

## 2. Quadratic expressions and equation 2

1

x	-4	-3	-2	-1	0	1	2
$2x^3$	-128	-54	-16	-2	0	2	16
$5x^2$	80	45	20	5	0	5	20
-x	4	3	2	1	0	-1	-2
-6	-6	-6	-6	-6	-6	-6	-6
y	-50	-12	0	-2	-6	0	28

B<sub>2</sub>

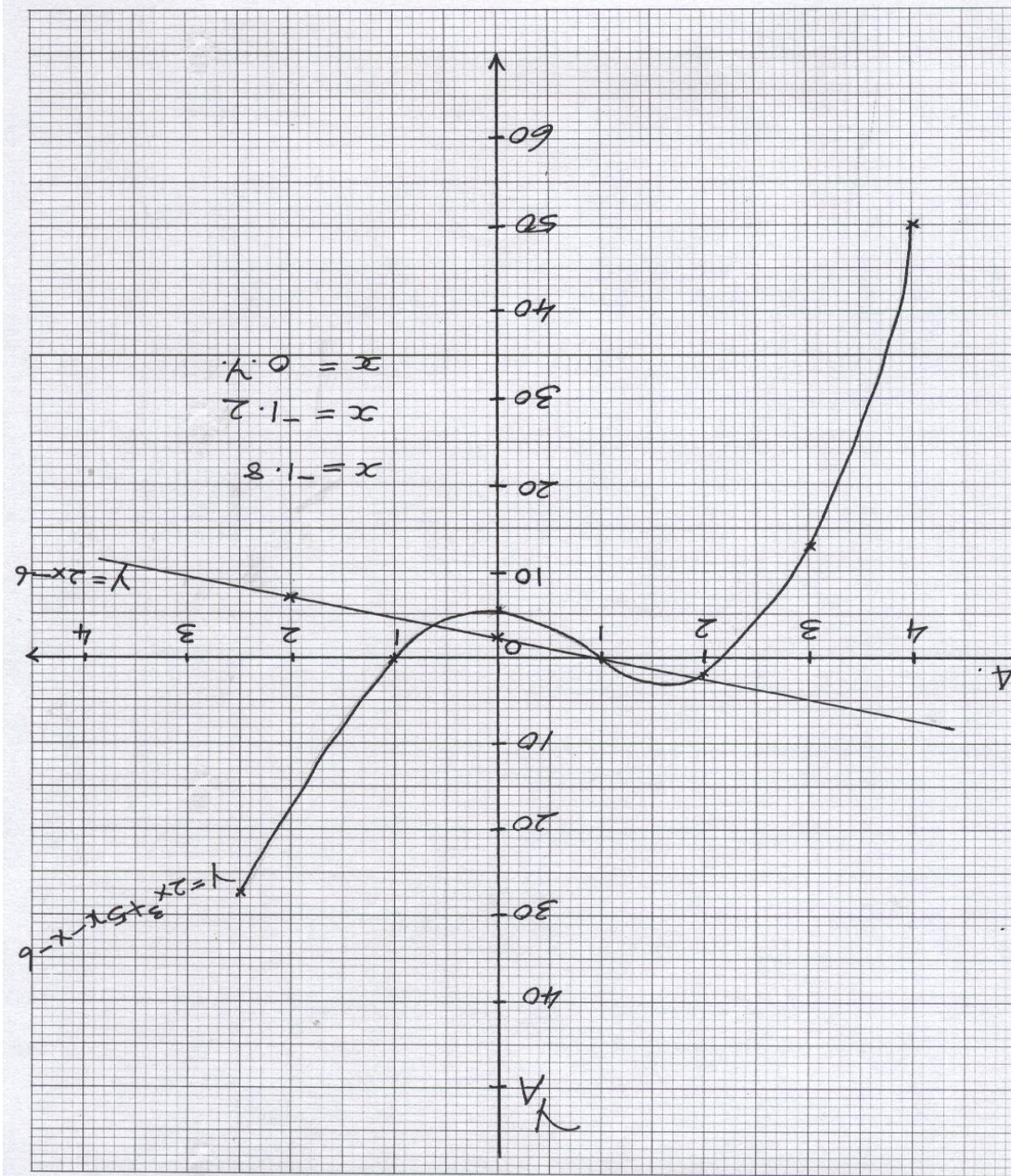
$$2x^3 + 5x^2 + x - 6 = y$$

$$2x^3 + 5x^2x - 4 = 0$$

$$-2x - 2 = y$$

$$y = -2x - 2$$

X	0	2
Y	-2	-6



		10										
2	<p>a)</p> $\frac{dy}{dx} = 4x - 6$ <p>b)</p> $4x - 6 = 0 \therefore x = 1.5$ $y = 2(1.5)^2 - 6(1.5) + 9$ $= 4.5$ <p><math>\therefore</math> Turning point (1.5, 4.5)</p> <table border="1" style="margin-left: auto; margin-right: auto;"> <tr> <td>1</td> <td>1.5</td> <td>2</td> </tr> <tr> <td>-</td> <td>0</td> <td>+2</td> </tr> <tr> <td>1</td> <td></td> <td></td> </tr> </table>	1	1.5	2	-	0	+2	1			<p>B1</p> <p>M1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p>	Equating to zero
1	1.5	2										
-	0	+2										
1												

	\   -   /		
	<p>Minimum point</p> <p>d) (i)</p> <p>Gradient = 2</p> $\frac{y-5}{x-2} = 2$ $\therefore y = 2x + 1$ <p>(ii)</p> $M_1 \times M_2 = -1$ $\therefore M_2 = -\frac{1}{2}$ $\frac{y-5}{x-2} = -\frac{1}{2}$ $\therefore y = -\frac{1}{2}x + 6$	B1  B1  B1	✓ gradient   ✓ gradient of normal
		10	
3.	$A = \frac{1}{2} \times \{(6+14) + 2(6 + 4 \ 16)\}$ $= \frac{1}{2} (20 + 32)$ $= 26 \text{ units}$	M1 M1 A1	Use of absolute values of y
		03	

1.  
(a)

