1. Vectors

1. Given that $4p - 3q = \begin{pmatrix} 10 \\ 5 \end{pmatrix}$ and $p + 2q = \begin{pmatrix} -14 \\ 15 \end{pmatrix}$ find

a) (i) $\underset{\sim}{p}$ *and* $\underset{\sim}{q}$ (3 mks)

(ii) $|p + 2q|$ (3 mks)

(b) Show that A (1, -1), B (3, 5) and C (5, 11) are collinear (4 mks)

2. Given the column vectors $\mathbf{a} = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$ $\mathbf{b} = \begin{pmatrix} 6 \\ -3 \\ 9 \end{pmatrix}$ $\mathbf{c} = \begin{pmatrix} -3 \\ 2 \\ 3 \end{pmatrix}$ and that $\mathbf{p} = 2\mathbf{a} - \frac{1}{3}\mathbf{b} + \mathbf{c}$

(c) (i) Express **p** as a column vector (2mks)

(d) (ii) Determine the magnitude of **p** (1mk)

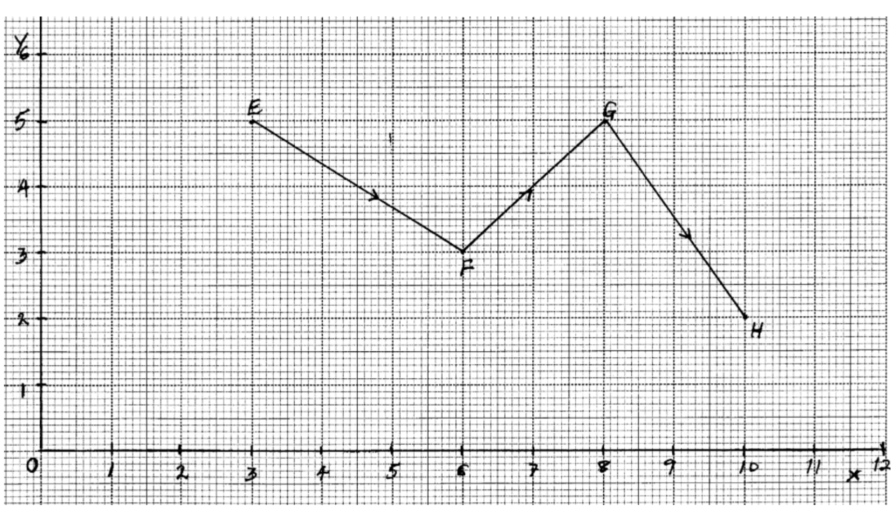
3. Given the points P(-6, -3), Q(-2, -1) and R(6, 3) express PQ and QR as column vectors. Hence show that the points P, Q and R are collinear. (3mks)

4. The position vectors of points x and y are $x = 2i + j - 3k$ and $y = 3i + 2j - 2k$ respectively. Find x y as a column vector (2 mks)

5. Given that $\underset{\sim}{\mathbf{a}} = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$, $\underset{\sim}{\mathbf{b}} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$, $\underset{\sim}{\mathbf{c}} = \begin{pmatrix} 3 \\ -2 \end{pmatrix}$ *and* $\underset{\sim}{\mathbf{P}} = 2\underset{\sim}{\mathbf{a}} + \underset{\sim}{\mathbf{b}} - 3\underset{\sim}{\mathbf{c}}$. *find* $\left|\underset{\sim}{\mathbf{p}}\right|$ (3mks)

6. The position vectors of A and B are $\begin{bmatrix} 2 \\ 5 \end{bmatrix}$ and $\begin{bmatrix} 8 \\ -7 \end{bmatrix}$ respectively. Find the coordinates of M which divides AB in the ratio 1:2. (3 marks)

7. The diagram shows the graph of vectors $\underset{\sim}{EF}$, $\underset{\sim}{FG}$ and $\underset{\sim}{GH}$.



Find the column vectors;

(a) $\underset{\sim}{EH}$ (1mk)

(b) $|\underset{\sim}{EH}|$ (2mks)

8. $\underset{\sim}{OA} = 2\underset{\sim}{i} - 4\underset{\sim}{k}$ and $\underset{\sim}{OB} = -2\underset{\sim}{i} + j - k$. Find $|\underset{\sim}{AB}|$ (2mks)

9. Show that P (4, 0 -4), Q (8, 2, -1) and R (24, 10, 11) are collinear. (3 mks)

10. Given that $\underset{\sim}{p}$ = 2i – j + k and q = i + j +2k, determine

a. $|p + q|$ (1 mk)

