COURSE TITLE: MATHEMATICS FOR INTEGRATED SCIENCE

COURSE CODE:
CREDIT UNITS: 2

COURSE MATERIALS DEVELOPED

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PREAMBLE

The course, mathematics for integrated Science is meant for 200 level B.Ed and B.Sc. (Ed) integrated Science Students who are purposely science teachers-in-training. As teachers of science, the course is aimed at acquainting them with some basic knowledge of higher mathematics that will help them understand and teach very well their teaching subjects.

MODULE 1: ALGEBRA

INTRODUCTION

In this module, you will be exposed to algebraic expressions and equations. Solutions of some equations will also be considered. Variations, Indices, and logarithms will be discussed.

The module is divided into three units, namely:

Unit 1 Algebraic expressions and equations

Unit 2 Indices and logarithms

Unit 3 Variation

Unit 1 Algebraic Expressions and Equations

CONTENT

1.0 Introduction
2.0 Objectives
3.0 Meaning and simplification of algebraic expressions and meaning and solution of algebraic equations
4.0 Conclusion
5.0 Summary
6.0 Tutored Marked Assignment
7.0 Reference/further Readings

1.0 INTRODUCTION

Algebraic expressions, equations and inequalities are very important to mathematics, science and technology. Scientific discoveries and relationship between quantities or variables are mostly expressed in the forms of algebraic equations or inequalities. It is therefore important to understand the meaning of some methods of simplifying and solving of algebraic expressions, equations and inequalities.

2.0 OBJECTIVES

After studying this unit, students should be able to

i. State the meanings of algebraic expressions, equations and inequality and distinguish the three
ii. Express word problems in the forms of algebraic expressions, equations and inequalities using alphabets.
iii. Understand law of algebra
iv. Simplify algebraic expressions
v. Solve some algebraic equations and inequalities using various methods

3.0 MEANING OF ALGEBRAIC EXPRESSION

In everyday life, we deal with statements involving quantities such as 1 book 5 pencils, 2 erasers, 10 sharpeners, 3 dusters and so on. Taking the firs letters of these items to represent them, they would be written as 1b, 5p, 2e, 10s, 3d arithmetical operations can be carried out with the above quantities to make sense. For example,

1. a student having 3 books, 5 pencils and 2 erasers can be algebraically represented as 3b + 5p + 2e
2. i am travelling with a brief case containing 3 shirts, 2 trousers, 4 handkerchiefs, 3 caps, 5 belts. Can be said that the briefcase contains 3s + 2t + 4L + 3c + 5b

Representing a statement using alphabetical symbols and arithmetic operations is said to be an algebraic expression. An algebraic expression may contain one or more quantities. Every algebraic expression is made up of a term and a coefficient. A term represents a quantity or a number in general, in the expression 5a; a is the term and 5 is the coefficient of a.

In the expression 3y – 2k + 7r; y, k and r are the terms and 3, 2 and 7 are the coefficients of y, k and r respectively.

SELF ASSESSMENT

1. define an algebraic expression
2. write 5 examples of algebraic expressions.

SIMPLIFICATION OF ALGEBRAIC EXPRESSIONS

Laws of Algebra

Algebraic expressions involve one or more arithmetic expressions of addition, subtraction, multiplication and division. These equations are carried out according to the following laws of algebra:

1- Commutative law of addition and multiplication. Two numbers a and b can be added or multiplied in any order without affecting the result.

i) a + b = b + a and ii) ab = ba

example

3 + 5 = 8 and 5 + 3 = 8 thus, 3 + 5 = 5 + 3 = 8

3 X 5 = 15 and 5 X 3 = 15 thus, 3 x 5 = 5 X 3 =15

It should be noted, however, that

I) a – b $\neq b - a$

But a – b = a +(-b) + (-b) +a = -b+a

ii) $a \div b \neq a \cdot \frac{1}{b} = \frac{1}{b} \cdot a$

example, 5 – 3 = 2 3 – 5 = -2, thus, $5 - 3 \neq 3 - 5$,

$$also \quad 6 \div 2 = 3 \; and \; 2 \div 6 = \frac{1}{3}, \; thus \; 6 \div 2 \neq 2 \div 6$$

2. Associative Law for addition and Multiplication: The way numbers are associated under the operation of addition or multiplication does not affect the result.

a + (b + c) = (a + b) +c = a + b +c

and a(bc) = (ab)c = abc

Example

2 + (7 +4) = 13
(2 + 7) = 4 =13
2 + 7 + 4 =13
Thus,
2 + (7 + 4) = (2+7)+4 = 2 + 7 + 4 = 13

2 ·$(7\cdot 4) = 56$
$(2\cdot 7)\cdot 4 = 56$
2· $7\cdot 4 = 56$
Thus,
2· $(7\cdot 4) = (2\cdot 7)\cdot 4 = 2\cdot 7\cdot 4 = 56$

3. Distributive law for multiplication: The operation of multiplication is distributive over addition without affecting the result.

   a (b+c) = ab + ac
   (a + b) c = ac + bc

Example
$5\cdot(4+7) = 5\cdot 4 + 5\cdot 7 = 20 + 35 = 55$
And 5· $(4+7) = 5\cdot 11 = 55$
Also,
(5+4) $\cdot 7 = 5\cdot 7 + 4\cdot 7 = 35 + 28 = 63$
And (5+4) $\cdot 7 = 9\cdot 7 = 63$

4. Distributive law for Division: It should be noted that the operation of division distributes over addition under the condition that if a,b and c are numbers, with c$\neq 0$,

   Then (a + b) ÷c = a $\div c + b \div c$
   This is because
   (a+b) $\div c = (a+b)\cdot\frac{1}{c} = a\cdot\frac{1}{c} + b\cdot\frac{1}{c} = \frac{a}{c} + \frac{b}{c}$

   Example,
   (9+ 6) $\div 3 = 9 \div 3 + 6 \div 3 = 3 + 2 = 5$
   And (9+6) $\div 3 = 15 \div 3 = 5$

   SELF ASSESSMENT
   1. Is the operation of subtraction commutative?
   2. Is the operation of division commutative?
   3. Test the distributive laws of multiplication and division for the following:
      i) 5 x (6 + 8)   ii) 8 X (100 + 20 + 4)   iii) (6 + 8) $\div 2$ iv) (4000 + 80 + 12) $\div 4$

in simplifying algebraic expressions, the laws of algebra are made use of, at the same time considering and bringing together like term, positive terms, negative terms and unlike terms are left as they are.

Examples;

1. $2x + 7x = 9x$
2. $31p - 9q = 31p - 9q$
3. $7l - 4 - 3l + 11 = 7l - 3l + 11 - 4 = 4l + 7$
4. $a - 2b - 4a + 3c + 4a + 5a = a + 4a - 4a + 5a - 2b + 3c = 5a - 4a + 5b - 2b + 3c = a + 3b + 3c$
5. $9ab + 4bc - 5ba + 3bc = 9ab + 4bc - 5ab + 3bc = 9ab - 5ab + 4bc + 3bc = 4ab + 7bc$

Concepts and Solutions of Equations

An algebraic equation is a statement equating two or more quantities.
Example;

1. My age is twice my son's age. If my age is x and my son's is y then it follows that $x = 2y \text{ is an equation}$
2. The sum of two numbers added to 10 is the same as their product. If the two numbers are a and b, then $a + b + 10 \text{ is an equation}$

The largest part of mathematics is about forming equations and solving them. There are many types of equations, among which are;
i) linear Equation ii) quadratic Equations iii) Cubic Equations etc

Equations in which the highest power of unknown is 1 are termed linear. Those in which the highest power is 2 are quadratic. Those in which the highest power is 3 are cubic and so on. Those in which the highest power is many are termed polynomials.

Solutions of equations are generally obtained by collecting like terms on the same side and simplifying or by making one unknown value the subject of the equation. But different types of equations have particular methods of solving them. Generally, in solving equation, whatever is done to one side of an equation must be done to the other side as well.

Solution of Linear Equations

1. If ten is added to a number it gives 12 what is the number?
   Solution
   Let the number be $x \text{ then } x + 10 = 12$
   To make the subject, we collect like terms on the same side by removing 10 from the side of X. This is done by subtracting 10 from both sides.
   $x + 10 - 10 = 12 - 10, \quad x = 2$
   The number is therefore 2

2. A number multiplied by 5 gives 15 what is the number?
   Solution:

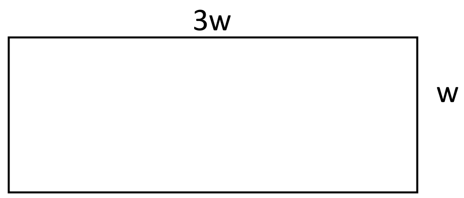
   Let the number be y
   Then $5y = 15$
   $\therefore \frac{5y}{5} = \frac{15}{5}$
   $y = 3$
   The number is therefore 3.

3. Solve the following equations;
   a) $3x + 11 - 8x = 36 - 9x - 13$
   b) $^{1}/_{3}\, a = 7$
   c) $^{2}/_{5}\, y = 14$

4. A rectangle is 3 times as long as it is wide, and its perimeter is 56cm. Find its length and width.

   Solution:

   Let the width of the rectangle be w. Then the length =3w



perimeter = w + w + 3w + 3w = 8w = 56

8w = 56

$$\frac{8w}{8} = \frac{56}{8}$$

W = 7

Thus, the width is 7cm. And length = 3w = 3 X 7 = 21

Solution of two linear equations simultaneously

Two linear equations in two unknowns can be solved at the same time using either one of i) substitution or ii) direct elimination methods.

Substitution Method

In this method, one of the unknowns is made the subject of one of the equations and it is substituted in the other to find its value. This is then substituted in any of the equations to find the value of the other unknown.

Examples;

1. Solve the following pairs of equations
   a) $5x + y = 0 \;\; and \; 3x - 2y = 13$
   b) $4a = 5b + 5 \; and \; 2a - 3b = 2$

Solution;

$$let\ 5x + y = 0\ be\ equation\ (i)$$
$$and\ 3x - 2y = 13\ be\ equation\ (ii)$$
$$from\ i)\ y = \ -5x - - - (iii)$$

Substitute for y in (ii)

$$3x - 2y = 13$$
$$3x - 2(-5x) = 13$$
$$3x + 10x = 13$$
$$13x = 13$$
$$x = 1$$

Substitute the value of x in (iii) to get the value of y, or in any of equations (i) and (ii).

$$y = -5x$$
$$\therefore y = -5\,X1 = -5$$
$$x = 1; y = -5$$

b. $4a = 5b + 5 - - - (i)$
$2a - 3b = 2 - - - (ii)$

From ii)

$2a - 3b = 2$
$2a = 2 + 3b$
$a = \frac{1}{2}(2 + 3b) - - - (iii)$

Substitute for a in (i)

$4a = 5b + 5$
$4(\frac{1}{2}(2 + 3b) = 5b + 5$
$2(2 + 3b) = 5b + 5$
$4 + 6b = 5b + 5$
$4 - 5b + 6b = 5b - 5b + 5$
$4 + b = 5$
$4 - 4 + b = 5 - 4$
$b = 1$

Substituting the value of b in (iii)

$a = \frac{1}{2}(2 + 3b)$
$= \frac{1}{2}(2 + 3X1) = \frac{1}{2}X5 = 2\frac{1}{2}$
a$\frac{1}{2}; b = 1$

Direct Elimination Method

In this method, one of the two unknowns or variables is eliminated directly from the two equations to leave one whose value is then determined. This value is then used in any of the 2 equations to find the value of the other variable.