x	-2	-1	0	1	2	3	4	5	6
y	-17	-9	-3	1	3	3	1	-3	-9

(b)  $y = 5x - x^2 - 3$

$$0 = 5x - x^2 - 3$$

$$y = 0$$

$$x = 0.75 \text{ or } 4.3 \pm 0.1$$

(c)  $y = 5x - x^2 - 3$

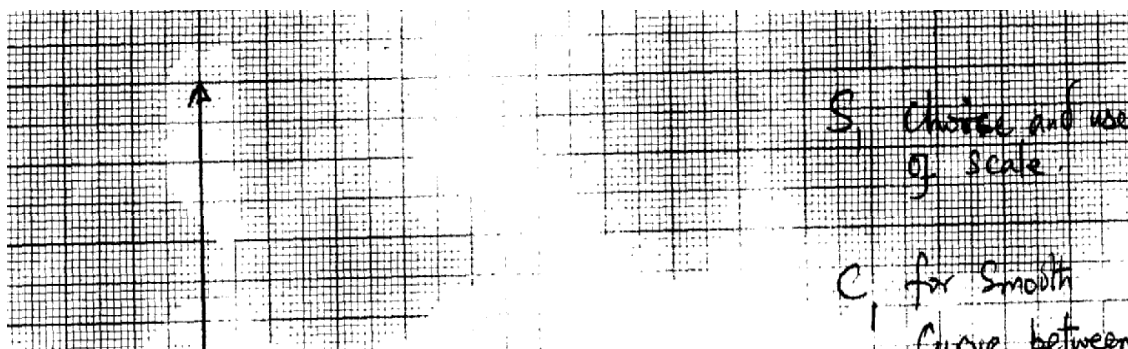
$$0 = 2x - x^2 + 3$$

$$y = 3x - 6$$

x	0	-1	2
y	-6	-9	0

*For all ✓ values of y B1  
for at least 5 ✓ values.*

$$x = -1 \text{ or } 3 \pm 0.1$$



$$\begin{aligned}
2. \quad & x - 2.5 - \sqrt{3} \quad x - 2.5 + \sqrt{3} = 0 \\
& x^2 - 2.5x + x\sqrt{3} - 2.5x + 6.25 - 2.5\sqrt{3} \\
& x\sqrt{3} + 2.5\sqrt{3} = 0 \\
& x^2 - 5x + 6.25 - 3 = 0 \\
& x^2 - 5x + 3.25 = 0 \\
& 4x^2 - 20x + 13 = 0
\end{aligned}$$

$$\begin{aligned}
3. \quad & 17.35 \times 13.85 = 240.3 \\
& 17.35 \times 13.75 = 237.2 \\
& \therefore 17.3 \times 13.8 = 238.7 \\
& \text{Max err} \quad 240.3 - 238.7 = 1.5 \\
& \text{Min err} \quad 238.7 - 237.2 = 1.6 \\
& \text{Max err} \quad = \frac{1.6 + 1.5}{2} = \frac{3.1}{2} = 1.55 \\
& \text{Product} \quad 238.7 \pm 1.55 \\
& \text{Last product} \quad 240 \\
& \text{Max err} = \quad 1.55
\end{aligned}$$

$$\begin{aligned}
& \text{Relative err} = \frac{1.55}{238.7} = 28.1\% \\
& \text{error} = \frac{1.55}{238.7} \times 100 = 0.6\% \quad 28.1 \\
& \text{Relative err} = \frac{1.55}{238.7}
\end{aligned}$$

4.

