. You have found how the enthalpy of a system relates to its internal energy. You have also noted that the entropy of any real-life process is always greater than zero.

5.0 SUMMARY

Having gone through this unit, you now know that:

- the first law of thermodynamics implies that

$$\delta Q = \delta U + \delta W,$$

where δQ is the change in the heat content of the system, δU is the internal energy of the system and δW is the element of work done by (or done on) the system.

- the element of work δW done by a system is given by

$$\delta W = p\delta V.$$

where p is the pressure of the gas and δV is the change in the volume of the gas.

- fon n moles of an ideal gas, the equation of state is

$$pV = nRT$$

where p is pressure, V is volume, T is temperature and R is the universal gas constant.

- the difference of the principal specific heat capacities of an ideal gas is given by

$$c_p - c_v = R$$

where c_p is the specific heat at constant pressure, c_v is the specific heat at constant volume and R is the gas constant.

- the ratio of the principal specific heat capacities is denoted by γ, where

$$\gamma = \frac{c_p}{c_v}.$$

- the equation of a reversible adiabatic process is

$$pV^\gamma = \text{constant},$$

where p is pressure and V is volume.

- the equation for an adiabatic temperature change is

$$RTV^{\gamma-1} = \text{constant},$$

where T is temperature.

- the work W done in an isothermal process is given by

$$W = RT\log_e\left(\frac{V_2}{V_1}\right),$$

where V_1 is the initial volume and V_2 is the final volume of the expanding the gas.

- The work W done in an adiabatic process is given by

$$W = \frac{1}{1-\gamma}\left(p_2V_2 - p_1V_1\right),$$

where γ is the ratio of the principal specific heat capacities.

- the change in the internal energy of an ideal gas is given by

$$\delta U = c_v\delta T$$

where δT is the change in temperature

- the enthalpy H of a system is given by

$$H = U + pV$$

where U is the internal energy of the system.

- the specific heat capacity at constant volume may be written as

$$c_v = \left(\frac{\partial U}{\partial T} \right)_v,$$

where U is the internal energy of the system
- the specific heat capacity at constant pressure may be written as

$$c_p = \left(\frac{\partial H}{\partial T} \right)_p,$$

where H is the enthalpy of the system
- a *constant-temperature* process is called an isothermal process.
- a *constant-heat* process is called an *adiabatic process.*
- a *constant-volume* process is called an *isochoric process.*
- a *constant-pressure* process is called an *isobaric process.*

6.0 TUTOR-MARKED ASSIGNMENT

1. The expression $\delta Q = \delta U + p\delta V$ represents a statement of
   a) the zeroth law of thermodynamics
   b) the first law of thermodynamics
   c) the second law of thermodynamics
   d) the third law of thermodynamics
2. The element of work δW done by an expanding gas is given by
   a) $\delta W = F\delta V,$ where F is force
   b) $\delta W = \rho\delta V,$ where ρ is density
   c) $\delta W = p\delta V,$ where p is pressure
   d) $\delta W = 0$ always
3. The equation of state of an ideal gas may be written as $pV = nRT,$ where n is the number of
   a) colliding particles
   b) non-interacting particles
   c) ions
   d) moles
4. The difference of the principal specific heat capacities is numerically equal to
   a) Avogadro's number
   b) the volume of a fixed mass of gas
   c) the gas constant
   d) the number of moles
5. The ratio of the principal specific heat capacities is usually denoted by
   a) $\gamma = \frac{c_p}{V}.$
   b) $\gamma = \frac{c_v}{c_p}.$

c) $\gamma = \frac{c_p}{c_v}.$

d) $\gamma = \frac{c_v}{p}.$

6. The expression $"pV^{\gamma} = \text{constant}"$ represents
   a) the general gas law
   b) the first law of thermodynamics
   c) the equations of reversible adiabatic change
   d) the equation of reversible isothermal change

7. For a fixed mass of gas, the expression $"TV^{\gamma-1} = \text{constant}"$ may be used to investigate how the temperature changes in
   a) an isothermal process
   b) an adiabatic process
   c) an isochoric process
   d) an isobaric process

8. The expression $W = RT\log_e\left(\frac{V_2}{V_1}\right)$ represents the work done in
   a) an isothermal process
   b) an adiabatic process
   c) an isochoric process
   d) an isobaric process

9. The expression $W = \frac{1}{1-\gamma}\left(p_2V_2 - p_1V_1\right)$ represents the work done in
   a) an isothermal process
   b) an adiabatic process
   c) an isochoric process
   d) an isobaric process

10. The change in the internal energy of a gas is given by the expression
    a) $\delta U = c_p \delta T.$
    b) $\delta U = c_v \delta T.$
    c) $\delta U = c_p \delta V.$
    d) $\delta U = c_v \delta p.$

11. The enthalpy of a system is given by
    a) $H = \delta U + p\delta V.$
    b) $H = \delta U - p\delta V.$
    c) $H = U + pV.$
    d) $H = U - pV.$

12. An expression for the specific heat capacity at constant pressure is

a) $c_p = \left(\frac{\partial H}{\partial T} \right)_V$.

b) $c_p = \left(\frac{\partial H}{\partial T} \right)_p$.

c) $c_p = \left(\frac{\partial H}{\partial p} \right)_T$.

d) $c_p = \left(\frac{\partial H}{\partial p} \right)_V$.

13. An expression for the specific heat capacity at constant volume is

a) $c_v = \left(\frac{\partial U}{\partial T} \right)_V$.

b) $c_v = \left(\frac{\partial U}{\partial T} \right)_P$.

c) $c_v = \left(\frac{\partial U}{\partial V} \right)_T$.

d) $c_v = \left(\frac{\partial U}{\partial V} \right)_V$.

14. A *constant-temperature* process is called
a) an isothermal process
b) an adiabatic process
c) an isochoric process
d) an isobaric process

15. A *constant-heat* process is called
a) an isothermal process
b) an adiabatic process
c) an isochoric process
d) an isobaric process

16. A constant-volume process is called
a) an isothermal process
b) an adiabatic process
c) an isochoric process
d) an isobaric process

17. A constant-pressure process is called
a) an isothermal process
b) an adiabatic process
c) an isochoric process
d) an isobaric process

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UNIT 3 AIR AND WATER VAPOUR

CONTENTS

1.0 INTRODUCTION

The air we breath is a very useful gas. The water we drink is a very useful liquid. Without air and water, it would be impossible for any living thing to survive on earth. In this unit, you shall be introduced to some of the most important properties of air, water and water vapour.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- describe the différent elements that make up the air
- describe the porperties of air and water vapour at room temperature
- discuss the saturation regions using a $p-T$ diagram
- distinguish between *moisture* and *quality* of the liquid-vapour saturation region
- relate the internal energy, enthalpy and entropy to the temperature and specific volume of a pure substance.

3.0 MAIN CONTENT

3.1 The Air around Us

The air we breathe is readily available near the surface of the earth. It is made up of nitrogen (78%), oxygen (21%), argon (0.9%) and other gases (0.1%). At room temperature, the mean molecular weight of air, $M = 28.97$; the specific heat capacity at constant pressure, $c_p = 1.005\,kJkg^{-1}K^{-1}$; the specific heat capacity at constant volume; $c_v = 0.718\,kJkg^{-1}K^{-1}$; the gas constant, $R = 287.1\,Jkg^{-1}K^{-1}$; and the ratio of the principal specific heat capacities, $g = \frac{c_p}{c_v} = 1.4$.

For water vapour at room temperature, the molecular weight, $M = 18.02$; the specific heat capacity at constant pressure $c_p = 1867\,Jkg^{-1}K^{-1}$; the specific heat capacity at constant volume, $c_v = 1407\,Jkg^{-1}K^{-1}$; the gas constant, $R = 460.6\,Jkg^{-1}K^{-1}$; and the ratio of the principal specific heat capacities, $g = \frac{c_p}{c_v} = 1.33$.

3.2 Water and its Vapour

Ordinary water may exist in seven phases, namely:

(i) a pure solid phase known as ice
(ii) a pure liquid phase commonly known as water
(iii) a pure vapour phase called steam
(iv) an equilibrium mixture of liquid and vapour phases
(v) an equilibrium mixture of liquid and solid phases
(vi) an equilibrium mixture of solid and vapor phases
(vii) an equilibrium mixture of solid, liquid and vapour phases.

Fusion takes place when a solid changes to a liquid; *vaporisation* takes place when a liquid changes to a vapour; *sublimation* takes place when a solid changes directly to a vapour. (At atmospheric pressure, the sublimation temperature for CO_2 is about -78^{0}C).

3.3 Macroscopic Properties of Pure Water

Consider a piece of ice at state point S. If the piece of ice is heated at constant pressure the temperature rises until the fusion line is reached. As soon as the fusion line is encountered, melting starts. The melting process continues at constant temperature until all the solid ice is changed to liquid. Further heating produces an increase in the temperature. The temperature of the liquid water continues to rise until the vaporisation line is encountered. As soon as the vaporisation line is reached, the liquid water begins to change to vapour.

Figure 2 demonstrates how water behaves when its temperature and pressure change. The pressure p_0 is atmospheric. The horisontal line, passing through the solid phase (S), the liquid phase (L) and the vapour phase (V), is a constant-pressure line. It shows that water at atmospheric pressure may be made to exist in solid, liquid and vapour phases.

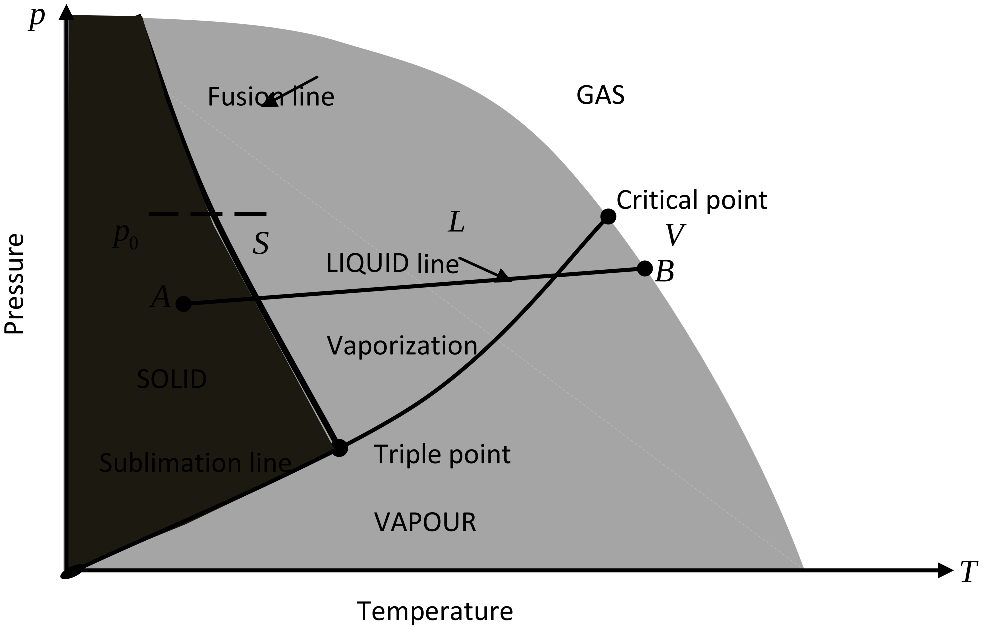

Fig. 3.1: Pressure-Temperature Diagram for Water (after Holman, 1980)

The vaporisation process continues at constant temperature until all the liquid water is changed to water vapour. Additional heating produces a rise in the temperature of the vapour. At atmospheric pressure, the fusion temperature for water is $0^\circ C$ and the vaporisation temperature is $100^\circ C$.

Figure 2 also shows that the freezing point is altered by a change in pressure. An increase in pressure lowers the freezing point. Figure 2 also shows that a change in pressure alters the vaporisation point. In particular, an increase in pressure raises the vaporisation temperature of water.

The p- T diagram shown in figure 2 is not drawn to scale. In practice, the fusion line is almost vertical and a large increase in pressure is needed to lower the melting temperature substantially. The solid-liquid mixture is represented by the fusion line; the liquid-vapour mixture is represented by the vapour line; and the solid-vapour mixture is represented by the sublimation line. The triple point is the *state point* where it is possible to maintain an equilibrium mixture of the solid phase, the liquid phase and the vapour phase. At the triple point, all the three phases of water co-exist in equilibrium.

At the critical point, the pure vapour phase has identical properties with a pure liquid phase at the same temperature and pressure. The surface meniscus separating the liquid phase from the vapour phase vanishes at the critical point. At super-critical temperatures and pressure, it is no longer possible to observe a distinction between the liquid phase and the vapour phase.

3.4 Saturation Regions

3.4.1 Liquid-Vapour Saturation Region

The fusion line, the vaporisation line and the sublimation line represent saturation regions. The fusion line represents the saturation region between the solid phase and the liquid phase; the vaporisation line designates the saturation region between the liquid phase and the vapour phase; the sublimation line represents the saturation region between the solid phase and the vapour phase. The vapour present in such a mixture is called a saturated vapour. In the saturation regions, the temperature and the pressure remain constant. However, there is a significant change in the specific volume (i.e. volume per unit mass) in the saturation regions.

To quantify the properties of the liquid-vapour saturation region, we define the *quality* x of the saturation region as the *fraction* of mass present in the vapour phase. We also define the *moisture* y as the *fraction* of mass present in the liquid phase. Obviously, the sum of the *two fractions* must be equal to unity. Therefore, we may write.

$$x + y = 1$$

or

$$y = 1 - x \tag{1}$$

or

$$\text{Moisture} = 1 - x,$$

where

$$x = \frac{\text{mass in the vapour phase}}{\text{total mass } m}$$

3.4.2 Wet Mixture

A *wet mixture* is a liquid-vapour mixture that has a *quality* x which is less than 100%. The subscripts s, l and v shall be used to represent solid phase, liquid phase and vapour phase, respectively. The lower-case "v" shall also be used to designate specific volume (i.e. $v = V/m$). This means that the specific volume of a saturated liquid becomes v_l while the specific volume of a saturated vapour becomes v_v. Of course, the total volume V is given by

$$V = m_l v_l + m_v v_v, \tag{2}$$

and the total mass m is given by

$$m = m_l + m_v,$$

so that the specific volume v of an equilibrium liquid-vapour mixture would be given by

$$v = \frac{V}{m}, \tag{3}$$

where m_l is mass of the liquid phase and m_v is mass of the vapour phase. Thus,

$$v = \frac{m_l v_l + m_v v_v}{m}$$

$$v = \frac{m_l}{m} v_l + \frac{m_v}{m} v_v$$

or

$$v = y v_l + x v_v.$$

$$v = (1 - x) v_l + x v_v,$$

where $x = m_v/m$ and $y = m_l/m$.