Examples:

1 solve the following pairs of equations

a) $5x + 2y = 2 \text{ and } 2x + 3y = -8$
b) $6l = 2k + 9 \text{ and } 3l + 4k = 12$

Solution;

a) $5x + 2y = 2 ----(i)$
$2x + 3y = -8 --- (ii)$

You can decide to eliminate x or y from the 2 equations. This done by balancing the coefficient of x or y and then adding or subtracting the results.

$2X5x + 2 \times 2y = 2 \times 2$
$5 \times 2x + 5 \times 3y = -8 \times 5$

Thus, $10x + 4y = 4$
$\underline{10x + 15y = -40}$
$0 \pm 11y = 4 — 40$
$-11 = 44$
$y = -4$

Thus, $5x + 2y = 2$
$5x + 2(-4) = 2$
$5x = 10$
$x = 2$
Thus $x = 2 \; and \; y = \; -4.$

b) 6h = 2k + 9 --------(i)
3h + 4k = 12 ------(ii)
To eliminate h, multiply (i) by 3 and (ii) by 6

Thus,

18h = 6k + 27
18h – 6h = 27
<u>18h + 24k = 72</u>
0 – 30k = -45
= k + 1 ½ 0r 3/2
Substituting for k in any of (i) or (ii)
6h = 2k + 9
6h = 2* 3/2 + 9
6h = 12
H = 2
Thus, solution is k = 2/3; h = 2.

Work problems:

i) In ten years' time father will be twice as old as his son; ten years ago he was six time as old. How?

Solution:
Let the father's and son's present ages be x and y respectively.
In ten years' time they will be x + 10 and y + 10 respectively.

Ten years ago they were

X – 10 and y – 10 respectively.

Now, in ten years' time the father's age will be twice the son's.

X + 10 = 2 (years)

X – 2y = 10 ------(i)

Also, ten years ago the father's age was 6 times as the son's.

X – 10 = 6(y – 10)

6y – x = 50 ------(ii)

From (i) x – 2y = 10

X = 10 + 2y ------(iii)

Substituting for x in (ii)

6y – x = 50

6y – 10 – 2y = 50

4y = 60

Y = 15

Substituting for y in (iii)

X = 10 + 2y

= 10 + 30 = 40

X = 40; y = 15

So the father's and son's present ages are 40 years and 15 years respectively.

ii) A bird laid a certain number of eggs in a rest, and a collector took a way two-thirds of them. The bird laid some more, and another collector took two-thirds of them. If the bird laid 12 eggs altogether and collectors took 10 altogether, how many eggs were laid to begin with?

Solution:

Let the number of eggs initially laid be x then the first collector took away $^2/_3\,x$.
There will be $^1/_3\,x$ eggs left.

Let y be the number of eggs laid again by the bird. There will now be $^1/_3\,x$ + y eggs in the rest second collector took away $^2/_3$ ($^1/_3\,x$+ y) eggs. Now, the bird laid altogether 12 eggs,

x + y = 12 -------(i)

the two collectors altogether collected 10 eggs,

$$^2/_3\,x + \, ^2/_3\left(\tfrac{1}{3}x + y\right) = 10$$
$$^2/_3\left(x + \tfrac{1}{3}x + y\right) = 10$$
$$^4/_3\,x + y = 15$$

4x + 3y = 45 ------(ii)

From (i), x + y = 12
X = 12 – y --------(iii)

Substituting for x in (ii)

4x + 3y = 45
4(12 – y) + 3y = 45
48 – 4y + 3y = 45
Y = 3

Substituting the value of y in (iii)

X = 12 – y
X = 12 – 3
X = 9

Thus, x = 9; and y = 3.
The bird first laid 9 eggs.

SELF ASSESSMENT:
1. Differentiate between an expression and an equation
2. Explain the meaning of a linear equation
3. Mention different methods of solving two linear equations in one variable simultaneously.
4. The sum of two numbers in19 and their difference is 5. Find the numbers
5. Six pencils and 3 erasers cost thirty naira. 2 pencils and 2 erasers cost 24 naira. How much does each cost?

Solution of Quadratic Equations

A quadratic equation is one in which the highest power of the variables is 2.

Examples:

1) $x^2 + x + xy = 1$
2) $2x^2 - 1 = 0$
3) $y^2 + 6y = 0$
4) $4x^2 - 10x + 2 = 0$

There are many types of quadratic equations. Some have only one variable and some have more than one variable. We shall consider, in this lesson only those quadratic equations whose general form can be expressed as $ax^2 + bx + c = 0$, where a, b and c are none zero integers.

The solution of quadratic equations of the form $ax^2 + bx + c = 0$ is called its roots. The roots of quadratic equation are usually 2 numbers that are either different or the same (and they are called repeated roots) or both imaginary. If a quadratic equation has repeated roots then it is said to be a perfect square. Any quadratic equation that is a perfect square can be factorized into the form $(x \neq m)^2 = 0$; where m is the root of the equation.

Examples of quadratic equations that are perfect squares:

i) $x^2 = 4x + 4 = 0; this\ can\ be\ factorized\ to\ (x-2)^2 = 0$

ii) $b^2 + 10b + 25 = 0; this\ can\ be\ factorised\ to\ (b+5)^2 = 0$

iii) $m^2 + 2m + 1 = 0; this\ can\ be\ factorised\ to\ (m+1)^2 = 0$

iv) $n^2 = n + \frac{1}{4} = 0; this\ can\ be\ factorised\ to\ (n - {}^{1}/_{2})^2 = 0$

Examples

Find what must be added to the expressions to make them perfect squares:

$i)\ s^2 + 6s \quad ii)\ p^2 - 8p \quad iii)\ g^2 + g$

i) $s^2\ can\ be\ written\ as\ s^2 + 6s + c$ where c is the term to make the expression a perfect square. Now if $s^2 + 6s + c\ is\ a\ perfect\ square\ it\ means\ it\ can\ be$ equated to $(s + a)^2; where\ a\ is\ some\ constant.$

$also, (s+a)^2 = s^2 + 2as + a^2\ and\ this\ is\ equal\ to\ s^2 + 6s + c$

By comparing the coefficient of s,

2a = 6

a = 3

$a^2 = 9$

but $a^2 = c$

Thus 9 must be added to the expression $s^2 + 6s$ to make it a perfect square.

ii) $p^2 - 8p\ can\ be\ written\ as\ p^2 - 8p + y. thus, p^2 - 8p + y = (p+a)^2 = p^2 + 2ap + a^2$

$compare\ coefficient\ of\ p\ and\ constant\ terms,$

$2ap = -8 \Rightarrow a = -4$

$y = a^2 = (-4)^2 = 16$

Thus, 16 must be added to make $p^2 - 8p$ a perfect square

iii) $g^2 + g\ can\ be\ written\ as\ g^2 + g + w$

$g^2 + g + w = (g+)^2 = g^2 + 2ag + a^2$

Comparing the coefficients of g and constant terms,

$2a = 1$

$\Rightarrow a = \frac{1}{2}$

And $w = a^2 = \frac{1}{4}$

Thus, $\frac{1}{4}$ must be added to make $g^2 + g$ a perfect square.

SELF ASSESSMENT

i) Differentiate between a linear equation and a quadratic equation.

ii) When is a quadratic equation said to be a perfect square?

iii) Find the terms that must be added to the expressions to make them perfect squares: $c^2 - 2c$ ii) $m^2 - m$ iii) $t^2 + 16t$

The roots of a quadratic equation of the form $ax^2 + bx + c = 0$ are formed through various methods, by factorization, by completing the square and by directly using the quadratic equation formula. These methods are based on the fact that the quadratic equation can be factorised into $(x+\alpha)(x+\beta) = 0\ where\ \alpha\ and\ \beta$ are the roots of the equation. This situation can be likened to a.b=0 and it follows that either a = 0 or b = 0 or both are 0.

Thus, to find the roots of a quadratic equation that is easily factorisable or through completing the square, this principle is made use of. And in the case of those equations that are not easily factorisable, the quadratic equation formula is used.

7.0 REFERENCE/FURTHER READINGS

Channon, B. et al (2008). New General Mathematics for West Africa, 1 – 4. Longman Educational Nigeria

Talbert, J.E.; Godman, A. & Ogun, G.E.O. (2000). Additional Mathematics for West Africa. Longman Group Ltd. Ibadan

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MODULE 2: DIFFERENTIAL CALCULUS AND INTEGRAL CALCULUS

Introduction

In this module you will be exposed to differential coefficients of functions, constants and applications of differential calculus. Also, you will be exposed to integration of functions by some methods and some applications of the integral calculus.

The module is divided into three units:

Unit 1: Differential calculus
Unit 2: Integral calculus
Unit 3: Some applications of differential and integral calculus

UNIT 1: DIFFERENTIAL CALCULUS

1.0 Introduction

In life there are many quantities that are related. This relationship is mathematically expresses in the forms of functions. The rate of change of one quantity with respect to another is measured by using differential calculus. Differential calculus is used in determining rates of change, in determining minimum and maximum values in arithmetic and in determining areas of plane shapes.

2.0 Objectives

After studying this unit, students should;

i. Understand the concept of differential coefficients;
ii. Be able to determine the differential coefficients of some functions and constants

3.0 Main Content

3.1 Differential Coefficients of Some Functions and Constants

Suppose y is a function of x or y is a function in terms of x, the $\frac{dy}{dx}(dy, dx)$ is called the differential coefficients of y with respect to x, and measures the rate of change of y compared with x. Differentiation is the process of finding the differential coefficient of a function and the resulting function is called the desired function.

Differentiation is done, among others, by the use of a general rule that says;

$if y = x^n, then\ \frac{dy}{dx} = nx^{n-1---(i)}$

And $if\ y = ax^n, then\ \frac{dy}{dx} = anx^{n-1} --- (ii)$

Examples:

Find the differential coefficients of the following functions with respect to x:

$i) - y = x^2$
$\frac{dy}{dx} = 2x^{2-1}$
$= 2x$

$ii) - y = 7x^2$
$\frac{dy}{dx} = 7 \times 2x^{2-1}$
$= 14x$

$iii) - y = 2x^3 - 3x^2$
$\frac{dy}{dx} = 6x^2 - 6x$
$= 6x(x-1)$

$iv)\ y = ax$
$\frac{dy}{dx} = a^{x1-1}$
$= ax^{\circ}$
$= a$

$v)\ y = cx^3$
$\frac{dy}{dx} = 3 \times cx^{3-1}$
$= 3cx^2$

It should be noted that the differential coefficient of a constant is 0.

Suppose $y = 2$

$\Rightarrow y = 2x^o$

Thus, $\frac{dy}{dx} = 0 \times 2x^{0-1} = 0 \times 2x^{-1} = 0$

SELF ASSESSMENT

1. Find the differential coefficients, with respect to x of the following functions:
   $i)\ \frac{1}{4^x} \quad ii)\ x + \frac{1}{x}$
2. Differentiate between differentiation and differential coefficient

4.0 Conclusion

With the forgoing lesson, it follows that differential coefficient is a measure of rate of change of one quantity compare with another. It also follow that when a variable is differentiated, the power of derive function is 1 degree less. The differential coefficient of a constant value is o.