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
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y		04	-2		-8	-8		-2	4	12	
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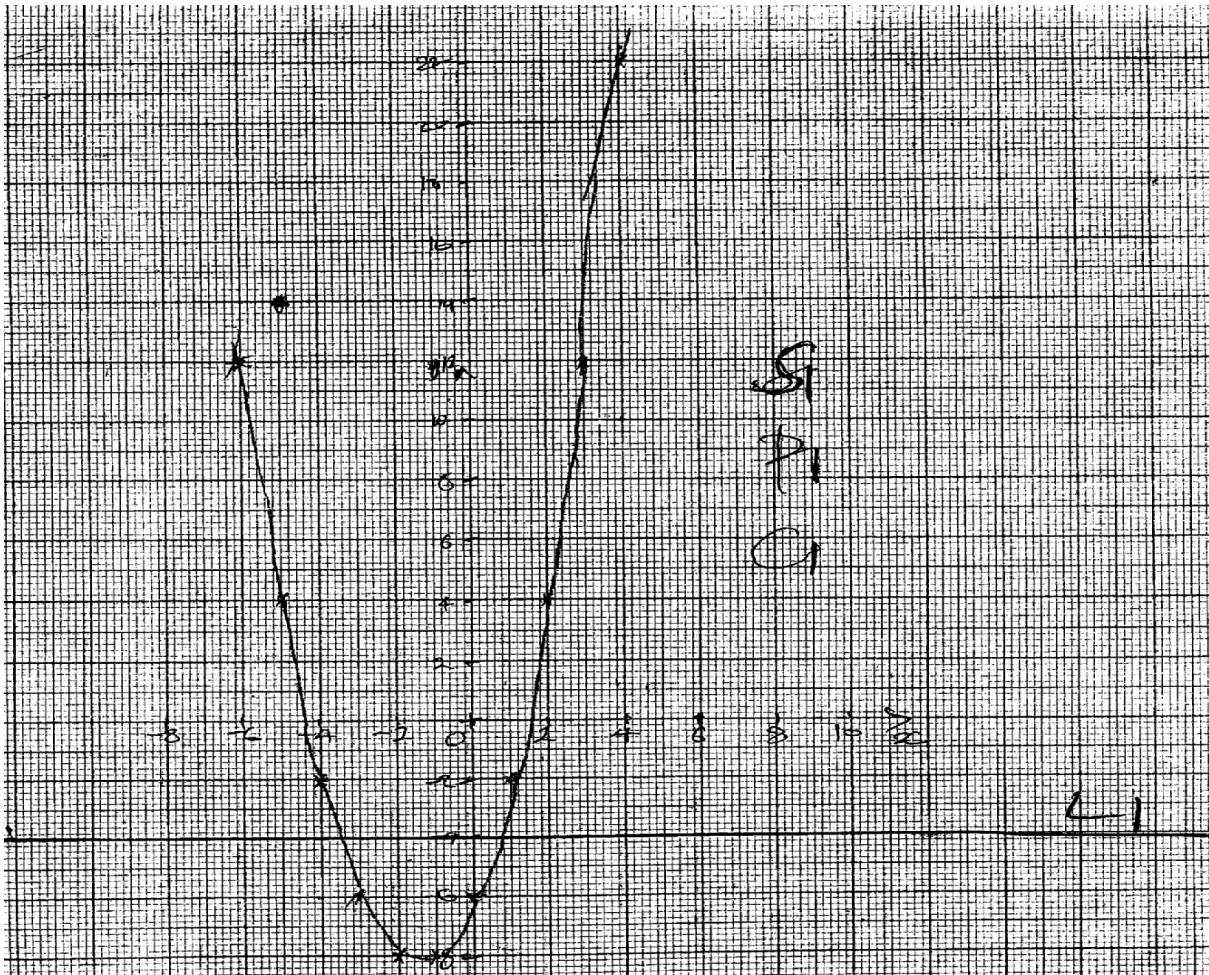
(c) (i)  $x^2 + 3x - 6 = 0$   
 $x = -4.5$  or  $1.5 \pm 0.2$   
(ii)  $y = x^2 + 3x - 6$   
 $x^2 + 3x - 2$   
 $y = -4$   
 $x = 5$  or  $4 \pm 0.2$

5.

x	-4	-3	-2	-1	0	1	2	3
y	21	10	3	0	1	6	15	28

(c)  $2x^2 + 3x + 1 = 0$   
 $2x + 4x - 3 = 0$   
 $-x + 2 = y$   
 $x = 0.6$  or  $x = -2.6 \pm 0.1$

(d)  $x = 0.30$   $-x = -1.8 \pm 0.1$



6. a) i)  $480,000 \neq$   
ii)  $\left( \frac{x}{480,000} \right) \neq$   
 $\left( \frac{480,000}{x-4} \right) \neq$

$$b) \quad \frac{480,000}{x-4} = \frac{480,000}{x} + 20,000$$

Multiply all hr' by L.C.M.

$$480,000x = 480,000(x-4) + 20,000(x^2 - 4x)$$

Dividing by 10,000

$$48x = 48x - 192 + 2x^2 - 4x$$

$$48x - 48x + 4x - 2x^2 + 192 = 0$$

$$4x - 2x^2 + 192 = 0$$

$$x = \frac{-b \pm \sqrt{(b^2 - 4ac)}}{2a}$$

$$= \frac{-4 \pm \sqrt{1552}}{-4}$$

$$= \frac{-4 \pm 39.3954}{-4}$$

$$x = \frac{-4 + 39.3954}{-4} \quad \text{or} \quad x = \frac{-4 - 39.3954}{-4}$$

But  $x$  cannot be -ve hence

$$x = \frac{-4 - 39.3954}{-4} = 10.8489$$

$$= 11$$

c) Original : new cont.

$$\frac{480,000}{11} : \frac{480,000}{7}$$

d) Size of land bought = 6 hectares

$$\frac{6}{7} = 0.857143$$

$$\approx 0.8571 \text{ hectares}$$

7.