Hence,

$$v = v_l - x v_l + x v_v,$$

or

$$v = v_l + (v_v - v_l)x.$$

Thus,

$$v = v_l + x v_{lv}, \tag{4}$$

where,

$$v_{lv} = v_v - v_l. \tag{5}$$

Equation (4) relates the specific volume (v) to the specific volume in the liquid phase (v_l), the specific volume in the vapour phase (v_v) and the *quality* (x) of the saturation region.

3.4.3 Super-Cooled Liquid and Super-Heated Vapour

A *compressed liquid* is one which exists at a temperature that is lower than the saturation temperature corresponding to its pressure. A compressed liquid is also called a *sub-cooled* liquid. A *Superheated* vapour is one that exists at a temperature greater than the saturation temperature corresponding to its pressure.

3.4.4 Energy Properties of a Pure Substance

How many independent properties are necessary to define the state of a simple, pure substance? *Only two independent properties are necessary to define the state of a simple, pure substance.* The two independent properties are *temperature* and specific *volume*. Why not *pressure?* In the saturation regions, temperature and pressure are not independent properties. Infact, a line of constant temperature is also a line of constant pressure in all the saturation regions.

Temperature, pressure and specific volume are not the only properties of interest when considering pure substances. Other important properties include the specific internal energy u and the specific enthalpy h. These energy properties may be expressed as functions of temperature T and specific volume v in the following why:

$$u = u(T, v),$$

and

$$h = h(T, v).$$

Equation (4) relates the specific volume v to v_l, v_v and x since

$$v = v_l + xv_{lv}$$

in the wet mixture region. By a similar argument, you may write

$$u = v_l + xv_{lv} \tag{6}$$

and

$$h = h_l + xh_{lv} \tag{7}$$

in the *wet mixture* region.
Here,

$$u_{lv} = u_v - u_l \tag{8}$$

and

$$h_{lv} = h_v - h_l, \tag{9}$$

where u_{lv} is the internal energy of vaporisation and h_{lv} is the enthalpy of vaporisation.

Table 3.1: Properties of Water-Saturation-Pressure Table (SI Units)*

$v \; in \; cm^3/g = m^3/kg : h \; and \; u \; in \; k\,J/kg.k; \; p \; in \; bars, 1\,bar = 10^5 \, pa$

Press bar p	Tem. ^oC T	Specific volume Sat. liquid u_l	Sat. vapor u_v	Internal energy Sat. liquid u_l	Sat. vapor u_v	Enthalpy Sat. liquid h_l	Evap. h_{lv}	Sat. vapor h_v	Entropy Sat. liquid s_l	Sat. vapor s_v
0.80	93.50	1.0380	2087.	391.58	2498.8	391.66	2274.1	2665.8	1.2329	7.4346
0.90	96.71	1.0410	1869.	405.06	2502.6	405.15	2265.7	2670.9	1.2695	7.3949
1.00	99.63	1.0432	1694.	417.36	2506.1	417.46	2258.0	2675.5	1.3026	7.3594
1.50	111.4	1.0528	1159.	466.94	2519.7	467.11	2226.5	2693.6	1.4336	7.2233
2.00	120.2	1.0605	885.7	504.49	2529.5	504.70	2201.9	2706.7	1.5301	7.1271

***Abridged from Holman (1980), Thermodynamics, McGraw-Hill Kogakusha Ltd, Tokyo**

Finally, the entropy s is another property that is extremely useful in solving practical problems. Like the specific internal energy u and the specific enthalpy h, the entropy s may be written in the characteristic form

$$s = s_l + xs_{lv}, \tag{10}$$

where

$$s_{lv} = s_v - s_l, \tag{11}$$

and s_{lv} is the entropy of vaporisation.

All these properties of a pure-substance are usually tabulated. In order to solve a practical problem, it is often necessary to consult appropriate tables. Typical values may be found in Table 1.

Exercise

Given that the pressure is 150kPa and that the quality is 1%, determine.

(i) the specific volume v,

(ii) the enthalpy h, and

(iii) the entropy s of water.

Solution

(a) The saturation region has 1% quality at 150kPa. We therefore consult Table 1 to obtain the following values:

$$v_l = 1.0528' \; 10^{-3} m^3 kg^{-1}$$

$$v_v = 1.159 m^3 kg^{-1}$$

$$h_l = 467.11 kJkg^{-1}$$

$$h_{lv} = 2226.5 kJkg^{-1}$$
$$s_l = 1.4336 kJkg^{-1}K^{-1}$$
$$s_v = 7.2233 kJkg^{-1}K^{-1}$$

The quality $x = 1\% = 0.01$.

(i) The specific volume v is

$$v = v_l + xv_{lv},$$

where

$$v_w = v_v - v_l.$$

Thus,

$$\therefore\ v = \left(1.0528 \times 10^{-3}\right) + (0.01)\left(1.159 - 1.0528 \times 10^{-3}\right)$$
$$v = \left(1.0528 \times 10^{-3}\right) + (0.01)(1.1579472)$$
$$v = 0.012632272$$
$$\therefore\ v \approx 0.0126.$$

(ii) The specific enthalpy h is

$$h = h_l + xh_{lv}$$
$$h = 467.11 + (0.01)(2226.5)$$
$$h = 467.11 + 22.265$$
$$\therefore\ h = 489.375$$

(iii) The entropy s is

$$s = s_l + xs_{lv}$$
$$s = 1.4336 + (0.01)(7.2233 - 1.4336)$$
$$s = 1.4336 + (0.01)(5.7897)$$
$$s = 1.4336 + 0.057897$$
$$s = 1.491497$$
$$\therefore\ s \approx 1.4915$$

4.0 CONCLUSION

In this unit, you have been introduced to the properties of air and ordinary water. You have seen that air is made up of nitrogen, oxygen and other gases. You have also seen that water can exist in threes pure phases and four equilibrium mixture phases. You have quantified the saturation regions and calculated the specific volume, the specific enthalpy and the entropy of a pure substance.

5.0 SUMMARY

Having gone through this unit, you now know that:

- the air is made up of nitrogen (78%), oxygen (21%), argon (0.9%), carbon dioxide and other gases (0.1%).
- the mean molecular weight of air at room temperature is $M = 28\cdot97$.
- the specific heat capacity of air at constant pressure $c_p = 1\cdot005\, kJkg^{-1}K^{-1}$.
- the specific heat capacity of air at constant volume $c_v = 0\cdot718\, kJkg^{-1}K^{-1}$.
- the gas constant for air $R = 287\cdot1\, Jkg^{-1}K^{-1}$.
- the ratio of the principal specific heat capacities for air $g = \frac{c_p}{c_v} = 1\cdot4$.
- ordinary water may exist in three pure phases:
  a) solid phase
  b) liquid phase, and
  c) vapour phase.

- ordinary water may also exist in four equilibrium mixture phases:
  a) an equilibrium mixture of liquid and vapour phases,
  b) an equilibrium mixture of liquid and solid phases,
  c) an equilibrium mixture of solid and vapour phases, and
  d) an equilibrium mixture of solid, liquid and vapour phases.
- the mean molecular weight of water vapour at room temperature is 18.02.
- fusion takes place when a solid changes to a liquid.
- vaporisation takes place when a liquid changes to a vapour.
- sublimation takes place when a solid changes directly to a vapour (without passing through the liquid phase).
- a $p - T$ diagram highlights the relative positions of the different phases of a pure substance.
- water at atmospheric pressure may be made to exist in all the three pure phases.
- at atmospheric pressure, the fusion temperature of water is 0^oC.
- at atmospheric pressure, the vaporisation temperature of water is 100^oC.
- an increase in pressure lowers, the freezing point of water.
- an increase in pressure raises the vaporisation temperature of water.

- the triple point is the *state point* where it is possible to maintain an equilibrium mixture of the solid phase, the liquid phase and the vapour phase.
- at the *triple point* of water, all the three phases co-exist in equilibrium.
- at the critical point, the pure vapour phase has identical properties with the pure liquid phase at the same temperature and pressure.
- the surface meniscus seperating the liquid phase from the vapour phase vanishes at the critical point.
- at super-critical temperatures and pressures, it is no longer possible to observe a distinction between the liquid phase and the vapour pahse.
- the fusion line represents the saturation region between the solid phase and the liquid phase.
- the vaporisation line represents the saturation region between the liquid phase and the vapour phase.
- the sublimation line represents the saturation region between the solid phase and the vapour phase.
- in the saturation regions, the temperature and the pressure remain constant.
- in the saturation regions, the specific volume (i.e. volume per unit mass) changes significantly.
- the *quality* x of a liquid-vapour saturation region, is defined as the fraction of mass present in the vapour phase.
- the *moisture* y of a liquid-vapour phase is defined as the fraction of mass present in the liquid phase.
- the sum of the *quality* x and the *moisture* y is always *unity* (i.e. $x + y = 1$).
- a wet mixture is a liquid-vapour mixture that has a *quality* x which is less than 100%.
- a compressed liquid is one which exists at a temperature that is lower than the saturation temperature corresponding to its pressure (a compressed liquid is also called a super-cooled liquid).
- a super-heated vapour is one which exists at a temperature higher than the saturation temperature corresponding to its pressure.
- the state of a simple, pure substance may be defined using only *two independent* properties (i.e. temperature and specific volume).
- energy properties (like specific internal energy, specific enthalpy and entropy) may be expressed in terms of temperature and specific volume i.e.

$$u = u(T, v),$$
$$h = h(T, v),$$

and

$$s = s(T, v).$$

6.0 TUTOR-MARKED ASSIGNMENT

1. The dominant element in the earth's atmosphere is
   a) argon
   b) carbon dioxide
   c) nitrogen
   d) oxygen
2. The mean molecular weight M of air is
   a) $M = 28\times97$
   b) $M = 18\times02$
   c) $M = 16\times00$
   d) $M = 14\times00$
3. The specific heat capacity of air at constant pressure is c_p, where
   a) $c_p = 0\times718\ kJkg^{-1}K^{-1}$
   b) $c_p = 1\times005\ kJkg^{-1}K^{-1}$
   c) $c_p = 1\times400\ kJkg^{-1}K^{-1}$
   d) $c_p = 287\times100\ Jkg^{-1}K^{-1}$
4. For air, the gas constant is R, where
   a) $R = 0\times718\ kJkg^{-1}K^{-1}$.
   b) $R = 1\times005\ kJkg^{-1}K^{-1}$.
   c) $R = 1\times400\ kJkg^{-1}K^{-1}$.
   d) $R = 287\times100\ kJkg^{-1}K^{-1}$.
5. For air, the ratio of the principal specific heat capacities $\frac{c_p}{c_v} = g$, where
   a) $g = 0\times718$.
   b) $g = 1\times005$.
   c) $g = 1\times400$.
   d) $g = 287\times100$.
6. Ordinary water may exist in
   a) solid phase only
   b) liquid phase only
   c) vapour phase only
   d) all of the above phases
7. Ordinary water may exist in an equilibrium mixture of
   a) solid and liquid phases only
   b) liquid and vapour pahses only
   c) solid and vapour phases only
   d) all of the above equilibrium mixture phases

8. The mean molecular weight of water vapour at room temperature is M, where
   a) $M = 28\times 97$.
   b) $M = 18\times 02$.
   c) $M = 16\times 00$.
   d) $M = 14\times 00$.
9. Fusion takes place when
   a) a liquid changes to a vapour.
   b) a solid changes to a liquid.
   c) a solid changes directly to a vapour.
   d) none of the above processes take place.
10. Vaporisation takes place when
   a) a liquid changes to a vapour
   b) a solid changes to a liquid
   c) a solid changes directly to a vapour
   d) none of the above processes take place
11. Sublimation takes place when
   a) a liquid changes to a vapour
   b) a solid changes to a liquid
   c) a solid changes directly to a vapour
   d) none of the above processes take place
12. At atmosphere pressure, the fusion temperature is
   a) 100^{o}C.
   b) 50^{o}C.
   c) 0^{o}C.
   d) none of the above
13. At atmospheric pressure, the vaporisation temperature is
   a) 100^{o}C
   b) 50^{o}C
   c) 0^{o}C
   d) none of the above
14. At the triple point of water
   a) only the solid and liquid phases coexist in equilibrium
   b) only the solid and vapour phases coexist in equilibrium
   c) only the liquid and vapour phases coexist in equilibrium
   d) all the solid, liquid and vapour phases coexist in equilibrium
15. At the critical point, the pure vapour phase has identical properties with
   a) the pure solid phase
   b) the pure liquid phase
   c) the pure gaseous phase
   d) none of the above phases
16. The critical point of water is marked by the total disappearance of the surface meniscus seperating
   a) the liquid phase and the vapour phase

b) the solid phase and the liquid phase
c) the solid phase and the vapour phase.
d) all the three phases of the water.

17. In the saturation region, both temperature and pressure
a) are decreasing
b) are increasing
c) are fluctuating
d) remain constant

18. The saturation region may be quantified using
a) *quantity* and *quality*
b) *quantity* and *moisture*
c) *quality* and *mixture*
d) *quality* and *moisture*

19. In the equilibrium mixture of the liquid and vapour phases, the fraction of mass present in the *liquid* phase is called the
a) moisture
b) mixture
c) quality
d) quantity

20. In the equilibrium mixture of the liquid and vapour phases, the fraction of mass present in the vapour phase is called the
a) moisture
b) mixture
c) quality
d) quantity

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MODULE 2

Unit 1 The Neutral Atmosphere
Unit 2 Radioactive Transfer
Unit 3 Line Profiles

UNIT 1 THE NEUTRAL ATMOSPHERE

CONTENTS

1.0 INTRODUCTION

Technically, the terrestrial atmosphere may be divided into two regions: The lower atmosphere and the upper atmosphere. The lower atmosphere starts from the ground and stops at a height of about 100 kilometres. The upper atmosphere starts from a height of about 100 kilometres to the near-earth spaces. However, each region may be sub-divided into different layers. In this unit, you shall be introduced to the dynamics of the neutral atmosphere.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- classify the atmosphere into various distinct layers
- derive an expression for the vertical diminution of pressure with height
- discuss the density profile in the lower atmosphere
- relate the partial pressure of a saturated water vapour to temperature, latent heat and specific gas constant of the water vapour
- discuss the effects of the troposphere on radio wave propagation.