5.0 Summary

1. The differential coefficient of y with respect to, $x\frac{dy}{dx}$ is a measure of the rate of change of y compare with x.
2. Differentiation is the process of finding the differential coefficient of a function
3. The general rule for differentiation is, $if\ y = x^n\ then, \frac{dy}{dx} = nx^{n-1}$
   $if\ y = ax^n, \frac{dy}{dx} = anx^{n-1}\ and\ if\ y = a\ then\ \frac{dy}{dx} = 0$

6.0 Tutor Marked Assignment

1. Find the rate of change of s compared with t $if\ i)\ s = t^2 - 3t\ \ ii)\ s = {}^5/_{t^2} + {}^3/_5 t^2 + 3$
2. Show that the differential coefficient of a constant is 0.
3. State the general rule of differentiation

7.0 Reference/further Readings

Channon, B. et al (2008). New General Mathematics for West Africa, 1 – 4. Longman Educational Nigeria

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UNIT 2: INTEGRAL CALCULUS

1.0 Introduction

Integral calculus or simply integration is equally important as differential calculus. It is also used in calculations involving rates of change, areas and volumes. It is therefore important for student to understand integral calculus as it is, in a way, linked to differential calculus.

2.0 Objectives

After studying this unit students should;

i) Understand the general rule of integration
ii) Carryout integration of ordinary function

3.0 Main Content
3.1 Integrals of Functions
Recall that the general rule for differentiation is that
$if\ y = x^n, then\ the\ derivative\ function\ is\ nx^{n-1}. thus, the\ derivative\ function\ of\ \frac{1}{n+1}x^{n+1}\ is\ x^n$
This follows that$if \frac{dy}{dx} = x^n, then\ y = \frac{x^{n+1}}{n+1}$. Therefore, the process of getting a function from a derivative function is actually the reverse of differentiation and is called integration. The symbol of integration is $\int$
The general rule for integration of functions is;

$if\ \frac{dy}{dx} = x^n, then\ y = \frac{x^{n+1}}{n+1} + c$ where c is an arbitrary constant.

Or it can be stated that;

$\int x^n dx = \frac{x^{n+1}}{n+1} + c$ this is the indefinite integral of x^n with respect (w.r.t) x. This rule applies to all functions of this form except when n = -1.

i.e $\int x^{-1} dx = \frac{x^{-1+1}}{-1+1} = \frac{x^o}{0} = \frac{1}{0} = \infty$ or senseless or meaningless. At all times, the function to be Integrated is called the integrand. For example; $\int x^n dx,\ the\ integrand\ is\ x^n$

Generally, $if \frac{dy}{dx} = f, then\ y = \int f(x)dx + c\ i.e\ if\ \frac{dy}{dx} = ax^n, then\ y = \int ax^n dx + c$
$= \frac{ax^{n+1}}{n+1} +$

Examples:
1. Find the functions of which the following are the derived functions:
i) x^2 $ii)\ 5x^4$ $iii)\ x^3/_4$ $iv)\ -2/_{x^3}$ $v)\ x^3 + x^2 + x + 1$ $vi)\ \frac{5}{x^2} - \frac{6}{x^3} + \frac{6}{x^4}$

Solution;
Using the general rule of integration;

i) $\int x^2 dx = \frac{x^{2+1}}{2+1} + c = \frac{x^3}{3} + c$

ii) $\int 5x^4 dx = \frac{5x^{4+1}}{4+1} + c = x^5 + c$

iii) $\int \frac{1}{4}x^3 dx = \frac{x^{3+1}}{4(3+1)} + c = \frac{x^4}{16} + c$

iv) $\int -2x^{-3}\, dx = \frac{-2x^{1-3}}{1-3} + c = x^{-2} + c$

v) $\int (x^3 + x^2 + x + 1)dx = \frac{x^4}{4} + \frac{x^3}{3} + \frac{x^2}{2} + x + c$

vi) $\int (\frac{5}{x^2} - \frac{6}{x^3} + \frac{6}{x^4} = \int (5x^{-2} - 6x^{-3} + 6x^{-4})dx = \frac{5x^{-1}}{-1} - \frac{6x^{-2}}{-2} + \frac{6x^{-3}}{-3} + c = \frac{-5}{x} + \frac{3}{x^2} - \frac{2}{x^3} + c$

SELF ASSESSMENT

1. $if\ \frac{dy}{dx} = ax^n, what\ is\ y?$
2. $integrate\ the\ following\ functions\ with\ respect\ to\ x$:
   $i) - 4x - 3 \quad ii) - x^2 - \frac{1}{x^2} \quad iii)\ \frac{x^3}{2} + \frac{x^2}{3} + \frac{x}{4} + \frac{1}{5}$

4.0 Conclusion

In this unit you learnt that integration is the reverse of differentiation and is equally important.

5.0 Summary

In this unit you learnt that:

i. Integration is the reverse of differentiation and its symbol is $\int$
ii. The general rule of integration is that $if\ \frac{dy}{dx} = f(x), then\ y = \int f(x)dx + c$
iii. $if\ \frac{dy}{dx} = ax^n, then\ y = \int ax^n dx + c = \frac{ax^{n+1}}{n+1} + c\ except\ for\ n = -1$

6.0 Tutor Marked Assignment

1. Explain why $\int x^n\ is\ not\ possible\ for\ n = -1$
2. Define an integrand
3. State the general rule of integration of a derived function

4. Explain the relationship between differentiation and integration.

7.0 Reference/further Readings

Channon, B. et al (2008). New General Mathematics for West Africa, 1 – 4. Longman Educational Nigeria

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UNIT 3: APPLICATIONS DIFFERENTIAL AND INTEGRAL CALCULUS

Content

1.0 Introduction

Application of differential calculus and integral calculus is what makes the topic a very important part of Mathematics, especially in applied Mathematics and engineering. It is therefore, important to not only understand how to differentiate and integrate functions but also to understand their applications.

2.0 Objectives

After studying this unit students should be able to;

i) understand the applications of differential and integral calculus to gradients of curves

ii) understand the application of differential and integral calculus to rate of change, especially as they relate to distance, velocity and acceleration of moving bodies.

3.0 Main Content

3.1 Application of Differential Calculus

It should be understood that, actually, the differential coefficient, $\frac{dy}{dx}$, is a measure of increment in y with an accompanying increment in x. Or it can be said to be the ratio of a change in y and a corresponding change in x. When this idea is compared with curves, it follows that by definition of the gradient of a curve m at a given point is $\frac{change\ in\ y}{change\ in\ x}, thus, m = \frac{change\ in\ y}{change\ in\ x} = \frac{dy}{dx}$

$thus, \frac{dy}{dx} = m$ i.e gradient of a curve is the same as differential coefficient. Again, the general equation of a curve at a point, with gradient m is;

$$y = mx + c, where\ c\ is\ a\ constant$$
$$\therefore \frac{dy}{dx} = mx^{1-1} + cx^{-1} + 0 = m + 0 = m, thus, \frac{dy}{dx} = m$$

i.e. differential coefficient is gradient.

Examples:

1. Calculate the gradient of the curve $y = x^3 - 5x + 3\ at\ the\ point\ where\ x = 3$
2. Find the coordinate of the point on the graph of $y = x^2 + 2x - 10$ at which the gradient is equal to 8.

Solution:

1. $if = x^3 - 5x + 3, then\ the\ gradient, \frac{dy}{dx} = 3x^2 - 5\ and\ at\ x = 3, \frac{dy}{dx} = 3 \times 3^2 - 5 = 27 - 5 = 22$

Thus, gradient of the curve $y = x^3 - 5x + 3\ at\ x = 3\ is\ 22$

2. $y = x^2 + 2x - 10$

$\frac{dy}{dx} = 2x + 2, if\ the\ gradient\ is\ 8, then\ 2x + 2 = 8 \Rightarrow x = 3$

$if\ x = 3, then\ y = x^2 + 2x - 10 = 3^2 + 2 \times 3 - 10 = 9 + 6 - 10 = 5$

Thus, the coordinates of the point on the graph of; $y = x^2 + 2x - 10\ at\ which\ the\ gradient\ is\ 8\ is\ (3,5)$

Similarly, if a particle is moving, its average velocity is define as the ratio of the change in the distance moved and the change in the time taken to make the move in a specific direction.

Thus, if a body moves a distance S in a time t, in a specified direction, then average velocity is;

$= \frac{change\ in\ S}{change\ in\ t} = \frac{ds}{dt}$

Or by the definition of velocity, which is the rate of change of displacement which is the same as $\frac{ds}{dt}$

Also, acceleration is the rate of change of velocity or change in velocity divided by change in time. This is the same as $\frac{dv}{dt}$

Thus, generally,

$v = \frac{ds}{dt}\ \ and\ a = \frac{dv}{dt}$

Where v is the average velocity of time t and the acceleration is a.

Examples:

1. A particle moves along a straight line in such a way that after t seconds it has gone s meters, where s = $3t^2 - 12t$. Find the velocity of the particle after 3 seconds. When is the particle momentarily at rest?

2. The velocity of a moving particle after t seconds is $vms^{-1}, where\ v = 30 - t^2$. find the acceleration of the particle after 1 second.

Solution:

1. $s = 3t^2 - 12t$

$$v = \frac{ds}{dt} = 6t - 12$$

$v = 6t - 12$

After t = 3secs.

$v = 6t - 12 = 18 - 12 = 6ms^{-1}$

$v = 6ms^{-1}\ \ after\ 3secs.$

The particle will be at rest when v=0.

Thus, v =6t -12 =0

$\Rightarrow t = 2; the\ particle\ will\ be\ at\ rest\ after\ 2\ secs.$

2 – v = 30 – t^2

$$a = \frac{dy}{dx} = -2t$$

After 1 $sec, a = -2ms^{-1}$
Thus, if is reducing its acceleration or it is moving to a stop or it is decelerating.

SELF ASSESSMENT
1. Define the following in terms of differential coefficient;
   i) gradient of a curve
   ii) Velocity of a moving particle
   iii) Acceleration of a moving particle

3.2: Application of Integral Calculus

If the gradient of a curve at a point is m, then the equation of the curve is $y = mx + c \text{ where } c \text{ is a constant or } y = \int mdx + c = mx + c \text{ i.e. } y = mx + c$

If v is the velocity with which a particle moves a distance in a time t, then;

$s = \int vdt + c \ \ and\ v = \int adt + c$ where a is the acceleration of the particle

You can always do the reverse of differentiation to get;
i) Equation of a curve if its gradient is provided;
ii) Equation of distance if velocity is provided
iii) Equation of velocity if acceleration is provided

Examples;

1. The gradient of a curve at any point is 5 – 6x. Find the equation of the curve if it passes through the point (1,2).
2. A particle is moving in a straight line in such a way that its velocity after t seconds is $(2t^2 – t)ms^{-1}$. Find the distance gone in the first 3 seconds.
3. A particle moves along a straight line in such a way that its acceleration after t seconds is $(2t + 1)ms^{-2}$. If its velocity after t seconds is vms^{-1}, find v in terms of t, given that v = 1 when t =2.