$x$	-3	-2	-1	0	1	2
$y$	13	4	-1	-2	1	8

19.

$$(iii) \quad y = 2x^2 + x - 2$$

$$0 = 2x^2 + 2x - 3$$

$$y = -x + 1$$

$x$	-3	-2	-1	0	1	2
$y$	5	3	2	1	0	-1

$$y = 2x^2 + x - 2$$

$$0 = 2x^2 + x - 5$$

$$y = 3$$

8. (a) Dist. traveled in 3hrs s. drawing

$$\text{Plane A} - 400 \times 3 = 1200\text{km} - \text{cm}$$

$$\text{Plane B} - 500 \times 3 = 7.5\text{cm}$$

$$\text{Plane C} - 300 \times 3 = 900\text{km} - 4.5\text{cm}$$

$$(b) \text{Dist. BA} = 12.8 \times 0.1 \times 200 = 2560\text{km} \quad 20\text{km}$$

$$T = \frac{D}{S} = \frac{2560 \text{ hrs}}{500}$$

$$= 5.12 \text{ hrs of } 5 \text{ hrs, } 7.2 \text{ mns}$$

$$\approx 5 \text{ hrs, } 7 \text{ min (nearest min)}$$

(c) Bearing of B from C =  $360^\circ - 20^\circ = 340^\circ$   
 Dist. BC =  $(10.9 \pm 0.1 \times 200) \text{ km}$   
 $= 2180 \text{ km} \pm 20 \text{ km}$

9. a)

x	-2	-1.5	-1	-0.5	0	0.5	1
x <sup>2</sup>	4	2.25	1	0.25	0	0.25	1
4x	-8	-6	4	-2	0	2	4

$$4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4$$

$$y \quad 0 \quad 0.25 \quad 9 \quad 2.25 \quad 4 \quad 6.25 \quad 9$$

$$A = \frac{1}{2} h \left\{ (y_1 + y_7) + 2(y_2 + \dots + y_6) \right\}$$

$$= \frac{1}{2} \times \frac{1}{2} \left\{ (0 + 9) + 2(0.25 + 9 + 2.25 + 4 + 0.25) \right\} \checkmark$$

$$= \frac{1}{4} \left\{ 9 + 4.25 \right\} \checkmark$$

$$= \underline{13.25 \text{ sq. units}} \checkmark$$

b)  $\int_{-2}^0 (x^2 + 4x + 4) dx + \int_0^1 (x^2 + 4x + u) dx$

$$\left[ \frac{x^3}{3} + 2x^2 + 4x \right]_{-2}^0 + \left[ \frac{x^3}{3} + 2x^2 + ux \right]_0^1 \checkmark$$

$$= \left( -\frac{8}{3} + 8 - 8 \right) + \left( \frac{1}{3} + 2 + 4 \right) \checkmark$$

$$= \underline{9} \checkmark$$

$$\text{Error} = 13.25 - 9 = 4.125$$

$$\% = \frac{4.125}{9} \checkmark \times 100$$

$$= \underline{45.84\%}$$

10. a)

x	-2	-1.5	-1	-0.5	0	0.5	1
x <sup>2</sup>	4	2.25	1	0.25	0	0.25	1
4x	-8	-6	4	-2	0	2	4

$$4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4 \quad 4$$

$$y \quad 0 \quad 0.25 \quad 9 \quad 2.25 \quad 4 \quad 6.25 \quad 9$$

$$A = \frac{1}{2} h \left\{ (y_1 + y_7) + 2(y_2 + \dots + y_6) \right\}$$

$$= \frac{1}{2} \times \frac{1}{2} \left\{ (0 + 9) + 2(0.25 + 9 + 2.25 + 4 + 0.25) \right\} \checkmark$$