(b) $|\tfrac{1}{2}p - 2q|$ (2 mks)

11. Express in surds form and rationalize the denominator.

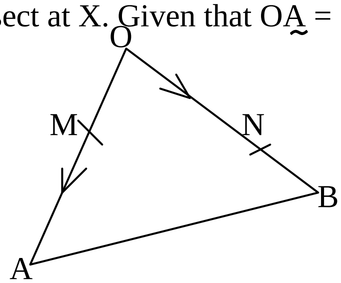
$$\frac{1}{\sin 60^o \sin 45^o - \sin 45^o}$$

12. If $\overrightarrow{OA} = 12\underset{\sim}{i} + 8\underset{\sim}{j}$ and $\overrightarrow{OB} = 16\underset{\sim}{i} + 4\underset{\sim}{j}$. Find the coordinates of the point which divides $\overrightarrow{\mathbf{AB}}$ internally in the ratio1:3

13. Find scalars **m** and **n** such that

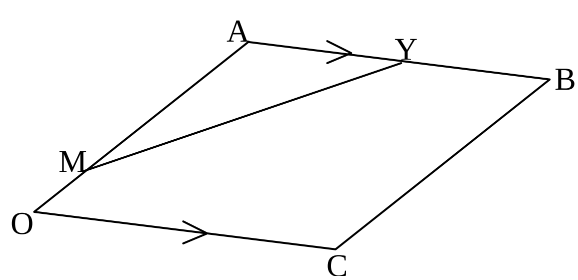
$$\mathbf{m}\begin{pmatrix}4\\3\end{pmatrix} + \mathbf{n}\begin{pmatrix}-3\\2\end{pmatrix} = \begin{pmatrix}5\\8\end{pmatrix}$$

14. In a triangle OAB, M and N are points on OA and OB respectively, such that OM: MA = 2:3 and ON: NB = 2:1. **AN** and **BM** intersect at X. Given that $\underset{\sim}{OA}$ = a and $\underset{\sim}{OB}$ = b



(a) Express in terms of $\underset{\sim}{a}$ and b
(i) BM
(ii) AN
(b) By taking **BX** = t and **AX** = **h** **AN**, where **t** and **h** are scalars, express **OX** in two different ways
(c) Find the values of the scalars **t** and **h**
(d) Determine the ratios in which **X** divides :-
(i) **BM**
(ii) **AN**

15. OABC is a parallelogram, M is the mid-point of OA and AX = $^2/_7$ AC, $\underset{\sim}{OA}$=a and $\underset{\sim}{OC}$ = c



(a) Express the following in terms of a and c
(i) MA
(ii) AB
(iii) AC
(iv) AX

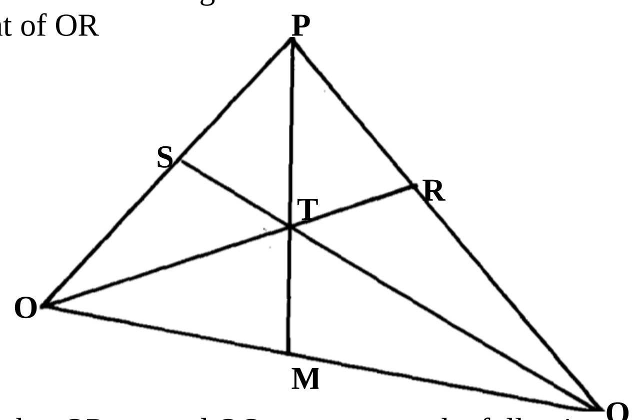
(b) Using triangle MAX, express MX in terms of a and c

(c)The co-ordinates of A and B are (1, 6, 8) and (3, 0, 4) respectively. If O is the origin and P the midpoint of AB. Find;
(i) Length of OP
(ii) How far are the midpoints of OA and OB?