Solution

1. $\frac{dy}{dx} = 5 - 6x$

$y = \int (5 - 6x)dx + c$

$y = 5x = 3x^2 + c$

If the curve passes through (1,2), then

$2 = 5 - 3 + c$

$\therefore c = 0$

So the equation of the curve is;

$y = 5x - 3x^2$

2 – $v = 2t^2 - t$

$\therefore s = \int (2t^2 - t)dt + c$

$s = {}^2/_3 t^3 - \frac{1}{2}t^2 + c, where\ s = 0, t = 0, \quad so\ 0 = 2/3 \times 0 - \frac{1}{2} \times 0 + c, thus, c = 0$

S= $\frac{2\times 27}{3} - \frac{1}{2} \times g + c$

$= 18 - 4\frac{1}{2} + 0 = 13.5$

$\therefore s = 13.5m$

3− $a = 2t + 1$

$v = \int a dt + c$

$v = t^2 + t + c$

$v = 1, where\ t = 2$

$1 = 2^2 + 2 + c$

C = -5

Thus, $v = t^2 + t - 5$

SELF ASSESSMENT

1. Express the following in terms if integral functions:
   i. equation of a curve given its gradient function;
   ii. distance moved by particle given its velocity function;
   iii. velocity of a moving particle given its acceleration function

4.0 Conclusion

After the lesson it can be concluded that;

i) Integral and differential calculus are a very important component of Mathematics;

ii) Integration is the inverse of differentiation;

iii) Differential and integral calculus have vast and reach applications in gradient of curves and rates of change in distance, velocity and acceleration of moving bodies among others

5.0 Summary

After studying this unit it can be seen that;

i) $\frac{dy}{dx}$ =gradient of a curve;

ii) $\frac{ds}{dt} = velocity\ of\ a\ moving\ body$

iii) $\frac{dv}{dt}$= acceleration of a moving body;

iv) the equation of a curve can be obtain from its gradient by integrating it;

v) the distance moving by a body can be obtain from the velocity with which it is moving by integrating it;

vi) the velocity of a moving body can be obtained from its acceleration by integrating it

6.0 Tutor Marked Assignment

1. The gradient of a curve is given by $3x^2 - 4x + {}^{2}/_{x^2}$. find the equation of the curve if the point (2, -4) lies on it.
2. The velocity of a particle moving in a straight line at time t seconds is given by $v = 2t^2 - 3t$. Find an expression for the distance travelled, if s = 0 when t = 0.
3. A particle is given an initial velocity of 18ms^{-1} and it travels in a straight line so that its retardation after t seconds is equal to 4tms^{-2} until it comes to rest. If the particle then remains stationary, calculate the distance travelled.

7.0 Reference/further Readings

Channon, B. et al (2008). New General Mathematics for West Africa, 1 – 4. Longman Educational Nigeria

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MODULE 3: STATISTICS AND PROBABILITY

INTRODUCTION:

As you go through this module, you will be exposed to some forms of data presentation, measures of central location and simple probability.

The module is divided into three units as follows;

Unit 1 Presentation of data
Unit 2 measures of central location
Unit 3 probability

Unit 1 Presentation of Data

Content
1.0 Introduction
2.0 Objectives
3.0 Data Collection, Collation and Presentation
4.0 Conclusion
5.0 Summary
6.0 Tutor Marked Assignment
7.0 References/Further Readings

1.0 **INTRODUCTION**

Data presentation is the act of transforming collected data in the forms of graphs and charts in order to make more sense. It is therefore important to understand the various ways data can be presented.

2.0 **OBJECTIVES**

After studying this unit, students should be able to;

i) list the various means of data presentation
ii) To prepare and interpret the various means of data presentation

3.0 Data Presentation

Statics is a field that deals with collection, collation, presentation and analysis of data in order to make inference or deduction or conclusion or generalization. Usually, data is collected from the field as a list of values or text that doesn't make much sense.

For example;

If the result of tossing a die a number of times is given as follows:
5, 2, 3, 1, 6, 5, 3, 1, 6, 4,
3, 2, 4, 3, 1, 5, 6, 1, 2, 3,
4, 1, 3, 2, 6, 5, 4, 3, 4, 1,
2, 4, 2, 6, 3, 3, 2, 3, 4, 2,

At a glance this data does not make much sense. It is just a jumble of meaningless numbers.

Rearranging these values in a table showing the different numbers and the member of times each appeared gives:

Number	Tally	Frequency
1	卌 I	6
2	卌 III	8
3	卌 卌	10
4	卌 II	7
5	IIII	4
6	卌	5

With this table, it can be seen for example, that the most frequently appearing number is 3 and the least in appearance is 5. This information may not be possible to discern from the list. This table is known as frequency distribution and the act of preparing it is known as collation, which is the organization of data in the form of a table to make more sense.

In order for data to make more sense and to clearly show patterns or trends in it, the process of data presentation is made use of. Data presentation is the representation of data in the forms of charts or graphs. There are many ways of data presentation, among what are:

(i) pictogram (ii) bar chart (iii) Pie-chart (iv) Frequency polygon

PICTOGRAM

This is a method of presenting data in the form of pictures. Usually this is done by choosing one picture to represent one or many items of the data. A key is always provided to explain the presentation. Objects that relate to the data are usually used in the presentation. For example, if the data is about humans, then a drawing of human is used. If the data is about vehicles then a drawing of a car is used, and so on.

Example 1;

The pictogram below shows the number of students offering arts, commerce, science and technical in a particular school.

Subject	No. Of students	Pictogram
Arts	30	
Commerce	25	
Science	15	
Technical	10	

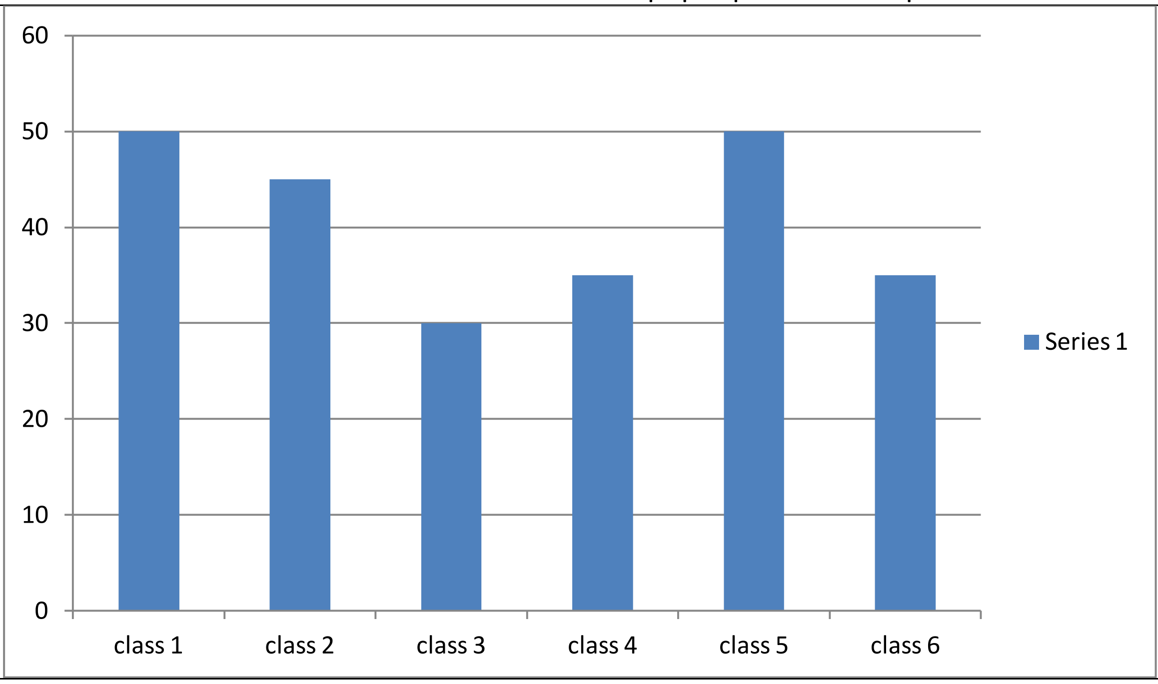
Key:  ≡5 Student

BAR CHART

This is a graphical way of presenting data in the form of bars. The frequencies are presented as bars, drawn vertically or horizontally. In the bar chart, the successive bars are separated but drawn of uniform size.

Example 2;

The bar chart below shows the number of pupils per class in a particular school.



class

PIE CHART

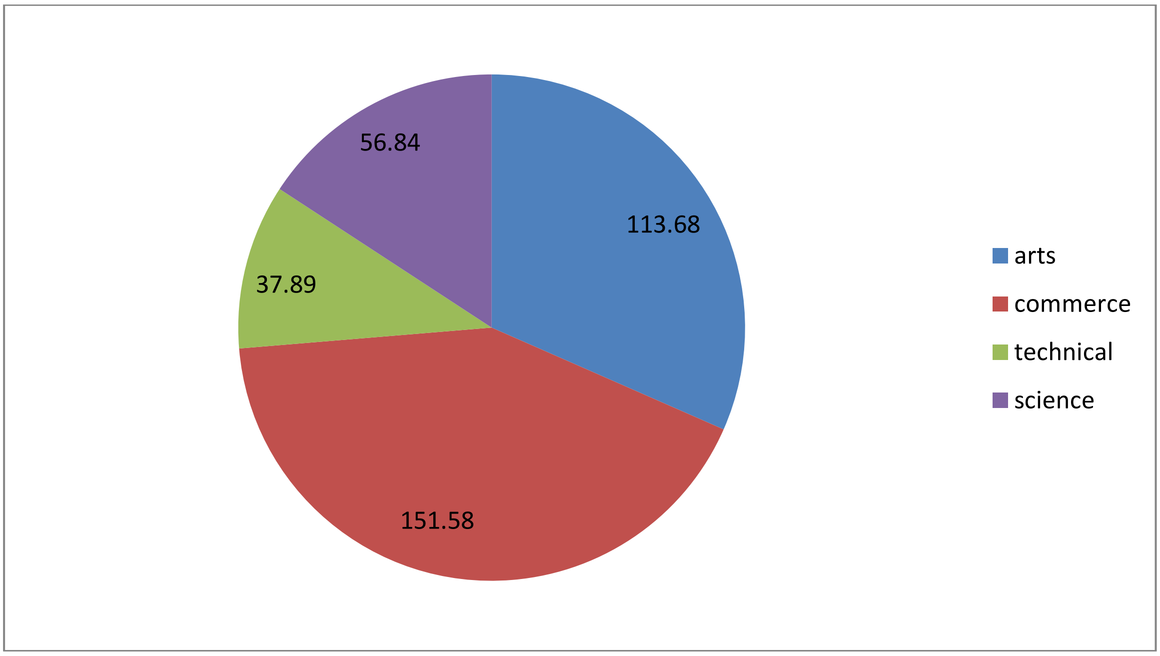
This is a way of presenting data on circle, with frequencies representing sectors of the circle. The frequencies are first converted to angles in degrees.

Example 3;

The number of student offering art, commerce, science and technical subjects in a particular school is presented on a pie chart below.

Subject	No. Of students	angles
Arts	30	$\frac{30}{95} \times 360^\circ = 113.68^\circ$
Commerce	40	$\frac{40}{95} \times 360^\circ = 151.58^\circ$
Science	15	$\frac{15}{95} \times 360^\circ = 56.84^\circ$
Technical	10	$\frac{10}{95} \times 360^\circ = 37.89^\circ$
		$359.99^\circ \approx 360^\circ$

Pie chart



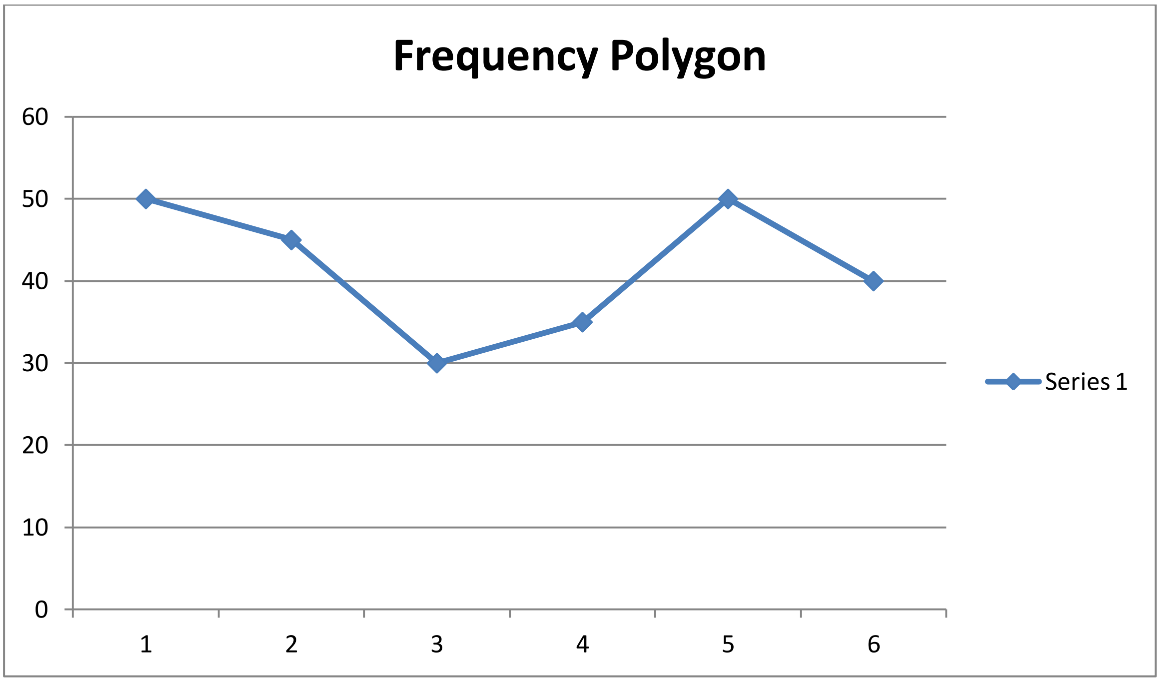
FREQUENCY P0LYGON

This is way of presenting data on a graph with the points joined successively by straight lines.