$$\left\{ \quad \quad \quad \right\}$$

$$= \frac{1}{4} \sqrt{9 + 4.25}$$

$$= \underline{13.25 \text{ sq. units}} \checkmark$$

$$b) \int_{-2}^0 (x^2 + 4x + 4) dx + \int_0^1 (x^2 + 4x + u) dx$$

$$\left[ \frac{x^3}{3} + 2x^2 + 4x \right]_{-2}^0 + \left[ \frac{x^3}{3} + 2x^2 + ux \right]_0^1 \checkmark$$

$$= (-\frac{8}{3} + 8 - 8) + (\frac{1}{3} + 2 + 4) \checkmark$$

$$= \underline{9} \checkmark$$

$$\text{Error} = 13.25 - 9 = 4.125$$

$$\% = \frac{4.125}{9} \checkmark \times 100$$

$$= \underline{45.84\%}$$

$$11. \quad y = 2x^2 - 4x - 5 \qquad y = 2x + 3$$

X	-3	-2	0	1	2	3	4	5			x	-4	-2	0	2
2x <sup>2</sup>	18	2	0	2	8	18	32	50			y	-5	-1	3	7
4x	-12	-8	-4	0	4	8	12	16	20						
5	5	5	5	5	5	5	5	5	5						
y	25	11	1	-5	-7	1	11	25	11	B <sub>2</sub>					

$$(a) x = 1$$

$$(b) -0.9 \times 2.8$$

$$x = -1 \text{ and } x = 4$$

12.

X	-1.5	-1	0	1.5	2	2.5	3.5
Y	-4	0	5	5	3	0	-9

$$(0.75, 6.125)$$

$$Y = -2$$

$$\text{Range of values } -1.3, < x < 2.75$$

$$\text{Integral values; } -1, 0, 1, 2$$

13. a)

x	-4	-3	-2	-1	0	1	2
2x <sup>2</sup>	32	18	8	2	0	2	8
4x - 3	-19	-15	-11	-7	-3	1	5
y	13	3	-3	-5	-3	3	13

$$(b) \text{Roots for } x = -2.6 \pm 0.1$$

$$x = 0.6 \pm 0.1$$

$$y = 2x^2 + 4x - 3$$

$$0 = 2x^2 + x - 5$$

$$y = 3x + 2$$



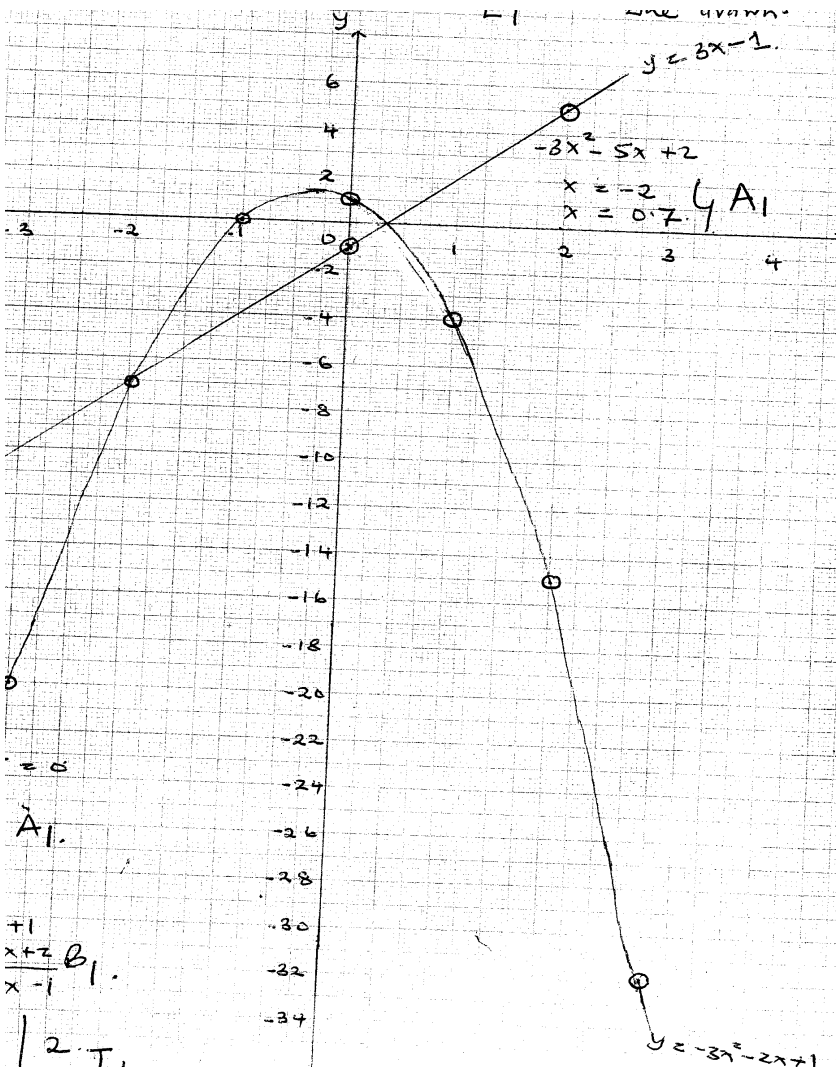
Roots read from the 2 pts of intersection of the line and curve.