3.0 MAIN CONTENT

3.1 Layers of the Atmosphere

The atmosphere of the earth may be classified in terms of:

(i) temperature
(ii) ionisation
(iii) magnetic field, and
(iv) radio wave propagation.

In terms of temperature, the atmosphere may be classified into four distinct layers, namely:

(i) the troposphere
(ii) the stratosphere
(iii) the mesosphere, and
(iv) the thermosphere.

In terms of magnetic field the atmosphere may be classified into two layers, namely:

(i) the dynamosphere, and
(ii) the magnetosphere.

In terms of radio wave propagation, the terrestrial atmosphere may be classified into two:

(i) the troposphere, and
(ii) the ionosphere.

The ionosphere is a dispersive medium in which *propagation* is frequency dependents. The troposphere is a non-dispersive medium that is largely neutral.

Table 1 shows one possible classification scheme for the terrestrial atmosphere.

Table 1. 1: A Possible Classification Scheme for the Terrestrial Atmosphere

Altitude [km]	Temperature	Ionisation	Magnetic field	Propagation	Technical	Escape Properties
	Thermosphere			Ionosphere		
10000 —		Protonosphere			Upper Atmosphere	
			Magnetosphere			
1000 —						
		Ionosphere				Exosphere
100 —						
	Mesosphere					
	Stratosphere		Dynamosphere		Lower Atmosphere	
10 —						
		Neutrosphere				
	Troposphere			Troposphere		

Abridged from Yilmaz (2006). *Atmosphere Physics.* Canakkale Onsekiz Mart University, Istanbul

3.2 The Governing Equations

3.2.1 Equation of State

In order to discuss the neutral atmosphere effectively, you need to have a good understanding of:

(i) the equation of state (gas equation)
(ii) the equation of hydrostatic equilibrium, and
(iii) Snell's law.

Charles' law states that the volume of a fixed mass of gas is directly proportional to its temperature, provided that the pressure remains constant. Thus, at constant-pressure, all gases expand by a constant amount (equal to $\frac{1}{273}$ of the volume at 0^{o}C) for every 1^{o}C rise in temperature. This is known as Charles' *constant-pressure* law. Conversely, if the volume is kept constant, all gases undergo an increase in pressure $\left(\text{equal to } \frac{1}{273} \text{ of their pressure at } 0\,^{o}C\right)$ for every $1\,^{o}C$ rise in temperature. This is known as Charles' *constant-volume* law. Thus, Charles' law states that the ratio of *volume* to *temperature* of a fixed mass of gas is constant i.e.

$$\frac{V}{T} = \text{constant},$$

provided that the pressure remains constant.

Boyles' law states that the volume of a fixed mass gas is inversely proportional to its pressure, provided that the temperature remains constant. Thus, at constant temperature the product of *pressure* and *volume* remains constant. This implies that

$$pV = \text{constant}.$$

In the light of these laws, the equation of state for an ideal gas may be formulated as follows:

$$pV = nkT' \qquad (1)$$

where p is pressure V is volume, n is number of molecules, T' is absolute temperature and k is Boltzmann constant. You may, however, choose to define the temperature T (in energy units) such that $T = kT'$. In this case, the equation of state reduces to

$$pV = nT,$$

Or

$$p = \left(\frac{n}{V}\right)T. \qquad (2)$$

By defining the number density N as the number of molecules per unit volume, you obtain

$$N = \frac{n}{V}. \qquad (3)$$

Hence, Eq. (2) may be written, in the light of Eq. (3), as

$$p = NT, \qquad (4)$$

where N is the number density.

3.2.2 Equation of Hydrostatic Equilibrium

The equation of hydrostatic equilibrium expresses the *balance* between the gravitational force at any point and the pressure gradient at that point. Thus, the hydrostatic equation may be expressed as

$$\frac{dp}{dz} = -\rho g, \qquad (5)$$

where p is the pressure, ρ is the mass density, g is the acceleration due to gravity and z is the height.

Equation (5) implies that the change in pressure dp may be expressed as

$$dp = -\rho g dz. \qquad (6)$$

Equations(4) and (6) imply that

$$\frac{dp}{p} = -\frac{\rho g dz}{NT}$$

or

$$\frac{dp}{p} = -\frac{dz}{H} \qquad (7)$$

where the *scale height* H is given by

$$H = \frac{NT}{\rho g}. \tag{8}$$

The scale height H may be defined in terms of the mean molecular mass m at any given height. Noting that the total mass M of the molecules is given by

$$M = \rho V,$$

you find that

$$nm = \rho V, \tag{9}$$

Since

$$M = nm.$$

Equation (9) indicates that the mean molecular mass m is given by

$$m = \rho\left(\frac{V}{n}\right) \tag{10}$$

Equation (3) and (10) show that the mean molecular mass m is given by

$$m = \frac{\rho}{N}. \tag{11}$$

Equation (8) may now be rewritten as

$$H = \left(\frac{N}{\rho}\right)\left(\frac{T}{g}\right)$$

or

$$H = \left(\frac{1}{m}\right)\left(\frac{T}{g}\right).$$

Therefore, the scale height H may be written as

$$H = \frac{T}{mg}, \tag{12}$$

where m is the mean molecular mass at a given height.

By integrating Eq.(7) from some reference height $z = z_o$ $\left(\text{at a pressure } p = p_o\right)$ to an arbitrary height $z\left(\text{at a pressure } p\right)$, you obtain the following:

$$\int_{p_o}^{p} \frac{dp}{p} = -\int_{z_o}^{z} \frac{dz}{H}$$

$$\left[\ln p\right]_{p_o}^{p} = -\int_{z_o}^{z} \frac{dz}{H}$$

$$\ln p - \text{In}\, p_o = -\int_{z_o}^{z} \frac{dz}{H}$$

$$\ln\left(\frac{p}{p_o}\right) = -\int_{z_o}^{z} \frac{dz}{H}$$

$$\frac{p}{p_o} = \exp\left(-\int_{z_o}^{z} \frac{dz}{H} \right)$$

$$\therefore p = p_o \exp\left(-\int_{z_o}^{z} \frac{dz}{H} \right).$$

Equation (13) shows how the pressure p varies with height z. It shows that the pressure decreases exponentially with height. Clearly, a large *scale height* is associated with the low-pressure regions of the atmosphere.

Equation (13) may also be used to find the distribution of the number density with height. You will recall that

$$p = NT \qquad (4)$$

so that , at some reference point where $p = p_o$ and $T = T_o$, the number density N_o is given by the expression

$$p_o = N_o T_o.$$

Hence, you can take appropriate ratios to obtain

$$\frac{p}{p_o} = \frac{NT}{N_o T_o},$$

so that

$$p = p_o \left(\frac{NT}{N_o T_o} \right). \qquad (14)$$

But, Eq (13) shows that the pressure p is given by

$$p = p_o \exp\left(-\int_{z_o}^{z} \frac{dz}{H} \right), \qquad (13)$$

so that Eq.(13) assumes the form

$$p_o \left(\frac{NT}{N_o T_o} \right) = p_o \exp\left(-\int_{z_o}^{z} \frac{dz}{H} \right)$$

Or

$$N = \left(\frac{N_o T_o}{T} \right) \exp\left(-\int_{z_o}^{z} \frac{dz}{H} \right) \qquad (15)$$

Equation (15) shows that the number density N diminishes exponentially with height too. It indicates that a *large scale* height is associated with a light gas.

In an isothermal atmosphere the temperature remains constant and the *scale height* becomes constant too. In consequence, Eq. (13) reduces to

$$p = p_0 \exp\left[-\frac{(z - z_0)}{H}\right], \qquad (16)$$

where p_0 is the pressure at some reference height z_0.
Also, in an isothermal atmosphere, the temperature remains constant so that $T = T_o$. In this case, Eq.(15) reduces to

$$N = N_o \exp\left[-\frac{(z - z_0)}{H}\right], \qquad (17)$$

where N is the number density at some reference height z_0.

3.2.3 Snell's Law

Snell's law states that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant. That *constant* is a measure of the *refractive index.* When a radio signal passes through the atmosphere its direction changes due to refraction.

Consider a neutral atmosphere that is horizontally stratified into m layers. If the ionospheric refraction is neglected then the total change in the direction of a radio signal passing through the atmosphere can be found by repeatedly applying Snell's law to each layer of the atmosphere (figure 2). In this case, Snell's law is given by $n_1 \sin\psi_1 = \sin\psi_m$.

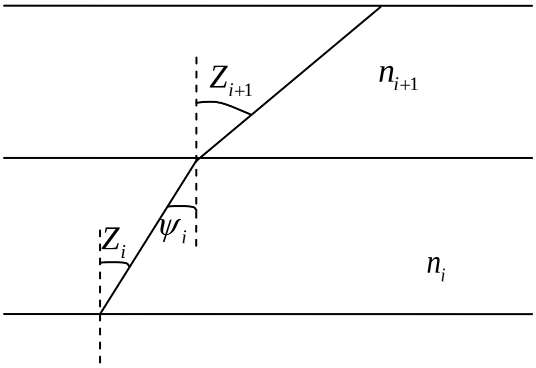

Fig. 1.2: **Snell's Law in Cartesian Coordinates for Horizontally Stratified Atmosphere (**Z_i **is Zenith Angle of Layer** i **;** n_i **is Refractive Index of Layer** i; ψ_m *is the angle of incidence at the near – Earth space*).

Abridged from Yilmaz (2006). *Atmosphere Physic.*, Canakkale Onsekiz Mart University, Istanbul

3.2.4 Water Vapour

The troposphere contains both water vapour and dry air. The amount of water vapour varies with time and location. On the other hand, the composition of dry air does not vary significantly with height. To understand water vapour one needs to have a good knowledge of some vapour-related quantities like:

(i) mixing ratio
(ii) partial pressure of vapour, and
(iii) relative humidity.

3.2.5 Mixing Ratio

A mixture of dry air and water vapour is called *moist air*. The mixing ratio gives a measure of the amount of moisture in the air. The mixing ratio w is defined as the ratio of the mass of water vapour to the mass of dry air. Thus,

$$w = \frac{m_v}{m_d} = \left(\frac{m_v}{V}\right)\left(\frac{V}{m_d}\right) = \left(\frac{m_v}{V}\right) \div \left(\frac{m_d}{V}\right) = \frac{\rho_v}{\rho_d}, \qquad (1)$$

where ρ_v is density of water vapour and ρ_d is density of dry air.

The equation of state holds for a fixed mass of gas. It also holds for a mixture of gases. Now, the equation of state

$$pV = RT \qquad (2)$$

can be written in terms of the specific volume and the specific gas constant for the i^{th} gas as follows:

$$pV = m_i R_i T \qquad (3)$$

where R_i is the specific gas constant and m_i is the mean molecular mass of the i^{th} gas. Of course, the specific gas constant $R_i = \frac{R}{m_i}$, where R is the universal gas constant.

Equation (2) can also be re-written in terms of the specific volume as follows:

$$pV = m_i R_i T,$$

or

$$p = \left(\frac{m_i}{V}\right) R_i T,$$

Or

$$p = \rho_i R_i T, \qquad (4)$$

where ρ_i is the density of the i^{th} gas defined by $\rho_i = \frac{m_i}{V}$. Thus, the pressure p may be written as

$$p = \rho_i R_i T.$$

(Note that the specific volume v_i is mass per unit volume i.e. $v_i = \frac{V}{m_i} = \frac{1}{\rho_i}$).

If this new version of the equation of state is applied to water vapour, you obtain

$$e = \rho_v R_v T \qquad (5)$$

where e is the partial pressure of water vapour. If the same version of the equation of state is applied to dry air, you get

$$p_d = p - e = \rho_d R_d T \qquad (6)$$

where p is the total pressure of the moist air and R_d is the specific gas constant of dry air.

The mixing ratio may therefore be defined in the light of equations (5) and (6). From Eq.(5), you see that

$$\rho_v = \frac{e}{R_v T}. \qquad (7)$$

From Eq. (6), you find that

$$\rho_d = \frac{p-e}{R_d T}. \qquad (8)$$

But, the mixing ratio

$$w = \frac{\rho_v}{\rho_d} \qquad (9)$$

or

$$w = \frac{e}{R_v T} \div \frac{p-e}{R_d T},$$

or

$$w = \frac{e}{R_v T} \times \frac{R_d T}{p-e},$$

or

$$w = \frac{e R_d}{R_v \left(p-e\right)},$$

Or

$$w = \varepsilon \frac{e}{p-e}, \qquad (10)$$

where ε is the ratio of the specific gas constant for dry air to the specific gas constant for vapour $\left(\text{i.e. } \varepsilon = \frac{R_d}{R_v}\right)$.

Experiments show that the specific gas constants R_d and R_v are given by

$$R_d = 237 \cdot 06 \pm 0 \cdot 01\, Jkg^{-1}K^{-1},$$

and

$$R_v = 461 \cdot 525 \pm 0 \cdot 003\, Jkg^{-1}K^{-1},$$

so that

$$\varepsilon = \frac{R_d}{R_v} = 0 \cdot 622. \qquad (11)$$

The partial pressure of water vapour is typically about 1% of the total pressure of the moist air. It can be shows that mixing ratio w is roughly equal to the specific humidity q $(i.e. \quad w \approx q)$.

3.3 Saturation Region

3.3.1 Partial Pressure of Saturated Air

In a closed system, *equilibrium* will be established when the number of water molecules passing from the liquid phase to the vapour phase is equal to the number of water molecules passing from the vapour to the liquid phase. Any vapour that meets this requirement is called a *saturated vapour*. A mixture of vapour and air under equilibrium conditions is called a saturated air. When a saturated air comes in contact with an unsaturated air, diffusion begins to take place. Molecules of water move from areas of higher concentration of water vapour to areas of lower concentration of water vapour.

The partial pressure of a saturated water vapour depends on the temperature. A warm air can contain large amounts of water vapour. When the warm air is cooled, the surplus of the water vapour (over and above the saturation value) at the new temperature condenses to form water. The condensation process leads to energy release. The energy released per unit mass is equal to the latent heat of vaporisation. Two more latent heats exist: the latent heat of fusion and the latent heat of sublimation. The latent heat of fusion is the energy required to change a unit mass of ice to liquid water at the same temperature. The latent heat of sublimation is the sum of the latent heat of fusion and the latent heat of vaporisation.