16. a) If A, B & C are the points (2, - 4), (4, 0) and (1, 6) respectively, use the vector method to find the coordinates of point D given that ABCD is a parallelogram.

b) The position vectors of points P and Q are **p** and **q** respectively. R is another point with position vector $r = {}^{3}/_{2}\,q - ½\,p$. Express in terms of P and q
(i) PR
(ii) PQ, hence show that P, Q & R are collinear.
(iii) Determine the ratio PQ : QR

17. The figure shows a triangle of vectors in which OS: SP = 1:3, PR:RQ = 2:1 and T is the midpoint of OR



a) Given that OP = p and OQ = q, express the following vectors in terms of P and q
i) OR
ii) QT

b) Express **TS** in terms of **p** and **q** and hence show that the points **Q, T** and **S** are collinear

c) M is a point on OQ such that OM = KOQ and PTM is a straight line. Given that PT: TM = 5:1, find the value of **k**

18. Given that a = , b = and c = and that $p = 3q – ½\,b + {}^{1}/_{10}c$
Express **p** as a column vector and hence calculate its magnitude /P/ correct to two decimal places

19. In a triangle OAB, M and N are points on OA and OB respectively, such that OM:MA= 2:3 and ON:NB= 2:1. AN and BM intersect at X. Given that OA = **a** and OB = **b**

(a) Express in terms of **a** and **b**:-
(i) BM
(ii) AN

(b) Taking BX = kBM and AX =hAN where **k** and **h** are constants express OX in terms of
(i) **a, b** and **k** only
(ii) **a, b,** and **h** only

(c) Use the expressions in **(b)** above to find values of **k** and **h**

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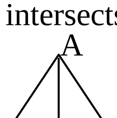
20. In the figure below OAB is a triangle in which M divides OA in the ratio 2:3 and N divides OB in the ratio 4:1. AN and BM intersects at X



(a) Given that OA = a and OB = b, express in terms of a and b

(i) AN

(ii) BM

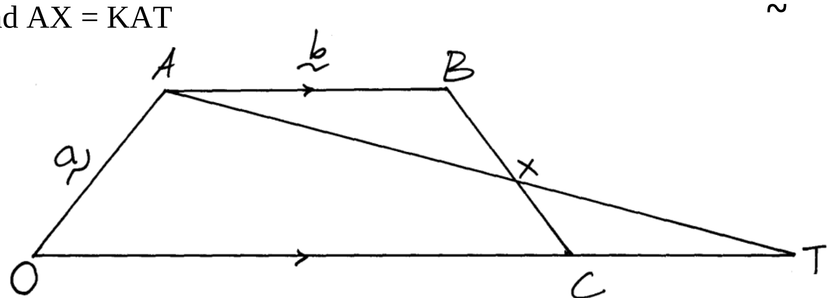
(iii) AB

(b) If AX = sAN and BX = tBM, where **s** and **t** are constants, write two expressions for OX in terms of **a**, **b**, **s** and **t**. Find the value of **s** and **t** hence write OX in terms of **a** and **b**

21. A student traveling abroad for further studies sets a side Kshs. 115800 to be converted into US dollars through a bank at the rate of 76.84 per dollar. The bank charges a commission of 2 ½ % of the amount exchanged. If he plans to purchase text books and stationery worth US$270, how much money, to the nearest dollar, will he be left with?

22. Given that:- r = 5i – 2j and m = -2i + 6j – k are the position vectors for R and M respectively. Find the length of vector RM

23. OABC is a trapezium in which OA = a and AB = b. AB is parallel to OC with 2AB = OC. T is a point on OC produced so that OC: CT = 2:1. At and BC intersect at X so that BX = hBC and AX = KAT



(a) Express the following in terms of a and b:-

(i) OB

(ii) BC

(b) Express **CX** in terms of a, b and h

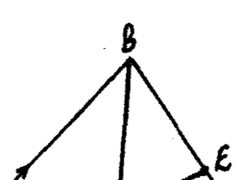
(c) Express **CX** in terms of a, b and k

(d) Hence calculate the values of h and **k**

24. Given that **a** = **2i** + **j** – **2k** and **b = -3i + 4j – k** find :-

| a + b|**.**

25. In the figure below, **E** is the mid-point of **BC**. **AD**:**DC**=3:2 and **F** is the meeting point of **BD** and **AE**



If **AB = b** and **AC = c;**

(i) Express **BD** and **AE** in terms of **b** and **c**

(ii) If **BF =*t*BD** and **AF =*n*AE**, find the values of ***t*** ad ***n***

(iii) State the ratios in which **F** divides **BD** and **AE**

26. The coordinates of point **O, A, B** and **C** are (**0, 0) (3, 4) (11, 6)** and **(8, 2)** respectively.
A point **P** is such that the vector **OP, BA, BC** satisfy the vector equation **OP = BA + ½ BC**
Find the coordinates of **P**

27. A point Q divides AB in the ratio 7:2. Given that A is (-3, 4) and B(2, -1).
Find the co-ordinates of Q