$$X = -1.9 \pm 0.1$$

$$X = 1.4 \pm 0.1$$

14.

$x$	-3	-2	-1	0	1	2	3
$-3x^2$	-27	-12	-3	0	-3*	-12	-27*
$-2x$	6	4	2	0	-2	-4	-6
$1$	1	1	1	1	1	1	1
$y$	-20	-7	0	1	-4	-15	-32



$$\left. \begin{array}{l} 1 - 2x - 3x^2 = 0 \\ x = -1 \\ \text{or } x = 0.7 \end{array} \right\} A_1$$

$$\left. \begin{array}{l} y = -3x^2 - 2x + 1 \\ 0 = -3x^2 - 5x + 2 \\ y = 0 + 3x - 1 \end{array} \right\} B_1$$

$x$	$0$	$2$
$y$	$-1$	$5$

 $T_1$

15.  $x^2 + ax - b = 0$   
 $(x-1)(x+5) = x^2 + ax - b$   
 $x^2 + 4x - 5 = x^2 + ax - b$   
 $a = 4, b = 5$

16. Let  $a = 1.5 + \sqrt{2}$   
 $b = 1.5 - \sqrt{2}$   
 $\therefore (x-a)(x-b) = 0$   
 $x^2 - xb - ax + ab = 0$   
 $x^2 - x(1.5 - \sqrt{2}) - x(1.5 + \sqrt{2}) + ab = 0$   
 $x^2 - 1.5x + x\sqrt{2} - x1.5x - \sqrt{2} = 0$   
 $x^2 - 3x + ab$   
 $x^2 - 3x + (1.5 + \sqrt{2})(1.5 - \sqrt{2}) = 0$   
 $x^2 - 3x + 2.25 - 2 = 0$   
 $x^2 - 3x + 1/4 = 0$   
 $4x^2 - 12x + 1 = 0$

17. a) i)  $a^2 + b^2 = 89$       $a + b = 13$   
 $a^2 + 2ab + b^2 = (a + b)^2 = 13^2 = 169$

ii)  $2ab = 169 - 89$   
 $= 80$

iii)  $a^2 - 2ab + b^2 = a^2 + b^2 - 2ab$   
 $= 89 - 80 = 9$

iv)  $(a - b)^2 = 9$   
 $a - b = \pm 3$

b)  $a + b = 13$   
 $\frac{a - b = 3}{2a = 16}$