By extension of Eq. (10), the saturation mixing ratio w_{sat} may be defined as

$$w_{\text{sat}} = \varepsilon \frac{e_{\text{sat}}}{p - e_{\text{sat}}} \qquad (13)$$

If the total pressure p is significantly higher than the saturated partial pressure e of the water vapour then the saturation mixing ratio becomes

$$w_{\text{sat}} \approx \varepsilon \frac{e}{p}. \qquad (14)$$

The relative humidity r_h is the ratio of the mixing ratio to the saturation mixing ratio. Thus, provided that p is large compared with both e and e_{sat}, the relative humidity r_h is given by

$$\begin{aligned} r_h &= \left(\varepsilon \frac{e}{p}\right) \div \left(\varepsilon \frac{e_{sat}}{p}\right) \\ &= \varepsilon \frac{e}{p} \times \frac{p}{\varepsilon e_{sat}} \\ &= \frac{e}{e_{sat}}. \end{aligned}$$

hence,

$$r_h = \frac{w}{w_{\text{sat}}} \approx \frac{e}{e_{\text{sat}}}. \qquad (15)$$

It is often necessary to express the relative humidity in *percentage*. In this case the two expressions for relative humidity become:

$$r_h = \frac{100w}{w_{\text{sat}}}\% \approx \frac{100e}{e_{\text{sat}}}\%, \qquad (16)$$

where r_h is the relative humidity and p is large compared with both e and e_{sat}.

3.3.2 Propagation Delay

The condition of the troposphere has a profound effect on radio wave propagation. A radio signal suffers *delay* by a neutral atmosphere. The total *delay* depends on the *refractivity* along the path of the radio signal. The refractivity itself depends on primary parameters like temperature and pressure.

Fermat's principle is the basic physical law governing radio wave propagation. Fermat's principle shows that light and other electromagnetic waves will follow a path between two points which involves the least travel time.

To understand this, let us define the optical (or electromagnetic) distance S between the source and the receiver as:

$$S = \int c\,dt\,, \qquad (19)$$

where c is the speed of light and t is time.

Let us also define the electromagnetic path s as:

$$s = \int v\,dt, \qquad (20)$$

where v is the propagation speed given by

$$v = \frac{ds}{dt}\,. \qquad (21)$$

Now, the refractive index n of a medium is the ratio of the speed of light in free space to the speed of light in the medium i.e.

$$n = \frac{c}{v}, \qquad (22a)$$

Or

$$v = \frac{c}{n}\,, \qquad (22b)$$

where c is the speed of light in vacuum and v is the speed of light in the medium.

Equations (21) and (22) indicate that

$$\frac{ds}{dt} = \frac{c}{n}\,,$$

so that

$$nds = cdt\,,$$

and, therefore,

$$\int n(s)\,ds \equiv \int \frac{c}{v}\,ds \equiv \int c\,dt. \qquad (23)$$

Equations (19), (22) and (23) show that the electromagnetic distance

$$S = \int c\,dt = \int n(s)\,ds = \int \frac{c}{v}\,ds. \qquad (24)$$

The excess path length D caused by the troposphere is given by

$$D = \left[\int n(s)\,ds - \int ds\right] + \left[\int_s ds - \int_l dl\right]. \qquad (25)$$

The excess path length D is also called the *delay*. It is measured in the *slant direction* caused by the troposphere.

The right-hand side of Eq.(25) is the sum of two excess path lengths: the first term represents the excess length caused by the propagation delay; the second term represents the excess path length caused by the bending of the radio signal.

3.3.3 Refractivity N

The refractive index n is always a small number (in the neighbourhood of 1). It is often more convenient to work with a parameter with large spread. For that reason, the *refractivity* N is often used. In relation to the refractive index n, the refractivity N is defined by the expression

$$N = (n-1) \times 10^6. \qquad (27)$$

Evidently, the range of values of the refractivity can be much more manageable than the corresponding range of values of the refractive index. For instance, in the ionosphere the values of refractivity N ranges from zero to about 300, whereas the values of the refractive index n remains clustered in the neighbourhood of unity.

The neutral atmosphere contains both dry air and water vapour. For this reason, the refractivity N can be split into a *dry air* component and a *water* vapour component. Thus, we may write

$$N = N_d + N_v, \qquad (28)$$

where N_d the refractivity of is dry air and N_v is the refractivity of water vapour. This is a wise decision since the *dry air* content remains nearly constant in time whereas the water vapour content varies widely in time and space.

For frequencies up to 20GHz, the refractivity may be written, as a function of temperature and partial pressure, in the following manner:

$$N_d = \left(k_1 \frac{p_d}{T} \right) Z_d^{-1}, \qquad (29)$$

and

$$N_v = \left(k_2 \frac{e}{T} + k_3 \frac{e}{T^2} \right) Z_v^{-1}, \qquad (30)$$

where Z_λ is the compressibility factor of dry air, Z_v is the compressibility factor for water vapour and $k_i \left(i = 1, 2, 3 \right)$ are dimensional constants. The unit of k_1 and k_2 is Kelvin per millibar; $\left(K\ mbar^{-1}\right)$. The unit of k_3 is Kelvin squared per millibar $\left(K^2\ mbar^{-1}\right)$. For an ideal gas, the compressibility factor is equal to 1. Any departure of Z from unity accounts for the non-ideal behaviour of the affected gas. Fortunately, the compressibility factor for both dry air and water vapour is roughly equal to 1, so that

$$Z_d \approx Z_v \approx 1.$$

Thus, dry air and water vapour may be treated as ideal gases. In this case, Eqs (29) and (30) reduce to

$$N_d = k_1 \frac{p_d}{T}, \qquad (31)$$

and

$$N_v = k_2 \frac{e}{T} + k_3 \frac{e}{T^2}, \qquad (32)$$

respectively.

Above the troposphere, there is practically no more water vapour. Thus, at high altitudes the contribution of the wet refractivity to the total refractivity becomes small. This is so because, the wet refractivity drops much faster than the dry refractivity as the height increases.

The propagation delay depends on both the refractivity and the path of the radio signal. In turn, the refractivity depends on both temperature and pressure. We therefore require adequate information on the temperature and the pressure in order to determine the propagation delay.

4.0 CONCLUSION

In this unit, you have been introduced to different ways of classifying the atmosphere into distinct layers. You have seen how the equation of state and the equation of hydrostatic equilibrium may be used to derive expressions that give the pressure and the density profiles. You found that the partial pressure of water vapour is about 1% of the total pressure of a moist air. You also found that the mixing ratio is roughly equal to the specific humidity. Finally, you have seen that the refractivity depends on both temperature and pressure. In turn, the propagation delay depends on both the refractivity and the path of the radio signal.

5.0 SUMMARY

Having gone through this unit, you now know that:

- the lower atmosphere starts from the mean sea level and stops at a height of about 100km
- the upper atmosphere starts from a height of about 100km and stops at the near-earth space
- the atmosphere may be classified (in terms of temperature, ionisation, magnetic field, escape properties, and radio wave propagation) into distinct layers
- the equation of state, $pV = nkT'$, may be written as $pV = nT$ provided that the temperature T' is defined in energy units such that $T = kT'$.
- by defining the number density N as the number of molecules per unit volume, the equation of state becomes

  $$p = NT, \text{ where } N = \frac{n}{V} \cdot$$
- the equation of hydrostatic equilibrium may be expressed as

  $$\frac{dp}{dz} = -\rho g,$$

  where p is pressure, ρ is mass density, g is acceleration due to gravity, and z is height.
- the *scale height* H may be defined in terms of the mean molecular mass m, at any given height, such that

  $$H = \frac{T}{mg} \cdot$$
- a large *scale height* is associated with the low-pressure regions of the atmosphere
- the pressure p is distributed according to the expression

  $$p = p_0 \exp\left(-\int_{z_0}^{z} \frac{dz}{H} \right).$$
- the number density N is distributed according to the relation

  $$N = N_0 \exp\left(-\int_{z_0}^{z} \frac{dz}{H} \right).$$
- Snell's law implies that the ratio of the sine of the angle of incidence to the sine of the angle of refraction is a constant n known as *refractive index*
- when a radio signal passes through the atmosphere, its direction changes due to refraction
- the troposphere contains both dry air and water vapour
- some of the vapour-related quantities needed to investigate the properties of water vapour are mixing ratio, relative humidity and partial pressure of the vapour
- a mixture of dry air and water vapour is called a moist air

- the mixing ratio of a moist air is the ratio of the mass of water vapour to the mass of dry air present in the moist air
- the partial pressure of water vapour is typically about 1% of the total pressure of the moist air
- the mixing ratio w is approximately equal to the specific humidity q.
- a saturated vapour is one in which the number of molecules passing from the liquid phase to the vapour phase is equal to the number of molecules returning from the vapour phase to the liquid phase
- diffusion begins to take place when an unsaturated vapour comes in contact with a sutured vapour (molecules from the areas of higher concentration of water vapour move to areas of lower concentration of water vapour)
- the saturation mixing ratio w_{sat} may be expressed as

  $$w_{sat} = \varepsilon \frac{e_{sat}}{p - e_{sat}},$$

  where e_{sat} is the partial pressure of the saturated water vapour, and ε is the ratio of the specific gas constant for dry air to the specific gas constant for water vapour
- the partial pressure of a saturated water vapour depends on the temperature
- the relative humidity r_h is the ratio of the mixing ratio to the saturation mixing ratio i.e.

  $$r_h = \frac{w}{w_{sat}} \approx \frac{e}{e_{sat}},$$

  where e is partial pressure and w is mixing ratio
- the neutral atmosphere contains both dry air and water vapour.
- in relation to the refractive index n, the refractivity N is defined by the expression $N = (n-1)\times 10^6$.
- the refractivity of the atmosphere (N) is the sum of the refractivity of dry air (N_d) and the refractivity of water vapour (N_v) i.e.

  $$N = N_d + N_v.$$
- the condition of the troposphere affects radio wave propagation significantly
- the propagation delay depends on both the refractivity and the path of the radio signal.

6.0 TUTOR-MARKED ASSIGNMENT

1. The lower atmosphere starts form the ground and stops at height of about

   a) 10km
   b) 20km
   c) 50km
   d) 100km

2. The upper limit of the upper atmosphere is the near-earth space while the lower limit of the upper atmosphere is about

   a) 10km above the ground
   b) 20km above the ground
   c) 50km above the ground
   d) 100km above the ground

3. The atmosphere may be classified in terms of

   a) temperature only
   b) ionisation only
   c) magnetic field only
   d) all of the above parameters

4. The equation of state may be written as $p = NT$, where N is the number density, provided that the temperature T is measured in

   a) Kelvin
   b) Celsius degree
   c) Energy unit
   d) Newton

5. The equation $\frac{dp}{dz} = -\rho g$ is known as the equation of

   a) state
   b) motion
   c) continuity
   d) hydrostatic equilibrium

6. The equation $pV = nkT'$ is known as the equation of

   a) state
   b) motion
   c) continuity
   d) hydrostatic equilibrium

7. In the equation $H = \frac{T}{mg}$, the quantity H represents

a) height
b) scale height
c) humidity
d) none of the above

8. A small *scale height* is associated with those regions of an isothermal atmosphere in which

a) the pressure is low
b) the pressure is high
c) the temperature is increasing
d) the temperature is decreasing

9. The pressure p is distributed according to the relation

$$p = p_0 \exp\left[-\frac{z - z_0}{H}\right],$$

provided that the atmosphere is

a) adiabatic
b) isothermal
c) isochoric
d) isobaric

10. The number density N is distributed according to the relation $N = N_0 \exp\left[-\frac{z - z_0}{H}\right]$, provided that the atmosphere is

a) adiabatic
b) isothermal
c) isochoric
d) isobaric

11. When a radio signal passes through the atmosphere it is

a) amplified
b) terminated
c) refracted
d) elliptically polarised

12. The troposphere contains

a) only dry air
b) only water vapour

c) dry air and water vapour
d) liquid hydrogen only

13. In order to investigate the behaviour of water vapour, one needs to know something about

a) mixing ratio only
b) Humidity only
c) partial pressure of the water vapour only
d) all of the above quantities

14. The ratio of the mass of water vapour to the mass of dry air in a moist air is called

a) mixing ratio
b) humidity
c) partial pressure of the water vapour
d) all of the above names

15. Typically, the ratio of the partial pressure of water vapour to the total pressure of a moist air is

a) 0.01
b) 0.10
c) 1.00
d) none of the above values

16. A vapour in which the number of molecules passing from the liquid phase to the vapour phase is equal to the number of molecules returning from the vapour phase to the liquid phase is called

a) an unsaturated vapour
b) a saturated vapour
c) a concentrated vapour
d) a diffusive vapour

17. The ratio of the *mixing ratio* to the *saturation mixing ratio* is called

a) relative humidity
b) absolute humidity
c) refractive index
d) refractivity

18. If N_d is the refractivity of dry air and N_v is the refractivity of water vapour then the refractivity N of the atmosphere is given by

a) $N = \frac{N_d}{N_v}.$
b) $N = N_d N_v.$
c) $N = N_d - N_v.$
d) $N = N_d + N_v.$

19. In relation to the refractive index $n,$ the refractivity N is defined by the expression

a) $N = n - 1.$
b) $N = n + 1.$
c) $N = 10^6 (n - 1).$
d) $N = 10^6 (n + 1).$

20. The propagation delay depends on

a) the refractive index alone
b) the refractivity alone
c) the path of the radio signal alone
d) the refractivity and the path of the radio signal

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UNIT 2 RADIATIVE TRANSFER

CONTENTS

1.0 INTRODUCTION

A wealth of information is available on the electromagnetic spectrum of gas clouds and stellar atmospheres. The electromagnetic spectrum of interest ranges from *metre radio waves* (i.e. photons with energies less than $10^{-6} eV$) to hard X-rays and g- *rays* (i.e. photons with energies greater than $10^{6} eV$). As radiation travels through a gas cloud or stellar atmosphere, its intensity changes. The change in intensity of the radiation depends on a number of factors. In this unit, you shall be introduced to the basic spectroscopic concepts and radiation transfer mechanisms.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- distinguish between intensity and flux, and relate luminosity to flux
- derive the equation of radioactive transfer
- discuss the concept of local thermodynamic equilibrium (LTE)
- apply the LTE approximation of the equation of radiative transfer to radio waves.

