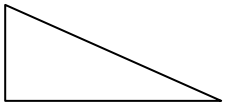
Three dimensional geometry

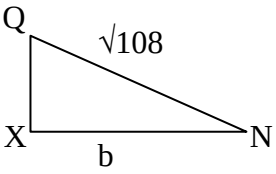
1. *a)*



$QN = \sqrt{12^2 - 6^2}$

$= 10.39$

b)

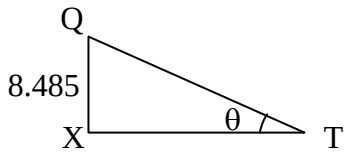


$QX = (\sqrt{108})^2 - 6^2$

$= \sqrt{72}$

$= 8.485$

c)



$\tan \theta = \frac{8.485}{6}$

$\theta = 54.73^0$

d) $\tan \theta = \frac{6}{10}$

$\theta = 30.96$

$\frac{6}{10}$ *obtuse* $= 180^0 - 30.96$

$= 149.04^0$

2. *a)* $\frac{Sin\ 36^0}{a} = \frac{5}{}$

Where a is the side

$a = \frac{5}{\sin 36} = 8.507$

$h^2 = 18.2 - 8.507$

$= 258.87$

$H = 16.09\ cm$

b) $\frac{1}{2} ab \sin \theta$

$\frac{1}{2} \times 8.507^2\ Sin\ 72 \times 5$

$= 172.06\ cm^2$

c) $\frac{Tan\ 36^0}{x} = \frac{5}{}$

$x = 6.882$

$Tan\ \theta = 16.09$

6.882

$\theta = 66.84^2$

d) $\frac{1}{3} \times 172.06 \times 16.09 = 922.8cm^3$

e) $S = 23.2$

$\sqrt{23.2\,(23.2 - 18.2)\,(23.2 - 10)}$

$= 87.50cm^3$

3. (i) $\frac{1}{3} \times 4.2 \times 7.5h = 52.5$

$h = \frac{52.5 \times 3}{4.2 \times 7.5} = 5.0cm$

(ii) $AC = \sqrt{4.2^2 + 7.5^2}$

$= \sqrt{17.64 + 56.25}$

$= \sqrt{73.89}$

$= 8.596$

$AO = 8.596 \div 2 = 4.298$

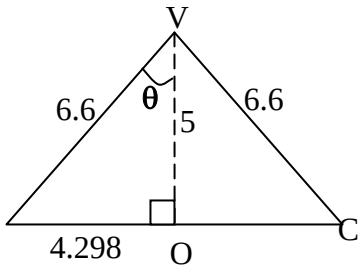
$AV = \sqrt{AO^2 + OV^2}$

$= \sqrt{4.298^2 + 5^2}$

$= 18.47 + 25$

$= \sqrt{43.47}$

$= 6.6cm$



(iii) $\tan\theta = \frac{4.298}{5}$

$= 0.8596$

$\theta = 40.68^o$

$\angle AVC = 40.68 \times 2$

$= 81.36$

Alternative

$\cos\theta = \frac{5}{6.6} = 0.7576$

$\theta = 40.749^oC$

$\angle AVO = 40.749^o$

$\angle AVC = 81.498^o$

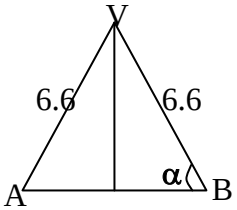
(iv) $\cos\alpha = \frac{2.1}{6.6}$

$= 0.3182$

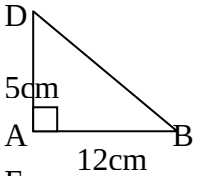
$\alpha = 71.45^o$ Acute angle

obtuse angle $= 180^o - 71.45^o$

$= 108.55^o$



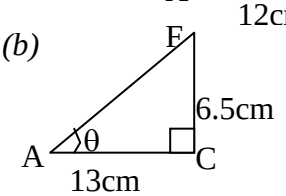
4. (a)



$BD^2 = 122 + 52 = 144 + 25 = 169$

$BD = \sqrt{169} = 13m$

(b)