Example 4

The number of pupils per class in a particular school is presented on a frequency polygon

Class	1	2	3	4	5	6
No. Of pupils	50	45	30	35	50	40



Class

SELF ASSESSMENT

1. Define data collation
2. Define data presentation
3. Collect raw data from within your classroom, organise it into a frequency distribution and indicate the possible different ways we can present it

4.0 **CONCLUSION**

Raw data does not make much sense and many attributes of the data cannot be determined unless it is collated and this makes it easier for data to be presented in the forms of charts or graphs. Data presentation is a way of making more sense of it and it makes it possible for many attributes of the data to be determined at a glance. Data presentation shows pattern and trends in a data very clearly.

5.0 **SUMMARY**

In order to present raw data, it is easier to organise it in tabular form. The pictogram, bar chart, pie chart and frequency are some form of data presentation. They indicate, at a glance attributes of the data such as the one with highest and lowest frequencies, patterns and trends.

Pictogram is a way of presenting data in the form of pictures. Bar chart is a way of presenting data in the form of bars on graph. Pie chart is a way of presenting data on a cycle and frequency polygon is a way presenting data on a graph with successive points joined by straight lines.

6.0 **TUTOR MARKED ASSIGNMENT**

1. Explain the aim of data presentation
2. Differentiate between pictogram, bar chart, pie chart and frequency polygon
3. Collect and organise any data in your school and present it in the form of (i) pictogram (ii) Bar chart (iii) Pie chart and (iv) Frequency polygon

3.0 THE MEANING AND CALCULATION OF MEAN, MEDIAN AND MODE

The measures of central location are the mean, median and mode. They perform the function of locating or identifying the centre of a data. The knowledge of the measures of central location help statisticians make comparisons between sets of data.

MEAN

Mean is the same as average. It is the sum of all values in a data divided by their number. The notation of mean is $\bar{x}$ or $\bar{X}$ or μ (mu), Mathematically,

Mean = $\bar{x}/\bar{X}/\mu = \frac{\sum X}{N} or \frac{\sum fx}{\sum f}$

It reads as the summation of all values x divided by their number or the summation of the product of every with its frequency divided by the summation of all frequencies of the values in the data.

Usually, $N = \sum fx$. The two formulae are respectively used when values are not rearranging and when there is frequency.

The mean of the values 0, 1,2,3,4,5,6,7,8,9 is $\bar{x} = \frac{0+1+2+3+4+5+6+7+8+9}{10}$

$= \frac{45}{10} = 4.5$

This is the same as

$$\bar{x} = \frac{\sum x}{N} = \frac{0+1+2+3+4+5+6+7+8+9}{10} = \frac{45}{10} = 4.5$$

Example 2

A die was cast 55 times and the outcomes recorded in the distribution:

No. on die	1	2	3	4	5	6
No. of times shown	5	7	13	15	10	5

Calculate, on the average, the number that showed up.

To solve this problem, a column of fx is created in order to determine$\sum fx$.

No. On die	F	Fx
1	5	4
2	7	14
3	13	39
4	15	60
5	10	50
6	5	30

$$\sum f = 55$$

$$\sum fx = 198$$

$$\therefore = \frac{\sum fx}{\sum f} = \frac{198}{55} = 3.6$$

$$\bar{x} = 3.6 \approx 4$$

The average of the numbers that showed up is 4.

MEDIAN

Determining the median of a set of values depends on whether they are odd or even numbered. The median of an odd number of values is the middle value when they are rearranged in order of increasing or decreasing size. And the median of an even number of values is the average of the two middle values when they are rearranged in order of increasing or decreasing size.

Example 1

Find the median of 7, 3, 2, 9, 8, 5, 1, 4, 6,
1 ,2, 3, 4, 5, 6, 7, 8, 9
The middle value is 5 since their number is 9, which is odd.

Example 2

Find the median of 7, 3, 2, 9, 8, 5, 0, 1, 4, 6,
The number of values is ten, which is even, so rearranging 9. 8. 7. 6. 5. 4. 3. 2. 1. 0
The two middle values are 5 and 4, so the median is $\frac{5+4}{2}$=$\frac{9}{2}$ $= 4.5$
So the median is 4.5

MODE

The mode of a set of values is the most frequently occurring value in the data or value with the highest frequency.

Example
In the case of tossing a die 55 times:

No. On die	1	2	3	4	5	6
No. of times	5	7	13	15	10	5

The number with the highest frequency is 4, with a frequency of 15. Thus mode = 4.

Note that if all values in a data have equal frequency then the data is said to have no mode or mode = 0.

A data could have
1 mode and it is uni-modal data
2 mode and it is bi-modal data
3 mode and it is tri-modal data and so on
Many modes are poly-modal data.

SELF ASSESSMENT

1. What is the function of the measures of the measures of central location?
2. Define each of mean, mode and median

4.0 **CONCLUSION**

The three measures of central location perform the same job of locating the centre or average of data means any of the three can be used as a measure of the centre of data. The mean is considered the strongest and most reliable measure of central location followed by the median. The mode is considered the weakest and unreliable measure.

5.0 **SUMMARY**

Mean = $\bar{x}/\bar{X}/\mu = \frac{\sum X}{N} or \frac{\sum fx}{\sum f}$

Mode is the most frequently occurring value

Median is the middle value for odd number of values or the average of the two middle values of an even number of values.

6.0 **TUTOR MARKED ASSIGNMENT**

1. Calculate the mean, median and mode of the following set of values;
   15, 2, 6,8, 4, 7, 3, 6, 9, 5, 10, 6, 7, 8, 9, 8, 7

2. Indicate the type of mode of the above data

7.0 Reference/Further Readings

Channon, B. et al (2008). New General Mathematics for West Africa, 1 – 4. Longman Educational Nigeria

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UNIT 3 PROBABILITY

CONTENT

1.0 **INTRODUCTION**

Probability is basically used in making predictions or forecasts. It is therefore important to study and understand some simple probabilities.

2.0 **OBJECTIVES**

After studying this unit, students should be able to;

2. Understand addition and multiplication laws of probabilities
3. Calculate simple probabilities

3.0 CONCEPTS OF SOME IMPORTANT TERMS IN PROBABILITY

Before going on to study the concept of probability and how to calculate the probability of events it is important to understand some terms that are of importance to the area of probabilities.

i) Experiment is an activity whose outcome can be predicted with certainty

ii) Event is the result of an experiment examples, pass in sitting for an exam. Head (H) or Tail (T) i tossing a coin 1, 2, 3, 4, 5, 6, in tossing a die. Win (W), Loss (L), Draw (D) in a competition.

iii) Sample space (S) is a list of all possible outcomes of an experiment. Examples: - (i) in tossing a coin S = {H,T} (ii) In tossing a die S = {1,2,3,4,5,6} (iii) In tossing two dice, sample space is;

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

4.0 Concepts of Probability, Addition and Multiplication Laws of Probability

The probability of an event, say, A, written
P(A) is the chance of the event

Or $P(A) = \frac{\textit{the number of times of A}}{\textit{the number of total possible outcomes of the experiment}}$

Thus, probability is a ratio of the number of times of a particular and the number of possible all possible outcomes for an experiment.

Examples

1. In a single toss of a fair die. Calculate the probability that what shows up is a (a) 2 (b) 5

First, write out the sample space for the experiment

S ={1,2,3,4,5,6}

a) P(2) $= \frac{number\ of\ time\ 2\ appears\ in\ S}{number\ of\ numbers\ in\ S}$

$\therefore P(2) = \frac{1}{6}$

b. P(5) $\frac{number\ of\ times\ 5\ appear\ in\ S}{number\ of\ numbers\ in\ S}$

$\therefore P(5) = \frac{1}{6}$

Examples

1. In a single of a coin, S = {H,T)
2. In a competition, W, D and L are mutually exclusive
3. In a single toss of a die; 1,2,3,4,5 and 6 are mutually exclusive.

Mutually exclusive events are those events of an experiment that cannot occur at the same time, naturally.

Addition law of probability states that in all computations with probabilities that of mutually exclusive events are added.

Example:

1. In a single toss of a fair die calculate the probability that what show up is

(a) an even number (b) at least 5

S = {1,2,3,4,5,6}

Solution

(a) An even number = 2 or 4 or 6

P (even no.) = P(2 or 4 or 6

The or is changed to addition since 2, 4 and 6 cannot show up in a single toss. If anyone of the three shows up, the rest will never show up.

So, P(2 or 4 or 6) = P(2) + P(4) + P(3)

$= {}^{1}/_{6} + {}^{1}/_{6} + {}^{1}/_{6} = \frac{3}{6} = \frac{1}{2}$

$\therefore P(even\ no.) = {}^{1}/_{2}$

(b) at least 5 is the same as 5 or more and this is 5 or 6

P(at least 5)=P(5 or 6) = p(5) + P(6)

$= {}^{1}/_{6} + {}^{1}/_{6} = {}^{2}/_{6} = \frac{1}{3}$

$\therefore P(at\ least 5) = {}^{1}/_{3}$

Independent events

If two or more events of an experiment are related in such a way that they can occur at the same without affecting each other, they are said to be independent.

Examples:

i. When two coins are tossed, the outcomes of each of them are independent.
1.e. S = {H,T), (T,H), (H,H), (T,T) in each of the four cells, the two entries are independent.

2. In tossing two dice once, in each of the 36 entries, the two entries are independent.
What shows up on the first die does not affect the outcome of the second die

Multiplication law of Probabilities states that in computations with probabilities, those of independent events are multiplied.