3.0 MAIN CONTENT

3.1 Meaning of Radiative Transfer

How does the flow of radioactive energy change as one moves through a gas that emits and absorbs radiation? How can one predict the spectrum that emerges from a given gas cloud or stellar atmosphere as a function of direction and frequency? How can one produce a model of the cloud or stellar atmosphere, for a given energy input, in order to be able to predict how the temperature runs through the cloud or stellar atmosphere? These are some the questions which scientists workings on *radioactive transfer* seek to answer.

3.2 Intensity, and Flux

Intensity and flux are the two fundamental measures of energy flow. *Intensity* is the energy flow in a particular direction through a unit area per unit time per unit solid angle, *where the unit area is perpendicular to the chosen direction defined by the solid angle. Flux is the net energy flow through a unit area per unit time, summed over all directions, where the unit area is fixed as being perpendicular to the direction in which the net flow of radiation is proceeding.* Thus, the flux is obtained by summing the intensity over all directions.

For an isotropic intensity, the positive and negative contributions will cancel out reducing the flux to zero. Quantitatively, the flux F and the intensity I are related by the expression

$$F = \int I \cos\theta \, d\Omega \qquad (1)$$

where $d\Omega$ is an element of the solid angle, and θ is the angle between the *fixed* direction defined by flux and the *variable direction* defined by the intensity.

The intensity is measured per unit area perpendicular to the line of sight. The flux is measured per unit area perpendicular to the fixed direction of the net flow of energy (Figure 1). This explains why a projection factor $(\cos\theta)$ appears in Eq. (1).

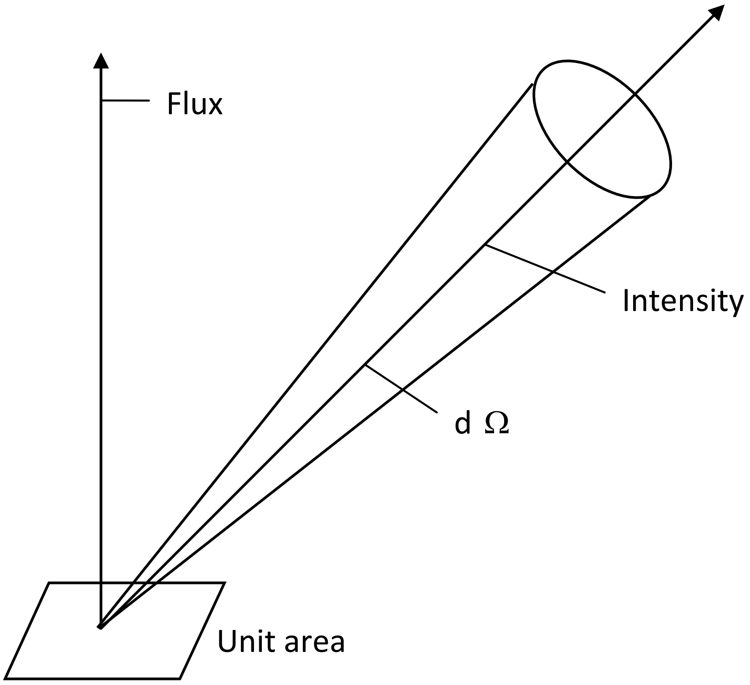

Fig.2. 1: Flux and Intensity

Scientists are usually interested in monochromatic intensities and fluxes e.g. intensity per unit frequency interval at the frequency ν. Here, such specific intensities and fluxes are denoted by I_ν and F_ν respectively. The quantities I and I_ν are related by the following integral:

$$I = \int I_\nu d\nu. \qquad (2)$$

By solving for intensity as a function of direction and integrating, you obtain the net flow of radiation (i.e. flux) proceeding from a stellar atmosphere or gas cloud.

3.3 Luminosity L

The luminosity L of a star is closely related to the flux F. Consider a spherical star of radius $R.$ If the flux evaluated at the surface is F, then the luminosity L is given by

$$L = 4\pi R^2 F$$

Thus, for a point object at a distance $d,$ the luminosity is given by

$$L = 4\pi d^2 F,$$

so that the flux becomes

$$F = \frac{L}{4\pi d^2}.$$

In the case of an extended object, resolved in angle by the telescope, the intensity received from various parts of the body is called the *surface* *brightness* of the object. If the object has uniform surface brightness, then the intensity is independent of distance.

3.4 Radioactive Transfer Equation

Consider a pencil of radiation of frequency ν travelling in the atmosphere. As the radiation travels a distance $ds,$ its specific intensity changes according to the expression.

$$dI_\nu = \varepsilon_\nu ds - \kappa_\nu ds, \qquad (3)$$

where ε_v is the emission coefficient, κ_v is the absorption coefficient and I_v is the specific intensity.

The *optical depth* τ_ν along a direction r is defined by the equation

$$d\tau_\nu = -\kappa_\nu dr, \qquad (4)$$

where r makes an angle θ with $ds.$
Now, you can write

$$-\kappa_\nu = \frac{d\tau_\nu}{dr}$$

on account of Eq (4).
Using Eq (5) in Eq (3), you get

$$dI_\nu = \varepsilon_\nu ds + \left(\frac{d\tau_\nu}{dr}\right) I_\nu ds \qquad (6)$$

The presence of the projection factor $\cos\theta$ implies that

$$dr = \cos\theta ds \qquad (7)$$

Using Eq (7) in Eq (6), you find that

$$dI_\nu = \varepsilon_\nu ds + \left(\frac{d\tau_\nu}{\cos\theta ds}\right) I_\nu ds$$

or

$$dI_\nu = \varepsilon_\nu ds + \left(\frac{1}{\cos\theta}\right) I_\nu d\tau_\nu$$

Dividing each term by $d\tau_\nu$, you obtain

$$\frac{dI_\nu}{d\tau_\nu} = \varepsilon_\nu \frac{ds}{d\tau_\nu} + \left(\frac{1}{\cos\theta}\right) I_\nu . \qquad (8)$$

Eq (7) implies that

$$ds = \left(\frac{1}{\cos\theta}\right) dr$$

Using Eq (9) in Eq (8), you get

$$\frac{dI_\nu}{d\tau_\nu} = \varepsilon_\nu \left(\frac{1}{\cos\theta}\right)\frac{d\rho}{d\tau_\nu} + \left(\frac{1}{\cos\theta}\right) I_\nu .$$

Multiplying across by $\cos\theta$, you get

$$\cos\theta \frac{dI_\nu}{d\tau_\nu} = \varepsilon_\nu \frac{d\rho}{d\tau_\nu} + I_\nu . \qquad (10)$$

Now, it is easy to see that

$$\frac{d\rho}{d\tau_\nu} = -\frac{1}{\kappa_\nu} \qquad (11)$$

on account of Eq. (5).

Using Eq. (11) in Eq. (10) you obtain

$$\cos\theta \frac{dI_\nu}{d\tau_\nu} = -\frac{\varepsilon_\nu}{\kappa_\nu} + I_\nu$$

or

$$\mu \frac{dI_\nu}{d\tau} = I_\nu - \frac{\varepsilon_\nu}{\kappa_\nu}, \qquad (12)$$

where the projection factor

$$\mu = \cos\theta. \qquad (13)$$

Equation (13) is the basic equation of radiative transfer. It has with the following sign convention:

(i) I_ν is positive outwards (towards the observer).

(ii) τ_ν is positive inwards (starting from zero at a surface "nearest" the observer).

(iii) s is positive inwards (staring from zero at a surface *nearest* the observer).

The ratio of the emission coefficient to the absorption coefficient defines the *source function* S_ν. Thus, the source function S_ν is given by

$$S_\nu = \frac{\varepsilon_\nu}{\kappa_\nu}. \qquad (14)$$

where ε_v and κ_v are the emission and absorption coefficients respectively.

3.5 Local Thermodynamic Equilibrium (LTE)

In the stellar atmosphere, it is impossible to achieve complete thermodynamic equilibrium. If there were complete thermodynamic equilibrium, the temperature would be the same everywhere with no temperature gradient to drive an outward flow of radiation. Besides, if the temperature were the same everywhere, then the radiation field would be isotropic so that the positive contributions and the negative contributions would cancel out, resulting in zero flux. No doubt, a complete thermodynamic equilibrium cannot occur in stellar atmosphere. However, a sufficiently small region of the stellar atmosphere may attain roughly the same temperature. In this case, the affected region may be characterised by a single local temperature. Such a small region is said to be in *local thermodynamic equilibrium (LTE).*

The LTE approximation greatly simplifies the solution of the equation of radiative transfer. For instance, when LTE reigns, the ratio ϵ_ν/κ_ν depends only on the temperature T, and the source function S_ν is simply the Planck function $B_\nu(T)$. Thus, under LTE conditions, the source function S_ν may be written as

$$S_\nu = B_\nu(T) = \frac{2h\nu^3}{c^2}\left[\exp\frac{h\nu}{kT} - 1\right]^{-1}, \qquad (15)$$

where ν is frequency, h is Planck's constant, k is Boltzmann's constant, and c is the speed of light.

Planck's law can also be written in terms of the wavelength λ. To do this, we require that

$$B_\nu d\nu = -B_\lambda d\lambda$$

or

$$\frac{d\nu}{d\lambda} = -\frac{B_\lambda}{B_\nu}. \qquad (16)$$

The negative sign indicates that the wavelength decreases with increasing frequency.

Now, the velocity of light is equal to the product of the frequency and the wavelength. Thus,

$$c = \nu\lambda$$

or

$$\nu = c\lambda^{-1}.$$

Differentiating the above expression, you find that

$$\frac{d\nu}{d\lambda} = -c\lambda^{-2} \qquad (17)$$

Comparing Eq (16) and Eq (17), you get

$$-\frac{B_\lambda}{B_\nu} = -c\lambda^{-2}$$

or

$$B_\lambda = \left(\frac{c}{\lambda^2}\right)B_\nu. \qquad (18)$$

But, Planck's function $B_\nu(T)$ is given by

$$B_\nu(T) = \frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]^{-1}$$

so that

$$B_\lambda(T) = \left(\frac{c}{\lambda^2}\right)\frac{2h\nu^3}{c^2}\left[\exp\left(\frac{h\nu}{kT}\right) - 1\right]^{-1}.$$

Also, the frequency ν may be written as $\nu = \frac{c}{\lambda}$ so that

$$B_\lambda(T) = \left(\frac{c}{\lambda^2}\right)\cdot\frac{2h}{c^2}\cdot\left(\frac{c}{\lambda}\right)^3\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1\right]^{-1}.$$

Simplifying, you find that $B_\lambda(T)$ is given by

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1\right]^{-1}. \qquad (19)$$

3.6 Application to Radio Waves

Radio waves have long wavelengths and low frequencies. For that reason, the quantity $\frac{h\nu}{kT}$ is very small. In particular, $h\nu \ll kT$.

Now, consider the exponential term in Eq (15). Rewrite the exponential function as

$$\exp\left(\frac{h\nu}{kT}\right) = e^x,$$

where

$$x = \frac{h\nu}{kT}.$$

The power series development of e^x is

$$e^x = 1 + x + \frac{x^2}{2!} + \cdots \qquad (20)$$

Recall that the following inequality holds:

$$h\nu \ll kT.$$

This implies that

$$\frac{h\nu}{kT} \ll 1,$$

or

$$x \ll 1. \qquad (21)$$

Since $x \ll 1$, you may retain only the first two terms of the power series expansion of e^x. Thus, you may write

$$e^x \approx 1 + x \qquad (22)$$

Or

$$\exp\left(\frac{h\nu}{kT}\right) \approx 1 + \frac{h\nu}{kT}. \qquad (23)$$

Using Eq. (23) in Eq (15), you find that $B_v(T)$ reduces to

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\left[\left(1 + \frac{h\nu}{kT}\right) - 1\right]^{-1},$$

or

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\left(\frac{h\nu}{kT}\right)^{-1}$$

or

$$B_\nu(T) \approx \frac{2h\nu^3}{c^2}\left(\frac{kT}{h\nu}\right)$$

Hence, the Planck's function $B_v(T)$ assumes the form

$$B_\nu(T) \approx \left(\frac{2kT}{c^2}\right)\nu^2. \qquad (24)$$

Equation (24) gives a particularly simple version of $B_v(T)$. It applies only when LTE reigns and the affected frequency is so low that $hv = kT$. You have seen that radio waves satisfy the required condition, $h\nu = kT$. For this reason, it is appropriate to use the expression for source function given by Eq.(24) when dealing with radio signals.

In terms of the wavelength, Eq. (19) may be processed as follows:

$$B_\lambda(T) = \frac{2hc^2}{\lambda^5}\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1\right]^{-1}$$

For radio signals,

$$hc = \lambda kT$$

so that

$$\exp\left(\frac{hc}{\lambda kT}\right) \approx 1 + \frac{hc}{\lambda kT}.$$

Thus, in terms of λ, the source function becomes:

$$B_\lambda(T) \approx \frac{2hc^2}{\lambda^5}\left[1 + \frac{hc}{\lambda kT} - 1\right]^{-1}$$

or

$$B_\lambda(T) \approx \frac{2hc^2}{\lambda^5}\cdot\left(\frac{hc}{\lambda kT}\right)^{-1}$$

or

$$B_\lambda(T) \approx \frac{2hc^2}{\lambda^5} \times \frac{\lambda kT}{hc}.$$

Again, the Planck's function $B_\lambda(T)$ assumes the following simple form:

$$B_\lambda(T) \approx \frac{2ckT}{\lambda^4}. \qquad (25)$$

Equations (3), (4), (12), (15), (19) (24) and (25) form the basis for discussion of transport of continuum radiation. However, the source function S_ν may depart significantly from the Planck's function $B_\nu(T)$ if strong spectral lines are involved. In that case, S_ν resembles the *direction-averaged* mean intensity J_ν defined by

$$J_\nu = \frac{1}{4\pi} \int I_\nu d\Omega,$$

where Ω is solid angle. For such strong spectral lines, the LTE approximation breaks down. For that reason, we must replace the LTE conditions by the assumption of a *statistically* steady state in each atomic energy level. We shall return to this case in your study of line profiles.

4.0 CONCLUSION

In this unit, you have been introduced to the concept of intensity, flux and luminosity. You have derived the equation of radioactive transfer. You have seen that LTE approximation can greatly simplify the expression for the source function in the equation of radiative transfer. Finally you have applied the LTE approximation of the equation of radiative transfer to radio waves.