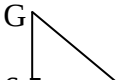


$AF^2 = 13^2 + 6.52 = 169 + 42.25$

$= 211.25$ $AF = \sqrt{211.25} = 14.53cm$ B1

$\tan\theta = \frac{6.5}{13} = 0.5$ M1

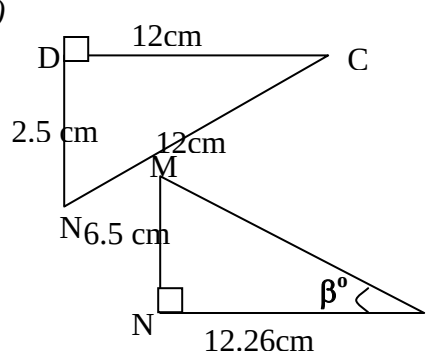
$\theta = 26.57^o$ A1



(c) $\tan\alpha^o = \frac{6.5}{5} = 1.3$ *M1*

$\alpha^o = 52.43$ *A1*

(d)



$NC^2 = 2.5^2 + 12^2 = 150.25$

$NC = \sqrt{150,25} = 12.26$ *B1*

$MC^2 = 6.52 + 150.25$

$= 42.75 + 150.25$

$= 192.5$

$MC = \sqrt{192.5} = 13.87$ *Ba*

$\tan\beta^o \frac{6.5}{12.26} = 0.5302$

$\beta^o = 27.93^o$ *B1*

5.

i) $Or = 16^2 - 5^2$

$= \sqrt{256 - 25}$

$= 15.198$ cm

ii) $\tan\theta = \frac{5.066}{4} = 1.2665$

$\therefore \theta 51.71^0$

6. *a) Height*

$AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{10^2 + 10^2}$

$= \sqrt{200}$

$= 14.142$

$\therefore OA = \frac{1}{2} AC = \frac{14.14^2}{2} = 7.71$

A θ 7.71 C

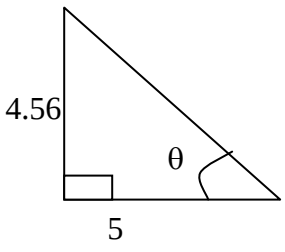
$OE = \sqrt{AE^2 - AO^2}$

5

$= \sqrt{64 - 59.44} = 4.56$

b)i) $\tan\theta = \frac{4.56}{5.00} = 0.912$

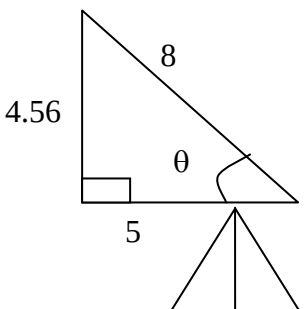
$\theta = 65.78°$



ii) $\tan\theta = \frac{4.56}{7.71} = 0.5914$

$\theta = 30.6°$

c)



