2. Vectors 2

1.	(a) BD = BA + AD $= -\underset{\sim}{b} + \frac{3}{5}\underset{\sim}{c}$	**B1**	
	$AE = AB + BE$ $= \underset{\sim}{b} + \frac{1}{2}BC$ $= \underset{\sim}{b} + \frac{1}{2}\left(\underset{\sim}{c} - \underset{\sim}{b}\right)$	**M1**	
	$= \frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{c}$	**A1**	
	(b) $BF = +\left(\frac{3}{5}\underset{\sim}{c} - \underset{\sim}{b}\right)$ $AF = n\left(\frac{1}{2}\underset{\sim}{b} + \frac{1}{2}\underset{\sim}{c}\right)$ $= \frac{n}{2}\left(\underset{\sim}{b} + \underset{\sim}{c}\right)$ $AF = AB + BF$ $= \underset{\sim}{b} + \frac{3}{5}t\underset{\sim}{c} - t\underset{\sim}{b}$ $= (1-t)\underset{\sim}{b} + \frac{3}{5} + \underset{\sim}{c}$	**M1**	AF and BF interms of n and t
	$(1-t)\underset{\sim}{b} + \frac{3}{5}t\underset{\sim}{c} = \frac{n}{2}\underset{\sim}{b} + \frac{n}{2}\underset{\sim}{c}$	**M1**	Equating the expressions
	$1 - t = \frac{n}{2}$...............(i) $2 - 2t = n$ $\frac{3}{5}t = \frac{n}{2}$...................$(ii)$	**M1**	Extraction of the coefficient
	$6t - 5n = 0$ $subt\ (i)\, in\, (ii)$ $6t - 5(2 - 2t) = 0$ $6t = 10$ $t = \frac{10}{16} = \frac{5}{8}$	**M1**	Substitution/its equivalent
	$n = 2 - 2\left(\frac{5}{8}\right)$	**A1**	Any of the unknown
	$n = \frac{3}{4}$	**B1**	The other unknown
	(c) BD:BF 8 : 5	**B1**	

		10	

1. a) (i) $AN = OA + ON$

$= -\underset{\sim}{a} + \frac{2}{7}\underset{\sim}{b}$

$= \frac{2}{7}\underset{\sim}{b} - \underset{\sim}{a}$

(ii) $\overrightarrow{AT} = \frac{7}{13}\overrightarrow{AN}$

$\frac{7}{13}\left(-\underset{\sim}{a} + \frac{2}{7}\underset{\sim}{b}\right)$

$\frac{2}{13}\underset{\sim}{b} - \frac{7}{13}\underset{\sim}{a}$

(iii) $AM = \frac{1}{4} AB$

$= \frac{1}{4}(\overrightarrow{AO} + \overrightarrow{OB})$

$= \frac{1}{4}(\underset{\sim}{b} - \underset{\sim}{a})$

(b) $OT = OA + AT$

$= \underset{\sim}{a} \left(\frac{2}{13}\underset{\sim}{b} - \frac{7}{13} a\right)$

$= \frac{2}{13}\left(3\underset{\sim}{a} + \underset{\sim}{b}\right)$

$\overrightarrow{OM} = \overrightarrow{OA} + \overrightarrow{AM}$

$= \underset{\sim}{a} + \left(-\frac{1}{4} a + \frac{1}{4}\underset{\sim}{b}\right)$

$\frac{3}{4}\underset{\sim}{a} + \frac{1}{4}\underset{\sim}{b}$

$\frac{1}{4}\left(3\underset{\sim}{a} + \underset{\sim}{b}\right)$

$\frac{OT}{OM} = \dfrac{\frac{2}{13}\,(3a + b)}{\frac{1}{4}\,(3a + b)}$

$\overrightarrow{OT} = \frac{8}{13} OM$

Or $\overrightarrow{OM} = \frac{13}{8}\overrightarrow{OT}$

√√Construction of $\angle 60^o$ and $\angle 90^o$

Bisect $\angle$ btw 90^o and 60^o to obtain $\angle 75^o$

√ Construction of the given sides

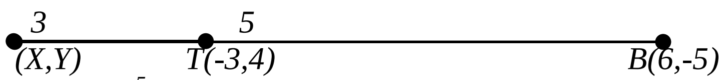
Construction of Δ XYZ

Since $\overrightarrow{OT} = \frac{8}{13}\overrightarrow{OM}$

Then $OT : TM = \frac{8}{13} : \frac{5}{13}$

$= 8 : 5$

2.



$\underset{\sim}{TB} = {}^{5}/_{8}\, AB$

$\begin{pmatrix} 6 \\ -5 \end{pmatrix} - \begin{pmatrix} -3 \\ 4 \end{pmatrix} = \frac{5}{8}\underset{\sim}{AB}$

$\begin{pmatrix} 9 \\ -9 \end{pmatrix} = \frac{5}{8}\left\{\begin{pmatrix} 6 \\ -5 \end{pmatrix} - \begin{pmatrix} x \\ y \end{pmatrix}\right\}$

$\begin{pmatrix} 9 \\ -9 \end{pmatrix} = \begin{matrix} {}^{5}/_{8}\,(6-x) \\ {}^{5}/_{8}\,(-5-y) \end{matrix}$

$\frac{30}{8} - \frac{5}{8}X = 9$

$-\frac{25}{8} - {}^{5}/_{8}\,y = -9$

$30 - 5x = 72$	$-5x = 42$
$-25 - 5y = -72$	$-5y = -47$
$X = -8.4 \quad y = 9.4$	

3. $\underset{\sim}{OX} = \frac{2}{3}(3\underset{\sim}{i} + 2j - 4k) + \frac{1}{3}(6\underset{\sim}{i} + 11j + 2k)$