Examples

i. in a single toss of two dice, calculate the probability that the numbers that show up
(a) give a total of 3 (b) give a total of 10 (c) are the same

Solution,

Numbers that give a total of 3 are (1, 2) or (2, 1)
∴ P(total) of 3) = P[(1,2) or (1,2)
= P(1,2) + P(2,1)
$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$
$\therefore P(total\ of\ 3) = \frac{1}{12}$

Alternatively,

P(total of 3) =P[(1,2) or (2,1)]
= P(1) XP(2)+P(2)XP(1)
$= \frac{1}{36} + \frac{1}{36} = \frac{2}{36} = \frac{1}{18}$

(b) Total of 10 = (4,6) or (6, 4) or (5,5)
thus, P (total of 10) = P[(4,6) or (4,6) or (5,5)
$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{3}{36} = \frac{1}{12}$

(c) The same numbers on the two dice means (1,1) or (2,2) or (3,3) or (4,4) or (5,5) or (6,6)
P(same numbers) =P[(1,1) or (2,2) or (3,3) or (4,4) or (5,5) or (6,6)]
= P(1,1) +P (2,2) +P (3,3) +P (4,4) P(5,5) + P(6,6)
$= \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} + \frac{1}{36} = \frac{26}{36} = \frac{1}{6}$
P (the same numbers)= $\frac{1}{6}$

SELF ASSESSMENT

2. Define the terms experiment, event, sample space and probability
3. When are probabilities of events added?
4. When are probabilities of events multiplied?

5.0 **CONCLUSION**

Probability of an event is the ratio of the number of times of the event and the number of total possible outcomes of experiment of the experiment. It is not every activity that can be termed an experiment, only those whose outcomes can be predicted with certainty are experiments. Some events occurring can naturally stop or affect the occurrence of others and there are those events that can occur at the same time without stopping or affecting the occurrence of the others. The former are mutually exclusive while the letter are independent.

6.0 **SUMMARY**

2. Any activity whose outcome can be predicted with certainty is experiment.
3. The result of an experiment is called an event
4. Probability of an event is the chance of the event i.e.

$$P(A) = \frac{number\ of\ times\ of\ A}{number\ of\ possible\ outcomes\ of\ the\ experiment}$$

5. The probabilities of mutually exclusive events are always added
6. The probabilities of independent events are always multiplied.

7.0 **TUTOR MARKED ASSIGNMENT**

2. Give three examples of an experiment and three examples of activities that are not experiments
3. List the sample space for tossing two (2) dice
4. Calculate the probability that in tossing a die what show up is a) 6 b) and odd number
5. Calculate the probability that in tossing two dice what show up is a) a sum of 5 b) odd numbers c) a sum of 12

8.0 REFERENCE/FURTHER READINGS

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MODULE 4: TRIGONOMETRIC RATIOS

Introduction

In this module, you will be exposed to trigonometric ratios and identities. The module is devided into 2 units:

Unit 1: Trigonometric Ratios
Unit 2: Trigonometric Identities

UNIT 1: TRIGONOMETRIC RATIOS

1.0 Introduction

Trigonometry is in essence, measurement of triangles in terms of the relationships between their sides and angles. It is about solutions of triangles based on their relationships. Trigonometry is important for its usefulness in solving real life problems.

2.0 Objectives

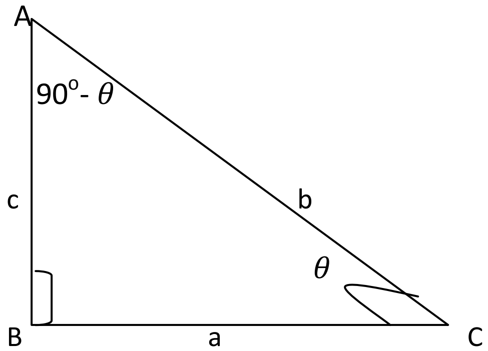
After studying this unit, students should be able to:

i. Distinguish between sine, cosine and tangent of an acute angle;
ii. Use sine, cosine and tangent tables to estimate trigonometric rations of acute angles in right angled triangles.
iii. Find solutions of triangles using trigonometric rations
iv. Estimate trigonometric ratios of the following angles by using the right angled triangle: 0^o, 30^o, 45^o, 60^o and 90^o.

3.0 Main Content
3.1 Trigonometric ratios of acute angles;
3.2 Trigonometric Ratios from Logarithm Table
3.3 Trigonometric Ratios of some Special Angles.

3.1 TRIGONOMETRIC RATIOS OF ACUTE ANGLES

Consider the right angled triangle ABC (Fig. 3.1). The angle ACB is denoted by θan acute angle. The side AB is opposite to θ and CB is adjacent to θ. The side AC, which is opposite to the right angle is called the hypotenuse of the triangle. Normally, the three angles of the triangle are denoted by corresponding letters A,B,C and side opposite these angles are denoted by the corresponding letters a,b and c. Since θ is acute, it follows that A = $90^o - \theta$.



Sine of the angle θ = sine $\theta = \frac{opposite\ side}{hypotenuse\ side} = \frac{c}{b}$

Cosine θ = cos $\theta = \frac{adjascent\ side}{hypotenuse\ side} = \frac{a}{b}$

Tangent of θ = tan $\theta = \frac{opposite\ side}{adjascent\ side} = \frac{c}{a}$

The three identities can be summarized as $s = \frac{O}{H}; C = \frac{A}{H}\ and\ T = \frac{O}{A}$

And this will give SOHCAHTOA for short. Now an important result shows a relationship between the three ratios:

$$tan\theta = \frac{c}{a} = {}^{c}/_{b} \times \frac{b}{a} = \frac{{}^{c}/_{b}}{{}^{a}/_{b}} = \frac{sin\theta}{cos\theta}\ thus, tan\theta = \frac{sin\theta}{cos\theta}$$

Again considering angle A = $90^{o} - \theta$, it is interesting to note that;

$$sin\theta = \frac{c}{b}\ and \cos(90^{o} - \theta) = \frac{c}{b}, thus, sin\theta = \cos(90^{o} - \theta) = {}^{c}/_{b}$$

Similarly, $\sin(90^{o} - \theta) = {}^{a}/_{b}\ but\ cos\theta = {}^{a}/_{b}\ thus, \sin(90^{o} - \theta) = cos\theta = {}^{a}/_{b}$

Trigonometric ratios of acute angles can be formed from logarithm tables.

Examples 1:

Find (i) sin 23.32° (ii) cos 21.28° (iii) tan 37.83°

Solution:

(i) Sin 23.3° = 0.3955	(ii) cos 21.1° = 0.9323	(iii) tan 37.8°
Mean difference f or 0.02 (to be added) 3	mean difference for 0.080 (to be added) 5	mean difference for 0.03 (to be added) 8
Sin 23.32° = 0.3958	cos 21.28° = 0.9318	tan 37.83° = 0.7765

Examples 2

(i) Find the value of sin 23°19'20"

Solution:

From the table sin $23^o19'$ = 0.3958

And $23^o20''$ = 0.3960

Difference for 1' or 60" = 0.0002

Difference for 20" = $\frac{0.0002}{60} \times 20 = 0.00007$

$\therefore sin23^o19'20'' = \frac{\begin{matrix}0.3958\\0.00007\end{matrix}+}{0.39587}$

Thus, sin $23^o19'20''$ = 0.39587

(ii) Find the of which 0.5616 is the sine.

sine 34.11^O = 0.5606 (from the table)

difference = 10

difference of 10 corresponds with an increase of 7 in angle, hence the angle 34.17^o.

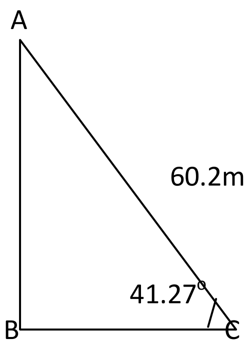
Trigonometric ratios can be used to solve any right-angled triangles. This is the use of the three trigonometric ratios, using some given values of the triangle to find the remaining values of the triangle.

Examples 3

i) Solve the right-angled triangle ABC when <BAC = 41.27^o, AC = 60.2m

ii) Solve the right-angled triangle ABC if a = 60m and b = 70m.

Solution:



A = $90^o - 41.27^o$ = 48.73

Sin 41.27^o = $\frac{AB}{AC} = \frac{AB}{60.2}$

$\Rightarrow AB = 60.2\ sin41.27^o$

= 60.2 X 0.6596

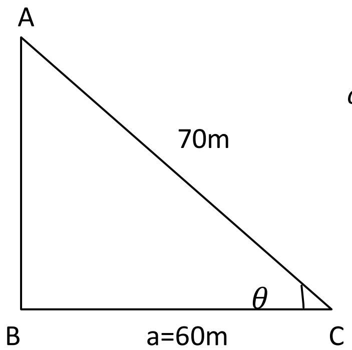
= 39.71m

Cos $41.27^o = \frac{BC}{AC} = \frac{BC}{60.2}$

$\Rightarrow BC = 60.2\ cos41.27^o$

$= 62.2\ \times 0.7521$

= 45.28m



$cos\theta = {}^a/_b = {}^{60}/_{70} = 0.8571$

θ =31^o

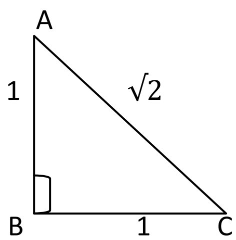
A = $90^o - 31^o = 59^o$

$sin\theta = \frac{c}{b} = \frac{AB}{70}$

AB = 70 sin 31^o

= 36.1m

Trigonometric ratios of some special angles: 0^o, 30^o, 45^o, 60^o, 90^o. Trigonometric ratios of these special angles are often used in the wider mathematics and here, their values are expressed. Consider the unit isosceles triangle that is right-angled.



AB = BC = unit length.

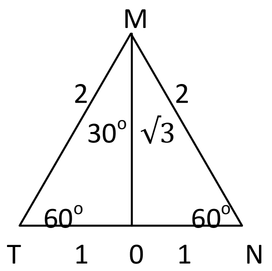
Since Δ ABC is isosceles, it follows that the base angles are 45°.

Thus, $\sin 45^o = \frac{AB}{AC} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\text{Cos } 45^o = \frac{BC}{AC} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\text{Tan } 45^o = \frac{AB}{BC} = 1$

Now, consider an equilateral triangle MTN whose sides are 2 units in length. Draw a perpendicular from M to TN at 0.



Now, MT = 2 units = MT = TN

M0 =√3

T0 = 1 = ON

$sin30^o = \frac{TO}{MT} = 1/2$

$$\sin 60^o = \frac{MO}{MT} = \frac{\sqrt{3}}{2}$$

$$\cos 30^o = \frac{MO}{MT} = \frac{\sqrt{3}}{2}$$

$$\text{Cos } 60^o = \frac{TO}{MT} 1/2$$

$$\text{Tan } 30^o = \frac{TO}{MO} = \frac{1}{\sqrt{3}}$$

$$tan60^o = \frac{MO}{TO} = \sqrt{3}$$

Sin 0° = 0 and cos 0° = 1

Thus, $\tan 0^o = \frac{sino^0}{cos0^o} = \frac{0}{1} = 0$

Sin 90° = 1; cos 90° = 0

Thus, $\tan 90^o = \frac{sin90^o}{cos90^0} = \frac{1}{0} = \infty$

These trigonometric ratios can be summarized in a table;

	0°	30°	45°	60°	90°
Sin	0	½	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1 \; or \; \frac{\sqrt{4}}{2}$
Cos	$1 \; or \; \frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	½	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

Self Assessment

1- Using a right-angled triangle show that tan $\theta = {sin\theta}/{cos\theta}$
2- From the logarithm tables of trigonometry ratios, find the sin, cosine and tangent of the following angles: (i) 15.41° (ii) 83.48° (iii) 50.17°
3- Solve the each of the following triangles. ABC, right angle of B given that (i) C = 30° and b=40cm (ii) A = 60° and b = 15cm

4.0 Conclusion

From the foregoing discussions about trigonometric ratios of sine, cosine and tangent. It can be concluded that;

i) Sine, cosine and tangent of an acute angle are related

ii) The sine of an acute angle is the same as the cosine of its complimentary angle;

iii) The cosine of an acute angle is the same as the sine if its complimentary angle;

iv) Trigonometric ratios of angles can be determined from logarithm tables of sine, cosine and tangent;

v) Trigonometric ratios of 0^o, 30^o, 45^o, 60^o and 90^o can be determined without using tables;

vi) Trigonometric ratios are an important tool for solving right angled triangles.