$< AEC = 30.6 \times 2$

$= 61.2°$

7. Let length of cut off pyramid be meters

Then $\frac{7 + h}{H} = \frac{5.5}{2.1}$

$14.7 + 2.1h = 5.5$

$3.4h = 14.7$

$h = 4.3$

Slant height of big pyramid

$= \sqrt{11.3^2 + 2.75^2} = 11.6$

Slant height of the pyramid cut off

$= \sqrt{4.3^2 + 1.05^2} = 4.4m$

Area of EFCD = ½ x11.6 x 5.5 – ½ x 4.4 x 2.1

$= 27.28\ m$

Total surface area = 4 x 27.28 + 2.1 x 2.1 = 113.5

b) ½ litre paint $10m^2$

4 litres paints $80m^2$

$\therefore 113.5m^2$ requires 2 tins

2 x 650 = Kshs.1300/=

8. (a) $PR = 12^2 + 9^2 = 144 + 81 = 225 = 15cm$

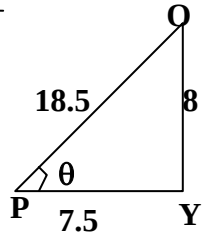
$h = 19.52 – 7.52$

$= \sqrt{380.25 – 56.25}$

$= \sqrt{324} = 18$

(b) $\tan\theta = \frac{18}{7.5} = 2.4$

$\theta^2 \tan 2.4 = 67.38^o$

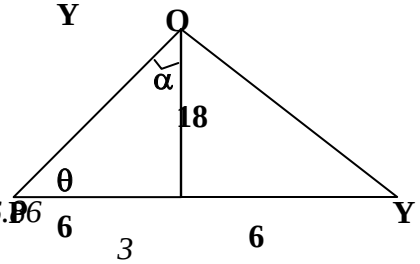


(c) $\tan \alpha = \frac{6}{18} = \frac{1}{3}$

$\alpha = \tan^{-1} 0.3333$

$= 18.43^o$

$\therefore \angle x\ OY = 2 \times 18.43 = 36.86$



(d) Volume $= \frac{1}{3} \times 12 \times 9 \times 18$

$= 648cm^3$

9. a) $AC^2 = 12^2 + 12^2 = 288$

$\therefore AC = \sqrt{288} = 16.97$

$VO^2 = h^2 = 24^2 – (\frac{16.97}{2})^2 = 504$

$h = \sqrt{504} = 22.45cm$

b) Base area = 12x12 = 144cm

$\therefore$ Volume $= \frac{1}{3} \times 144 \times 22.45$

$= 1077.6cm^3$

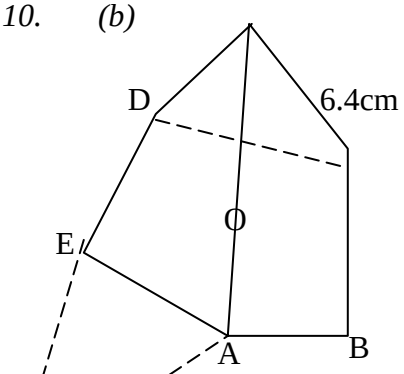
c) Slanting surface = $\sqrt{}$30(30-24) (30-24) (30-12)

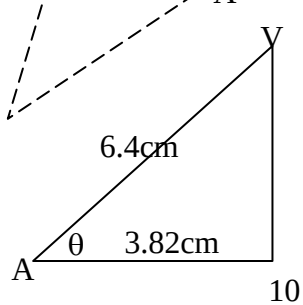
= 139.44cm^2

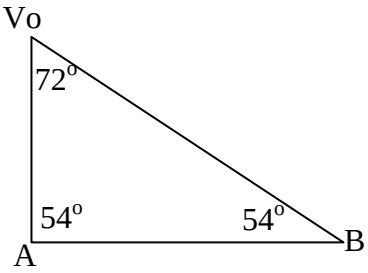
Total curved S.A = 139.44cm^2 x 4 + 144cm^2

= 701.6cm^2

10. (b)







10

AO = $\frac{4.5 \times \sin 54^o}{\sin 72^o}$ = 3.82cm

= cos^{-1} $\frac{(3.82)}{6.4}$ = 53.35^o

(c) Vo = $\sqrt{}$6.42 – 3.82

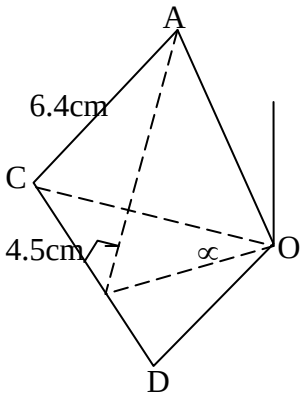
= 5.13

VX = $\sqrt{}$6.4^2 – 2.55^2

= 5.99cm

$\propto$= Sin^{-1} ($^{Vo}/_{Vx}$) Sin-1($^{5.13}/_{5.99}$)

$\propto$= 58.91^o



11. a) Longitude difference = 139 ° + 41 °

= 180 °

b) Distance along latitude = $^{\emptyset}/_{360}$ x 2 π r cos θ

= $^{180}/_{360}$ x 2 x $^{22}/_{7}$ x 6370 cos 60°

= 22 x 910 x 0.5

= 10,010 Km

Or via north pole (great circle)

Latitude difference = 60°

Distance = $^{60}/_{360}$ x 2 x $^{22}/_{7}$ x 6370

= 6673.33 Km

c) Distance = $^{long\ diff}/_{360}$ x 2πR cos 60°

420 = $^{\emptyset}/_{360}$ x 2 x $^{22}/_{7}$ x 6370 cos 60°

θ = $\frac{420 \times 360 \times 7}{2 \times 22 \times 6370 \cos 60°}$

$= 7.552°$

$Longitude\ of\ C = 41° - 7.55° \quad = 33.45°N$