5.0 SUMMARY

Having gone through this unit, you now know that:

- the intensity of a radiation changes as the radiation travels through a gas cloud or stellar atmosphere.
- *intensity* is the energy flow in any chosen direction through a unit area per unit time per unit solid angle, where the unit area is perpendicular to the chosen direction defined by the solid angle.
- flux is the net flow of energy through a unit area per unit time, summed over all directions, where the unit area is fixed as being perpendicular to the direction in which the net flow of radiation is proceeding.
- for an isotropic intensity, the positive and negative contributions will cancel out reducing the flux to zero .
- the flux F and the intensity I are related by the expression

$$F = \int I \cos\theta d\Omega$$

where $d\Omega$ is an element of the solid angle and θ is the angle between the fixed direction (defined by the flux) and the variable direction (defined by the intensity).

- the intensity is measured per unit area perpendicular to the *line of sight* while the flux is measured per unit area perpendicular to the fixed direction of the net flow of energy.
- the intensity I and the specific intensity I_ν are related by the expression

  $$I = \int I_\nu d\nu,$$

  where I_ν the specific intensity (i.e. I_ν is the intensity per unit frequency interval at the frequency ν).
- the luminosity L of a spherical star is related to the flux according to the expression.

  $$L = 4\pi R^2 F,$$

  where R is the radius of the star and F is the flux .
- in the case of an extended object, resolved in angle by the telescope, the intensity received from various parts of the body is called the *surface brightness* of the body.
- if the surface brightness of a body is uniform then the intensity is independent of distance.
- as radiation travels a distance ds, its specific intensity I_ν will change according to the expression

  $$dI_\nu = \varepsilon_\nu ds - \kappa_\nu ds,$$

  where ε_ν is the emission coefficient, κ_ν is the absorption coefficient, and I_ν is the specific intensity.
- the optical depth τ_ν along a direction r is defined by the expression

  $$d\tau_\nu = \kappa_\nu dr,$$

  where r makes an angle θ with the direction in which the distance ds is traveled by the radiation.
- the basic equation of radiative transfer is

  $$\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{\varepsilon_\nu}{\kappa_\nu},$$

  where the projection factor $\mu = \cos\theta.$
- the equation of transfer goes with the following sign convention:

  a) I_ν is positive outwards (starting from zero at a surface nearest the source and increasing toward the observer).

b) τ_v is positive inwards (starting from zero at a surface nearest the observer and increasing further away from the observer).

c) s is positive inwards (starting from zero at a surface nearest the observer and increasing further away from the observer).

- the ratio of the emission coefficient ε_v to the absorption coefficient κ_v defines the source function S_v, such that
$$S_v = \frac{\varepsilon_v}{\kappa_v}.$$
- in the stellar atmosphere, it is impossible to achieve complete thermodynamic equilibrium (otherwise, the temperature would be the same everywhere with no *temperature gradient* to drive an outward flow of energy)
- a sufficiently small region of a stellar atmosphere may attain roughly the same temperature (characterised by a single local temperature) thereby achieving *local thermodynamic equilibrium* (LTE).
- the LTE approximation can greatly simplify the source function in the equation of transfer.
- under LTE conditions, the source function S_v may be written (in terms of frequency) as
$$S_v = B_v(T) = \frac{2hv^3}{c^2}\left[\exp\left(\frac{hv}{kT}\right) - 1\right]^{-1},$$
where $B_v(T)$ is the Planck's function, h is Planck's constant, k is Boltzmann's constant and c is the speed of light.
- under LTE conditions, the source function S_v may be written (in terms of wavelength) as
$$S_\lambda = B_\lambda(T) = \frac{2hc^3}{\lambda^5}\left[\exp\left(\frac{hc}{\lambda kT}\right) - 1\right]^{-1},$$
where λ is the wavelength and T is the temperature.
- radio waves have long wavelengths and low frequencies so that $hv << kT \left(\text{i.e } \frac{hv}{kT} << 1\right)$ which implies that the source function S_v is given by

a) $S_v = B_v(T) \approx \dfrac{2kTv^2}{c^2}$ (in terms of frequency), or

b) $S_\lambda = B_\lambda(T) \approx \dfrac{2ckT}{\lambda^4}$ (in terms of wavelength).

- for strong spectral lines, the LTE approximation breaks down and the source function S_ν departs significantly from the Planck's function $B_\nu(T)$.
- we must replace the LTE conditions by the assumption of a *statistically* steady state in each atomic energy level if the LTE approximation breaks down.

6.0 TUTOR-MARKED ASSIGNMENT

1) As radiation travels through a gas cloud or stellar atmosphere, its intensity
   a) remains constant
   b) becomes zero
   c) becomes infinite
   d) changes
2) The energy flow in a particular direction through a unit area (perpendicular to the chosen direction) per unit time per unit solid angle (defining the chosen direction) is called
   a) luminosity
   b) flux
   c) intensity
   d) specific intensity
3) The net energy flow through a unit area (perpendicular to the direction of the net flow of radiation) summed over all directions is called
   a) luminosity
   b) flux
   c) intensity
   d) specific intensity
4) If the luminosity L of a spherical star of radius R is given by

   $$L = \left(\frac{3\rho}{R}\right)F,$$

   then the quantity ρ must be the
   a) number density of the star
   b) the surface area of the star
   c) the volume of the star
   d) mass density of the star
5) If the surface brightness of a body is uniform then the intensity
   a) depends on distance
   b) does not depend on distance
   c) is always constant
   d) is always zero

6) If radiation travels a distance ds through a gas cloud with an absorption coefficient κ_ν and an emission coefficient ε_ν, the intensity of the radiation changes by an amount dI_ν given by

a) $dI_\nu = \varepsilon_\nu ds + \kappa_\nu ds.$
b) $dI_\nu = \kappa_\nu ds - \varepsilon_\nu ds.$
c) $dI_\nu = -(\kappa_\nu - \varepsilon_\nu)ds$
d) none of the above expressions

7) In the expression $d\tau_\nu = -\kappa_\nu dr,$ the optical depth is denoted by

a) $r.$
b) $\tau_\nu.$
c) $\kappa_\nu.$
d) $dr.$

8) If the angle between the *line-of-sight* and the direction of *net flow* of radiation is zero, then the equation of radiative transfer becomes

a) $\mu \frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{\varepsilon_\nu}{\kappa_\nu}.$
b) $\frac{dI_\nu}{d\tau_\nu} = I_\nu - \frac{\varepsilon_\nu}{\kappa_\nu}.$
c) $0 = I_\nu - \frac{\varepsilon_\nu}{\kappa_\nu}.$
d) $\frac{dI_\nu}{d\tau_\nu} = I_\nu + \frac{\varepsilon_\nu}{\kappa_\nu}.$

9) The sign convention for the equation of transfer is/are

a) I_ν is positive outwards only
b) τ_ν is positive inwards only
c) s is positive inwards only
d) all of the above statements

(10) The ratio of the emission coefficient to the absorption coefficient is called the

a) optical depth
b) source function
c) projection factor
d) Planck’s function

(11) Under local thermodynamic equilibrium (LTE) conditions, the source function becomes approximately equal to

a) the optical depth
b) the projection factor
c) the Planck’s function
d) none of the above

12) Radio waves have

a) long wavelength

b) short wavelength
c) high frequency
d) none of the above descriptions

13) We must replace the LTE conditions by the assumption of a statistically steady state in each atomic level if
a) LTE approximation holds good
b) LTE approximation breaks down
c) the spectral line is very weak
d) none of the above is the case

14) In terms of frequency, the Planck's function $B_\nu(T)$ approximates to
a) $B_\nu(T) \approx \left(\frac{2kT}{c^2}\right)\nu^2.$
b) $B_\nu(T) \approx \left(\frac{2kT}{c^2}\right)\lambda^2.$
c) $B_\nu(T) \approx \frac{2ckT}{\lambda^3}.$
d) $B_\nu(T) \approx \frac{h\nu}{kT}.$

15) In terms of wavelength, the Planck's function $B_\lambda(T)$ approximates to
a) $B_\lambda(T) \approx \left(\frac{2kT}{c^2}\right)\nu^3.$
b) $B_\lambda(T) \approx \left(\frac{2kT}{c^2}\right)\lambda^2.$
c) $B_\lambda(T) \approx \frac{2ckT}{\lambda^4}.$
d) $B_\lambda(T) = \frac{h\nu}{kT}.$

16) Radio waves are photons with energies
a) less than $10^{-6}eV$.
b) greater than $10^{-6}eV$ but less than $1MeV$.
c) greater than $10^{-6}eV$ but less than $10MeV$.
d) greater than $10MeV$.

17) Hard X-rays and $\gamma-$rays are photons with energies
a) less than $10^{-6}eV$
a) less than $10^{-6}eV$.
b) greater than $10^{-6}eV$ but less than $1eV$.
c) greater than $1eV$ but less than $1MeV$.
d) greater than $1MeV$.

18) The intensity I and the relative intensity I_v are related by the expression

a) $I_v = \frac{dI}{dv}$.

b) $I_v = \int I dv$.

c) $v = \int I_v dI$.

d) none of the above expressions.

19) If the emission coefficient happens to be zero then the source function S_v becomes

a) 0

b) 1

c) 10

d) undefined

20) In the expression $B_v dv = -B_\lambda d\lambda$, the negative sign indicates that the wavelength

a) remains constant

b) is always zero

c) increases with increasing frequency

d) deceases with increasing frequency

7.0 REFERENCES/FURTHER READING

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UNIT 3 LINE PROFILES

CONTENTS

1.0 INTRODUCTION

In your study of radioactive transfer, you saw that the source function S_v may depart considerably from the Planck's function $B_v(T)$. That is often the case when strong spectral lines are involved. For such strong spectral lines, we replace the LTE by the assumption of a *statistically* steady state in each atomic energy level. A spectral line is formed whenever an electron jumps from a higher energy level to a lower energy level. In this unit, you shall be introduced to the transition rules, the formation of spectral lines and the profile of such spectral lines.

2.0 OBJECTIVES

At the end of this unit, you should be able to:

- describe *transition* from one atomic energy level to another
- apply the transition rules to atomic energy levels
- discuss degeneracy of atomic levels
- deduce the shape of a spectral line, given its *normalised* profile
- discuss line broadening.

3.0 MAIN CONTENT

3.1 Transition

A transition is said to occur when an electron jumps from one energy level to another. When a transition occurs, a spectral line is formed. In the case of strong spectral lines, the source function may depart significantly from the Planck's function, resembling more the direction-averaged mean intensity J_ν which is given by

$$J_\nu = \frac{1}{4\pi}\int I_\nu d\Omega.$$

For such spectral lines, we replace the conditions of local thermodynamic equilibrium (LTE) by the assumption of a *statistically* steady state in each atomic energy level. Recall that a spectral line is formed whenever an electron jumps from a higher energy level to a lower energy level. Conversely, an absorption line is formed when an electron jumps from a lower energy level to a higher energy level.

Consider a radiating atom. Suppose that a line is formed by transition between two bound levels i and j, where $j > i$. Let κ denote the continuum. For an optically thin emission line (i.e. a case where τ_ν is small), the emergent intensity I_{ji} of such an emission line (after passing through a plasma) is given by

$$I_{ji} = n_j A_{ij} \nu_{ji} dV,$$

where n_j is the population of the upper atomic level from where the electrons jump to the lower level i during the emission of the quanta (photons) $h\nu_{ji}$, and V is the volume of the plasma radiating the line ν_{ji}. The population n_j of the energy level j in an atom (or ion) depends on

a) the ratio $\frac{n_j}{n_{ion}}$, where n_{ion} is the total number of atoms (or ions) in that particular ionisation state;

b) the ionisation equilibrium $\frac{n_{ion}}{n_{elem}}$, where n_{elem} is the elemental abundance;

c) the abundance of the element relative to hydrogen, $\frac{n_{elem}}{n_H}$, where n_H is the number density of hydrogen; and

d) the number density of hydrogen, n_H.

Each energy level in an atom (or ion) is characterised by four quantum numbers:

(i) the principal quantum number n,
(ii) the spin quantum number s,
(iii) the angular momentum quantum number l, and
(iv) the total angular momentum quantum number j.

A photon is emitted when an electron jumps from a higher energy level characterised by (n,s,l,j) to a lower energy level characterised by (n',s',l',j'). Conversely, a photon is absorbed when an electron jumps from a lower energy level (n',s',l',j') to a higher energy level (n,s,l,j). If the atom contains more than one electron, then you use the lower-case letters s,l and j to denote the angular momenta for a particular electron and the upper-case letters S,L and J to denote the angular momenta of the entire atom. In this case, the electrons are arranged in shells. According to Pauli's exclusion principle, no two electrons, may have the same set of quantum numbers. Thus, the energy level of the entire atom may be symbolised by (n,S,L,J) while the energy level of any single electron may be denoted by (n,s,l,j). The principal quantum number is always denoted by the lower-case letter n.

Suppose that electrons jump from an upper level j denoted by (n,S,L,J) to a lower level i denoted by (n',S',L',J'). Electrons that jump from level j to level i will emit a spectral line of frequency ν_{ji} given by

$$\nu_{ji} = \frac{E_j - E_i}{h},$$

where E_j is the energy of the upper level j and E_i is the energy of the lower level i. On the other hand, an absorption line is formed when electrons jump from the lower level i to the upper level j. In the case of absorption, the photon is absorbed by the electron and the energy of the photon is used to raise the electron to a higher energy level.

3.2 Transition Rules

3.2.1 Transition Rules in the Absence of a Magnetic Field

A *transition* occurs when an electron jumps from one energy level to another. In spectroscopy, it is quite possible to predict the transitions that will occur under a given condition. In this way, the frequencies of the resulting set of emission (or absorption) lines can also be predicted. This is possible because the quantum numbers (n, S, L, J) obey certain transition rules. In particular, an electron can jump, only if the change in quantum numbers is consistent with the following rules:

(a) $S' = S$

$\Rightarrow S' - S = 0$

$\Rightarrow \Delta S = 0.$

(b) $L' = L \quad or \quad L' = L \pm 1$

$\Rightarrow L' - L = 0 \quad \Rightarrow \quad L' - L = \pm 1$

$\Rightarrow \Delta L = 0 \quad \Rightarrow \quad \Delta L = \pm 1$

Combining the two results, you obtain

$\Delta L = 0 \; or \pm 1$.