5.0 Summary

What has been learned in this unit can be summarised as follows:

i) If s stands for sine, C for cosine and T for tangent of an angle θ, of a right angled triangle with A standing for adjacent side, O for opposite and H for hypotenuse side of the triangle, then the three ratios can be abbreviated to SOHCAHTOA, meaning; $sin\theta = \frac{opposite\ side}{hypotenuse\ side}$, $cos\theta = \frac{adjascent\ side}{hypotenuse\ side}$; $tangent\theta = \frac{opposide\ side}{adjascent\ side}$

ii) $tan\theta = \frac{sin\theta}{cos\theta}$

iii) $sin\theta = \cos(90^o - \theta)\ and\ cos\theta = \sin(90^o - \theta)\ \ for\ any\ acute\ angle\ \theta$

iv)

	0^o	30^o	45^o	60^o	90^o
Sin	0	½	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	$1\ or\ \frac{\sqrt{4}}{2}$
Cos	$1\ or\ \frac{\sqrt{4}}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	½	0
Tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	∞

6.0 Tutored Marked Assignment

1. Find the base and altitude of an isosceles triangle whose vertical angle is 65^o and whose equal sides are 415cm.

2. A plane rise from a take off and flies at a fixed angle of 12.5^o with the horizontal ground. When it has gained 500 in altitude, fine, to the nearest meter, (a) the horizontal distance flown (b) the distance the plane has actually flown

7.0 Reference/Further reading

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UNIT 2: TRIGONOMETRIC IDENTITIES

1.0 Introduction

The relationships between trigonometric ratios lead to more identities that lead to the determination of trigonometric ratios of not just acute angle but obtuse and reflex angles as well. This relation also leads to formation of trigonometric equations and their solutions. In this unit, students will be exposed to more trigonometric identities, trigonometric ratios of obtuse and reflex angles and solutions of trigonometric equations.

2.0 Objectives

After studying this unit, students should be able to:

1- State the different relation between trigonometric ratios that leads to the trigonometric identities;
2- Define a positive and negative angle;
3- State the sign of the trigonometric ratios of acute angles
4- Define the trigonometric ratios of obtuse and reflex angles;
5- Obtain the values of the trigonometric ratios of obtuse and reflex angles.

3.0 Main Content

3.1 TRIGONOMETRIC IDENTITIES

The three trigonometric ratios are related in such a way that the different relations give rise to other trigonometric identities. These relations can be classified as;

i) The reciprocals of the three ratios are as follows;

a) $cosec\theta = \frac{1}{sin\theta}$ b) $sec\theta = \frac{1}{cos\theta}$ $c)\ cot\theta = \frac{1}{tan\theta}$

Thus, given rise to cosecant, secant and cotangent respectively.

ii) The quotient relations are that

a) $tan\theta = \frac{sin\theta}{cos\theta}\ \ and\ \ cot\theta \frac{cos\theta}{sin\theta}$

iii) The squared relations are that;

a) $sin^2\theta + cos^2\theta = 1$ $b)\ sec^2\theta = 1 + tan^2\theta$ $c)\ cosec^2\theta = 1 + cot^2\theta$

these relations between trigonometric ratios, termed trigonometric identities, make it possible for one trigonometric ratio be expressed in terms of another.

Examples 4

1. Simplify the following;

a) $cos\theta tan\theta$ $\quad b)\ \frac{1}{1-sin\theta}+\frac{1}{1+sin\theta}$ $\quad c)\ sec\theta - sin\theta tan\theta$

2. show that;

a) $tan\theta + cot\theta = \frac{1}{sin\theta cos\theta}$ $\quad b)\ \frac{cos^2\theta}{sin^2\theta} - \frac{cos^{2\theta}}{tan^2\theta} = cos^2\theta$

Solution:

1. a. $cos\theta tan\theta = \frac{cos\theta sin\theta}{cos\theta} = sin\theta$

$b)\ \frac{1}{1-sin\theta}+\frac{1}{1+sin\theta}$

$\frac{1+sin\theta+1-sin\theta}{(1-sin\theta)(1+sin\theta)} = \frac{2}{1-sin^2\theta} = \frac{2}{cos^2\theta} = 2sec^2\theta$

c) $sec\theta - sin\theta tan\theta = \frac{1}{cos\theta} - \frac{sin\theta sin\theta}{cos\theta} = \frac{1-sin^2\theta}{cos\theta} = \frac{cos^2\theta}{cos\theta} = cos\theta$

2.a. $tan\theta + cot\theta = \frac{1}{sin\theta cos\theta}$

$l.h.s = tan\theta + cot\theta$

$= \frac{sin\theta}{cos\theta} + \frac{cos\theta}{sin\theta} = \frac{sin^2\theta+cos^2\theta}{sin\theta+cos\theta} = \frac{1}{sin\theta cos\theta}$ =r.h.s

b. $\frac{cos^2\theta}{sin^2\theta} - \frac{cos^2\theta}{tan^2\theta} = cos^2\theta$

$l.h.s = \frac{cos^2\theta}{sin^2\theta} - \frac{cos^2\theta}{tan^2\theta}$

$= cos^2\theta\left(\frac{1}{sin^2\theta} - \frac{1}{tan^2\theta}\right) = cos^2\theta\left(\frac{1}{sin^2\theta} - \frac{cos^2\theta}{sin^2\theta}\right)$

$= cos^2\theta\left(\frac{1-cos^2\theta}{sin^2\theta}\right) = cos^2\theta \times \frac{sin^2\theta}{sin^2\theta} = cos^2\theta = r.h.s$

Self Assessment

1- Write the reciprocals of $sin\theta, cos\theta\ and\ tan\theta$

2- Express the relationship between the squares of the following pairs of trigonometric ratios $a)\ sin\theta\ and\ cos\theta \quad b)\ sec\theta\ and\ tan\theta \quad c)\ cosec\theta\ and\ cot\theta$

3.2. TRIGONOMETRIC RATIOS OF THE GENERAL ANGLES

So far, the trigonometric ratios learned (sine, cosine and tangent) were defined in terms of the sides of right-angled triangle with acute angles. The logarithm tables of these ratios are for acute angle only. It is therefore, not possible to directly find trigonometric ratios of angles that are higher than go. That is, obtuse and reflex angles or the general angles. It is therefore important to find a relationship between the trigonometric ratios of acute and general angles in order to find the values of the latter.

Now, consider a circle drawn on a graph with its centre at the origin (fig. 2). An angle is formed by moving the radius OP in the anticlockwise direction are termed positive and all those formed by moving the radius in the clockwise direction are termed negative.

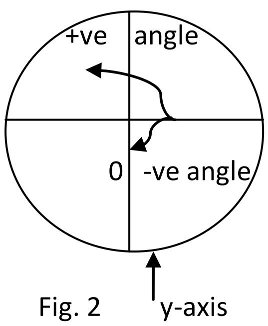


Fig. 2

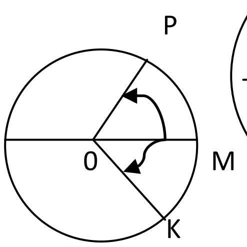


fig. 3

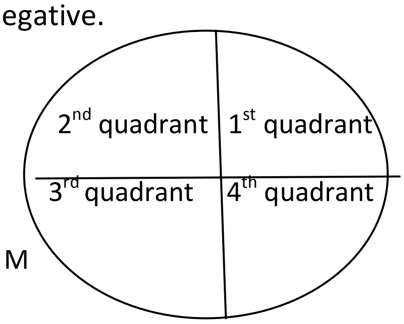


fig. 4

If an angle is acute then it is less 90◦. If is therefore in the first quadrant (fig.4). if an angle is obtuse then it is between 90◦ and 180◦. If is therefore in the second quadrant. The angle is reflex, then it is either (i) between 180◦ and 270◦ or ii) between 270◦ and 360◦. In the case of (i) the angle is in the third quadrant and in the case of (ii) the angle is in the fourth quadrant.

Since all the trigonometric ratios of sin, cosine and tangent were for acute angles using the right-angled triangle, it follows that it is not possible to determine these ratios directly from the tables for angles that are more than 90^o. It has been discovered however, that it is always possible to relate the trigonometric ratios of angles that are more than 90^o, called the general angle, to the ratios of some acute angle. This idea is the basis for determining the signs and values of trigonometric ratios of the general angle.

1- The sign of trigonometric ratios of an Acute Angle ($0^o \le \theta \le 90^o$). Consider a circle with centre 0 and diameter AOB (fig. 5)

Draw a radius OP to make an acute angle θ with AOB.
Draw a perpendicular from P To M on AOB
Let line OP b r, this is a positive distance
Line OM be x, this is also positive
Line PM be y, this is also positive

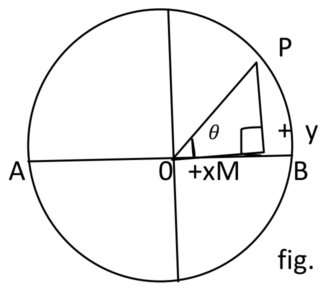


fig. 5

Then $sin\theta = \frac{OM}{OP} = \frac{+y}{y} = +sin\theta$

$$cos\theta = \frac{OM}{OP} = \frac{+x}{y} + cos\theta$$

$$tan\theta = \frac{PM}{OM} = \frac{+y}{+x} = +tan\theta$$

Thus, in the first quadrant all trigonometric ratios are positive.

2- The sign of trigonometric Ratios of an Obtuse Angle ($90^o < \propto \le 180^o$)

Consider a circle of diameter AOB, centre 0 (fig. 6)
Draw radii OP and OM to make
Angle θ with AOB, with θ being acute
Draw lines PT and MS
Perpendicular to line AOB

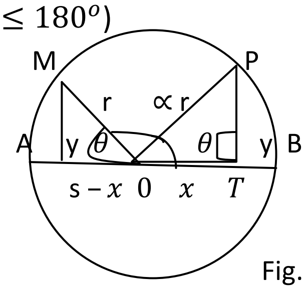


Fig. 6

Draw line PT and MS perpendicular to line AOB. Line MS, PT, OM, OP and OT are positive but line OS is negative.

Clearly, triangles OPT and OMS are congruent and corresponding sides are equal. OM = OP, SM = TP and OS = -OT. Also, angle BOM = $180^0 - \theta$

Then $sin \propto = \sin(180^o - \theta) = \frac{SM}{OM} = \frac{TP}{OP} = sin\theta$

$$cos \propto = \cos(180^o - \theta) = \frac{OS}{OM} = \frac{-OT}{OP} = -cos\theta$$

$$tan \propto = \tan(180^o - \theta) = \frac{SM}{OS} = \frac{TP}{-OT} - tan\theta$$

Thus, in the second quadrant, the sine of an obtuse angle is the positive sine of its supplement the cosine of an obtuse angle is the negative cosine of its supplement; the tangent of an obtuse angle is the negative tangent of its supplement.

3- The trigonometric Ratios of a Reflex Angle that is between 180° and 270° ($180^o \leq\propto\leq 270^o$)

Consider a circle with centre 0 and diameter AOB fig. 7)

Draw radii OP and OM to make angles θ, acute angle, with Diameter AOB.