$= 2\underset{\sim}{i} + 4j - \frac{8}{3}\underset{\sim}{k} + 2i + \frac{11}{3}j + \frac{2}{3}$

$= 4\underset{\sim}{i} + 5\underset{\sim}{j} - 2k$

$|\underset{\sim}{OX}| = \sqrt{16 + 25 + 4}$

$= 6.71$units

4. a) $2^5 - 5(2^4)({}^{1}/_{5}) + 10(2^3)({}^{1}/_{5}x)^2 - 10(2^2)({}^{1}/_{5}x)^3 + 5(2)({}^{1}/_{5}x)^4 - ({}^{1}/_{5}x)^5$

$32 - 16x + {}^{16}/_{5}x^2 - {}^{8}/_{25}x^3 + {}^{2}/_{125}x^4 - {}^{1}/_{3125}x^5$

$-{}^{1}/_{5}x = -0.04$

$x = 0.2$

b) $32 - 16(0.2) + {}^{16}/_{5}(0.2)^2 - {}^{8}/_{25}(0.2)^3 + \ldots\ldots$

$= 32 - 3.2 + 0.128 - 0.00256$

$= 28.92544$

$= 29.925$

5. $AS = AO + OS$

$= -a + 2(3c)$

$= 2c - a\ldots\ldots\ldots$

$BC = BA + AC$

$= a - b + AC$

But $AC = AO + OC = -a + 3c$

$= 3c - a\ldots\ldots\ldots$

$AB + \frac{2}{\ }\,OC = \frac{2}{\ }\,3c = 2c$

$$3 \qquad 3$$

$BA = 2c$…….

$BC = -12c + 3c - a = c - a$.

b) (i) $AT = \eta AS = \eta (2c - a)$

$= 2\eta c - \eta a$

$AT = AB + BT = 2c + K(c - a)$

$= 2c + Kc - Ka$

$= (2 + k)c - Ka$

(ii) $2 + K = 2\eta$ (i) $K = \eta$ (ii)

$2 + \eta = 2\eta$

$2 = 2\eta - \eta$

$2 = \eta, K = 2$

(c) $BT : BC$

$BT = 2\,BC$

6. (a) (i) $\underset{\sim}{PQ} = \underset{\sim}{PO} + OQ$

$= \underset{\sim}{P} + \underset{\sim}{q}$ or $\underset{\sim}{q} - p$

(ii) $\underset{\sim}{QR} = \underset{\sim}{QP} + \underset{\sim}{PR}$

$= \underset{\sim}{P} + \frac{2}{3} \underset{\sim}{PQ}$

$= \underset{\sim}{P} + \frac{2}{3} (\underset{\sim}{q} - \underset{\sim}{p})$

$= \underset{\sim}{P} + \frac{2}{3} \underset{\sim}{q} - \frac{2}{3} \underset{\sim}{p}$

$= \frac{1}{3} \underset{\sim}{p} + \frac{2}{3} \underset{\sim}{q}$

(iii) $\underset{\sim}{SQ} = \underset{\sim}{SO} + \underset{\sim}{OQ}$

$= -\frac{3}{4} \underset{\sim}{OP} + \underset{\sim}{OQ}$

$= -\frac{3}{4} \underset{\sim}{p} + \underset{\sim}{q}$ or $\underset{\sim}{q} - \frac{3}{4} \underset{\sim}{p}$

For √PQ or P and q
For √exp. Of OR
For √OR in p & q
For √ SQ in P & Q
For √OT or p & q Multiply this by 12
For √OT in p,q
For √eq. both expr.Multiply these by 12
For √equations both express.
For √ elimisn. of m
Both ans.

(b) Express OT in two different ways:

Given $\underset{\sim}{OT} = n\,\underset{\sim}{QR}$

$= n\left(\frac{1}{3}\underset{\sim}{P} + \frac{2}{3}\underset{\sim}{q}\right)$

$= \frac{n}{3}\underset{\sim}{p} + \frac{2n}{3}\underset{\sim}{q}$

From $\triangle OST$,

$OT = \underset{\sim}{OS} + \underset{\sim}{ST}$

$= \frac{3}{4} \underset{\sim}{OP} + M\,\underset{\sim}{SQ}$

$= \frac{3}{4}\underset{\sim}{P} + M\left(-\frac{3}{4}\underset{\sim}{P} + \underset{\sim}{q}\right)$

$= \left(\frac{3}{4} - \frac{3m}{4}\right)\underset{\sim}{p} + m\underset{\sim}{q}$

$\therefore \frac{n}{}\,\underset{\sim}{p} + \frac{2n}{}\,\underset{\sim}{q} = \left(\frac{3}{} - \frac{3m}{}\right)\underset{\sim}{p} + m\underset{\sim}{q}$

3 3 4 4

Compare the coefficients of $\underset{\sim}{p}$ and $\underset{\sim}{q}$

$$\frac{n}{3} = \frac{3}{4} - \frac{3}{4}m$$

$$4n = 9 - 9m$$

$$4n + 9m = 9 \text{ ………………..eq (1)}$$

$$\frac{2n}{3} = m$$

$$m = \frac{2n}{3} \text{ ……………eq. (2)}$$

Substitutes form in equation (1)

$$4n + 9\left(\frac{2n}{3}\right) = 9$$

$$4n + 6n = 9$$

$$10n = 9$$

$$n = \frac{9}{10}$$

Substitute for n in equation (2)

$$m = \frac{2}{3} \text{ x } \frac{9}{10} = \frac{3}{5}$$