$\therefore \Delta L = -1, 0, +1.$

(c) $J' = J \quad or \quad J' = J \pm 1$

$\Rightarrow J' - J = 0 \quad \Rightarrow \quad J' - J = \pm 1$

$\Rightarrow \Delta J = 0 \quad \Rightarrow \quad \Delta J = \pm 1$

Combining these two results, you obtain

$\Delta J = 0 \; or \pm 1$

$\therefore \Delta J = -1, 0, +1.$

Note

1. There is no restriction on the principal quantum number n.
2. The transition from $J' = 0$ is for bidden.
3. Parity must change by1. (Parity is the sum of the l-values of all the electrons).
4. The fact that $\Delta s = 0$ implies that spin is conserved.

3.2.2 Degeneracy of Atomic Levels

The transition rules stated above are sufficient in the absence of magnetic field. If a magnetic field is applied, however, the total angular momentum J may be pointed only in certain "*quantized*" directions with respect to the direction of the magnetic field. In this case, J may be pointed in such a way that its projection along the applied magnetic field can only take one of the following values: $J, J-1, J-2, \cdots, -J$. This gives a total of $2J+1$ sub-levels. These 2J+1 sub-levels are often indistinguishable.

The projection of J along the magnetic field is called the magnetic quantum number denoted by M_J. It is the *fifth quantum number*. The magnetic quantum number M_J indicates that a level is composed of $2J+1$ sub-levels. Such a level is said to be *degenerate*. The *degree* of the degeneracy is the number $2J+1$. The degree of degeneracy is also called the *statistical weight* g of that particular level (i.e. $g = 2J + 1$). The statistical weight g represents the number of electrons that can occupy a given level without violating Pauli's exclusion principle.

When there is a transition between one degenerate state A and another degenerate state B, there will be transition between *certain sub-levels* of state A and *certain sub-levels* of state B. However, there will be no transition between *some* sub-levels of state B and *some* sub-levels of state B. The magnetic quantum number M_J shall communicate with each other only when certain rules for M_J are obeyed. These rules for M_J are stated below:

$$M'_J = M_J \qquad or \qquad M'_J = M_J \pm 1,.$$

$$\Rightarrow M'_J - M_J = 0 \qquad\qquad \Rightarrow M'_J - M_J = \pm 1$$

$$\Rightarrow \Delta M_J = 0 \qquad\qquad \Rightarrow \Delta M_J = \pm 1$$

Combing the two results, you obtain

$$\Rightarrow \Delta M_J = 0 \ \ or \ \pm 1$$

$$\Rightarrow \Delta M_J = 0, \pm 1$$

$$\Rightarrow \Delta M_J = -1, 0, +1.$$

Example

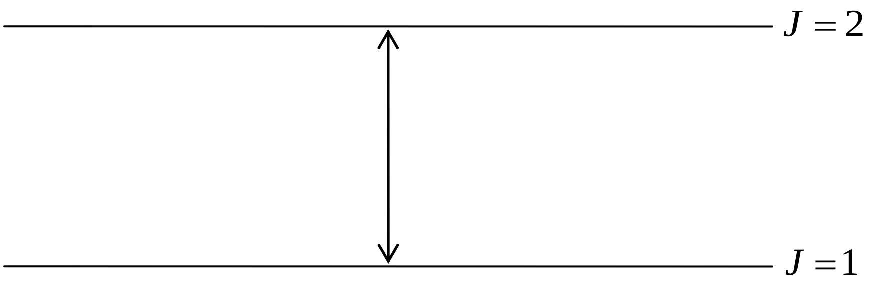
Make a schematic representation of:

a) aspectral line between two degenerate levels $(J = 2 \ and \ J = 1)$, in the absence of a magnetic field.

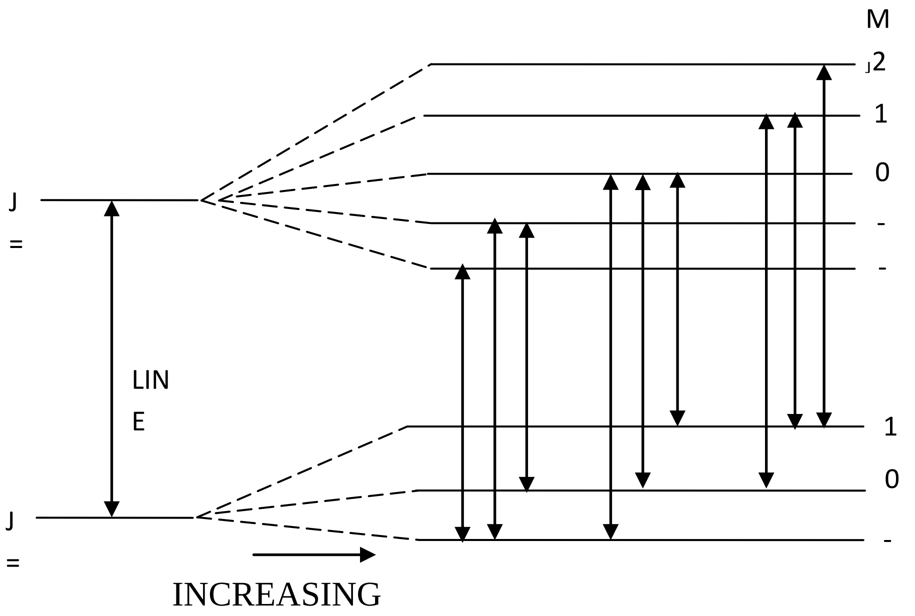
b) the allowed components (of the spectral line) when degeneracy is lifted due to the application of a magnetic field.

Solution

a) In the absence of a magnetic field, *one* spectral line is observed (figure 1a).



b) In the presence of a magnetic field, the degeneracy is lifted and the sub-levels appear separately as shown in figure 1b.



3.2.3 Forbidden Transitions

In the classical treatment, a *series expansion* of the radiation field can be obtained. The dominant term is the electric dipole contribution. This is a good first approximation if other terms are neglected. In quantum theory, the dipole term corresponds to the electron transitions. It is governed by the quantum number rules of Eqs (4) and (5).

In the classical treatment, there are other less important terms such as the electric quadrupole term and higher (electric and magnetic) terms. In the quantum picture, there are “forbidden” transitions which do not obey the rules set down in Eqs (4) and (5). Such transitions my still give rise to observable spectral lines. These spectral lines may not be important under the laboratory (high-density) conditions. However, under low-density conditions, the spectral lines resulting from the “forbidden” transitions become quite significant.

The selection rules for *magnetic dipole* radiation are

$$\left.\begin{array}{lll} S' = S, & \text{which implies that} & \Delta S = 0 \\ L' = L, & \text{which implies that} & \Delta L = 0 \\ J' = J \text{ or } J' = J \pm 1, & \text{which imply that} & \Delta J = 0, \pm 1 \\ n'l' = nl, & \text{which implies that} & \Delta(nl) = 0 \end{array}\right\} \qquad (6)$$

Thus, the transitions which give rise to magnetic dipole radiation must take place between levels within one and the same term. (The only parameter that may differ by ±1 is the total angular momentum quantum number J).

3.3 Line Profile

3.3.1 Absorption Lines

All the information about the physical properties of astronomical objects comes from the study of the spectra. In particular, by studying the strengths of the absorption lines, we can deduce stellar masses, temperatures and compositions. Light from a source can be dispersed into a spectrum by means of a prism or a diffraction grating. Different sources have different spectra. For instance, the spectra of a quasar consist of a continuous spectrum (or continuum) with narrow spectral lines superimposed. These lines are mostly absorption (Fraunhofer) lines, but sometimes bright emission lines, are also observed. The continuous spectrum comes from the hot surface of the source nucleus. Atoms in the atmosphere absorb certain characteristic wavelengths of this radiation leaving dark "gaps" at the corresponding points in the spectrum.

3.3.2 Electron Lifetime and Levels

Let us now take a closer look at the equation

$$h\nu_{ji} = E_j - E_i, \qquad E_j > E_i.$$

If the energy levels E_j and E_i were sharply defined, an infinitely narrow spectral line would be emitted when an electron jumps from energy level j to energy level i. This would correspond to an infinitely long lifetime of the electron in that energy level. The frequency spread $\delta\omega$ and the lifetime δt are related by the expression:

$$\delta\omega\delta t \approx 2\pi, \qquad (7a)$$

or

Dividing both sides of Eq.(7a) by 2π, you get

$$\left(\frac{\delta\omega}{2\pi}\right)\delta t \approx 1.$$

But, by *definition* of angular frequency, you know that:

$$\omega = 2\pi\nu,$$

so that

$$\delta\omega = 2\pi\delta\nu,$$

and

$$\frac{\delta\omega}{2\pi} = \delta\nu.$$

Hence, Eq (7a) reduces to

$$\left(\frac{\delta\omega}{2\pi}\right)\delta t \approx 1.$$

or

$$\left(\frac{2\pi\delta\nu}{2\pi}\right)\delta t \approx 1$$

so that

$$\delta\nu\delta t \approx 1. \qquad (7b)$$

Multiplying both sides of Eq (7b) by the Planck's constant h, you get

$$(h\delta\nu)\delta t \approx h. \qquad (8)$$

But,

$$E = h\nu,$$

so that

$$\delta E = h\delta\nu. \qquad (9)$$

Hence, using Eq (9) in Eq (8), you find that

$$\delta E\delta t \approx h. \qquad (10)$$

Here, δE is a measure of the sharpness of the energies; δt is a measure of the electron lifetime; h is Planck's constant; $\delta\nu$ represents the spread of a spectral line of frequency ν. At this stage, you may wish to compare Heisenberg's uncertainty principle with the foregoing discussion. Equation (9) shows that a sharply defined energy δE corresponds to an infinitely narrow spectral line $\delta\nu,$ and Equation (10) shows that an infinitely narrow spectral line $\delta\nu$ corresponds to an infinitely long lifetime $\delta t.$ The conclusion here is obvious: if a particular atomic energy level is sharply defined then any electron in that energy level will have an infinitely long lifetime.

3.3.3 The Lorentzian Line Profile

Evidently, the lifetime of an electron in any energy level is finite. Therefore, the energy level must also have a finite width. This fact was expressed quantum mechanically by Weisskopet and Wigner in the following probability distribution law:

$$P\left(E_j\right)dE = \frac{A_j}{h}\frac{dE}{\frac{1}{4}A_j^2 + \frac{4\pi^2}{h^2}\left(E - E_j\right)^2}\ , \qquad (11)$$

where E_j is the mean energy of the energy level j and A_j is the probability of transition from level j to all other possible levels $E.$

Thus, the energy of a level with mean energy E_j is distributed as shown in Eq (11).

Equation (11) defines the line profile. Its curve is called a *damping profile* or a *dispersion curve*. Clearly, the probability distribution $P(E_j)$ has a maximum value when $E - E_j = 0(\text{i.e. when } E = E_j)$. This implies that most of the atoms have energies in the neighbourhood of the mean energy E_j.

When an electron jumps from level j to level i, the intensity of the resulting spectral line is given by

$$I_{ji} = h\nu_{ji}A_{ji}\frac{A_j + A_i}{\frac{1}{4}\left(A_j + A_i\right)^2 + \frac{4\pi^2}{h^2}\left[E - \left(E_i - E_j\right)^2\right]}\ , \qquad (12)$$

where A_{ji} is the probability of transition from level j level i.

Equation (12) can be simplified by setting $\Gamma = A_j + A_i$ and noting that $E = h\nu$ while $E_i - E_j = E_j - E_i = h\nu_{ji}$. Equation (12) can then be processed as follows:

$$I_{ji} = E_{ji}A_{ji}\frac{\Gamma}{\frac{1}{4}\left(\Gamma^2\right) + \frac{4\pi^2}{h^2}\left[E - \left(E_i - E_j\right)\right]^2}$$

$$I_{ji} = E_{ji}A_{ji}\frac{\Gamma}{\frac{1}{4}\left(\Gamma^2\right) + \frac{4\pi^2}{h^2}\left(h\nu - h\nu_{ij}\right)^2}$$

$$= E_{ji}A_{ji}\frac{\Gamma}{\frac{1}{4}\left(\Gamma^2\right) + \frac{4\pi^2}{h^2}\left(h^2\nu^2 - 2h^2\nu_{ji}\nu + h^2\nu_{ij}^2\right)}$$

$$= \frac{E_{ji}A_{ji}\Gamma}{\frac{\Gamma^2}{4} + \frac{4\pi^2}{h^2}\cdot h^2\left(\nu^2 - 2\nu_{ji}\nu + \nu_{ij}^2\right)}$$

so that

$$I_{ji} = \frac{E_{ji}A_{ji}\Gamma}{\frac{\Gamma^2}{4} + 4\pi^2\left(\nu - \nu_{ji}\right)^2}.$$

Dividing the numerator and the denominator $4\pi^2$, you obtain

$$I_{ji} = \frac{\left(\frac{E_{ji}A_{ji}\Gamma}{4\pi^2}\right)}{\left(\frac{\Gamma^2}{16\pi^2}\right) + \left(\nu - \nu_{ji}\right)^2}. \qquad (13)$$

For a constant decay probability A_{ji}, Eq (13) may be simplified further by setting

$$E_{ji}A_{ji} = 1 \qquad (14)$$

and

$$I_{ji} = \phi_v \qquad (15)$$

so that

$$\phi_v = \frac{\Gamma}{4\pi^2}\frac{1}{\left(v - v_0\right)^2 + \left(\frac{\Gamma}{4\pi}\right)^2}. \qquad (16)$$

Here, ϕ_v is the normalised profile and $\Gamma = \frac{2}{T_o}$, where T_o is the average time interval between successive collisions of the radiating particles. The line shape given by Equation (16) is called *Lorentzian* profile and the parameter Γ is called the Lorentzian width. The damping constant $\Gamma = A_j + A_i$ is the sum of the decay constants of the two energy levels involved in the electron jump.

3.3.4 Line Broadening

The wings of a spectral line will be weak while the core of the spectral line remains strong. Line broadening is a measure of the spread of the frequency v of the emitted spectral line on either side of the core. Line broadening can be divided into three classes:

1) Natural line broadening
2) Doppler line broadening; and
3) Collisional (or pressure) line broadening.

The natural line broadening is always present. The Doppler line broadening is due to the motion of the observed atoms in different directions with different velocities. The resulting Doppler shift produces a spread in the frequency of the observed line. Collisional (or pressure) line broadening is due to the effects of other particles on the radiating atom. Doppler broadening is always proportional (in wavelength units), to the wavelength of the *centre* line. Natural and collisional broadenings do not show any systematic trend with the centre wavelength. However, collisional broadening is always proportional to the number of colliding particles per unit volume (and, by extension to pressure). Let us consider the Doppler line broadening and the collisional (or pressure) line broadening in detail.