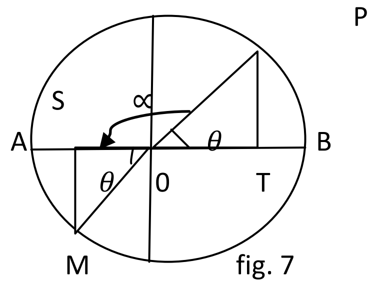


fig. 7

Clearly, triangles OPT and OMS are congruent and Corresponding sides are equal.

OM = OP, MS = - PT, OS = -OT and <BOM=$\propto = 180^o + \theta$,

$$sin \propto = \sin(180^o + \theta) = \frac{MS}{OM} = \frac{-PT}{OP} = -sin\theta$$

$$cos \propto = \cos(180^0) = \frac{OS}{OM} = \frac{-OT}{OP} = -cos\theta$$

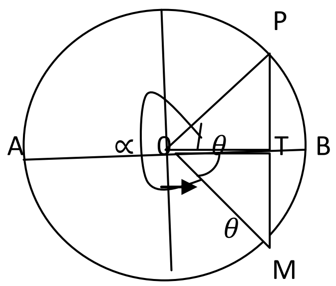
$$tan \propto = \tan(180^o - \theta) = \frac{MS}{OS} = \frac{-PT}{-OT} = \frac{PT}{OT} = tan\theta$$

Thus, the trigonometric ratios of a reflex angle that is between 180° and 270° are negative except for tangent.

4- The trigonometric Ratios of a Reflex Angle that is Between 270° and 360° ($270^o \leq\propto\leq 360^o$)

Consider a circle with centre 0 and diameter AOB (fig. 8).

Draw radii OP and OM to make Angles θ, an acute angle, with the Diameter AOB



Draw perpendicular lines TP and TM to AOB.

Clearly, triangles OPT and OMT are congruent and corresponding sides are equal.

OT = OT (common), OM = OP; TM = - TP and $B\hat{O}M = \propto = (360^o - \theta)$

$$Thus, sin \propto = \sin(360^o - \theta) = \frac{TM}{OM} = \frac{-TP}{OP} = -sin\theta = \sin(-\theta)$$

$$cos \propto = \cos(360^o - \theta) = \frac{OT}{OM} = \frac{OT}{OP} = cos\theta = (-\theta)$$

$$tan \propto = \tan(360^o - \theta) = \frac{TM}{OT} = \frac{-TP}{OT} = -tan\theta = \tan(-\theta)$$

Thus, trigonometric ratios of a reflex angle that is between 270° and 360° are negative except for cosine.

So far, it has been shown that;

i) In the first quadrant, all trigonometric ratios are positive;
ii) In the second quadrant, only sine is positive;
iii) In the third quadrant, only tangent is positive;
iv) In the fourth quadrant, only cosine is positive.

v) The above four results can be simplified to all Sine Tan Cos or CAST if one decided to start considering first the fourth quadrant and go on in an anticlockwise direction to the other quadrants.

These results can be used to find the trigonometric ratios of the general angle.

Examples 5

1- Find the values of (a) sin 130^0; cos 140^o and tan 95^o (b) sin 210^o; cos 220^o and tan 250^o (c) sin 300^o; Cos 290^o and tan 345^o
2- Find θ given that (a) sin θ = - 0.1537 (b) Cos θ = 0.2764 (c) tan θ = - 1.271

Solutions:

1. a. 130^o is in the second quadrant
$\therefore 130^o = 180^o - 50^0$
Thus, sin 130^o = sin(180^o – 50^o) = sin50^o=0.766 from the table

140^o is in the second quadrant
$140^o = 180^o - 40^o$
Thus, cos 140^o = Cos (180^o -40^o) = - Cos 40^o = - 0.766 from the tables.

95^o is in the second quadrant
$95^o = 180^o - 85^o$
Thus, tan 95^o = tan(180^o -85^o) = - tan85^o = - 11.43 from the tables.

b. 210^o is in the third quadrant
$210^o = (180^o + 30^o)$
Thus, sin 210^o = sin(180^o + 30^o) = - sin 30^o = - 0.5 from the tables.

220^o is from the third quadrant
$220^o = (180 + 40)^o$
Thus, Cos$(180 +40)^o$ = - Cos 40^o = -0.766 from the tables.

250^o is in the third quadrant
$250^o = 180^0 + 70^o$
Thus, tan 250^o = tan $(180 + 70)^o$ = tan 70^o = 2.747 from the tables.

c. 300^o is in the fourth quadrant
$300^o = 360^o - 60^o$
Thus, sin 300^o = sin $(360 -60)^o$ = - sin 60^o = 0.866 from the tables.

290^o is in the fourth quadrant
$290^o = 360^o - 70^o$
Thus, Cos 290^o = $(360 - 70)^o$ = cos 70^o =0.342 from the tables.

345^o is in the fourth quadrant
$345^0 = (360 - 15)^o$
Thus, tan 345^o = tan $(360 - 15)^o$ = - tan 15^o = - 0.2679 from the tables.

2. (a) $sin\theta$ = - 0.1537

Note that, $sin\theta$ is negative in the third and fourth quadrants, which imply that θ must be (180^o + θ) 0r (360^o – θ). From the tables 0.1537 correspond with 8.84^o therefore,

θ =$(180 + 8.84)^o$ = 188.84^o or θ = $(360 – 8.84)^o$ = 351.84^o

(b) Cos θ = 0.2764

Note that Cos θ is positive in the 1^{st} and 4^{th} quadrants, i.e. θ or $(360 – \theta)^o$.

From the tables 0.2764 corresponds with 73.95^o

therefore, θ =73.95^o 0r $(360 – 73.95)^o$ = 286.05^o

(c) Tan θ = - 1.271

Note that tan θ is negative in the second and fourth quadrants i.e. $(18 – \theta\ )^o$ and $(360 – \theta)^o$.

From the tables 1.271 corresponds with 51.8^o

θ = $(180 – 51.8)^o$ = 128.2^o or $(3609 – 51.8)^o$ = 308.2^o

Self Assessment

1- What is the sign of an angle that made by moving (i) anticlockwise (ii) clockwise on a plane?
2- How many quadrants are there in a circle?
3- What is the signs of the three trigonometric ratios of an angle in each of the quadrants of a circle?

3.3 SOLUTIONS OF TRIGONOMETRIC EQUATIONS

Many equations can be raised involving 1 or 2 or many trigonometric ratios and these can be solved mathematically.

For example, we have seen that the solution of the equation tan θ = - 1.271 is 128.2^o or 308.2^o. There are instances with more than one ratio.

Examples 6:

1- Find the values of θ which satisfy (a) 5 sin θ = 4 Cos θ (b) $\sin^2\theta$ + sin θ -2 = 0

Solution

(a) 5 sin θ = 4 Cos $\theta \Longrightarrow \frac{sin\theta}{cos\theta} = \frac{4}{5} = 0.8$

$\Rightarrow \tan\theta = 0.8\ from\ the\ tables,$

But tan θ corresponds with 38.66^0 is positive in the 1^{st} and 3^{rd} quadrants.

θ = 38.66^o and $(180 + 38.66)^o$ = 218.66^o

(b) $\text{Sin}^2\theta$ + Sin θ -2 = 0

$\Longrightarrow (sin\theta + 2)(sin\theta - 1) = 0$

$\Longrightarrow sin\theta + 2 = 0\ or\ sin\theta - 1 = 0$

$\Longrightarrow sin\theta = -2, which\ is\ possible\ as\ all\ sine\ values\ cannot\ exceed\ 1$

Or $\sin\theta = 1$

If sin θ = 1, then it follows that $\theta = 90^o$.

Self Assessment.

1- Calculate the values of θ between 0^o and 360^o which satisfy the equation $6\sin^2\theta + \cos\theta - 4 = 0$
2- Solve the following equations for θ between 0^o and 360^o: $5\cos^2\theta + 9\sin\theta = 9$

Solution

(i) $6\sin^2\theta + \cos\theta - 4 = 0$

$6(1-\cos^2\theta) + \cos\theta - 4 = 0$

$6 - 6\cos^2\theta + \cos\theta - 4 = 0$

$2 + \cos\theta - 6\cos^2\theta = 0$

$(2 - 3\cos\theta)(1 + 2\cos\theta) = 0$

Thus, $\cos\theta = 2/3$ or ½

If $\cos\theta = 2/3$; $\theta = 48.19^o$ or 311.81^o

$\cos\theta = -1/2$; $\theta = 120^o$ or 240^o

The solution are: 48.19^o, 120^o, 240^o, 311.81^o.

(ii) $5\cos^2\theta + 9\sin\theta = 9$

$5(1 - \sin^2\theta) + 9\sin\theta - 9 = 0$

$5 - 5\sin^2\theta + 9\sin\theta - 9 = 0$

$-5\sin^2\theta + 9\sin\theta - 4 = 0$

$5\sin^2\theta - 9\sin\theta + 4 = 0$

$(5\sin\theta - 4)(sin\theta - 1 = 0$

$\sin\theta = {}^4/_5 \; or \; 1$

$if \; sin\theta = {}^4/_5 = 0.8, \theta = 53.13 \; or \; 126.87^o$

$sin\theta = 1; \; \theta = 90^o$

Thus, the solutions are 53.13^0, 90^0, 126.87^0

4.0 Conclusion

From what was learnt in this unit it can be concluded that;

i. Relationships between the sine, Cosine and tangent of an acute angle give rise to a number of useful trigonometric identities
ii. Angles formed by moving in anticlockwise direction are positive and those formed by moving in the clockwise direction are negative.
iii. Trigonometric ratios of angles more than 90^o can be obtained by relating then to an appropriate acute angle;
iv. Trigonometric identities can be used in solving trigonometric equations.

5.0 Summary

What has so far been learnt in this unit can be summarised as follows:

1- $\sin(90^0 - \theta) = \cos\theta$ and $\cos(90^o - \theta) = \sin\theta$
2- $\sin(180^o - \theta) = \sin\theta$; $\cos(180^o - \theta) = -\cos\theta$ and $\tan(180^o - \theta) = -\tan\theta$
3- $\sin(180^o + \theta) = -\sin\theta$; $\cos(180^o + \theta) = -\cos\theta$ and $\tan(180^o - \theta) = -\tan\theta$
4- $\sin(360^o - \theta) = -\sin\theta$; $\cos(180^o - \theta) = \cos\theta$ and $\tan(360^o - \theta) = -\tan\theta$
5- cosec $\theta = \frac{1}{sin\theta}; sec\theta = \frac{1}{cos\theta} \; and \; cot\theta = \frac{1}{tan\theta} = \frac{cos\theta}{sin\theta}$
6- the signs of sine, cosine and tangent of an angle in the first quadrant is positive for all of them;
7- in the second quadrant only the sine of an angle is positive;
8- in the third quadrant only the tangent of an angle is positive;
9- in the fourth quadrant only the cosine of an angle is positive;
10- numbers 6 to 9 can be shortened to All Sine Tan Cos or CAST

6.0 Tutor Marked Assignment

1. Simplify the following:
   a) $\frac{sec\theta}{cot\theta} - \frac{tan\theta}{cot\theta}$ $\quad (b) sec\theta - sin\theta tan\theta$

2. $Show\ that\ (a) cot\theta + \frac{sin\theta}{cos\theta} = cosec\theta \quad (b)(sin\theta + cos\theta)(tan\theta + cot\theta) = sec\theta + cosec\theta)$
3. Find the values of θ, between 0^o and 360^o for which (i) sin θ=0.25 (ii) cos θ= 0.3124 (iii) tan θ = 0.75
4. Find the values of the following, from the tables: (i) sin 110^o (ii) Cos 250^o (iii) tan 100^0

7.0 Reference/Further Reading

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