3.3.5 Doppler Line Broadening

Doppler broadening is due to the motion of individual atoms in a hot gas. The thermal motions of the atoms (or ions) produce particles which are moving away from an observer, and particles moving toward the observer. This situation gives rise to Doppler effect. The frequency v of a spectral line emitted by an atom moving with velocity v_r along the *line of sight* is given by

$$\Delta v = v - v_o = -\left(\frac{v_r}{c}\right).v_o,$$

where v_o is the rest frequency of the spectral line and c is the speed of light. Clearly, Δv is positive when v_r is negative. Also, Δv is negative when v_r is positive. A negative value of Δv implies that the radiating atom is moving away from the observer (i.e. v_r is positive). This gives rise to a shift in the wavelength of a visible line towards the *'red end'* of the optical spectrum. In this case, a *redshift* is said to have taken place. The observed frequency is *lower* than the rest frequency of the emitted line (i.e. the observed wavelength is longer than the rest wavelength of the spectral line).

If the radiating atom is moving toward the observer then v_r is negative. In this case, the shift Δv in frequency of the spectral line is positive. A positive value of Δv gives rise to a shift in the wavelength of a visible spectral line toward the *'blue end'* of the electromagnetic spectrum. The observed frequency is *higher* than the 'rest frequency of the spectral line (i.e. the observed wavelength is *shorter* than the rest wavelength of the line).

When the Doppler broadening of a spectral line is entirely due to the thermal motion of the radiating particles, the *width* of such a line is called the *Doppler width* Δv_D.

In terms of the frequency, the Doppler width Δv_D is given by

$$\Delta v_D = \frac{v_o}{c}\left(\frac{2kT}{m}\right)^{1/2},$$

where v_o is the frequency of the undisturbed line centre, T is temperature, m is the mass of the particle and k is Boltzmann's constant.

In terms of the wavelength, the Doppler width $\Delta\lambda_D$ is given by

$$\Delta\lambda_D = \frac{\lambda_o}{c}\left(\frac{2kT}{m}\right)^{1/2},$$

where λ_o is the wavelength of the undisturbed line centre, $\Delta\lambda_D = \lambda - \lambda_o$, and the profile ϕ_λ is now normalised with respect to $\Delta\lambda$.

Notice that the expression for the shift in wavelength $\Delta\lambda_D$ (or frequency shift Δv_D) is closely related to the expression for the thermal velocity v_o (written in terms of the temperature T) of the radiating particles.

3.3.6 Gaussian Profile

The thermal motions of individual atoms are not the only *motion of interest* here. Sometimes, the radiating atoms may possess a *thermal* velocity component as well as a *nonthermal* (turbulent) velocity component. In this case, the Doppler width is modified to accommodate the nonthermal component in the following way:

$$\Delta\lambda_D = \frac{\lambda_o}{c}\left(\frac{2kT}{m} + \xi^2\right)^{½}.$$

The profile ϕ_v of the Doppler broadened line is then given by ϕ_v

$$\phi_v = \frac{1}{\sqrt{\pi}\Delta v_D}\exp\left[-\left(\frac{\Delta v}{\Delta v_D}\right)^2\right],$$

where the Doppler width is again defined by

$$\Delta v_D = \frac{v_o}{c}\sqrt{\frac{2kT}{m}}.$$

In terms of the wavelength, the profile assumes the form

$$\phi_\lambda = \frac{1}{\sqrt{\pi}\Delta\lambda_D}\exp\left[-\left(\frac{\Delta\lambda}{\Delta_D}\right)^2\right].$$

This is a bell-shaped profile called the *Gaussian profile.* The Gaussian profile falls to half its maximum height (or depth) when the exponential term is equal to half i.e. when

$$\exp\left[-\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2\right] = \frac{1}{2}.$$

In this case,

$$-\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2 = \log_e\left(\frac{1}{2}\right),$$

$$-\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2 = \log_e 1 - \log_e 2,$$

$$\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2 = \log_e 2 - \log_e 1,$$

$$\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2 = \log_e\left(\frac{2}{1}\right),$$

$$\left(\frac{\Delta\lambda}{\Delta\lambda_D}\right)^2 = \log_e 2,$$

$$\frac{\Delta\lambda}{\Delta\lambda_D} = \sqrt{\log_e 2},$$

and

$$\Delta\lambda = \Delta\lambda_D\sqrt{\log_e 2}\cdot$$

The *full width at half height* (FWHH) of the line profile is defined as the wavelength separation between the points when the profile drops to half its maximum height (or depth) on *EITHER* side of the line centre.

On one side of the line centre, $\Delta\lambda = \Delta\lambda_D\sqrt{\log_e 2}\cdot$ On the other side of the line centre, $\Delta\lambda = \Delta\lambda_D\sqrt{\log_e 2}$ also. Thus, the *full width at half height* is given by the sum of the two equal values of $\Delta\lambda$ i.e.

$$FWHH = \Delta\lambda_D\sqrt{\log_e 2} + \Delta\lambda_D\sqrt{\log_e 2}$$

or

$$FWHH = 2\Delta\lambda_D\sqrt{\log_e 2}\cdot$$

Substituting the value of $\sqrt{\log_e 2},$ you obtain

$$FWHH = 1.667\Delta\lambda_D.$$

This result is applicable to both the absorption and emission line profiles. However, the actual line will suffer radiative transfer effects (like saturation) which may reduce the dip in an absorption line and hence increase the observed $FWHH$.

4.0 CONCLUSION

In this unit, you were introduced to the basic transition and selection rules that are useful in understanding the formation of spectral lines. You studied the Lorentzian profile and the Gaussian profile. You were able to use the normalised profile to deduce the shape of a spectral line. You have seen the three mechanisms by which a spectral line may be broadened.

5.0 SUMMARY

Having gone through this unit, you now know that:

(i) a spectral line is formed whenever an electron jumps from a higher energy level to a lower energy level.

(ii) the emergent intensity I_ν of an optically thin emission line is given by

$$I_\nu = n_j A_{ij} h\nu_{ji} dV,$$

where n_j is the population of the upper atomic level j (from where the electrons jump to the lower level i during the emission of the photons), $h\nu_{ji}$ is the energy of the emitted photons, and V is the volume of the plasma radiating the line of frequency ν_{ji}.

(iii) the energy level in an atom is characterised by

- (a) the principal quantum number n;
- (b) the spin quantum number s;
- (c) the angular momentum quantum number l; and
- (d) the total angular momentum quantum number j.

(iv) a photon is *emitted* whenever an electron jumps from a higher energy level defined by the set of quantum numbers (n,s,l,j) to a lower energy level defined by a new set of quantum numbers (n',s',l',j').

(v) a photon is *absorbed* whenever an electron jumps from a lower energy level (n',s',l',j') to a higher energy level (n,s,l,j).

(vi) if the atom contains more than one electron, then, the lower-case letters s,l,j are used to denote the angular momenta of a particular electron and the upper-case letters S,L,J are used to denote the angular momenta of the entire atom. (In all cases, the principal quantum number is denoted by the lower-case letter, n).

(vii) the energy level of a particular electron is symbolised by (n,s,l,j) while the energy level of the entire atom is symbolised by (n,S,L,J).

(viii) according to Pauli's exclusion principle, no two electrons can have the same set of quantum numbers.

(ix) an electron that jumps from an upper level j to a lower level i will emit a spectral line of frequency ν_{ji} given by

$$\nu_{ji} = \frac{E_j - E_i}{h},$$

where E_j is the energy of the upper level j and E_i is the energy of the lower level i. (In this case, an emission line is produced)

(x) an electron that jumps from the lower level i to the upper level j will absorb a photon of frequency ν_{ij} given by

$$v_{ij} = v_{ji} = \frac{E_j - E_i}{h},$$

where h is Planck's constant. (In this case, an absorption line is produced).

(xi) a transition is said to occur when an electron jumps from one energy level to another.

(xii) an electron cannot jump from one energy level to another unless the following conditions are met:

a) $S' = S$, which implies that $\Delta S = 0$

b) $L' = L$ *or* $L' = L \pm 1$, which imply that $\Delta L = 0, \pm 1$

c) $J' = J$ *or* $J' = J \pm 1$, which imply that $\Delta J = 0, \pm 1$

(xiii) the transition from $J' = 0$ *to* $J = 0$ is forbidden since *parity* must change by 1.

(xiv) there is no restriction on the principal quantum number, n.

(xiv) if a magnetic field is applied, the total angular momentum J must be pointed in such a way that its projection along the magnetic field can only take one of the following values:
$J,\ J-1,\ J-2, \cdots, -J,$
giving a total of $(2J+1)$ sub-levels that are often *indistinguishable*.

(xv) the projection of J along the magnetic field is called the magnetic quantum number denoted by $M_J \cdot$

(xvi) the magnetic quantum number M_J indicates that a degenerate level is composed of $(2J+1)$ sub-levels.

(xvii) the *degree* of *degeneracy* (or *statistical weight*) g of a degenerate level is given by $g = 2J + 1$.

(xviii) the degree of degeneracy (i.e. statistical weight) g represents the number of electrons than can occupy a particular level without violating Pauli's exclusion principle.

(xix) when there is a transition between one degenerate state A and another degenerate state B, there will be transition between *only* certain sub-levels of state A and certain sub-levels of state B which obey the selection rule for the magnetic quantum number.

(xx) the selection rule for the magnetic quantum number is
$M'_J = M_J$ *or* $M'_J = M_J \pm 1$. This implies that $\Delta M_J = 0, \pm 1$

(xxi) light from a source can be dispersed into a spectrum by means of a prism or diffraction grating.

(xxii) no spectral line is infinitely narrow (as that would imply that the life-time of the electron emitting the line is infinitely long).

(xxiii) according to Weisskopf and Wigner, the fact that the life-time of an electron is finite implies that the following probability distribution law holds:

$$P\left(E_j\right)dE = \frac{A_j}{h}\ \frac{dE}{\frac{1}{4}A_j^2 + \frac{4\pi^2}{h^2}\left(E - E_j\right)^2},$$

where E_j is the mean energy of the energy level j and A_j is the probability of transition from level j to all other possible levels E.

(xxiv) the normalised profile ϕ_v of the spectral line produced when an electron jumps from level j to level i is given by

$$\phi_v = \frac{\Gamma}{4\pi^2}\ \frac{1}{\left(v - v_0\right)^2 + \left(\frac{\Gamma}{2\pi}\right)^2}\ ,$$

where Γ is the Lorentzian width.

(xxv) line broadening can be divided into three classes:

1) natural line broadening,
2) doppler line broadening, and
3) collisional (or pressure) line broadening.

(xxvi) Doppler broadening is due to the thermal motions of indiving atoms (or ions) toward (and away from) the observer.

(xxvii) in terms the frequency $v,$ the Doppler width Δv_D is given by

$$\Delta v_D = \frac{v_0}{c}\left(\frac{2kT}{m}\right)^{\frac{1}{2}},$$

where v_0 is the frequency of the undisturbed line, T is temperature, m is mass and k is Boltzmann’s constant.

(xxviii) the natural broadening is always present.

(xxix) collisional (or pressure) broadening is due to the effects of other particles on the radiating atom.

6.0 TUTOR-MARKED ASSIGNMENT

1. When an electron jumps from a higher energy level to a lower energy level,
   (a) dispersion occurs
   (b) emission of a spectral line occurs
   (c) absorption of a spectral line occurs
   (d) diffraction occurs
2. When an electron jumps from a lower energy level to a higher energy level,
   a) dispersion occurs

b) emission of a spectral line occurs
c) absorption of a spectral line occurs
d) diffraction occurs

3. The energy level in an atom is characterised by
a) the principal and the spin quantum numbers only
b) the angular momentum quantum number only
c) the total angular momentum quantum number only
d) all of the above quantum numbers

4. The energy level of a particular electron in an atom is symbolised by
(a) (n,s,l,j).
(b) (n,S,L,J).
(c) (n,S',L',J').
(d) all of the above

5. The energy level of the entire atom is symbolised by
(a) (n,s,l,j).
(b) (n',s',l',j').
(c) (n,S,L,J).
(d) all of the above

6. The transition from $J'=0 \ to \ J=0$ is
(a) frequent
(b) allowed
(c) forbidden
(d) quantized

7. The projection of the total angular momentum relative to an applied magnetic field is called the
(a) principal quantum number
(b) spin quantum number
(c) angular momentum quantum number
(d) magnetic quantum number

8. The number of electrons that can occupy a particular energy level without violating Pauli's exclusion principle is represented by
(a) the strength of the magnetic field
(b) the magnitude of the angular momentum
(c) the degree of degeneracy
(d) the strength of the emission line

9. The degree of degeneracy (or statistical weight) g is given by
(a) $g=J\pm1$.
(b) $g=L\pm1$.
(c) $g=2J-1$.
(d) $g=2J+1$.

10. In the expression for the normalised profile

$\phi_v = \frac{\Gamma}{4\pi^2}\,\frac{1}{(v-v_0)^2+\left(\frac{\Gamma}{4\pi}\right)^2}$, the parameter Γ is known as

(a) the Gaussian width
(b) the Lorentzian width
(c) the probability of transition
(d) the line profile

11. A diffraction grating may be used to

(a) reflect light from a spectrum
(b) polarise light in a spectrum
(c) disperse light into a spectrum
(d) modulate light from a spectrum

12. An infinitely narrow spectral line would demand that the life-time of the emitting electrons on the particular energy level be

(a) infinitely long
(b) infinitely short
(c) finite (but short)
(d) zero

13. The line broadening which is always present is known as

(a) natural broadening
(b) Doppler broadening
(c) collisional (or pressure) broadening
(d) Gaussian profile

14. The line broadening which is due to the individual thermal motions of atoms (or ions) is known as

(a) natural broadening
(b) Doppler broadening
(c) collisional (or pressure) broadening
(d) Gaussian profile

15. The line broadening which is due to the effects of other particles on the radiating atom is known as

(a) natural broadening
(b) Doppler broadening
(c) collisional (or pressure) broadening
(d) Gaussian profile

16. In terms of the frequency the Doppler width Δv_D is given by

$$\Delta v_D = \frac{v_0}{c}\left(\frac{2kT}{m}\right)^{\frac{1}{2}},$$

where v_0 is the frequency of

(a) a weak emission line
(b) a strong emission line
(c) a strong absorption line
(d) the undisturbed line centre

7.0 REFERENCES/FURTHER READING

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